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COUPLING OF MAGNETIZED PLASMA
OSCILLATIONS TO ELECTROMAGNETIC RADIATION

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COUPLING OF MAGNETIZED PLASMA OSCILLATIONS TO ELECTROMAGNETIC RADIATION

ABSTRACT

We report the results of a preliminary theoretical study of the coupling of magnetized plasma oscillations with frequency ω less than electron-cyclotron frequency ω_{ce} to circularly polarized electromagnetic (EM) waves in an inhomogeneous plasma modeled by a cold electron fluid. The external magnetic field makes an angle θ with the density gradient ∇n_0 . The differential equations which determine the fields are decoupled by an approximation which requires that the scale length of the density inhomogeneity be small compared to the free space wavelength of the EM waves. Via WKB approximation we solve for the magnetized plasma wave away from a resonance at $\omega_{pe}^2/\omega^2 = (1 - \omega^2/\omega_{ce}^2)/(\cos^2 \theta - \omega^2/\omega_{ce}^2)$, where ω_{pe} is electron plasma frequency, and match this solution to an integral solution in the resonance region in which the density is taken as a linear function of distance. The magnetized plasma wave in the resonance region determines the amplitude of the EM radiation. The coupling is strongest when the magnetized plasma wave propagates down the density gradient to give an EM wave amplitude conversion ratio of $\pi^{1/2} \sin \theta (\cos^2 \theta + \omega/\omega_{ce})^{1/2} / (\cos^2 \theta - \omega^2/\omega_{ce}^2)^{1/2}$.

INTRODUCTION

Recently, Baldwin¹ suggested the existence of high frequency turbulence (just below the electron-cyclotron frequency ω_{ce}) in mirror-confined plasmas of sufficient density $\omega_{pe}^2/\omega_{ce}^2 \gtrsim 1$, where ω_{pe} is the electron plasma frequency. The existence of this turbulence was conjectured to suggest a mechanism for the apparent stability of the drift-cyclotron loss cone mode which is predicted theoretically of 2XII conditions but which — at least under certain experimental conditions — is seemingly absent. Because it is intended to attempt measurement of the electromagnetic radiation from 2XIIB resulting from this turbulence, it becomes important to assess the strength of coupling of the longitudinal turbulence to the EM field. In this note we present calculations describing this coupling under the conditions at which it is strongest.

The plasma model is a cold pressureless electron fluid in which the ions form a static background of density n_0 . The scale length of the spatial variation of n_0 is assumed to be short compared to the free space EM wavelength c/ω . The external magnetic field $\underline{B}_0$ is along the z axis and the density gradient ∇n_0 makes an angle θ with $\underline{B}_0$ as shown in Fig. 1.

1. D. E. Baldwin, H. L. Burk, L. D. Pearlstein, T. Kammash, and T. Uckan, "Stabilizing Mechanisms for Loss Cone Modes," IAEA 5th Conference on Plasma Physics and Controlled Nuclear Fusion, Japan. Paper No. CN-33/01-1, in press (1975).

Fig. 1. Coordinate system showing the orientation of the external magnetic field B_0 to the density gradient ∇n_0 . The dielectric tensor $\underline{K}$ is usually expressed in the y-z system, but the s-t coordinates are more convenient for expressing the differential equations.

We choose a longitudinal wave propagating with arbitrary angle with respect to $\underline{B}_0$ and ∇n_0 from the region of dense plasma towards the vacuum. This wave may reach a point of resonance where its electric field, parallel to ∇n_0 , becomes very large. This large electric field is then capable of generating transverse EM waves that propagate away from the resonant region. We shall see that a necessary condition for this process is that ∇n_0 be neither parallel to nor at too large an angle from the direction of $\underline{B}_0$.

In our analysis we obtain a solution for the longitudinal wave which allows for propagation in any direction. The coupling to circularly polarized transverse waves is found to be strongest when the direction of propagation of the plasma oscillation parallels $-\nabla n_0$. If the longitudinal wave propagates in a direction which is not close to $-\nabla n_0$, then the resonance occurs where the transverse waves are evanescent. In this case energy cannot be carried away by the EM field and the coupling is weak.

Description of the Assumed Model

Some simplifications can be made before writing down Maxwell's equations determining the fields. First, the time-dependence of all quantities is removed via a Laplace transformation which requires that $\text{Im}(\omega) > 0$ and implies that $\partial/\partial t \rightarrow -i\omega$. Next, in order to separate the longitudinal and transverse components of the field, we use the Coulomb Gauge. Thus,

$$\underline{E}(\underline{r}) = -\nabla\phi(\underline{r}) + \frac{i\omega}{c} \underline{A}(\underline{r}), \quad (1)$$

$$\nabla \cdot \underline{A}(\underline{r}) = 0. \quad (2)$$

When we combine this with Maxwell's equations and our model of the plasma in which the electrons are treated as a cold fluid and the ions are a static background, we obtain²

$$\nabla \cdot [\underline{K}(\underline{r}) \cdot \nabla\phi(\underline{r})] = \frac{i\omega}{c} \nabla \cdot [\underline{K}(\underline{r}) \cdot \underline{A}(\underline{r})], \quad (3)$$

$$\nabla^2 \underline{A} + \frac{\omega^2}{c^2} \underline{K}(\underline{r}) \cdot \underline{A}(\underline{r}) = -\frac{i\omega}{c} \underline{K}(\underline{r}) \cdot \nabla\phi, \quad (4)$$

where $\underline{K}(\underline{r})$ is the equivalent dielectric tensor for a drift-free, cold plasma. The elements of $\underline{K}(\underline{r})$ in a coordinate system with $\underline{B}_0$ along the z axis are given by

2. D. E. Baldwin, D. M. Henderson, and D. W. Ignat, J. Appl. Phys. **42**, 2431 (1971).

$$\begin{aligned}
K_{xx} &= K_{yy} = 1 + \frac{\omega_{pe}^2(\underline{r})}{\Delta^2}, \\
K_{xy} &= K_{yx} = 1 \left(\frac{\omega_{ce}}{\omega} \right) \frac{\omega_{pe}^2(\underline{r})}{\Delta^2}, \\
K_{zz} &= 1 - \frac{\omega_{pe}^2(\underline{r})}{\omega^2}, \\
K_{xz} &= K_{zx} = K_{yz} = K_{zy} = 0,
\end{aligned} \tag{5}$$

where

$$\Delta^2 = \omega_{ce}^2 - \omega^2 > 0.$$

Equations 3 and 4 are intractable as they stand. However, we obtain an approximate solution to the set of Eqs. 3 and 4 by solving instead

$$\nabla \cdot [\underline{\underline{K}}(\underline{r}) \cdot \nabla \phi_0] = 0 \tag{6}$$

$$\nabla^2 \underline{A}_1 + \frac{\omega^2}{c^2} \underline{\underline{K}}(\underline{r}) \cdot \underline{A}_1(\underline{r}) = -\frac{i\omega}{c} \underline{\underline{K}}(\underline{r}) \cdot \nabla \phi_0. \tag{7}$$

The two functions $\phi_0(\underline{r})$ and $\underline{A}_1(\underline{r})$ involved in these simpler equations are good approximations to the actual fields $\phi(\underline{r})$ and $\underline{A}_1(\underline{r})$ provided that the free space EM wavelength c/ω is much greater than the scale length of the inhomogeneity and provided also that ω is not close to ω_{ce} .

Let us first look at Eq. 6 to see the nature of the magnetized plasma wave represented by ϕ_0 . In a coordinate system aligned with $\hat{s}$ (see Fig. 1), Eq. 6 takes the form

$$\frac{d}{ds} \left(K_{ss} \frac{d\phi_0}{ds} \right) + 2ik_t K_{st} \frac{d\phi_0}{ds} - \left[k_y^2 K_{yy} + k_t^2 K_{tt} \right] \phi_0(s) = 0, \tag{8}$$

where we have transformed in $k_\perp = (k_t^2 + k_y^2)^{1/2}$ and where

$$\begin{aligned}
K_{ss} &= K_{xx} \sin^2 \theta + K_{zz} \cos^2 \theta, \\
K_{st} &= (K_{zz} - K_{xx}) \sin \theta \cos \theta, \\
K_{tt} &= K_{xx} \cos^2 \theta + K_{zz} \sin^2 \theta.
\end{aligned} \tag{9}$$

To obtain the WKB solution of Eq. 8 we first rewrite Eq. 8 in the form

$$\phi_0(s) = \exp \left\{ - \int \frac{\alpha(s)}{2} ds \right\} u(s), \tag{10}$$

where

$$\alpha(s) = (K'_{ss} + 2ik_t K_{st})/K_{ss}, \quad (11)$$

$$u'' + k^2(s)u(s) = 0, \quad (12)$$

$$k^2(s) = \beta(s) - \left(\frac{\alpha}{2}\right)^2 - \left(\frac{\alpha}{2}\right)^1, \quad (13)$$

$$\beta(s) = -\frac{k_y^2 K_{yy} + k_t^2 K_{tt}}{K_{ss}}. \quad (14)$$

Application of the WKBJ method to (12) combined with (10) yields,

$$\phi_0(s) \approx N \exp \left\{ -\int \frac{\alpha(s)}{2} ds \right\} k(s)^{-1/2} \exp \left\{ -i \int k(s) ds \right\} \quad (15)$$

which is valid only away from the point where $k^2(s)$ becomes infinite.

To determine the solution in the resonance region where the WKBJ solution fails, we approximate K_{ss} as a linear function of s

$$K_{ss}(s) = -s/L \equiv -\xi, \quad (16)$$

where L characterizes the scale length of the inhomogeneity. This determines the spatial dependence of the plasma frequency

$$\omega_{pe}^2(s)/\omega^2 = \frac{1 - \omega^2/\omega_{ce}^2}{\cos^2 \theta - \omega^2/\omega_{ce}^2} (1 + \xi). \quad (17)$$

The s -dependence of all of the components of $\underline{K}$ is thus determined. Putting this into Eq. 8 yields

$$\frac{d}{d\xi} \left(\xi \frac{d\phi_0}{d\xi} \right) + ib \frac{d\phi_0}{d\xi} + c\phi_0(\xi) = 0, \quad (18)$$

where

$$b = \frac{2k_t L \sin \theta \cos \theta}{\cos^2 \theta - \omega^2/\omega_{ce}^2}, \quad (19)$$

$$c = \frac{k_y^2 L^2 \cos^2 \theta + k_t^2 L^2 (\cos^2 \theta - \sin^2 \theta)}{\cos^2 \theta - \omega^2/\omega_{ce}^2}. \quad (20)$$

In writing b and c , we have made the approximation of neglecting ξ compared to unity because we seek the solution of Eq. 8 only in the vicinity of resonance at $\xi = 0$. The solution of Eq. 18 giving the behavior of ϕ_0 near the resonance is the subject of the next section.

Now we return to Eq. 7, rewriting it in our coordinate system along $\hat{s}$ and transformed in $k_{\perp}$. Including the components of $\underline{K}$ written in terms of ξ according to Eq. 17, we obtain

$$A_{\pm}'' + K_{\pm}^2 A_{\pm}(\xi) = \Omega_{\pm} \phi_0' \quad (21)$$

$$A_t'' + K_t^2 A_t(\xi) = \Omega_t \phi_0' \quad (22)$$

where primes denote $d/d\xi$ and where

$$A_{\pm} = A_x \pm iA_y$$

which shows that Eq. 21 describes circularly polarized waves. Also,

$$K_{\pm}^2 = (\omega L/c)^2 \left(\frac{\cos^2 \theta \pm \omega/\omega_{ce}}{\cos^2 \theta - \omega^2/\omega_{ce}^2} - \frac{c^2 k_{\perp}^2}{\omega^2} \right), \quad (23)$$

$$\Omega_{\pm} = -i \left(\frac{\omega L}{c} \right) \sin \theta \frac{\cos^2 \theta \pm \omega/\omega_{ce}}{\cos^2 \theta - \omega^2/\omega_{ce}^2}, \quad (24)$$

$$K_z^2 = - \left(\frac{\omega L}{c} \right)^2 \left(\frac{\sin^2 \theta}{\cos^2 \theta - \omega^2/\omega_{ce}^2} + \frac{c^2 k_{\perp}^2}{\omega^2} \right) \quad (25)$$

$$\Omega_z = i \frac{\omega L}{c} \frac{\cos \theta \sin^2 \theta}{\cos^2 \theta - \omega^2/\omega_{ce}^2}. \quad (26)$$

Equations 21 and 22 are approximate because we have again neglected ξ compared to unity in the expressions for $K_{\pm}^2$ and K_z^2 . This still leaves us with good equations in the vicinity of resonance.

Solution for the Longitudinal Wave

We now must solve Eq. 18 in order to describe the longitudinal wave near resonance. This solution will then be joined up with the WKBJ solution (Eq. 15) which is valid away from resonance. The longitudinal wave function ϕ_0 can be expressed by integral solutions of Eq. 18 by writing

$$\phi_0(\xi) = \int_{\Gamma} dk e^{ik\xi} \hat{\phi}_0(k). \quad (27)$$

Substitution of (27) into (18) determines $\hat{\phi}(k)$,

$$\hat{\phi}_0(k) = \frac{k^{ib} e^{ic/k}}{k} \quad (28)$$

and the contour Γ which can be either Γ_1 or Γ_2 as shown in Fig. 2. As the figure indicates, Γ_1 depends on the sign of ξ . These two contours give the two independent solutions for $\phi_0(\xi)$:

$$\phi_0(\xi) = A \int_{\Gamma_1} dk e^{ik\xi} \hat{\phi}_0(k) + B \int_{\Gamma_2} dk e^{ik\xi} \hat{\phi}_0(k). \quad (29)$$

This solution can be determined to within one overall constant by applying boundary conditions. For the case of no bounding surface close to the left of the resonant point, $\xi = 0$, then the boundary condition requires elimination of the solution that grows as $\xi \rightarrow -\infty$. The behavior of $\phi_0(\xi)$ in this limit is determined by the saddle-point method. The saddle points are indicated in Fig. 2 and the results are

$$\int_{\Gamma_1} dk e^{ik\xi} \hat{\phi}_0(k) \xrightarrow{\xi \rightarrow -\infty} \frac{i\pi^{1/2} e^{\pi b/2}}{c^{1/4} (-\xi)^{1/4}} \left(\frac{c}{-\xi}\right)^{ib/2} \exp \left\{ -2c^{1/2} (-\xi)^{1/2} \right\} \quad (30)$$

$$\int_{\Gamma_2} dk e^{ik\xi} \hat{\phi}_0(k) \xrightarrow{\xi \rightarrow -\infty} \frac{\pi^{1/2} e^{-\pi b/2}}{c^{1/4} (-\xi)^{1/4}} \left(\frac{c}{-\xi}\right)^{ib/2} \exp \left\{ +2c^{1/2} (-\xi)^{1/2} \right\}. \quad (31)$$

The integral over Γ_2 gives a growing solution and therefore we must put $B = 0$ in Eq. 29.

The saddle-point method is now applied for $\xi \rightarrow +\infty$ to show that the integral over Γ_1 represents only waves propagating in the direction of decreasing values of ξ .

$$\int_{\Gamma_1} dk e^{ik\xi} \hat{\phi}_0(k) \xrightarrow{\xi \rightarrow \infty} \frac{e^{i3\pi/4} \pi^{1/2} e^{b\pi}}{c^{1/4} \xi^{1/4}} \left(\frac{c}{\xi}\right)^{ib/2} \exp \left\{ -i2c^{1/2} \xi^{1/2} \right\}. \quad (32)$$

We are now in a position to compare this solution for $\phi_0(\xi)$ with the WKBJ solution expressed in the linear region. The result of this comparison determines A , as follows, so that the two solutions are the same in their common region of validity,

$$A = N e^{-i3\pi/4} \pi^{-1/2} e^{-b\pi} c^{-ib/2}. \quad (33)$$

For a wave of roughly unit amplitude away from the resonance region, we put $N = 1$.

As stated in the introduction, in solving for $\underline{A}$, we use our solution ϕ_0 in the form in which the direction of propagation is the direction of $-\nabla n_0$, that is, we put $k_{\perp} = 0$. In terms of the parameters appearing in Eq. 18 this means that $b = 0 = c$. Since Eqs. 21 and 22 require the derivative of ϕ_0 , it can be shown from Eq. 29 that

$$\frac{d\phi_0}{d\xi} \xrightarrow[b \rightarrow 0]{c \rightarrow 0} \frac{A}{\xi} \quad (34)$$

just as expected from Eq. 18 if you put $b = 0 = c$ immediately.

Fig. 2. The contours of integration used in obtaining the two independent solutions for ϕ_0 . The branch cut along the negative imaginary axis is shown. The saddle points and the direction of steepest descent through the saddle points are shown also.

Solution for the Transverse Waves

In this section we investigate Eqs. 21 and 22 to determine whether or not the individual components of the transverse EM waves are strongly coupled to the longitudinal wave. Before looking at the solutions of these equations, we can draw some conclusions about this coupling. To begin with, we can demonstrate that the coupling of ϕ_0 to A_z is always weak. From Eq. 25 we see that $K_z^2 \leq 0$, thus A_z is generated in a region of evanescence. The coupling will be weak because the transverse wave will have to tunnel through to a propagating region. From this argument it looks like the coupling becomes strong as $\theta \rightarrow 0$, because then $K_z^2 = 0$ and the tunneling distance vanishes. However, the coupling remains weak, because as $\theta \rightarrow 0$, the right side of Eq. 22 vanishes. In other words as the tunneling distance decreases so does the term that drives A_z . Thus the coupling to A_z is always weak and we can dispense with solving Eq. 22.

There is no coupling to the circularly polarized waves A_+ and A_- for ∇n_0 along B_0 ($\theta = 0$) because the driving terms on the right side of Eq. 21 vanish. Furthermore as θ approaches $\cos^{-1} \omega/\omega_c$, the resonance takes place only for very high plasma densities. There is no resonance at all for $\theta > \cos^{-1} \omega/\omega_c$. Thus, A_+ and A_- are only weakly coupled to the longitudinal wave when ∇n_0 makes an angle with B_0 near to or greater than $\cos^{-1} \omega/\omega_c$.

The same reasoning that was used to eliminate A_z can be used to show that the existence of $k_\perp^2$ tends to weaken the coupling to A_+ and A_- . The strongest coupling of ϕ_0 to A_+ and A_- occurs for $k_\perp = 0$. This means that the strongest coupling occurs for ϕ_0 propagating along $-\nabla n_0$. Putting $k_\perp = 0$ greatly simplifies the ϕ_0 solution and allows us to obtain simple expressions for A_+ and A_- .

Before deriving these expressions for A_+ and A_- we first recall that the Laplace transformation requires $\text{Im } \omega > 0$. By Eq. 17 this implies that $\text{Im } \xi < 0$. Thus we write

$$\xi \rightarrow \xi - i\delta,$$

where δ is positive and vanishingly small.

Equation 21 for A_+ now becomes

$$A_+'' + K_+^2 A_+ = \Omega_+ \frac{A}{\xi - i\delta}, \quad (35)$$

where we have used Eq. 34 to replace ϕ_0 . This equation is easily solved using a Green's Function. The result is

$$A_+(\xi) = e^{i3\pi/4} \pi^{1/2} \sin\theta \left(\frac{\cos^2\theta + \omega/\omega_{ce}}{\cos^2\theta - \omega^2/\omega_{ce}^2} \right)^{1/2} \begin{cases} e^{ik+\xi} & \xi > 0 \\ e^{-ik+\xi} & \xi < 0 \end{cases}. \quad (36)$$

This gives the amplitude of the resultant right hand circularly polarized EM wave generated by an outgoing plasma wave of unit amplitude.

We solve for A_- in a similar manner obtaining

$$A_-(\xi) = e^{i3\pi/4} \pi^{1/2} \sin\theta \left(\frac{\cos^2\theta - \omega/\omega_{ce}}{\cos^2\theta - \omega^2/\omega_{ce}^2} \right)^{1/2} \begin{cases} e^{ik_-\xi} & \xi > 0 \\ e^{-ik_-\xi} & \xi < 0 \end{cases} \quad (37)$$

The amplitude of A_- is smaller than A_+ and furthermore, for $\omega^2/\omega_e^2 < \cos^2\theta < \omega/\omega_c$, k_- is purely imaginary. This means that A_- waves are evanescent in the resonant region and then the coupling to A_- is much weaker than the coupling to A_+ .

CONCLUSION AND DISCUSSION

In summary we find that a magnetized plasma wave propagating out of the plasma along the density gradient couples strongly to circularly polarized EM radiation as expressed in Eqs. 36 and 37. The strong coupling depends on the angle θ between ∇n_0 and B_0 with the requirement that $0 < \theta < \cos^{-1} \omega/\omega_{ce}$. According to Eqs. 36 and 37 the coupling appears to be more effective as θ increases, but one has to keep in mind that as θ increases towards $\cos^{-1} \omega/\omega_e$ the plasma density required for resonance also increases and without limit. Thus the coupling depends implicitly on the density profile of the plasma in such a way that for a real plasma there will be some optimum angle θ which yields the strongest coupling.

Further work on this problem could include (1) a power-flow calculation to determine how much power is absorbed from the longitudinal wave to check that the absorbed power is not exceeded by the power flow of the transverse EM waves, and (2) a more general solution for the transverse EM waves, this time allowing the longitudinal wave to propagate in an arbitrary direction.

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