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6-4-12-09

SPECIAL REVIEW FINAL DETERMINATION	Reviewed	Class.	Date
	JRP	U	5-18-82
Class: U			

WANL-TME-534

TITLE: Detailed Model Status Report
(Title Unclassified)

MASTER

DATE: September 25, 1963

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Appendix

1.0 Introduction

1.1 Purpose of This Report

The main purposes of this report are to provide 1) a reference source for the interchange of analog engine model information with AGC, and 2) a reference point for future WANL analog model development. Being a model status report, the present state of the models are presented. These models have been used in various NERVA and NRX dynamics studies. As is usually the case, the models are still being developed -- the drum actuator pre-presentation will probably be the next to be upgraded by taking advantage of the present capability of running the physical actuator connected to the analog computer.

The format of this report reflects its intended uses. Each major component or system is discussed in a separate section with equations numbered sequentially within each section. The continued development work should result in changes to only one discrete section at a time, allowing simpler upgrading of the model status report. This report represents part of an analog model handbook which is being written as a working document for the analog effort.

1.2 Application of the Analog Models

The engine analog simulation has been used in design of a reference reactor power control system. This established a basis for early hardware development. A breadboard reactor power controller was tied into the analog simulation to verify its equivalence to the analog design.

Besides providing a tool for reactor controller design the engine simulation was used to study the combined operation of the turbopump and reactor to search for potential

problem areas. These studies generally relate to the power range (i.e. after chilldown), or to the subpower range in which only the neutronics effects are important. The chill-down period and the long range shutdown period are not studied on the analog computer. Bootstrap startup cannot be adequately studied with the present analog models.

The analog models for the NERVA and NRX reactors are essentially the same, thus many of the engine analog models reported here are used in NRX analysis work.

2.0 Overall Engine Simulation

2.1 Block Diagram

The block diagram of the NERVA engine simulation is shown in Figure 2-1. The bulk of this report discusses the analog models used block by block. The emphasis at WANL is on the development of models for the reactor portions while AGC is developing the models for the turbopump and associated portions. This block diagram shows how all the pieces fit together.

The signals into and out of each block reflect the particular analog mechanization chosen. The direction of information flow can be altered by changing the analog mechanization. This would not alter the basic physical representations presented in this report.

2.2 Nomenclature

To facilitate the exchange of information between AGC and WANL the nomenclature in this report has been modified to approach that used by REON in ref. 8. There are some differences resulting from different component descriptions. For internal WANL purposes, the working level WANL designations are included. Where no analogous station designation is available, the WANL symbol was used. A cycle schematic is shown in Figure 2-2. This serves to identify the station locations as well as give physical substance to the engine description.

The more frequently used symbols are listed below. Additional variables are defined and used in several sections of the report. The stations numbered higher than 9 are discussed in Section 12.

(Ref. 734:M0296:bs)

FIGURE 2-2
CYCLE SCHEMATIC

(a) Frequently Used Variables

P	Pressure of fluid	psia
H	Enthalpy of fluid	BTU/lb _m
ρ	Density of fluid	lb _m /ft ³
$\overline{\rho H}$	Average energy density of fluid	BTU/ft ³
T	Temperature of fluid	°R
T_m	Temperature of material	°R
N	Relative neutron level	%
N	Turbopump speed	rpm
Q	Heat power generated or transferred	BTU/sec
hA	Heat transfer coefficient	BTU/sec-R°
W	Mass flow rate	lb _m /sec
W_7	Mass flow rate into fuel channels	lb _m /sec
M	Torque	lb _f -ft
θ	Drum angular position	degrees
δ_k	Total reactivity	----
C_p	Specified heat	BTU/lb _m -R°

(b) Subscripts for Fluid Conditions at Particular Locations

Station Symbol	Alternate Symbol	Location
1		Tank exit, inlet to TSOV
2		Pump inlet
3	d	Pump discharge

(b) Subscripts for Fluid Conditions at Particular Locations Continued

Station Symbol	Alternate Symbol	Location
4	nci	Nozzle cooling region inlet
5	ri	Reflector inlet
6	re	Reflector exit
7	ci	Fuel and support element inlets, above orifice
	ce	Fuel element exit
	tre	Support element exit
8	n	Mixed condition in core exit plenum
9		Nozzle throat

(c) Other Subscripts

nc	Average fluid condition for one section nozzle cooling region
r	Average fluid condition for one section reflector
c	Average fluid condition for one section fuel element
tr	Average fluid condition for one section support element
m	Material
t	Turbine
p	Pump
e	Exit
i	Inlet
f	Film, fuel

3.0 Nozzle Cooling Region

The full flow from the feedsystem passes thru the nozzle cooling region cooling the nozzle and in the process is preheated before entering the reflector. (The nozzle cooling region is simulated to permit a better description of the reflector inlet conditions and to permit a more accurate heat balance about the engine.) The nozzle wall temperature is not a desired output and is not calculated explicitly by the analog computer.

The energy input to the nozzle cooling comes mainly from heat transfer from the hot gas on the inside of the nozzle through the nozzle wall. Other sources are heat radiation from the core exit surfaces and nuclear power generation in the nozzle itself. The nozzle loses energy by radiation as well

as by transfer to the coolant. The radiation effects

are quite significant during shutdown but play a lesser role in the power range. These influences are implicitly accounted for in the power range since the coefficients calculated for the simple model are based on digital calculations which do explicitly consider such effects.

Heat transfer equations were written about the nozzle cooling region with heat transfer coefficients considered functions of flow. Transfer functions were derived from these equations and then the groupings of constants calculated from steady state data from digital codes. The digital work utilized detailed spatial descriptions of heat transfer and other energy sources. By comparing coefficients it was found that the following expression gives reasonably accurate results at from 20% to 100% power.

Block Diagram of Nozzle Cooling Region

$$T_5 = T_3 + .2435 (10^{-3}) W_8 T_8 / (1 + 3s) \quad (3.1)$$

The details of this derivation are given in the Appendix.

This gives values of T_5 within $\pm 5R^\circ$ of the digital calculation. The calculated time constant at several flows varies from 3.13 to 2.6 seconds over the range 20% to 100%. Since the digital calculations are quite sensitive to choice of heat transfer coefficient it was not felt necessary to fit the digital data better than this. Essentially, this equation accounts for the temperature rise in the nozzle cooling to help in the energy balance and allows for time delay due to heat capacity.

Depending on what flow and power profiles are chosen for startup, two-phase flow may exist in the nozzle cooling passages. No two-phase pressure drop or heat transfer correlations are explicitly simulated in the analog models and checks against digital calculations are necessary to ensure that today's reasonable assumptions are still reasonable tomorrow. The pressure drop equation used is

$$P_3 - P_5 = K_{40} W_3^2 / \rho_5 \quad (3.2)$$

The mass storage effects in the nozzle cooling passages and piping between the pump exit and the nozzle cooling inlet are lumped with those of the reflector and are discussed in the reflector section. This lumping of terms in the reflector implies making the assumptions

$$W_3 = W_4 \quad (3.3)$$

$$W_4 = W_5 \quad (3.4)$$

4.0 Reflector Region (Reflector Inlet to Core Inlet)

4.1 Introduction

The reflector is essentially a beryllium cylinder surrounding the core. Control drums are placed in this cylinder to control the reflector effectiveness and thus core reactivity. Besides providing a flow path between the nozzle cooling region and the core inlet, the reflector requires specific coolant flows through its different regions because of the internal heat generation due to radiation. There is little heat conduction from the core radially to the beryllium reflector due to the insulating effects of the pyrographite strips and graphite barrel.

During startup and when the system pressures are below the critical pressure, two-phase flow will generally exist in the nozzle cooling passages and generally only single-phase fluid will exist in the reflector. However, two-phase flow can occur in the reflector for certain startup profiles. Digital codes are being used to study the parallel channel flow balances and heat transfer. Their detailed description of the many different size reflector channels is necessary for providing design information on channel sizing and providing the details of reflector temperatures.

Digital computer studies have shown that a single channel model does not describe the reflector dynamics well. Multi-channel codes have now been developed and are now used for detailed studies. The analog model is based on a single channel concept, with coefficients based on comparisons with latest digital runs. The range of validity is thus closely limited to those ranges used for comparison with digital data. This is roughly

10 to 100% power.

Below 10% power the absolute values of pressure and temperature calculated for the reflector on the analog computer are not very accurate. However, the general character of reactor system dynamics appears comparable between the digital and analog data, and the limited KIWI experience indicates the dynamics description to be reasonable. Thus, study of control stability during startup and power range operation can be carried out with the model but demonstration of bootstrap capability, which depends more on the absolute values of temperature and pressure, must be treated as preliminary.

Block Diagram: Reflector Region

4.2 Reflector Equations

The reflector is represented by a single heat capacity associated with one coolant channel. The radial heat transfer to the coolant is

$$Q_r - hA_r (T_{mr} - T_r) = \frac{d}{dt} (M_r C_r T_{mr}) \quad (4.1)$$

Axial heat transfer is neglected as well as radial heat conduction from the core.

The internal heat generation, Q_r , is assumed directly proportional to the average neutron flux:

$$Q_r = K_{41} N \quad (4.2)$$

This is quite reasonable during startup and power operation but the build up of fission products can make this assumption poor for shutdown analysis. The initial, rapid dynamics of shutdown from power will be adequately simulated. The extended but slower dynamics

of shutdown are studied by use of digital codes. Analog studies of the long term shutdown transients are not now envisioned.

The energy balance about the hydrogen is

$$W_5(H_6 - H_5) = hA_r(T_{mr} - T_r) \quad (4.3)$$

This neglects the changes of stored heat in the hydrogen, and uses the inlet flow rate for the heat balance. Startups and most transients to be studied are slow relative to the flow readjustments within the reflector, thus supporting these assumptions.

The heat transfer coefficient, hA_r , is assumed to be a linear function of the inlet flow, W_5 . This has been found to yield reasonable fits to the digital data and simplifies the simulation.

$$hA_r = K_{42} W_5 \quad (4.4)$$

The hydrogen properties are quite variable. By determining reasonable ranges of temperature and pressure at the reflector inlet and exit, based on digital cycle calculations, and using tables of hydrogen properties, the following fits were obtained:

$$H_5 = -162.17 + 3.669 T_5 - 0.0652 P_5 \quad (4.5)$$

$$T_6 = \frac{H_6}{3.772} + 45.16 + 0.03218 P_6 \quad (4.6)$$

$$\rho_5 = \frac{P_5}{5.359 T_5 - 0.158 P_5} \quad (4.7)$$

It was found convenient to use the simple average:

$$T_r = (T_5 + T_6)/2 \quad (4.8)$$

The specific heats of the reflector materials are strong functions of temperature at LH_2 temperatures. Because there are several materials involved and because the temperature distributions are not very uniform, the assignment of a heat capacity to the one lump model is difficult. Material in which flow rates were largest were combined and an average heat capacity vs. temperature relation obtained. This is shown in Table A4-1.

$$T_{mr} = f_{r3} \left[\int_0^{T_{mr}} M_r C_r dT_{mr} \right] \quad (4.9)$$

The pressure drop through the reflector is quite variable when two-phase flow exists -- it is reasonably well behaved in the single-phase regime. A detailed simulation of reflector pressure drops was found unworkable with the overall engine simulation and the present model assumes the following pressure drop relation:

$$P_5 = P_7 + K_{43} \frac{W_7^2}{\rho_7} \quad (4.10)$$

where the core inlet flow and density are chosen for convenience, and the shield and reflector pressure drops combined.

The flow volume in the reflector and nozzle cooling passages has a significant influence on the flow dynamics. Since explicit flow equations were not written at the inlet and exit of these two regions, a simpler approximation was used. A time constant was associated with the flow

$$W_6 = W_5 / (1 + K_{44} s) \quad (4.11)$$

This time constant was calculated from the flow volumes and compared with rough test data obtained on KIWI tests. A reference value for this time constant was established but it was considered a parameter to be varied in detailed control studies.

4.3 Shield

In some forms of heated bleed cycles the turbine drive fluid comes from the reflector exit through the shield in the core inlet plenum. For such cycles, explicit simulation of the shield was used. In the present hot bleed cycle, however, the turbine feed is obtained elsewhere in the system and the shield has a much smaller effect on the engine dynamic response. For this reason, and to simplify the simulation, the explicit simulation of the shield has been eliminated. This was done by combining stations 6 and 7, reflector exit and core inlet, into a single station.

$$W_6 = W_7 \quad (4.12)$$

$$T_6 = T_7 \quad (4.13)$$

$$H_6 = H_7 \quad (4.14)$$

$$P_6 = P_7 \quad (4.15)$$

$$\rho_6 = \rho_7 \quad (4.16)$$

5.0 Core Heat Transfer and Flow

5.1 Physical Description

The core is a right circular cylinder made up of full length hexagonal graphite elements arranged in some 280 hexagonal clusters. Each fueled graphite element has 19 axial propellant channels about 0.1 inch in diameter. Each cluster has 6 fuel elements surrounding a central hexagonal unfueled graphite support element. A stainless steel tie rod passes through the support element and is supported so as to keep the cluster under compression.

5.2 Model Description

The many fuel elements and propellant flow channels have been combined in a single fuel element with all the active core heat capacity, heat generation, and flow rate. The model thus represents average elements, and heat transfer coefficients are calculated for the average conditions. Variations with radial position which exist are assumed to play a negligible role in determining average dynamic response.

One radial section has been taken in the fuel, which implies a uniform radial fuel temperature. This is reasonable for most transients considered but can introduce errors in malfunction studies involving faster transients.

No axial conduction is assumed. This is generally an acceptable approximation.

One axial section of the core has been taken, based on a compromise between equipment limitations and the desire to incorporate transient flow relations. More axial sections,

with simpler flow relations, would allow better matching of steady-state data and improve some transient temperature calculations, but would lose some transient flow calculation capability.

Core exit conditions are used to calculate core pressure drops and the coefficients are calculated accordingly.

5.3 Relation to Overall System

The overall engine system block diagram is shown in Figure 2.1. Discussed in this section is the block shown below. The input and output variables are shown. Which are input and which are output is dictated by the method of analog mechanization chosen, not by the physical system.

Block Diagram: Core Heat Transfer and Flow

5.4 Physical Equations

The heat transferred from the fuel to the propellant is given by:

$$Q_c = hA_c (T_f - T_c) \quad (5.1)$$

where all terms are average values for the one section.

The heat generated in the fuel is proportional to the neutron level, N . Writing an energy balance about the fuel:

$$\frac{d(M_f C_f T_f)}{dt} = K_1 N - Q_c \quad (5.2)$$

where numerical constants are written as K's and a separate listing included in the Appendix.

The specific heat of the fuel, C_f , is a function of the average fuel temperature, T_f , leading to

$$\int_0^{T_f} C_f(T_f) dT_f = T_f \quad (5.3)$$

The function $C_f(T_f)$ is presented on Figure A5-1 of the Appendix, and is the specific heat of unfueled graphite. The mass of fuel, M_f , was calculated from physical dimensions using the density of unfueled graphite. Since the product of density times specific heat is relatively constant regardless of fuel loading it is reasonable to apply this consistent set of unfueled graphite properties to the fueled graphite case.

The average heat transfer coefficient, hA_c , is a combination of graphite and cladding conductivities and a film coefficient. Calculations in digital codes use detailed correlations for hA_c involving functions of fluid temperatures, pressures, and flow rates. The most important single factor is a term proportional to flow to the 0.8 power. From digital data, plots were made of $hA_c / (\text{flow})^{0.8}$ as a function of axial position with different flow rates as a parameter. These plots fell very close together indicating that the analog representation can properly be considered a function of $(\text{flow})^{0.8}$ only and not a function of temperature also. This conclusion is based on data from 20% to 100% power steady state digital data, as shown in the Appendix.

$$hA_c = K_{12} W_c^{0.8} \quad (5.4)$$

The heat energy into the propellant in the core is Q_c from before. The heat energy removed from the propellant is just the difference between enthalpy out and enthalpy in.

$$\text{Energy removed} = W_{ce} H_{ce} - W_7 H_7.$$

Using the relation $W_c = (W_{ce} + W_7)/2$ and realizing that
 $(H_{ce} - H_7) \gg (W_c - W_7)$ or $(W_{ce} - W_c)$ we can assume that

$$\text{Energy removed} = W_c (H_{ce} - H_7).$$

Now writing an energy balance about the hydrogen

$$V_c \frac{d}{dt} [\bar{\rho} H_c - K_2 P_7 - K_3 P_{ce}] = Q_c - W_c (H_{ce} - H_7) \quad (5.5)$$

where the pressure terms arise from considering a change in enthalpy to come from a change in energy content plus a volume times change-of-pressure term.

A hydrogen mass balance can be written about the core

$$V_c \frac{d}{dt} (\rho_c) = W_7 - W_{ce} \quad (5.6)$$

This equation must be solved in an entirely different fashion when the flow through the core is assumed constant. The equivalent time constant of this term is short and is of concern only during transients in which the density is changing rapidly. Because of the large immediate reactivity effect of hydrogen density in the core, this method of calculating density was chosen.

A similar mass balance can be written about the core exit plenum

$$V_{ce} \frac{d}{dt} (\rho_{c2}) = W_{ce} - W_n - W_{tb} \quad (5.7)$$

The pressure drop through the core is a combination of the distributed drop through the propellant channels and the drop at the core inlet orifice. These have been combined into a single expression, $K_5 \frac{W_{ce}^2}{\rho_{ce}}$.

The net pressure drop acting on the fluid acts as an acceleration term

$$(P_7 - P_{ce}) - K_5 \frac{W_{ce}^2}{\rho_{ce}} = K_{13} \frac{d}{dt} W_c \quad (5.8)$$

Core exit conditions were chosen for simpler calculation of K_5 from comparisons with digital data.

To relate the several variables, equations of state were required. These were either the perfect gas law which is valid for normal core conditions or linear relations between temperature and enthalpy

$$P_7 = K_6 \rho_7 T_{ci} \quad (5.9)$$

$$P_{ce} = K_7 \rho_{ce} T_{ce} \quad (5.10)$$

$$T_{ce} = K_8 H_{ce} + K_9 \quad (5.11)$$

$$H_7 = K_{10} T_7 - K_{11} \quad (5.12)$$

The coefficients K_8 , K_9 , K_{10} , and K_{11} implicitly account for the small influence of pressure since they were determined by linear fits in the predicted ranges of temperature and pressure at the core inlet and exit.

The following linear averages were used to relate variables:

$$W_c = (W_{ce} + W_7)/2 \quad (5.13)$$

$$H_c = (H_{ce} + H_7)/2 \quad (5.14)$$

$$T_c = (T_{ce} + T_7)/2 \quad (5.15)$$

Linear averages are exact but are reasonable for the above variables. However, the average hydrogen density is far from a linear average of inlet and exit densities.

A much better fit can be obtained by assuming an exponential shape for density,

$$\rho(x) = a e^{-Ax} + b$$

where $\rho(x)$ = hydrogen density as a function of axial position

a , A , and b = empirical constants.

This assumption leads to the following equation for average density (the derivation is given in the Appendix).

$$\rho_c = \frac{1}{K_{20}} \left[\rho_7 + (K_{20} - 1) \rho_{ce} \right] \quad (5.16)$$

The value of K_{20} used was a best average of 34 values of K_{20} calculated from transient calculations of ρ_c , ρ_7 , and ρ_{ce} in the normal ranges of operating conditions from 10% to 100% power. The RMS error was 5.4%.

Combining Equation 5.16 with the linear relation for temperature and enthalpy we also obtain

$$\overline{\rho H}_c = \rho_c H_c + \frac{1}{K_{20}} (H_{ce} - H_7) \left[\rho_c - \frac{(\rho_7 + \rho_{ce})}{2} \right] \quad (5.17)$$

where

$$\overline{\rho H}_c \equiv \frac{\int (\rho H) dx}{\int dx}$$

This equation is used to solve for H_c .

6.0 Support Element

6.1 Introduction

The purpose of this section is: (1) to provide all the information necessary for the reader to duplicate the WANL analog computer heat transfer and coolant flow model of the KIWI-B4 reactor support element, (2) to show how the support element model was developed, and thus give an idea of how much detail is included in the model, and (3) to show the relationship between the analog computer and the digital computer models of the support element.

The following procedure is followed: (1) the physical model of the support element is developed, (2) basic heat transfer equations are written, and (3) the method of calculating the coefficients is discussed.

6.2 Background

One of the basic features of the B4 reactor core is the use of a support element to supply axial support to the core fuel elements. The key load bearing member of the support element is the tie rod.

The tie rod must be protected at all times from the extremely high temperatures of the core in order to maintain its structural integrity. This is done in two ways: (1) The tie rod and its coolant channel are surrounded by heat resistant materials of low thermal conductivity. (2) Some of the hydrogen coolant is forced to flow around the tie rod to remove heat. Tie rod temperature during reactor operation is therefore related basically to the combination of reactor power level and hydrogen coolant flow. This particular flow-power relationship required for tie rod protection must be considered at all times during

reactor power range operation. In fact, this flow-power relationship limits the hydrogen propellant core exit temperature, at reactor operating levels below 100% rated power.

Any reactor control system must be designed to consider this critical power-flow relationship limitation. Thus, the need for a good analog computer model of the tie rod becomes evident.

The support element simulation presented in this report is based to some extent on an earlier simulation by A. F. Maguire (WANL-TME-335). The present support element model contains one axial section as does the previous model. However, the present model includes a more detailed simulation of the radial components of the support element.

6.3 Physical Description

The support element provides the axial force required to hold its six neighboring fuel elements against the upper support plate of the core. The pressure difference across the core during reactor operation tends to drive the fuel elements axially out of the exhaust end of the core. The support element resists this force by placing the basic member of the support element, the tie rod, in tension.

The high temperatures encountered in the core during reactor operation are greatly above the maximum allowable tie rod temperature. Tie rod temperatures above this allowable temperature limit would weaken it and prevent it from adequately supporting its six fuel elements. Therefore, some means of keeping the tie rod cool must be provided. The remaining components of the support element are designed to perform this service.

Figure 6-1 shows an end view of the support element drawn to scale (dimensions are given in the lower left corner). The cross section of the support element agrees with this drawing over its entire length. The outer shell of the support element is of structural graphite. Its outer surface is hexagonal to mate with the six adjacent hexagonal fuel elements. The inner surface of the structural graphite shell is cylindrical. A cylindrical shell fits inside the structural graphite. This shell is of pyrolytic graphite; a material which has a much lower thermal conductivity than the structural graphite. A thin stainless steel cylindrical shell lines the inner surface of the pyrolytic graphite. The tie rod exists in the center of the void formed by the stainless steel sleeve. Thus, all components of the support element have a common cylindrical axis. A cylindrical void between the tie rod and the stainless steel sleeve provides a flow path for the hydrogen coolant. The function of the hydrogen is to carry away heat from the tie rod and the surrounding structure.

The inlet to the tie rod coolant channel is orificed to provide some degree of control over the division of hydrogen flow to the fuel elements and tie rods during reactor operation.

For the sake of simplicity, the outer surface of the support element was assumed to be cylindrical instead of hexagonal, the diameter of the cylinder being that diameter which gave the same volume of structural graphite as the hexagon. Such an approximation greatly simplifies the development of the analog equations and should have little effect on the correctness of the analog model.

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FIGURE 6-1

END VIEW OF SUPPORT ELEMENT

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6.4 Physical Equations

The basic equations used for the simulation of the support element are given in this section.

In order to show the relationship of the support element to the reactor, the block diagram below is shown. The block is assumed to represent the support element, or the equations which describe its behavior. The arrows shown pointing into the block represent the reactor variables, which are required for the solution of the support element equations. The arrows leaving the block show those variables which are generated in the block by the support element equations. These variables are needed by other parts of the reactor.

Block Diagram: Support Element

Energy Balance Equation for the Structural Graphite:

The rate of heat energy generation in the structural graphite Q_s , due to the reactor neutron flux is assumed to be directly proportional to that neutron flux level and is given by

$$Q_s = K_{22} N \quad (6.1)$$

The heat flow rate into the structural graphite from the fuel element is given by:

$$K_{21} (T_f - T_s)$$

where K_{21} is assumed to be constant.

The heat energy flow rate from the structural graphite to the pyrolytic graphite is given by:

$$K_{sp} (T_s - T_p)$$

where K_{sp} represents a temperature sensitive thermal conductance term which is assumed to vary in proportion to the thermal conductivity of pyrolytic graphite,

$$K_{sp} = K_{30} f_p \left(\frac{T_s + T_p}{2} \right) \quad (6.2)$$

The function $f_p(T)$ is given in Figure A6.1 of the Appendix.

The rate of heat energy accumulation in the structural graphite can be written as

$M_s C_{ps} \frac{dT_s}{dt}$. Since this expression is equivalent to the net heat flow rate into the structural graphite, we can write

$$M_s C_{ps} \frac{dT_s}{dt} = Q_s + K_{21} (T_f - T_s) - K_{sp} (T_s - T_p) \quad (6.3)$$

C_{ps} is assumed to be constant.

Energy Balance Equation for Pyrolytic Graphite:

The neutron heat generation rate in the pyrolytic graphite can be assumed to be direct by proportional to the neutron flux level. Thus,

$$Q_p = K_{23} N \quad (6.4)$$

The heat energy flow conducted into the pyrolytic graphite from the structural graphite is given by $K_{sp} (T_s - T_p)$.

The heat energy flow conducted from the pyrolytic graphite to the inner cylindrical surface of the pyrolytic graphite is given by $K_{pi} (T_p - T_i)$. K_{pi} is a quantity assumed to vary directly with the thermal conductivity curve of pyrolytic graphite,

$$K_{pi} = K_{31} f_p \left(\frac{T_p + T_i}{2} \right) \quad (6.5)$$

The function $f_p(T)$ is given in Figure A6.1 of the Appendix.

The rate of heat energy accumulation in the pyrolytic graphite is given by $M_p C_p \frac{dT_p}{dt}$.

Since this expression is equivalent to the net heat energy flow rate into the pyrolytic graphite we can write,

$$M_p C_p \frac{dT_p}{dt} = Q_p + K_{sp} (T_s - T_p) - K_{pi} (T_p - T_i) \quad (6.6)$$

C_p is assumed to be constant.

Energy Balance Equation for the Inner Surface of the Pyrolytic Graphite:

The rate of heat energy flow into the inner surface of the pyrolytic graphite is given by

$$K_{pi} (T_p - T_i).$$

The rate of heat energy flow from the inner pyrolytic graphite surface to the stainless steel sleeve is given by $K_{bi} (T_i - T_b)$. The term K_{bi} is assumed to vary directly as the thermal conductivity of hydrogen,

$$K_{bi} = K_{32} f_b \left(\frac{T_b + T_i}{2} \right) \quad (6.7)$$

where the function $f_b(T)$ is given in Figure A6.2 of the Appendix.

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Since the inner surface of the pyrolytic graphite can be considered to have zero mass, we can assume zero storage of heat energy. Therefore, the heat balance equation becomes

$$0 = K_{pi} (T_p - T_i) - K_{bi} (T_i - T_b) \quad (6.8)$$

Energy Balance Equation for the Stainless Steel Sleeve:

The rate of heat energy generation in the stainless steel sleeve is assumed to be directly proportional to the neutron flux level,

$$Q_b = K_{25} N \quad (6.9)$$

The rate of heat energy flow into the stainless steel sleeve from the pyrolytic graphite is given by $K_{bi} (T_i - T_b)$.

The rate of heat energy flow into the hydrogen from the stainless steel sleeve is given by $K_f (T_b - T_{gs})$. The term K_f is assumed to vary as a function of hydrogen flow given by

$$K_f = K_{29} (W_g)^{0.8} \quad (6.10)$$

The accumulation of heat energy in the stainless steel sleeve can be written as

$M_{ss} C_{pss} \frac{dT_b}{dt}$. This term is equivalent to the net heat flow into the stainless steel.

$$M_{ss} C_{pss} \frac{dT_b}{dt} = Q_b + K_{bi} (T_i - T_b) - K_f (T_b - T_{gs}) \quad (6.11)$$

C_{pss} is assumed to be constant.

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Energy Balance Equation for the Tie Rod:

The neutron heat generation rate in the tie rod is assumed to be directly proportional to the neutron flux level,

$$Q_{tr} = K_{24} N \quad (6.12)$$

The rate of heat energy flow from the tie rod to the hydrogen is given by $K_{ft}(T_{mtr} - T_{gs})$.

The term K_{ft} is assumed to vary directly with hydrogen flow, $K_{ft} \propto (W_g)^{0.8}$. Since K_f is also proportional to $(W_g)^{0.8}$ (Eq. 6.10), we can write

$$K_{ft} = K_{28} K_f \quad (6.13)$$

The rate of heat energy accumulation in the tie rod can be written as $M_{tr} C_{ptr} \frac{dT_{mtr}}{dt}$.

Since this is equivalent to the net heat flow rate to the tie rod, we can write

$$M_{tr} C_{ptr} \frac{dT_{mtr}}{dt} = Q_{tr} - K_{ft}(T_{mtr} - T_{gs}) \quad (6.14)$$

C_{ptr} is assumed to be constant.

Energy Balance Equation for the Hydrogen Coolant in the Tie Rod Flow Channel:

The rate of heat flow into the hydrogen coolant from the tie rod is $K_{ft}(T_{mtr} - T_{gs})$.

The rate of heat flow into the hydrogen coolant from the stainless steel sleeve is expressed as $K_f(T_{mtr} - T_{gs})$. Both K_{ft} and K_f are functions of flow (Equations 6.10 and 6.13).

The rate at which heat energy is convected out of the hydrogen coolant channel is given by the expression $C_{pg} W_g (T_{gse} - T_g)$. If it is assumed that the temperature of the

hydrogen coolant at the flow channel exit, T_{gse} can be obtained from

$$T_{gse} = 2T_{gs} - T_7 \quad (6.15)$$

The rate of heat energy convection from the tie rod coolant channel can be written as

$2 C_{pg} W_g (T_{gs} - T_7)$. C_{pg} in this expression is assumed constant.

Generally, the rate of heat storage in the hydrogen can be assumed zero. Therefore, the net heat flow rate into the hydrogen can be equated to zero,

$$0 = K_f (T_b - T_{gs}) + K_{ft} (T_{mtr} - T_{gs}) - 2 C_{pg} W_g (T_{gs} - T_7) \quad (6.16)$$

Hydrogen coolant flow into the tie rod channel can be calculated from the knowledge of the pressure drop across the tie rod channel inlet orifice and the inlet density of the hydrogen.

$$P_7 - P_{tri} = K_{26} \frac{W_{\theta}^2}{\rho_7} \quad (6.17)$$

A similar relation is written for the tie rod hydrogen coolant channel

$$P_{tri} - P_8 = K_{27} \frac{W_g^2}{\rho_8} \quad (6.18)$$

The density of the hydrogen at the outlet of the tie rod coolant channel can be obtained from the equation of state,

$$\rho_8 = \frac{P_8}{R T_{gse}} \quad (6.19)$$

The basic simplifying assumptions which were made in the development of these equations are listed in Table 6.1.

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These support element equations were all written on the basis of a single support element whose behavior can be considered equivalent to that of the average support element.

In order to obtain the total tie rod flow W_{tre} , from the single channel flow W_{gs} , the equation

$$W_{tre} = 289 W_{gs} \quad (6.20)$$

must be used where 289 is the number of support elements in the core.

6.5 Evaluation of Constants and Variable Coefficients

The evaluation of the constants and the evaluation of the 100% values of the variable coefficients (such as heat conductance terms) was greatly facilitated by the use of digital data. A digital model of the support element produced by the Reactor Analysis Group provided all the heat flow rates, the internal temperatures and the hydrogen pressures and flows for the case of 100% steady state reactor operation.

The digital data provided values for each of the variables (such as temperature, heat flow rate, etc.) for every 3" length interval in the direction of the core axis. This data was averaged for each variable, along the length of the core to provide 100% steady state values for the variables of a single axial section analog model. Table A6.1 of the Appendix presents this averaged data.

Figure 6-2 shows the averaged digital data applied to a scale drawing of an axial section of the physical model, i.e. a temperature profile. The arrows in this figure show the heat flow rates.

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FIGURE 6-2

TEMPERATURE PROFILE OF SUPPORT ELEMENT

TABLE 6.1
BASIC ASSUMPTIONS - SUPPORT ELEMENT
PHYSICAL EQUATIONS

1. The support element, with a hexagonal outer surface, can be represented for analytical purposes by a cylindrical surface of cross sectional area equal to that of the hexagon.
2. The support element can be represented by one axial section.
3. The thermal conductivity of the stagnant hydrogen in the gap between the fuel element and the support element is constant.
4. The thermal conductivity of the structural graphite is very high.
5. The thermal conductivity of the gap between structural and pyrolytic graphite is much higher than the conductivity of the pyrolytic graphite between its outer surface and its center.
6. The thermal conductivity of the stainless steel sleeve is very high.
7. The thermal conductivity of the tie rod is very high.
8. The hydrogen coolant has no heat capacity.
9. All specific heats (for the hydrogen coolant and the various materials in the support element) are constant.

In order to simplify the calculation of the constants and the variable coefficients in the physical equations, and also to insure agreement between analog and digital data at 100% steady state operation, the digital data was applied to the physical equations. The physical equations were then solved for the constants and the variable coefficients (such as heat conductances, hydrogen specific heat, neutron heat generation ratio, flow impedences, etc.). When the physical equations are rewritten in terms of the numerical values of these constants, one can see the relationship between the digital data and the analog equations. In effect, the analog equations have been "adjusted" to the digital data. Such adjustment guarantees perfect correspondence between the analog and digital models for 100% steady state operation.

Table A6.2 of the Appendix lists the 100% steady state value of each of the variable coefficients calculated in the above manner. Table A6.3 of the Appendix lists the value of each of the constants calculated as described above.

Those constants which were not calculated from digital data are listed in Table A6.4 of the Appendix. These terms in general could not be calculated as described above because they are coefficients of derivative terms (which go to zero in the steady state).

6.6 Additional Nomenclature Pertaining to the Support Element

Variables:

<u>Symbol</u>	<u>Definition</u>
T_s	Instantaneous temperature of the structural graphite at the center of its wall, ($^{\circ}\text{R}$).
T_p	Instantaneous temperature of the pyrolytic graphite at the center of its wall, ($^{\circ}\text{R}$).
T_i	Instantaneous inside wall temperature of the pyrolytic graphite, ($^{\circ}\text{R}$).
T_b	Instantaneous inside wall temperature of the stainless steel liner, ($^{\circ}\text{R}$).
T_{gs}	Instantaneous temperature of the hydrogen coolant flowing between the tie rod and the stainless steel sleeve, ($^{\circ}\text{R}$).
T_{mtr}	Instantaneous temperature of the tie rod center, ($^{\circ}\text{R}$).
T_7	Instantaneous temperature of the hydrogen coolant entering the tie rod channel at the core inlet, ($^{\circ}\text{R}$).
T_{gse}	Instantaneous temperature of the hydrogen coolant* leaving the tie rod channel at the core exit, ($^{\circ}\text{R}$).
Q_s	Instantaneous neutron heat generation rate in the structural graphite, (BTU/Sec).
Q_p	Instantaneous neutron heat generation rate in the pyrolytic graphite, (BTU/Sec).
Q_b	Instantaneous neutron heat generation rate in the stainless steel sleeve, (BTU/Sec).
Q_{tr}	Instantaneous neutron heat generation rate in the tie rod, (BTU/Sec).

Variables Continued:

<u>Symbol</u>	<u>Definition</u>
W_g	Instantaneous hydrogen flow in the tie rod coolant channel, (lb/sec).
P_{tri}	Instantaneous pressure at the inlet to the tie* rod coolant channel (at the exit of the core inlet orifice), (psia).
ρ_{tre}	Instantaneous hydrogen density at the tie rod coolant channel exit, (lb/ft ³).*
K_{sp}	Instantaneous heat conductance between the center of the structural graphite wall and the center of the pyrolytic graphite wall, (BTU/Sec °R).
K_{pi}	Instantaneous heat conductance between the center of the pyrolytic graphite wall cross section and the inside surface of the pyrolytic graphite wall, (BTU/Sec °R).
K_{bi}	Instantaneous heat conductance between the inner surface of the pyrolytic graphite sleeve and the inner surface of the stainless steel sleeve, (BTU/Sec °R).
K_f	Instantaneous heat conductance between the inner surface of the stainless steel sleeve and the hydrogen coolant, (BTU/Sec °R).
K_{ft}	Instantaneous heat conductance between the tie rod center and the hydrogen coolant, (BTU/Sec °R).

Constants:

K_{21}	Heat conductance between the cross sectional center of the structural graphite wall and the adjacent wall of the fueled element, (BTU/Sec °R)
M_s	Mass of the structural graphite sleeve in the support element, (lb).
C_{ps}	Specific heat of the structural graphite, (BTU/lb °R)

Constants Continued:

<u>Symbol</u>	<u>Definition</u>
M_p	Mass of the pyrolytic graphite, (lb).
C_p	Specific heat of the pyrolytic graphite, (BTU/lb °R).
M_{ss}	Mass of the stainless steel sleeve, (lb).
C_{pss}	Specific heat of the stainless steel sleeve, (BTU/lb °R).
C_{pg}	Specific heat of hydrogen in tie rod coolant channel, (BTU/lb °R).
M_{tr}	Mass of tie rod, (lb).
C_{ptr}	Specific heat of tie rod, (BTU/lb °R).
K_{22}	Ratio of the heat generation rate in the structural graphite with respect to per cent reactor neutron level, (BTU/Sec %).
K_{23}	Ratio of the heat generation rate in the pyrolytic graphite with respect to per cent reactor neutron level, (BTU/Sec %).
K_{24}	Ratio of the heat generation rate in the tie rod with respect to per cent reactor neutron level, (BTU/Sec %).
K_{25}	Ratio of the heat generation rate in the stainless steel sleeve with respect to per cent reactor neutron level, (BTU/Sec %).
$f_p(T)$	A function which relates the thermal conductivity of pyrolytic graphite to its temperature, (Figure A6.1, Appendix).
$f_b(T)$	A function which relates the thermal conductivity of hydrogen to its temperature, (Figure A6.2, Appendix).
K_{26}	Flow impedance offered to tie rod coolant by the orifice, (Sec ² /in ² ft ³).*
K_{27}	Flow impedance offered to the tie rod coolant by the tie rod coolant channel, (Sec ² /in ² /ft ³).

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Constants Continued:

<u>Symbol</u>	<u>Definition</u>
R	Gas constant for hydrogen divided by 144, (ft ³ /in ² °R).*
K ₂₈	The factor which relates the conductance of heat from the tie rod to the hydrogen (K _{ft}), to the conductance of heat from the stainless steel sleeve to the hydrogen (K _f).
K ₂₉	The factor which relates the conductance of heat from the stainless steel sleeve to the hydrogen (K _f), to the hydrogen flow taken to the 0.8 power.
K ₃₀	The factor relating the conductance of heat from the structural graphite to the pyrolytic graphite (K _{sp}), to the heat conductivity curve for pyrolytic graphite f _p (T), (ft hr/sec).
K ₃₁	The factor relating the conductance of heat from the center of the pyrolytic graphite to its inside wall (K _{pi}), to the heat conductivity curve for pyrolytic graphite f _p (T), (ft hr/sec).
K ₃₂	The factor relating the conductance of heat from the inside wall of the pyrolytic graphite to the stainless steel sleeve (K _{bi}), to the heat conductivity curve for hydrogen f _b (T), (ft hr/sec).

Subscripts

s	Structural graphite or support element.
p	Pyrolytic graphite.
i	Inner surface of the pyrolytic graphite shell.
b,ss	stainless steel sleeve liner.
tr	tie rod.
g	Hydrogen gas coolant.

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Subscripts Continued:

<u>Symbol</u>	<u>Definition</u>
i	Inlet
e	Exit
f	Film or fuel
m	Material

Superscripts

o	Pertaining to the 100% steady state condition.
---	--

NOTE:

In general, all the constants and variables described here represent an average value of that constant or variable taken along the length of the core. This is necessary for a single axial section analog simulation of the support element. Those terms marked with an asterisk (*) do not represent axially averaged quantities.

7.0 Core Exit Plenum and Nozzle

The hydrogen leaves the propellant channels at about Mach 0.2 and expands into the core exit plenum. Similarly, the tie rod coolant leaves the tie rod channel at about the same mach number and mixes with the other hydrogen. This tie rod flow is generally less than 5% of the total flow but is much cooler (800°R vs. 4000°R).

To calculate the mixed fluid temperature we write an energy balance

$$\frac{W_{ce} H_{ce} + W_{tr} H_{tre}}{W_{ce} + W_{tr}} = H_8 \quad (a)$$

and

$$T_8 = f(H_8) \quad (b)$$

However, as an expedient to save computer equipment, the analog mechanization solved the following equation:

$$\frac{W_{ce} C_{pce} T_{ce} + W_{tr} C_{ptre} T_{tre}}{W_{ce} C_{pce} + W_{tr} C_{ptre}} = T_8 \quad (7.1)$$

This equation assumes different but constant specific heats for the different temperature hydrogen flows. The specific heat is relatively constant over much of the temperature range but starts to increase considerably around 4000°R. If equipment is available it would be preferable to use Equations (a) and (b) rather than (1).

The nozzle equation used is the common form for choked flow

$$W_8 = K_{48} \frac{P_{ce}}{\sqrt{T_8}} \quad (7.2)$$

Block Diagram:
Core Exit Plenum and Nozzle

Since the hydrogen is moving relatively slowly through the core exit plenum, static and total pressures will be equal. If no losses are assumed then the static pressure in the fuel channel at the exit can be used. Actually, some losses occur and are calculated in digital steady state codes. The constant K_{48} is obtained directly from the digital code values of W_g , P_{ce} , and T_g at rated conditions. Any errors arising from using the same coefficient for all conditions are neglected.

At present, thrust is not simulated since no vehicle studies are being made.

8.0 Neutronics

The usual representations of nuclear reactor kinetics are used in NERVA analog analysis. These representations assume that the neutron and power generation spatial distributions are invariant. Actually, minor changes in distribution do occur due to changes in control drum position, hydrogen mass distribution, and core temperature.

These changes in power distribution are found to be small, based on nuclear steady-state calculations, and are assumed to have little influence on reactor dynamics. A long range effort is aimed at incorporating these effects in the analog but all studies to date use the standard shape-independent neutronics representations.

Block Diagram: Neutronics

KIWI test experience indicates that reactor dynamics analysis based on these representations is valid for the type of reactors being used for NERVA.

The linear neutronics equations are:

$$\frac{dN}{dt} = \frac{\delta K - \beta}{\Lambda} N - \sum \lambda_i C_i + S \quad (8.1)$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\Lambda} N - \lambda_i C_i \quad (8.2)$$

where

N = Neutron density

δK = Total reactivity

β_i = Effective fraction of neutrons in i^{th} delayed group

β = $\sum \beta_i$ = Total effective delayed neutron fraction

- λ_i = Decay constant of the i^{th} group of delayed neutron emitters
 C_i = Concentration of the i^{th} group of delayed neutron emitters
 Λ = Mean effective neutron lifetime
 S = External source density
 t = Time

The values for λ_i and β_i/β used are the latest published data (Neutron Data for Reactor Kinetics, G. R. Kepin, Nucleonics, Vol. 20, No. 8).

Group	λ_i	β_i/β
1	0.0127	.038
2	0.0317	.213
3	0.115	.188
4	0.311	.407
5	1.40	.128
6	3.87	<u>.026</u>
		1.000 Check

The values for β and Λ are dependent on the particular core design. The latest values used for engine analysis are given in the Appendix.

For most power range studies the source temperature S is negligible.

The 6-group linear neutronics just described is one of two forms used in the engine simulation.

For many studies it is found convenient to use a logarithmic neutronics form. This essentially is just a rearrangement of Equations (1) and (2) which takes the form:

$$U = \frac{\delta K - \beta}{\Lambda} - \sum \lambda_i \left(\frac{C_i}{N} \right) + \frac{S}{N} \quad (8.3)$$

$$\frac{d}{dt} \left(\frac{C_i}{N} \right) = \frac{\beta_i}{\Lambda} - \lambda_i \left(\frac{C_i}{N} \right) - \left(\frac{C_i}{N} \right) U \quad (8.4)$$

$$\int_0^t U dt = \int_{N_0}^{N(t)} \frac{dN}{N} = \ln \left(\frac{N(t)}{N_0} \right) \quad (8.5)$$

where

$$\frac{d \ln N}{dt} \equiv U = \text{Inverse reactor period.}$$

This form of neutronics allows simulation over many decades of power and is useful in subpower range studies and in malfunction studies. It is also useful when the power controller is based on log power.

For simulating these equations one multiplication is required per delayed group. In order to simplify the analog setup it is convenient to reduce the number of delayed groups. Equivalent 3-group values of λ_i and β_i can be obtained by curve fitting transfer functions.

T. Springer of Los Alamos (in "Delayed Neutron Reduced Groups by Least Squares" N-4-1027U, 19 December 1961) has done this using a least squared error criterion and his values are used here:

$\lambda_1 = .0276$	$\beta_1/\beta = .274$
$\lambda_2 = .235$	$\beta_2/\beta = .559$
$\lambda_3 = 1.65$	$\beta_3/\beta = .167$

The fit is good for startup and power operation but starts to produce errors if used for long term shutdown studies, where the longest delayed group becomes most significant. Alternate values for λ_i and β_i would be used for shutdown studies on the analog or else all six groups

would be simulated. However, the detailed long term shutdown studies are presently performed by digital analyses which not only include the six groups but also account for fission product decay.

9.0 Reactivity Calculations

9.1 Inherent Reactivity Feedbacks

There are two significant inherent reactivity feedbacks to be accounted for in the dynamic analysis of NERVA reactors. These are the hydrogen density effects and the material temperature effects.

Hydrogen has several properties which affect core reactivity. Neutrons can collide with hydrogen atoms and can thus undergo both a change in direction and an energy loss.

Neutrons can also react with the hydrogen and be removed from the system. Increasing the number of hydrogen atoms around the outside of the core (reflector, core inlet plenum) will reflect some of the neutrons leaking away

from the core and also absorb some which would have been reflected back by other materials. These two processes compete and in the reflector the net effect is small and may be that of a poison. At the core inlet there is a definite positive effect, i.e. increasing hydrogen density increases core reactivity.

In the core proper increasing hydrogen density increases the population of lower energy neutrons which are more effective in causing fissioning. This definitely overbalances the increase in absorption of neutrons for NERVA type cores.

Block Diagram: Reactivity Calculations

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This simplified picture of hydrogen effects aids in understanding the results of nuclear calculation. From interpretation of many steady-state calculations the following relations were obtained

$$\delta k_{H1} = \frac{0.078 \rho_c}{\rho_c + 0.964} \quad \pm 20\% \quad (a)$$

$$\delta k_{H2} = 0.0320 \rho_c \quad \pm 20\% \quad (b)$$

where

$$\rho_c = \text{Average core hydrogen density} = \frac{\int \rho(x) dx}{\int dx}$$

$$\delta k_{H1} = \text{Reactivity associated with hydrogen in the core}$$

$$\delta k_{H2} = \text{Reactivity associated with hydrogen external to the core}$$

$$x = \text{Axial position.}$$

It was possible to describe δk_{H2} as a function of ρ_c since there is an implicit correlation between ρ_c and the hydrogen densities in the rest of the system.

These relations were plotted and a straight line approximation made for the combined hydrogen reactivity as a function of average core hydrogen density.

$$\delta k_H = 0.1054 \rho_c \quad (9.1)$$

where

$$\delta k_H = \text{reactivity associated with all hydrogen in reactor.}$$

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The material temperature effects can be separated into two major components. There is an effect rising from the thermal expansion of the core and change of thermal base. There is the Doppler effect relating to the fuel temperature. Both these terms lead to reductions in reactivity with increases in temperature. Combining these terms an overall temperature reactivity effect is determined. The approximation used in analog studies is

$$\begin{aligned}\delta k_{Tf} &= 0.0035 - 5.88 \times 10^{-6} T_f, & 0 < T_f < 2500^\circ\text{R} \\ &= 0.01055 - 8.70 \times 10^{-6} T_f, & 2500^\circ\text{R} < T_f\end{aligned}\tag{9.2}$$

where

δk_{Tf} = Reactivity associated with average material temperature

T_f = Average material temperature.

In the present analog model one axial section is used and thus all the temperature reactivity is associated with the one average temperature. Where more axial sections are used the weighting of this reactivity would be taken as uniform in light of present data.

9.2 Drum Reactivity

The angular rotation of the control drums in the reflector determines their reactivity contribution. The general relation is

$$\delta k_r = f(\theta)$$

where

θ is the drum position.

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The zero reference is where the poison strip on the drum is as near the core as possible.

The full 180° rotation of the drum then puts the poison as far away as possible and the full reactivity span of the drums is realized.

Calculated and experimental results have indicated that when all the drums are moved together, the functional relation is very closely approximated by

$$\delta k_r = K_{49} \sin^2 \left(\frac{\theta}{2} \right) \quad (9.3)$$

The present reference value of K_{49} is 0.0569.

9.3 Total Reactivity

The total reactivity, δk , is just the sum of the component reactivities plus a reference value which represents the inherent reactivity of the cold core with no hydrogen and with the drums at 0°. This reference value is negative since the core must not be critical under these conditions. The value is negative since the core must not be critical under these conditions. The value is termed the shutdown reactivity, δk_s . Thus,

$$\delta k = \delta k_H + \delta k_{Tf} + \delta k_r + \delta k_s \quad (9.4)$$

The present reference value of δk_s for engine studies is -0.0156.

10.0 Reactor Controls and Instrumentation

10.1 Introduction

The purpose of the reactor control system is to automatically bring the reactor from a sub-critical state at very low power to a steady state condition at rated power within a specified interval of time; and to shut down and restart the reactor automatically from any intermediate power level. Furthermore, the control system must regulate the transient operation during startup, shutdown and restart within the restrictions imposed by the physical characteristics of the engine. This is accomplished by a dual-mode control system consisting of a subpower range section, source level to 1% power and a power range section, 1% to 100% power. A more detailed description of the design criteria, operation and analysis of the reactor control system is given in reference 7.

Block Diagram: Reactor Control System

10.2 Subpower Range Control Section

Figure 10-1a is a block diagram of the subpower range section of the reactor control system. The starting signal is a fixed voltage applied to block A. Block A consists of an integrating amplifier with a feedback network to the amplifier grid. For a fixed startup signal voltage, the output of this block is a ramp in demanded Log ND₁ signal corresponding to a two second reactor period.

The feedback circuit of block A consists of a comparison of the measured and demanded Log N signals plus a bias "a" applied to the amplifier grid through a diode. The effect of this circuit is to decrease the Log ND₁ ramp rate when the Log ND₁ signal exceeds the Log NM signal by the amount of the bias "a".

The Log ND₁ and Log NM signals are compared and the error signal $\epsilon_{\text{Log } N_1}$ is applied to block B. Block B consists of a non-symmetric gain network with $\epsilon_{\text{Log } N_1}$ input and $\dot{\theta}_D$ output defined by

$$\dot{\theta}_D = 12.5 \epsilon_{\text{Log } N_1} \quad -0.375 \leq \epsilon_{\text{Log } N_1} \quad (10.1)$$

$$\dot{\theta}_D = 25 \epsilon_{\text{Log } N_1} + 4.7 \quad -0.375 > \epsilon_{\text{Log } N_1} \quad (10.2)$$

Upper and lower limits are placed on the output of block B to prevent excessive control drum velocity demand signals, $\dot{\theta}_D$.

The control drum velocity demand signal is integrated in block C to produce the demanded control drum position signal θ_D . This signal is applied to the control drum actuator system to position the drums at the demanded value.

A coincidence circuit terminates the subpower range startup ramp at 1% power by removing the startup signal.

The Log NM signal is obtained from three channels of nuclear instrumentation, two sub-power range channels and one power range channel. The equation describing the nuclear instrumentation dynamics in the subpower ranges is

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$$\tau_0 \frac{d \log NM}{dt} = \frac{N}{N_0} - \epsilon^{\log NM} \quad (10.3)$$

$$\tau_0 = 1 \text{ second}$$

The control drum actuator dynamics are described by a second-order equation as follows:

$$\frac{\theta}{\theta_D} = \frac{1}{\left(\frac{s}{10\pi}\right)^2 + 2(0.5)\left(\frac{s}{10\pi}\right) + 1} \quad (10.4)$$

10.3 Power Range Control Section

Figure 10-1b is a block diagram of the power range section of the reactor control system.

Block D represents the exit chamber pressure transducer with output $P_c M$ applied to block E. The output of block E is a signal proportional to the demanded exit chamber gas temperature. This function, given on Figure 10-2 in the Appendix, defines the relationship between temperature and flow such that the tie rod exit gas temperature will be maintained essentially constant throughout the power range. Block F is a rate limiter circuit to prevent temperature demand signal rates from exceeding $+150^\circ\text{R}/\text{sec}$ and $-300^\circ\text{R}/\text{sec}$.

The temperature demand signal $T_c D$ is the summation of $T_c D^i$ and the output of block G. Block G is a lead network which is blocked for an increasing $T_c D^i$ signal. For decreasing $T_c D^i$ signals, block G reduces the $T_c D$ signal and hence the power level so that the tie rod limitations are met.

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The measured ($T_c M$) and the demanded temperature signals are compared and applied to block I, the temperature controller. Block I is a proportional-plus-integral controller as follows.

$$\frac{\text{Log ND}_2}{\epsilon_T} = 0.3250 + \frac{0.0325}{S} \quad (10.5)$$

The measured chamber signal $T_c M$ is obtained from the exit gas thermocouple, block H, defined by:

$$\frac{T_c M}{T_8} = \frac{1}{\tau S + 1} \quad (10.6)$$

$$\tau = \frac{0.3}{\sqrt{\frac{W_8}{70 \text{ lb/sec}}}} \quad (10.7)$$

The power demand signal Log ND_2 is compared to the measured log power signal Log NM and the error is applied to block J, the power controller. Block J is a proportional-plus-integral controller as follows:

$$\frac{\theta_D}{\epsilon_{\text{Log ND}_2}} = 1.0 + \frac{10}{S} \quad (10.8)$$

The demanded control drum position θ_D is applied to the control drum actuator system to position the drums.

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FIGURE 10-1a
SUBPOWER RANGE BLOCK DIAGRAM

FIGURE 10-1b
POWER RANGE BLOCK DIAGRAM

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11.0 Turbopump System

11.1 Introduction

The NERVA turbopump system, consisting of turbine, pump, turbine power control valve, control system, bleed system and associated piping and valving, is being developed by AGC. The mathematical descriptions used in the analog are based on information provided by AGC before March 1963. Since then a later design has been formulated and reported by AGC in ref. 8. This later model will be used in future engine studies.

The equations presented here are the ones used by WANL in the last engine studies. The block diagram below shows how this system fits into the overall system shown in Figure 2.1. The control portion of the system is treated separately in Section 12.0.

Block Diagram: Turbopump System

11.2 Physical Equations for Turbopump

The turbine and pump operation are typically described by a series of operating characteristics or maps determined experimentally and theoretically. These maps may then be reduced to equation form or simple function form in most cases. The following equations are the reduced form of the design operating maps.

Turbine Torque

$$M_t = (K_{60} \sqrt{T_{15}} - K_{61} N) W_{15} \quad (11.1)$$

Pump Torque

$$M_p = N^2 \left(f_{t1} \frac{W_3}{N} \right) \quad (11.2)$$

Pump Efficiency

$$\frac{1}{\eta_p} = f_{t3} \left(\frac{W_3}{N} \right) \quad (11.3)$$

Pump Head

$$\Delta H = N^2 f_{t2} \left(\frac{W_3}{N} \right) \quad (11.4)$$

These functions and constants are presented in the Appendix.

A torque balance is used to calculate turbopump speed

$$\frac{dN}{dt} = K_{62} (M_t - M_p) \quad (11.5)$$

The pump efficiency represents the fraction of the total power input that is converted to hydraulic power. The rest of the power goes into heat. The temperature rise can thus be related to the pressure rise and efficiency by

$$T_3 - T_2 = \left(\frac{1 - \eta_p}{\eta_p} \right) \frac{\Delta H}{778 C_{pd}} \quad (11.6)$$

The exit pressure is simply

$$P_3 = P_2 + K_{63} \Delta H \quad (11.7)$$

This assumes a constant fluid density in the pump at all operating levels. Equation 11.6 shows that generally both pump exit temperature and pressure will increase together.

These have opposing influences on density which thus tends to remain relatively constant.

The specific heat used in Equation 11.6 changes considerably at the pump conditions and is treated as a function of temperature and pressure

$$C_{pd} = T_3 f_{t4}(\Delta H) \quad (11.8)$$

The temperature T_3 varies only a little during most runs and such changes are negligible for power range studies. The simulation was included as a step toward refinement which would extend the range of validity of the models.

12.0 Turbopump Controls and Bleed System

12.1 Control System

The hydrogen flow in the NERVA engine is regulated by controlling the amount of bleed flow to the turbine. The exit chamber pressure is the control variable used to adjust the turbine power control valve (TPCV) blade angle, thus adjusting turbine inlet flow and the system total flow.

A block diagram of the pressure controller for the turbine is given on Figure 12-1a, (reference 6). The pressure transducer measures the exit chamber pressure P_8 . The transfer function of this transducer is:

$$\frac{P_c M}{P_8} = \frac{0.008}{1 + 0.004 S} \quad (12.1)$$

The measured pressure, $P_c M$, is compared to the demanded chamber pressure, $P_c D$, forming the pressure error, ϵ_p . The error is applied to the pressure controller which consists of the following transfer function:

$$\frac{a_D}{\epsilon_p} = \frac{6.18 (1 + 0.107 S)}{S} \quad (12.2)$$

The overall loop gain given above is a factor of four lower than the gain given in reference 6.

Block Diagram:

Bleed System and Turbopump Controls

FIGURE 12-1a

PRESSURE CONTROL SYSTEM BLOCK DIAGRAM

FIGURE 12-1b

HOT-BLEED SYSTEM SIMPLIFIED DIAGRAM

The demanded TPCV angle signal α_D is applied to the TPCV actuator. The transfer function of the actuator is:

$$\frac{\alpha}{\alpha_D} = \frac{1}{\left(\frac{s}{10\pi}\right)^2 + 2(0.5)\left(\frac{s}{10\pi}\right) + 1} \quad (12.3)$$

where α is the TPCV position. The TPCV flow area vs. angle is given on Figure A12-1 in the Appendix.

12.2 Bleed System

A simplified diagram of the hot-bleed system piping used for the engine studies is given on Figure 12-1b.

Hot gas is bled from the exhaust chamber to run the turbine. This hot gas is mixed with cooler gas from the nozzle cooling jacket at the reflector inlet. The mixed gas flow to the turbine is regulated by the turbine power control valve to control turbine speed and hence the reactor flow rate.

The pressures and temperatures in the hot and cold bleed lines are assumed equal to the pressures and temperatures in the exit chamber and the reflector inlet, respectively.

Thus,

$$P_{11} = P_5 \quad (12.4)$$

$$T_{11} = T_5 \quad (12.5)$$

$$P_{12} = P_8 \quad (12.6)$$

$$T_{12} = T_8 \quad (12.7)$$

The gas flow for the cold (W_{11}) and hot (W_{12}) bleed points are given by the orifice equations:

$$W_{11} = K_{50} \frac{P_5}{\sqrt{T_5}} \Psi \left(\frac{P_{13}}{P_5} \right) \quad (12.8)$$

$$W_{12} = K_{51} \frac{P_8}{\sqrt{T_8}} \Psi \left(\frac{P_{13}}{P_8} \right) \quad (12.9)$$

where Ψ is defined and given on Figure A12-2 in the Appendix.

The total flow to the turbine is the sum of the hot and cold bleed flow:

$$W_{15} = W_{11} + W_{12} \quad (12.10)$$

The mixed gas bleed temperature is given by the energy balance equation:

$$T_{13} = \frac{K_{52} W_{11} T_5 + K_{53} W_{12} T_8}{W_{15}} \quad (12.11)$$

The pressure drop and heat input to the bleed gas in the line from the mixing point to the TPCV inlet is neglected, thus

$$T_{13} = T_{14} = T_{15} \quad (12.12)$$

$$P_{13} = P_{14} \quad (12.13)$$

The ratio of the TPCV exit-to-inlet pressures is a function of the valve area A_{TV} as follows:

$$\frac{P_{15}}{P_{14}} = \frac{K_{54} A_{TV} \Psi \left(\frac{P_{15}}{P_{14}} \right) \sqrt{T_{14}}}{\Psi \left(\frac{P_{16}}{P_{15}} \right) \sqrt{T_{15}}} \quad (12.14)$$

The bleed mixing point pressure is determined by the equation

$$P_{13} = \frac{K_{55} W_{15} \sqrt{T_{15}}}{A_{TV} \Psi \left(\frac{P_{15}}{P_{14}} \right)} \quad (12.15)$$

The ratio of the turbine exit-to-inlet pressures is assumed to be constant:

$$\frac{P_{16}}{P_{15}} = 0.1 \quad (12.16)$$

The above bleed equations are obtained from reference 2.

13.0 Conclusions

This report has presented the engine analog models in their present state. Changes will be made to both the physical design and to the analytic representations, thus presenting a continually changing analog simulation. This model status report is a useful reference point for future model development.

The direction of development has been and will be to obtain digital computer data, based on quite detailed models, for comparison with analog data. The aim will be to determine those simple model coefficients which result in the best fit to the more refined digital data, and to enable the magnitude of error to be expected.

NRX and other test data will be used when possible to evaluate and develop models.

LIST OF REFERENCES

The analog models are based on data obtained from various sources. Early work is almost always based on informal undocumented data, but as design work expands documentation of source data becomes possible. This is a slow process but should be done when there are a variety of dynamic analyses performed by different groups and the differences are to be understood. The list included here is a starting point. Sources for future changes in design and future modification of the models based on later comparisons should be identified and listed.

Included in this list are some reports of analyses based on the analog models. The end product of model building is analysis of the systems and the two must not be divorced. Evaluation of the models will rest on how well the analyses are supported by experimental results.

It is anticipated that future analyses reports will refer to some model report and will spell out any variations from the referenced model. This will facilitate the comparison of analyses.

<u>Number</u>	<u>Description</u>
1	WANL-TME-103, "NERVA Power Range Analysis", A. F. Maguire, August 15, 1962 This presents an earlier model of the entire engine which used explicit shield and nozzle cooling jacket representations.
2	AGC 7082.3, "Hot Bleed Equations", N. E. Maclean, September 24, 1962 This early document presents the turbopump equations discussed in the model status report.
3	WANL-TME-334, "Preliminary Analysis of an Analog Simulation of the B-4 Core", G. L. Hohmann, May 2, 1963 This is an external issue of WANL-TMI -333, October 22, 1962. The basic core heat transfer and flow models are presented here and comparisons between digital and analog data made.

<u>Number</u>	<u>Description</u>
4	<p>WANL-TME-188, "Functional Operations of the Reference Reactor Control System", November 9, 1962</p> <p>This is an early published description of the control system. The present design is functionally similar but differs in some details. A more recent description appears in Reference 5.</p>
5	<p>WANL-TME-335, "Preliminary Fuel Element Support Simulation", A. F. Maguire, May 6, 1963</p> <p>This is an external issue of WANL-TMI-369, November 12, 1962. An earlier version of the support element simulation is presented here, along with results of a dynamic study.</p>
6	<p>732:L0056:dj, AGC Letter, N. L. Irvine, January 8, 1963</p> <p>This letter transmits turbopump control system data.</p>
7	<p>WANL-TME-270, "NERVA Engine Reactor Control System: Preliminary Synthesis and Analysis", February 8, 1963</p> <p>This is the most recent WANL control study report. It presents the reference control design along with analyses of the system operation.</p>
8	<p>TM 738:63-202, "Equations for Analog Simulation of the AJ30-5 NERVA Engine in the Power Range (U)", N. E. Maclean, May 29, 1963</p> <p>This is an AGC document which presents the engine physical models developed by AGC. It is (together with its Supplement) prime source for turbopump and associated systems information which will be used in our next engine studies.</p>
9	<p>TM 738:63-202, Supplement, July 9, 1963</p> <p>This Supplement presents the analog models used to mechanize the previously reported physical models. Later turbopump data is included.</p>
10	<p>734:M0296:bs, "Hot Bleed Cycle Schematic Diagram", June 5, 1963</p> <p>The cycle information is used in recalculation of coefficients so that steady state operating conditions are reproduced. The station designations serve as a basis for the nomenclature in the WANL model status report.</p>

APPENDIX

This Appendix contains details, such as special derivations, lists of coefficients, steady state data, and various functional relations, referred to in the main body. These details are presented by section number of the main body.

3.0 Nozzle Cooling Region

Derivation of Equation 3.1:

A one-section model of the nozzle cooling region is described by the following equations:

$$M_z C_z \frac{dT_z}{dt} = K_{fh} (T_9 - T_z) - K_{fc} (T_z - T_{nc}) \quad (A3.1)$$

$$T_{nc} = (T_5 + T_4)/2 \quad (A3.2)$$

$$K_{fh} = f_{z1} (W_4) \quad (A3.3)$$

$$K_{fc} = f_{z2} (W_9) \quad (A3.4)$$

$$W_4 (H_5 - H_4) = K_{fc} (T_z - T_{nc}) \quad (A3.5)$$

$$H_5 = f_{z3} (T_5, P_5) = C_{p5} T_5 \quad (A3.6)$$

$$H_4 = f_{z4} (T_3, P_3) = C_{p3} T_3 \quad (A3.7)$$

where the subscripts refer to

- 3 = Pump exit
- 4 = Nozzle cooling inlet
- 5 = Reflector inlet
- 9 = Nozzle throat (hot side)
- z = Nozzle material

and

$M_z C_z$	=	Heat capacity of the nozzle material, BTU/R ^o
K_{fh}	=	Heat transfer coefficient between the nozzle material and the hot hydrogen, BTU/sec-R ^o
K_{fc}	=	Heat transfer coefficient between the nozzle material and the cold nozzle coolant, BTU/sec-R ^o
T_{nc}	=	Average temperature of the nozzle coolant, °R
T	=	Temperature, °R
H	=	Enthalpy, BTU/lb
C_p	=	Specific heat, BTU/lb-R ^o
W	=	Mass flow rate, lb/sec
P	=	Pressure, psia

Defining

$$\tau_1 \equiv \frac{M_z C_z}{K_{fh} + K_{fc}} \quad (A3.8)$$

$$K_{f2} \equiv \frac{K_{fc} + K_{fh}}{K_{fc}} \quad (A3.9)$$

and noting that digital data indicates $1 < K_{f2} < 2$ (A3.10)

we obtain

$$\begin{aligned}
 T_5 = T_3 & \left[\frac{K_{f2} \left(\frac{2 W_3 C_{p3}}{K_{fc}} \right) + S \left(\tau_1 K_{f2} \frac{2 W_3 C_{p3}}{K_{fc}} \right)}{K_{f2} \left(\frac{2 W_3 C_{p5}}{K_{fc}} \right) + S \left(\tau_1 K_{f2} \frac{2 W_3 C_{p5}}{K_{fc}} \right)} \right] \\
 & + T_9 \left[\frac{2 (K_{fh}/K_{fc})}{K_{f2} \left(\frac{2 W_3 C_{p5}}{K_{fc}} \right) + S \left(\tau_1 K_{f2} \frac{2 W_3 C_{p5}}{K_{fc}} \right)} \right] \quad (A3.11)
 \end{aligned}$$

Now we assume

$$C_{p3} = K_A C_{p5} \quad (A3.12)$$

Then

$$T_5 = T_3 K_A + T_9 K_B \frac{1}{1 + \tau_1 s} \quad (A3.13)$$

where

$$K_B \equiv \frac{K_{fh} K_{fc}}{K_{fh} + K_{fc}}$$

In this equation the K_A and K_B may be variables. If the heat transfer coefficients K_{fh} and K_{fc} are assumed to be proportional to flow a simpler form results

$$T_5 = T_3 K_A + T_9 K_C W_3 \left(\frac{1}{1 + \tau_1 s} \right) \quad (A3.14)$$

From digital computer data at two steady states (50% flow, 20% power and 100% flow, 100% power) the time constant τ_1 was calculated to vary from 3.16 to 2.60. It was felt justifiable to approximate τ_1 with a constant 3.0 second value. Because of the limited data to be fit, simple constants were determined for K_A and K_C . K_A was set equal to one -- this is reasonable since at very low flows one expects $T_5 \approx T_3$.

Thus

$$T_5 = T_3 + 0.2435 \times 10^{-3} W_3 \frac{T_9}{(1 + 3 s)} \quad (A3.15)$$

Calculation of coefficient in Equation 3.2:

Steady State Values

$$P_3 = 915 \text{ psia}$$

$$P_5 = 730 \text{ psia}$$

$$W_3 = 75.1 \text{ lb}_m/\text{sec}$$

$$T_5 = 12.6 \text{ }^\circ\text{R}$$

$$\rho_5 = 1.09 \text{ lb}_m/\text{ft}^3 \text{ from perfect gas law}$$

$$\text{Calculated value: } K_{40} = 0.0358$$

4.0 Reflector Region

Table A4-1 gives the integrated heat capacity of the reflector as a function of the average reflector material temperature. For this function it was assumed that 2000 lbs of beryllium and 1000 lbs of aluminum were effective for fast transients and that one average temperature applied to both. This is not rigorous but since the model is only a one section, single channel model some such assumption is necessary.

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TABLE A4-1

T_{mr} (°R)	H_{mr} (BTU × 10 ³)	T_{MR} (°R)	H_{MR} (BTU × 10 ³)
0	0	420	163
100	18	480	218
180	32	540	280
240	48	600	348
300	75		
360	115		

where T_{mr} = Average temperature of reflector material

$$H_{mr} \equiv \int_0^{T_{mr}} M_{mr} C_{mr} dT_{mr}$$

List of Coefficients

$$\begin{aligned} K_{41} &= 217.5 \\ K_{42} &= 6.57 \\ K_{43} &= 0.00572 \\ K_{44} &= 2 \end{aligned}$$

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5.0 Core Heat Transfer and Flow

Derivation of Equation 5.16:

From various plots of core hydrogen density as a function of distance along the axis from the inlet, one can roughly assume the shape to be of the form

$$\rho_c(x) = ae^{-Ax} + b \quad (A5-1)$$

where $\rho_c(x)$ is the local density at x

a , A , and b are constants.

Defining

$$\bar{\rho}_c \equiv \frac{\int_0^L \rho_c(x) dx}{\int_0^L dx} \quad (A5-2)$$

where L = length of core

and performing the integration

$$\bar{\rho}_c = \frac{1}{L} \left[\frac{a}{A} (1 - e^{-AL}) + bL \right] \quad (A5.3)$$

Applying the boundary conditions

$$\rho_{ci} = a + b \quad (A5.4)$$

$$\rho_{ce} = ae^{-AL} + b$$

and making the approximation that $ae^{-AL} \ll b$

$$\bar{\rho}_c = \frac{\rho_{ci}}{AL} + \frac{AL - 1}{AL} \rho_{ce}$$

or

$$\bar{\rho}_c = \frac{1}{K_{20}} \left[\rho_7 + (K_{20} - 1) \rho_{ce} \right] \quad (A5.5)$$

where

$$K_{20} \equiv \frac{1}{AL} \quad (A5.6)$$

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The specific heat of unfueled graphite is given in Figure A5-1.

The heat transfer coefficient, hA_c , between the fuel and the propellant can be assumed to be a function of $(\text{flow})^{0.8}$ along with other factors. Treating this as the major factor, values of $hA_c/(W_c)^{0.8}$ were calculated from digital data and are presented in Figure A5-1. This figure shows a significant change of this coefficient with axial position but surprisingly does not show much change with operating power. This implies that the effective heat transfer coefficient is not explicitly dependent upon the propellant temperature or pressure and justifies taking an average axial shape. For the one axial section model, use of a single constant value for $hA_c/(W_c)^{0.8}$ is justified.

List of Constants

V_{ce}	9.84 ft^3
V_c	6.55 ft^3
M_f	2652 lb
K_1	$1.037 \times 10^4 \text{ BTU/sec}$
K_2	$0.1203 \text{ BTU in}^2/\text{lb ft}^3$
K_3	$0.06478 \text{ BTU in}^2/\text{lb ft}^3$
K_5	$0.0004375 \text{ sec}^2/\text{ft}^3 \text{ in}^2$
K_6	$5.32 \text{ ft}^3/\text{in}^2 \text{ } ^\circ\text{R}$
K_7	$5.32 \text{ ft}^3/\text{in}^2 \text{ } ^\circ\text{R}$
K_8	$0.2873 \text{ lb } ^\circ\text{R}/\text{BTU}$

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FIGURE A5-1
 SPECIFIC HEAT OF UNFUELED GRAPHITE

FIGURE A5-2
 HEAT TRANSFER COEFFICIENT/(FLOW)^{0.8}

List of Constants

K_9	22.03°R
K_{10}	$3.48 \text{ BTU/lb }^{\circ}\text{R}$
K_{11}	76.67 BTU/lb
K_{12}	$5.172 \text{ BTU ft}^3/\text{lb }^{\circ}\text{R}$
K_{13}	$0.0005836 \text{ sec}^2/\text{in}^2$
K_{20}	7.28

6.0 Support Element

TABLE A6.1

100% Steady State Value of the Support Element
Variables Averaged from Digital Data

Variable	Value
T_f°	2950°R
T_s°	3050°R
T_p°	2296°R
T_i°	1503°R
T_6°	721°R
T_7°	210°R
T_{gse}°	880°R
Q_s°	51.30 BTU/sec
Q_p°	8.58 BTU/sec

TABLE A6.1

Q_b°	2.09 BTU/sec
Q_{tr}°	5.61 BTU/sec
W_g°	0.01313 lb/sec
N°	100%
P_7°	689 psi
P_{tri}°	586 psi
P_8°	560 psi
ρ_7°	0.604 lb/ft ³
ρ_{tre}°	0.131 lb/ft ³

TABLE A6.2

100% Steady State Value of the Support Element
Variables and Variable Coefficients
(Calculated from Physical Equations and Digital Data)

Variable	Value
T_{gs}°	505°R
K_{sp}°	0.01610 BTU/sec °R
K_{pi}°	0.0261 BTU/sec °R
K_{bi}°	0.02653 BTU/sec °R
K_f°	0.1055 BTU/sec °R
K_{ft}°	0.06164 BTU/sec °R

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TABLE A6.3

Constants for Support Element Simulation Equation Calculated
from Digital Data and the Physical Equations

Constant	Value
K_{21}	0.3918 BTU/sec °R
C_{pg}	3.666 BTU/lb °R
K_{22}	0.513 BTU/sec %
K_{23}	0.0858 BTU/sec %
K_{24}	0.0561 BTU/sec %
K_{25}	0.0209 BTU/sec %
K_{28}	0.5843
K_{26}	36.1×10^4
K_{27}	1.98×10^4
K_{29}	3.378
K_{30}	0.0278 ft hr/sec
K_{31}	0.036 ft hr/sec
K_{32}	0.139 ft hr/sec

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TABLE A6.4
Constants for Support Element Equations

	Volume	Density	Mass	Spec. Heat	Mass x Spec. Heat
Tie Rod	0.513 in ³	0.273 lb/in ³ *	0.14 lb	0.12 BTU/lb °R*	0.0168 BTU/°R
S.S. Sleeve	0.1959 in ³	0.3 lb/in ³ *	0.0588 lb	0.12 BTU/lb °R*	0.00705 BTU/°R
Pyro. Graphite	2.78 in ³	2.2 gm/cc*	0.221 lb	0.5 BTU/lb °R*	0.1105 BTU/°R
Struct. Graphite	20.9 in ³	1.75 gm/cc*	1.322 lb	0.445 BTU/lb °R*	0.5883 BTU/°R

Pyro. Graphite Thermal Conductivity = $f_p(T)$ (Figure A6.1, Appendix)

Hydrogen Thermal Conductivity = $f_b(T)$ (Figure A6.2, Appendix)

Hydrogen Gas Constant:

$$R = 767 \text{ ft}^2/\text{°R} \cdot \frac{1}{144} \text{ ft}^2/\text{in}^2 = 5.33 \text{ ft}^3/\text{°R in}^2$$

* Taken from WANL-TNR-033

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FIGURE A6-1

PYROLYTIC GRAPHITE THERMAL CONDUCTIVITY

FIGURE A6-2
THERMAL CONDUCTIVITY OF HYDROGEN

7.0 Core Exit Plenum and Nozzle

List of Constants

C_{pce}	3.48
C_{ptre}	3.667
K_{48}	8.14

8.0 Neutronics

0.0078
 25×10^{-6} seconds

9.0 Reactivity Calculations

K_{49}	0.0569
----------	--------

11.0 Turbopump System

K_{60}	11.37
K_{61}	0.0043
K_{62}	57.3
K_{63}	0.03042

12.0 Turbopump Controls and Bleed System

K_{50}	0.0753	K_{53}	1.088
K_{51}	0.865	K_{54}	0.451
K_{52}	1.0	K_{55}	3.69

FIGURE A11-1
 PUMP TORQUE CURVE

FIGURE A11-2
PUMP HEAD CURVE

FIGURE A11-3
PUMP EFFICIENCY

FIGURE A11-4
 SPECIFIC HEAT FUNCTION

FIGURE A12-1
TPCV FLOW AREA VS. BLADE ANGLE

FIGURE A12-2
 Ψ FUNCTION