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MASTER

The enclosed Engineering Operations Report documents the requirements established for the Common Digital Model and the equations used in Phase I. Originally, a joint document was going to be issued with WANL, however, with the rapid progress on Phase II, it was agreed with M. Wright to have the joint document cover Phase II of the Common Digital Model.

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ENGINEERING OPERATIONS REPORT

N8600R:72-004

COMMON DIGITAL MODEL REQUIREMENTS

October 1971

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Section 1. SCOPE

This document establishes the requirements for performance, design, test, and qualification of a computer program identified as Common Digital Model, hereafter referred to as CDM. This computer program is used to accomplish analysis of the baseline NERVA nuclear rocket flight engine system but will retain flexibility to analyze variations in the baseline engine system.

The CDM is used to analyze all phases of engine behavior including engine conditioning, thrust buildup, steady state, shutdown and cooldown.

Section 2. APPLICABLE DOCUMENTS

The input data for the engine system analysis used for engine and component specifications will be incorporated into the S-001-CP090290-F1, Systems Analysis Summary Report. Data for other analytical studies will be documented in internal memorandums at ANSC and WANL.

Section 3. REQUIREMENTS

3.1 Performance

The CDM shall meet the functional requirements specified herein. These requirements are divided into three phases. Phase I is the initial CDM program suitable for scoping engine studies. Phase II provides a detailed model limited by the capacity of the Sigma 5 Computer. Phase III is a model capable of detailed analyses of all phases of operation designed for analysis using a large-size digital computer such as IBM 370/165, CDC 6600 or UNIVAC 1108.

3.1.1 System Requirements

The CDM shall be capable of performing the operational functional requirements specified herein. These shall cover the following operating modes as defined in CP 90290.

- (a) Normal Mode
- (b) Malfunction Mode
 - (1) Single Turbopump Operation
 - (2) Component Malfunction
- (c) Emergency Modes

3.1.1.1 Startup. Startup consists of engine conditioning (temperature conditioning, nuclear startup, and bootstrap) and thrust buildup. Startup is initiated upon receipt of a command signal to initiate propellant flow or nuclear startup and is completed when rated performance has been achieved. Chamber

temperature ramp rates shall not exceed 175°R/sec and chamber pressure ramp rates shall not exceed 60 psi/sec. The CDM program shall be capable of analyzing the engine behavior for the engine programmed operation (for flow rates, pressures, and temperatures at component interfaces, reactivity feedback, power, speed and drum) with an accumulated error of less than:

	<u>Phase I</u>	<u>Phase II</u>	<u>Phase III</u>
Engine Conditioning	Correct Trends	5%	2%
Thrust Buildup	Correct Trends	2%	1%

The standard for evaluation of accuracy shall be the 1137400E NETAP analysis as specified in S001-CP090290-F1, superceding detailed component data or test data.

3.1.1.2 Steady-state Operation

The engine is capable of operating for a continuing period of time at any point within the operating map defined in Figure 1. The CDM shall be capable of analyzing the engine behavior of major components (for flow rates, pressures and temperatures at component interfaces, reactivity feedback, drum position, power and speed) throughout the operating map within the following accuracy:

	<u>Phase I</u>	<u>Phase II</u>	<u>Phase III</u>
Calibrated at design	1/2%	1/2%	1/2%
Throttle point	10%	1/2%	1/2%

The standard for evaluation of accuracy shall be the 1137400E NETAP analysis as specified in S001-CP090290-F1, superceding detailed component data or test data.

3.1.1.3 Shutdown and Cooldown

Shutdown consists of throttling, throttle hold, temperature retreat and pump tailoff, and is initiated by a command signal to depart from rated conditions and is completed upon termination of the pump tailoff phase and initiation of cooldown. Cooldown is initiated upon completion of engine shutdown and is completed upon termination of propellant flow or the receipt of a command signal for restart. Cooldown propellant is supplied at tank pressure conditions. The CDM shall be capable of analyzing the engine behavior throughout shutdown and cooldown (for flow rates, pressures and temperatures at component interfaces, reactivity feedback, drums, power and speed) with an accumulation error of less than:

	<u>Phase I</u>	<u>Phase II</u>	<u>Phase III</u>
Shutdown	Correct Trends	5%	2%
Cooldown	Correct Trends	5%	2%

3.1.1.4 Performance Range

The CDM program shall be capable of analyzing system performance within specified limits of accuracy and stability within the following range:

(1)	Pressure	0 to 1000 psia
(2)	Temperature	36.5 to 5500°R

3.1.1.5 Control System Requirements

The CDM shall include control system options for the following modes:

3.1.1.5.1 Steady-state operation specifying tank conditions (temperature and pressure or enthalpy and pressure), stage pressurization flow and chamber temperature and pressure. The CDM shall be capable of analysis for specified drum position, control valve position, pump discharge control valve (PDCV) and structural support control valve (SSCV) positions; also, stem flow and turbine bypass flow can be specified.

3.1.1.5.2 Startup operation specifying tank conditions, (temperature and pressure or enthalpy and pressure), stage pressurization flow and control system dynamics with a band width to 1 cps for specified chamber temperature and pressure ramps or turbopump speed using control elements of drums, SSCV, PDCV and BCV.

3.1.1.5.3 Shutdown operation specifying tank conditions (temperature and pressure or enthalpy and pressure), stage pressurization flow and control system dynamics with a band width to 1 cps for specified chamber temperature and pressure ramps using control elements of drums, SSCV, PDCV, Bypass Control valve (BCV). In addition, the CDM shall be capable of analysis when structural support temperatures are controlled by the SSCV.

3.1.1.5.4 Cooldown operation specifying tank conditions (temperature and pressure or enthalpy and pressure), and control system dynamics with a band width to 1 cps using chamber, in-core material, reflector, or structural support temperatures controlled by CSCV, PDCV or SSCV. .

3.1.1.5.5 Open loop control shall be possible of all engine valves and reactor control drums.

3.1.2 Operational Requirements

The CDM shall be capable of representing engines similar to Figure 2. All parts shall be modeled thermodynamically to meet the performance requirements specified in 3.1.1 (System Requirements).

3.1.2.1 Source and Type of Inputs

The source of CDM input shall be S001-CP090290-F1 (Systems Analysis Summary Report). Input shall be in the form of punched cards. The input data shall include information as specified in Table 1. Input shall be in a format using standard engineering terminology that can be read by engineers not familiar with computer codes. ANSC shall provide input on non-nuclear components; WANL shall provide input on the nuclear subsystem.

3.1.2.2 Destination and Type of Outputs

CDM output shall be in the form of summary listings, detailed listings and graphical plots. The graphical plots can be prepared by a separate program step. The summary listing of key engine performance parameters in the format as specified in Table 2 shall be available as an option. The detailed listing shall list the performance data for each part and mode in the engine model. The graphical plots shall graphically display as a function of time by parameters of each part in the engine model. In addition, summary plots up to 20 can be defined at the option of the user for display. Each plot will include up to six parameters. The destination of the output shall be engine specification, component specifications, S-001 System Analysis Summary Report and Test Predictions. All output will be formatted in engineering terminology.

3.1.2.3 Main Program Processing

The CDM program shall represent a trade-off between the need for accurately modeling the system in great detail and the simplifications

required to keep computer running time at a reasonable level. The basic unit modeled in the program is called a "part". Each part consists of a single flow path through a solid material in which heat is generated. The part shall be divided into a number of length increments called nodes, in which finite-difference calculations of heat transfer and pressure drop are made. All nodes in a particular part shall be of equal length. Only one set of material thermal properties shall be input for each part, and that set applies to all nodes in the part. The properties can be made a function of temperature, however. The calculation procedure used in each part shall be as follows:

The initial temperature for each node in the part is known. The working fluid undergoes a pressure drop due to an inlet loss coefficient before entering the first node. A heat-transfer coefficient and friction factor are calculated on the basis of conditions at the inlet to the first node. These values are then used to calculate the pressure and temperature change through that node. The resulting conditions become inlet conditions for the next node. When computations are complete for all nodes in a part, the fluid receives an additional pressure drop due to an exit loss coefficient. The equations shall neglect any fluid-acceleration terms in the pressure-drop equations and mass storage effects due to variations of pressure and temperature with time. When the fluid calculations are complete, the film coefficient and fluid temperature calculated above, together with the internal heat-generation rate and material thermal properties, shall be used to calculate the material temperature change over some time interval, ΔT . Since a new initial temperature distribution is known, the entire calculation procedure can be repeated for the next time interval.

Those portions of the engine that cannot be modeled conveniently by using the "part" format shall be represented by special routines. Examples of special routines might be the structural support system, turbopump, nozzle, valves, and tanks.

3.1.2.4 Program Operation

After a particular engine system has been described in terms of the part input, additional data shall be used to describe the flow path through these parts. In addition, specifications shall be inputted to link the special routines into the flow path. These specifications shall contain such data as the locations for special valves and tankage information. In all cases, the fluid enters the nozzle chamber after it leaves the last part in the flow system described above.

Figure 3 shows a typical engine system that might be analyzed with this program. For this example case, the string of parts starting with the tank and including the nozzle, reflector, core, etc., is the main flow path. Two branching, parallel flow circuits are also shown. The first contains only the structural support system (a special routine) while the second contains two basic parts with input set up to model a simple line and one special part - a control valve.

Two basic types of systems shall be analyzed, open- and closed-loop systems. In the open-loop system, fluid is supplied at an arbitrary pressure from a source not controlled by the system (e.g., a pressurized tank), and this pressure is the only driving force on the fluid. In the closed-loop system, some of the fluid in the system is used to power a turbine. This turbine

drives a pump that produces a pressure rise in the fluid at some point in the system.

The example system shown in Figure 3 is a closed-loop system. All the work produced by the turbine will be input to the pump and will cause a head rise there. The function of the control valve in the second parallel bypass circuit is to regulate the amount of flow through the turbine and therefore the amount of power produced. It is normally called the "Turbine Power Control Valve" or TPCV. In other possible systems it could be in the main flow path just upstream or just downstream of the turbine or at the pump discharge or various other places.

The open-loop system has two variables, flow rate and inlet pressure; one must be specified, and the other is calculated by the program. If inlet pressure is specified, a guess is made of the flow rate. Pressure, temperature, and flow are calculated throughout the system. With this calculated pressure and temperature, the choked flow is calculated at the minimum-area point in the nozzle tubes and the main nozzle. Pressure at the nozzle is also compared with the input ambient pressure. Another flow guess is then made and the procedure is repeated until the flow is equal to the lower of the two choke flows or until the exit pressure equals the back pressure and the flow is less than choked flow. If flow is specified in the input, this flow is held constant and the inlet-pressure guess is varied until the choked-flow closure constraint is met as before.

In the closed-loop system, inlet pressure must be specified. Turbine-power-control-valve position is also input, and flow-rate guesses are made until a closure is found on choked flow as in the open-loop cases.

A predictor-corrector routine shall be used to provide a minimum-computer-time closure of the system.

The predictor subroutine, itself, shall be a complete, though simplified engine program. Each major engine component shall be represented by simplified pressure drop, enthalpy rise, and heat capacitance equations. The engine flow balances shall be performed within the predictor, along with the engine controller calculations and the turbopump calculations.

The corrector subroutine shall be used to update the predictor to improve its accuracy. The corrector contains the detailed heat transfer and energy balance relationships, and includes a capability to use a multi-node representation for each engine part. The corrector hydraulic model is also more rigorous than the predictor.

Figure 4 shows a simplified schematic of the program logic used in this predictor-corrector calculation scheme. Under normal operation the predictor and corrector calculations will be done alternately until they agree.

Computation speeds are greatly decreased with this predictor-corrector calculation scheme. A few of the more important factors which have contributed to decreasing program execution times are listed below:

- a. Predictor flow rates can be used in the corrector, thereby obviating the need for doing flow balances in the complex model.

- b. It is possible to segregate the controller iterations from the corrector material node calculations by incorporating the controllers in the predictor.
- c. Upgrading of material temperatures in steady-state options is done selectively on the basis of changes in component flow rates or changes in engine power levels.

In all of the above cases, the internal heat generation shall be specified as a function of position in the system and as a function of time.

A set of routines shall be incorporated to allow simulation of the reactor neutronics. The neutronics shall be solved either by: (1) power being given to find the required drum position, or (2) drum position being given to find the resulting power. In either case, the desired parameter shall be input as a function of time. When the program is to close on a demanded chamber temperature, the problem is handled in the same manner as for a given-power case, with the power calculated by the program from the demanded chamber temperature, the problem is handled in the same manner as for a given-power case, with the power calculated by the program from the demanded chamber temperature. In addition, the reactivity caused by the fluid in the structural support system can be varied by changing the position of the structural support control valve (SSCV). Either the drum position or the SSCV position can be varied by a controller.

3.1.2.5 Equation Formulation

The equations defined in this section have been incorporated into the Phase I program completed 30 September 1971.

3.1.2.5.1 General Heat-Transfer Equations

3.1.2.5.1.1 Predictor

The general predictor heat-transfer model consists of two energy balances, one on the material and one on the fluid. A control volume is assumed as follows:

The energy into the fluid control volume is set equal to the energy out of the control volume.

$$\frac{\partial E_{in}}{\partial t} = \frac{\partial E_{out}}{\partial t}$$

The energy balance on the liquid assumes steady flow and steady state over the node or part and therefore no energy stored in the fluid.

$$h_f * F_p * L(\theta - T_F) = \dot{W}(H_{out} - H_{in})$$

Letting h_f be a constant and adding a ratio of flows from predictor to corrector yields:

$$h_f * \frac{\dot{w}_p}{\dot{w}_c} * F_p * L(\theta - T_F) = \dot{w}(H_{out} - H_{in})$$

Now: $T_F \equiv \phi_2 (T_{in} + T_{out})$

where ϕ_2 is found in the corrector and T_{out} is a function of P_{out} and H_{out} .

So:
$$H_{out} = H_{in} + \frac{\frac{\dot{w}_p}{\dot{w}_c} h_f * F_p * L(\theta - T_F)}{\dot{w}_p}$$

where $L = \text{length}$ and $F_p = \text{Average weighted perimeter}$.

The energy balance on the solid node is more complex than that of the fluid; the energy stored is set equal to the nuclear energy generated plus the energy conducted minus the energy transferred to the fluid.

$$\frac{\partial E_{in}}{\partial t} - \frac{\partial E_{out}}{\partial t} = \frac{\partial E_{storage}}{\partial t}$$

Substituting equivalent value yields:

$$Q_{cond} + Q_{gen} - q_f = \frac{\partial E_{storage}}{\partial t}$$

or substituting:

$$Q_{cond} + HG * f * Vol - h_f * F_p * L * \frac{\dot{w}_p}{\dot{w}_c} (\theta - T_F) = \frac{\Delta \theta}{\Delta t} \rho_R C_R Vol$$

Rewriting and using temperatures averaged over the time step yields:

$$(\theta_N - \theta_{N-1}) \frac{\rho_R C_R \text{Vol}}{\Delta t} = Q_{\text{cond}} + \text{HG} * f * \text{Vol} - h_f * F_p * L * \frac{\dot{w}_p}{\dot{w}_c} \left(\frac{\theta_N + \theta_{N+1}}{2} - \frac{T_{\text{FN}} + T_{\text{FN-1}}}{2} \right)$$

$$\text{Define } \phi' \equiv \frac{\Delta t}{\rho_R C_R} \quad \text{and} \quad \phi'' \equiv \frac{h_f * \frac{\dot{w}_p}{\dot{w}_c} * F_p}{2 A_F}$$

Then:

$$(\theta_N - \theta_{N-1}) \frac{1}{\phi'} A_F * L = Q_{\text{cond}} + \text{HG} * f * \overline{A_F} * L - \phi'' * L * \overline{A_F} (\theta_N + \theta_{N-1} - T_{\text{FN}} - T_{\text{FN-1}})$$

Solving for θ_N

$$\theta_N = \left[\theta_{N-1} + \phi' \left\{ f * \text{HG} - \phi'' (\theta_{N-1} - T_{\text{FN-1}} - T_{\text{FN}}) + \frac{Q_{\text{cond}}}{\overline{A_F} L} \right\} \right] \frac{1}{1 + \phi' \phi''}$$

To allow for correction by the corrector:

$$\phi' \text{ becomes } \phi' = \frac{\phi_1 \Delta t}{\rho_R C_R} \quad \text{and} \quad \phi'' \text{ becomes } \phi'' = \frac{\overline{h_f} * \frac{\dot{w}_p}{\dot{w}_c} * \overline{F_p}}{2 \overline{A_F}}$$

And the basic equation for material temperature becomes:

$$\theta_N = \theta_{N-1} + \phi' \left[f * HG - \phi'' (\theta_{N-1} - T_{FN-1} - T_{FN}) + \frac{Q_{cond}}{A_F L} \right] \frac{1}{1 + \phi' \phi''}$$

This equation is used to calculate the material temperature in the parts. The complete logic is presented in the following description, together with the equations solved.

Engine-Parts Heat Transfer:

(1) Obtain initial values such as flow rate ($\dot{w}$) times t and Δt , temperatures T and θ .

(2) Guess H_{out} .

(3) Calculate outlet pressure:

$$P_{out} = P_{in} - \frac{\alpha_P \dot{w}^2}{\rho_{avg}}$$

(4) Calculate coefficients for heat transfer equation.

(5) Calculate material temperature.

(6) Calculate outlet enthalpy and close on assumed enthalpy.

3.1.2.5.1.2 Nozzle and Structural Support

Both the nozzle tubes and the structural support system are counterflow heat exchangers and thus cannot be properly represented by the standard predictor part model. A separate model has been developed

which consists of two fluid flow paths and two material nodes as shown below:

If the inlet conditions to flow channel (3) are found from the exit conditions of flow channel (1), the system can represent the stems. If channels (3) and (1) are each given their proper inlet conditions from the predictor flow calculations, the system can represent the nozzle tubes. The calculation sequence and equations are as shown below.

List of Symbols Used in this Section

A	Material cross-sectional area	in. ²
CR	Material specific heat	Btu/lb-°R
DEN	Material density	lb/in. ³
F	Wetted perimeter	in.
f	Heat generation distribution factor	--

H	Enthalpy	Btu/lb
h	Heat transfer coefficient	Btu/in. ² -sec-°R
HG	Internal heat generation rate	Btu/in. ³ -sec
K1	Heating factor in stem turnaround area	Btu/lb-°R
KC	Conduction factor for material 5 to 4	Btu/sec-°R
L	Length	in.
P	Pressure	lb/in. ²
p	Indicates table lookup from NBS hydrogen properties	
QF	Fluid heat pickup	Btu/sec
T	Temperature	°R
ΔT	Calculation time increment	sec
$\dot{w}$	Flow rate	lb/sec
α	Friction loss factor $\frac{\Delta P \cdot \rho_{avg}}{\dot{w}^2}$	lb _f -sec ² /lb _m -in. ⁵
θ	Material temperature	°R
ϕ	Correction factor (input of from corrector)	--
ρ	Density	lb _m /in. ³

NOTE: Number subscripts refer to a material node or flow path number shown on the figure at the start of this section. Double number subscripts indicate a parameter along a boundary between the two numbered nodes or flow paths. C subscript indicates value from corrector.

1. Assume θ_2 and θ_4 .

2. Assume $H_1(\text{out})$.

$$3. \quad \rho_1(\text{avg}) = p\left(P_1(\text{in}), \frac{H_1(\text{in}) + H_1(\text{out})}{2}\right)$$

$$4. \quad P_1(\text{out}) = P_1(\text{in}) - \frac{\alpha_1 \dot{w}_1^2}{\rho_1(\text{avg})}$$

$$5. \quad T_1(\text{out}) = p[P_1(\text{out}), H_1(\text{out})] \quad [H_1(\text{out}) \text{ from last iteration}]$$

$$6. \quad T_{1F} = \phi_1 [T_1(\text{in}) + T_1(\text{out})]$$

$$7. \quad QF_{21} = \left[h_{21} \cdot \frac{\dot{w}_1}{\dot{w}_{1C}} \cdot F_{21} \cdot L \right] (\theta_2 - T_{1F})$$

$$8. \quad H_1'(\text{out}) = H_1(\text{in}) + \frac{QF_{21} + HG \cdot f_1 \cdot A_1 \cdot L \cdot \frac{\rho_1(\text{in}) + \rho_1(\text{out})}{2}}{\dot{w}_1}$$

$$9. \quad \text{Return to step (2) and iterate until } H_1(\text{out}) = H_1'(\text{out}).$$

$$10. \quad \text{For stems only: } H_3(\text{in}) = H_1(\text{out}) + K_1 T_C$$

$$P_3(\text{in}) = P_1(\text{out})$$

$$\dot{w}_3 = \dot{w}_1$$

For nozzle - get inlet conditions from predictor flow calculations.

$$11. \quad \text{Assume } H_3(\text{out}).$$

$$12. \quad \rho_3(\text{avg}) = p\left(P_3(\text{in}), \frac{H_3(\text{in}) + H_3(\text{out})}{2}\right)$$

$$13. \quad P_3(\text{out}) = P_3(\text{in}) - \frac{\alpha_3 \dot{w}_3^2}{\rho_3(\text{avg})}$$

$$14. \quad T_3(\text{out}) = p[P_3(\text{out}), H_3(\text{out})] \quad [H_3(\text{out}) \text{ from last iteration}]$$

$$15. \quad T_{3F} = \phi_3 [T_3(\text{in}) + T_3(\text{out})]$$

$$16. \quad QF_{23} = \left(h_{23} \cdot \frac{\dot{w}_3}{\dot{w}_{3C}} \cdot F_{23} \cdot L \right) (\theta_2 - T_{3F})$$

$$17. \quad QF_{43} = \left(h_{43} \frac{\dot{w}_3}{\dot{w}_{3C}} \cdot F_{43} \cdot L \right) (\theta_4 - T_{3F})$$

$$18. \quad H_3'(\text{out}) = H_3(\text{in}) + \frac{[QF_{23} + QF_{43} + HG \cdot f_3 \cdot A_3 \cdot L \left(\frac{\rho_3(\text{in}) + \rho_3(\text{out})}{2} \right)]}{\dot{w}_3}$$

19. Return to step (11) and iterate until $H_3'(\text{out}) = H_3(\text{out})$.

20. If transient calculations are desired, skip to step (25).

If steady state, continue.

$$21. \quad \theta_2^1 = \theta_2 + HG \cdot f_2 - \frac{QF_{23} + QF_{21}}{A_2 L}$$

$$22. \quad \theta_4^1 = \theta_4 + HG \cdot f_4 - \frac{QF_{43} - K_C (T_5 - \theta_4)}{A_4 L}$$

23. Return to step (21) and iterate until $\theta_2^1 = \theta_2$ and $\theta_4^1 = \theta_4$.

24. Steady-state calculations complete. Return to predictor.

25. If time = 0, and no material calculations, return to predictor.

$$26. \quad QF_{21}(\text{avg}) = \frac{QF_{21}(N) + QF_{21}(N-1)}{2}$$

$$27. \quad QF_{23}(\text{avg}) = \frac{QF_{23}(N) + QF_{23}(N-1)}{2}$$

$$28. \quad QF_{43}(\text{avg}) = \frac{QF_{43}(N) + QF_{43}(N-1)}{2}$$

$$29. \quad HG(\text{avg}) = \frac{HG(N) + HG(N-1)}{2}$$

$$30. \quad \theta_2(\text{avg}) = \frac{\theta_2(N) + \theta_2(N-1)}{2}$$

$$31. \quad \theta_4(\text{avg}) = \frac{\theta_4(N) + \theta_4(N-1)}{2}$$

$$32. \quad Vol = L \cdot A$$

$$33. \quad \theta_2' = \theta_2 + HG(\text{avg}) F_2 - \frac{QF_{21}(\text{avg}) + QF_{23}(\text{avg})}{vol_2} - DEN_2 \cdot CR_2 \cdot \Delta T[\theta_2 - \theta_2(N-1)]$$

$$34. \quad \theta_4' = \theta_4 + HG(\text{avg}) F_4 - \frac{QF_{43}(\text{avg}) + KC[T_5 - \theta_4(\text{avg})]}{vol_4}$$

$$- DEN_4 \cdot CR_4 \cdot \Delta T[\theta_4 - \theta_4(N-1)]$$

$$35. \quad \text{Return to step (33). Iterate until } \theta_2' = \theta_2 \text{ and } \theta_4' = \theta_4.$$

$$36. \quad \text{Calculations complete. Return to predictor.}$$

3.1.2.5.1.4 TURBOPUMP

The turbopump is mathematically modeled using (1) pump work equation, (2) turbine work equation, and (3) the energy balance on the turbopump assembly. These are:

(1) Pump work required is the flow times the isentropic enthalpy difference across the pump divided by the pump efficiency.

$$W_{P \text{ act}} = \frac{w_p \Delta h_{(S)}}{\eta_p}$$

where $W_{P \text{ act}}$ = pump work

w_p = pump flow rate

$\Delta h_{(S)}$ = isentropic enthalpy difference across pump

η_p = pump efficiency

Performance curves of $\eta_p = f(Q/N, N, \frac{HSVP}{N^2})$, $\epsilon = f(Q/N)$, and $\frac{\Delta H}{N^2} = f(Q/N, N, \frac{HSVP}{N^2})$ are used in calculating the pump work where Q = volumetric flow rate, N = speed, ΔH = pump head rise, and ϵ is bearing friction parameter, $HSVP$ = difference between pump inlet pressure and vapor pressure at the same temperature.

(2) Turbine work is found by calculating the isentropic turbine work and multiplying it by a turbine efficiency factor.

$$W_{T \text{ act}} = \eta_t [w_T (h_{11} - h_{12})_{\text{isen}}]$$

where $W_{T \text{ act}}$ = turbine work

η_t = turbine efficiency

w_t = turbine flow rate

h_{11} = turbine inlet enthalpy

$h_{12(s)}$ = turbine exit enthalpy

$$\text{Performance curves of } \eta_t = f\left(\frac{U_m}{C_o}, N, N/\sqrt{T_{ti}}\right) \text{ and } \frac{w_n \sqrt{T_{ti}}}{P_{ti} A_{SI}} = f(P_r, N/\sqrt{T_{ti}})$$

are used in calculating the turbine work where

U_m = turbine tip speed

C_o = isentropic spouting velocity

T_{ti} = turbine inlet temperature

N = speed

W_{ti} = turbine inlet flow

P_{ti} = turbine inlet pressure

A_{SI} = turbine inlet area

(3) Energy Balance on the Turbopump Assembly is calculated from the rotational energy differences that result between pump and turbine work.

$$\frac{d(I\alpha^2)}{dt} = W_{P \text{ act}} - W_{T \text{ act}}$$

where I = moment of inertia

α = angular velocity

t = time

$W_{P \text{ act}}$ = Actual pump work required

$W_{T \text{ act}}$ = Actual turbine work delivered

Converting to finite-difference form, solving for $W_{P \text{ act}}$, rearranging terms yields the equation

$$W_{P \text{ act}}(i+1) = -W_{P \text{ act}}(i) - \left[\frac{I}{12g \Delta t} \left(\frac{2\pi}{60} \right)^2 \left(\frac{N_{i+1}^2 - N_i^2}{778} \right) \right] + W_{T \text{ act}}(i+1) + W_{T \text{ act}}(i)$$

where i and $i+1$ represent the TPA parameters and time i and new time $i+1$, respectively.

Δt = Time Grid or $t_{i+1} - t_i$

N = TPA speed
 I = Moment of Inertia
 $W_{P \text{ act}}$ = Actual pump work required
 $W_{T \text{ act}}$ = Actual turbine work delivered

I. Pump Equations and Logic

The definitions to be used in the following pump equations set are:

Subscripts and superscripts used are:

o indicates stagnation conditions
 s indicates isentropic conditions
 (1) pump inlet
 (2) Pump discharge
 v vapor condition of fluid
 ta indicated table look up valve

Definition of terms:

h = Enthalpy BTU/lb
 s = Entropy BTU/lb $^{\circ}R$
 P = Pressure #/ft²
 T = Temperature $^{\circ}R$
 g_c = Constant $\frac{ft-lb}{\# \text{ sec}^2}$
 J = Conversion Factor ft-#/BTU
 N = Pump Speed R.P.M.
 η_p = Pump Efficiency

w = Pump Flow Rate
 ρ = Density lb/ft³
 H = Head ft
 Q = Flow Rate gal/min

1. From the engine program we obtain $T^\circ, P^\circ, h^\circ, w, N_{\text{guess}}, S_1^\circ$.

2. Compute $\text{HSVP} = P_1^\circ - P_v$

Note ρP_v is the vapor pressure of the fluid at temperature T_1°

3. A guess of pump outlet density, ρ_2 , is made by the program.

4. Compute $Q = \frac{w}{2} \left(\frac{60}{231} \right)$

60/231 is a conversion factor = $\frac{\text{gal-sec}}{\text{in}^3\text{-min}}$

5. Compute Q/N

6. Look up from TPA input Table

$$\left[\frac{\Delta H}{N^2} \right]_{\text{ta}} = f \left(\frac{Q}{N}, \text{HSVP}, N \right)$$

ta = stands for table look up value

7. Compute pump outlet head

$$H_2 = H_1 + \left[\frac{\Delta H}{N^2} \right]_{\text{ta}} N^2$$

8. Compute pump outlet enthalpy

$$h_2 \textcircled{S} = \frac{H_2}{J}$$

9. Since the NBS properties deck contains no h-s entry the computer code guesses a P_2 and with h_{2s} as well as the NBS properties deck looks up $S_2 = f(P_2, h_{2s})$. A P_2 is assumed and a S_2 is founded; $S_2 = F(P_2, h_{2s})$.
10. This S_2 is compared to S_1° , Step (1) and P_2 is changed until S_2 and S_1° agree within some predetermined value.

NOTE: A special interpolation routine, along with the fine grid NBS properties deck and an extremely tight closure tolerance on S are required to assure the answer is accurate and repeatable.

11. Look up pump theoretical efficiency from TPA input curves

$$\eta'_p = f\left(\frac{Q}{N}, \text{HSVP}, N\right)$$

12. Look up bearing friction parameter

$$\epsilon = f\left(\frac{Q}{N}\right)$$

13. Compute actual turbine efficiency

$$\eta_p = \frac{\eta'_p}{1 + \frac{\epsilon}{2\rho_2(1728)N^2}}$$

14. Compute actual pump discharge enthalpy

$$h_2 = h_1^\circ + \frac{[h_{2s} - h_1^\circ]}{\eta_p}$$

15. Using NBS properties deck, look up actual pump outlet density

$$\rho_2 = f(P_2, h_2)$$

16. Compare ρ_2 Step (15) vs. ρ_2 guess Step (3). Change ρ_2 Guess and redo calculations Step (4) through (16) until the densities agree within some predetermined value.
17. Compute actual pump work $W_{P \text{ act}} = w [h_2 - h_1^\circ]$
18. The speed guess N Step (1) is closed by comparing $W_{P \text{ act}}$ from step 17 with $W_{P \text{ act}}$ computed by equation

$$W_{P \text{ act } (i+1)} = -W_{P \text{ act } (i)} - \left[\frac{I}{2g \Delta t} \left(\frac{2\pi}{60} \right)^2 \left(\frac{N_{i+1}^2 - N_i^2}{778} \right) \right] + W_{T \text{ act } (i+1)} + W_{T \text{ act } (i)}$$

This equation was shown on Page 2 of this write-up. This closure takes place after the turbine power for time $i+1$ is computed.

II. Turbine Equations and Logic

Below is a simple flow schematic and definitions of terms to be used in this write-up.

Subscripts Used:

- (1) Inlet to Turbine Inlet Line and Turbine Bypass Line
- (2) Turbine Inlet
- (3) Turbine Discharge
- (4) Outlet of Turbine Discharge Line and Bypass Line
- s Indicates isentropic conditions
- o Indicates stagnation conditions
- ta Indicates table look up value

Definition of Terms:

- h = Enthalpy BTU/lb
- S = Entropy BTU/lb^{°R}

P = Pressure $\#/ft^2$
 T = Temperature $^{\circ}R$
 k = Ratio of Specific Heats
 R = Gas Constant $ft\text{-}lb/1b^{\circ}R$
 g_c = Conversion factor $ft\text{-}lb_f/lb_m \text{-} sec^2$
 C_p = Specific Heat at constant pressure $BTU/lb^{\circ}R$
 J = Conversion factor $\frac{ft \cdot lb}{BTU}$
 C_1 = Conversion factor TPA speed (N) to tip speed (V)
 U = Turbine tip speed
 N = Turbine speed R.P.M.
 A_t = Turbine inlet area ft^2
 M_t = Turbine efficiency
 w = Turbine flow rate lb/sec
 P = Turbine pressure ratio

1. Other parts of the program provide turbine inlet conditions including P_2° , h_2° , N , w_{Guess} .
2. From the NBS Para Hydrogen properties deck, look up R_2 , k_2 and $T_2^{\circ} = f(P_2^{\circ}, h_2^{\circ})$.
3. A guess is made by the program of turbine outlet pressure, P_3° .
4. From this, an initial value of P_R is computed.

$$P_R = \frac{P_2^{\circ}}{P_3^{\circ}}$$

5. Next, compute $N/\sqrt{T_2^\circ}$
6. Look up from the turbopump curve is made on:

$$\left[\frac{w \sqrt{T}}{P A_t} \right]_{ta} = f(P_R, N/\sqrt{T_2^\circ}, N)$$

7. An area is computed using:

$$A_t = \left[\frac{w \sqrt{T}}{P} \right]_{ta} \sqrt{\frac{k g_c}{R} \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}}}$$

NOTE: A_t here is not the actual area of the turbine but is only a psuedo area for choke.

8. Again, look up R_2 , k_2 , T_2° , S_2° , C_{P_2} , using appropriate properties deck; i.e., Para Ortho, Equilibrium, or some percentage of Ortho to Para Hydrogen.

NOTE: Either the NBS fine grid deck or the generating codes is used for all TPA properties look ups.

9. Compute $P_2^\circ = \frac{w \sqrt{T_2^\circ}}{A_t} \sqrt{\frac{k_2 g_c}{R_2} \left(\frac{2}{k_2+1} \right)^{\frac{k_2+1}{k_2-1}}}$

$$\text{and } P_3^\circ = P_3^\circ / P_R$$

NOTE: This A_t calculated in Step (7). This gives a method analogous to that shown in Section III below of this report. This method, nowever, uses paranydrogen as the reference gas instead of air at standard conditions.

10. Compare P_3° vs. P_3° and if they are not equal go back to Step (1) and try a new P_3° until they are within a prescribed tolerance.
11. Compute $T_3^\circ = T_2^\circ \left(\frac{1}{P_R} \right)^{\frac{R_2}{C_{P2} J}}$. This is only used to get a better first guess for h_3° .
12. Use T_3° and P_3° to look up S_3°, h_3° from appropriate NBS properties look up.
13. Compare S_3° vs. S_2° and if not equal, guess new h_3° and look up $S_3^\circ = f(P_3^\circ, h_3^\circ)$. Continue until $S_3^\circ = S_2^\circ$.
14. $\Delta h_{(S)} = h_2^\circ - h_3^\circ$
15. Compute $C_o = \sqrt{2g_c J \Delta h_{(S)}}$
16. Compute $U/C_o = \frac{C_1 N}{C_o}$
17. Look up $\eta_t = f\left(N/\sqrt{T_2^\circ}, N, U/C_o\right)$
18. $\Delta h = \eta_t \Delta h_{(S)}$
19. $h_3^\circ = h_2^\circ - \Delta h$
20. Look up $S_3^\circ, T_3^\circ, R_3^\circ$, etc. = $f(P_3^\circ, h_3^\circ)$ from appropriate NBS properties deck.

21. Use standard ΔP , Δh relationships on the turbine discharge line to compute fluid conditions at 4.

22. Compare P_4° (Step 21) vs. P_4° computed from the bypass line. If they differ, go back to Step 1 and guess a new w . Continue iteration until these two pressures are equal.

23. Compute turbine power output $W_T = w\Delta h$.

24. This work is compared to that required by the pump (including acceleration), and TPA speed (N) is changed to effect a closure.

This completes the main program logic. There are other program options for NERVA, however, such as fixing turbine bypass flow percentage and then computing the turbine area required to obtain this. Hydrogen properties across the turbine can be obtained from para, ortho, equilibrium, or any input fixed percentage of ortho to para hydrogen. This latter option has been used for the hydrogen conversion analysis.

3.1.2.5.1.5 Neutronics

3.1.2.5.1.5.1 Nomenclature

Following is a list of nomenclature used in modeling the system neutronics.

a	Decay precursor fraction
A	Area or feedback coefficient
B	Decay precursor concentration
c	Decay precursor time constant
C	Heat capacitance or delayed neutron precursor concentration
C_p	Specific heat
CZ	Heat transfer film correlation coefficient
D	Hydraulic diameter
DP	Decay power
E	Energy or feedback coefficient
f	Lanning friction factor
F	Feedback coefficient
FN	Fission power
F_p	Wetted perimeter
g	gravitational constant
h	Heat transfer film coefficient
H	Enthalpy
k	Thermal conductivity
ℓ	length
ℓ^*	Mean effective neutron lifetime
L	Length of part
M	Effective hydrogen mass
P	Pressure
Pr	Prandtl number
q"	Heat generated per unit volume
Q	Energy
QT	Total thermal power
Re	Reynolds number
S	Neutron rate source
t	Time

T_F	Fluid temperature
V	Volume
W	Flow rate
ΔX	Length increment
Y	Admittance
α_p	Defined by Eqn 33
β_i	Delayed neutron fraction
β	Defined by Eqn 48
λ_i	Delayed neutron time constant
$\mathcal{E}$	Roughness
ρ	Density or feedback
Ψ_i	Defined by Eqn. 12
$\bar{\Psi}_i$	Defined by Eqn. 15
ϕ_1	Defined by Eqn. 37
ϕ_2	Defined by Eqn. 32
ϕ_3	Defined by Eqn. 36
θ	Material temperature
μ	Viscosity

SUBSCRIPTS OR SUPERSCRIPTS

b	Bulk	M	Material
c	Corrector value	n	Space index
$cond$	Conducted	o	Outlet
f	Fluid	p	Predictor value
gen	Generated	s	Surface
i	Inlet		

3.1.2.5.1.5.2 DECAY POWER CALCULATIONS

A common method of calculating decay power is straightforward integration of the convolution integral. This method is inefficient for some digital calculations since the integration must always extend back over the entire fission history. However an alternate method³, where the decay power can be expressed in terms of differential equations, transforms the time consuming evaluation of the convolution integral to the integration of a series of differential equation. This method as applicable to CDM, is probably 2 orders of magnitude faster than the convolution method.

Assume that the beta and gamma decay function can be represented by a series of exponentials.

$$g(t) = \sum_{i=1}^N a_i e^{-c_i t} \quad (38)$$

then

$$DP(t) = \int_{t_0}^t P(T) g(t-T) dT \quad (39)$$

or

$$DP(t) = \sum_{i=1}^N \frac{a_i}{c_i} \left[c_i \int_{t_0}^t P(T) e^{-c_i (t-T)} dT \right] \quad (40)$$

where

- $g(t)$ = delayed energy release rate due to instantaneous burst of fissions
- $P(t)$ = fission rate
- $DP(t)$ = decay energy release rate

Let

$$B_i(t) = c_i \int_{t_0}^t P(T) e^{-c_i (t-T)} dT \quad (41)$$

differentiate with respect to t:

$$\frac{d B_i(t)}{dt} = c_i \left[P(t) - B_i(t) \right] \quad (42)$$

then

$$DP(t) = \sum_{i=1}^N \frac{\sigma_i}{c_i} B_i(t) \quad (43)$$

The time consuming evaluation of the convolution integral (equation 39) has been transformed to the integration of a series of differential equations (equation 42) and a summation of results (equation 43) to obtain the decay power.

Equation 42 may be solved explicitly by using the analytic continuation method or implicitly as follows (assuming power is linear over the time step)

$$B_i = B_{i0} E_i + P_o F_i + (P - P_o) G_i, i = 1, 10 \quad (44)$$

where

B_i = concentration at end of time step

B_{i0} = concentration at start of time step

P = fission rate at end of time step

P_o = fission rate at start of time step

E_i = $e^{-c_i \Delta t}$

F_i = $1 - E_i$

G_i = $1 - F_i / (c_i \Delta t)$

Then the decay power at the end of the time step is $DP = \sum_{i=1}^{10} \frac{\sigma_i}{c_i} B_i$

Reactor Neutron Kinetics

Fission power may be calculated either by specifying the total thermal power or by specifying the reactivity (drum position, SSCV position, etc). When the total thermal power is specified the fission power calculation is as follows:

The derivative of decay power is approximated by

$$\frac{d DP}{dt} = \sum_{i=1}^{10} \alpha_i (FN_o - B_i) \quad (45)$$

then the fission power is:

$$FN = QT - DP - \frac{d}{dt} DP \Delta t \quad (46)$$

the delayed neutron precursor concentration

$$C_i = C_{i0} e^{-\lambda_i \Delta t} + \frac{\beta_i}{\lambda_i \ell^*} \left[FN_o (1 - e^{-\lambda_i \Delta t}) + \right. \\ \left. + [FN - FN_o] \left(1 - \frac{1 - e^{-\lambda_i \Delta t}}{\lambda_i \Delta t} \right) \right] \quad (47)$$

$$i = 1 \text{ to } 6$$

Define

$$\beta = \sum_{i=1}^6 \beta_i \quad (48)$$

The total reactivity is then

$$\rho = \rho_o + \frac{\ell^*}{FN} \left(\frac{FN - FN_o}{\Delta t} - \sum_{i=1}^6 \lambda_i C_i - S \right) / \beta \quad (49)$$

The drum reactivity is then

$$\rho_D = \rho - \rho_f$$

where ρ_f is the feedback reactivity so that the drum position may be found

$$\theta_D = 2. \sin^{-1} \sqrt{\left[\frac{\rho_D}{1 + F_\theta} + C1 \right] / C2} \quad (50)$$

where C1 is determined from the cold critical drum position and C2 is the drum span worth.

The calculation of fission power when the drum position is specified or controlled is as follows:

First calculate total reactivity

$$\rho = \rho_f + \left[C2 \sin^2 \left(\frac{\theta_D}{2} \right) - C1 \right] \left[1 + F_\theta \right] \quad (51)$$

The point reactor kinetic equations

$$\frac{dN}{dt} = \frac{\rho - \beta}{\ell^*} N + \sum_{i=1}^6 \lambda_i C_i + S \quad (52)$$

$$\frac{dC_i}{dt} = \frac{\beta_i N}{\ell^*} - \lambda_i C_i \quad (53)$$

$$i = 1 \text{ to } 6$$

are solved simultaneously.

3, 1, 2, 5, 1.5, 4

Reactivity Feedback

OK

The reactivity feedback equations are calculated as follows:

The total reactivity feedback

$$\rho_f = \sum_{i=1}^8 A_i F_i \quad (54)$$

where the A_i 's are calculated in the corrector and the F_i 's are calculated in the predictor.

To calculate the feedback due to hydrogen in the core obtain the effective hydrogen mass in the fuel

$$M_s = A_f \Delta X \sum W_i \rho_i$$

in the support stems downcomer

$$M_D = A_D \Delta X \sum W_i \rho_i \quad (55)$$

in the support stems upcomer

$$M_U = A_U \Delta X \sum W_i \rho_i$$

$$D_1 = 1 - .0212 (M_s + M_D + M_U)$$

$$A_1 = 2.53 M_s D_1 \frac{1.}{[E_1 \rho_i^P + E_2 \rho_o^P]} \quad (56)$$

$$A_2 = 2.38 M_D D_1 \frac{1.}{[E_3 \rho_i^P + E_4 \bar{\rho}^P + E_5 \rho^P]} \quad (57)$$

$$A_3 = 2.38 M_U D_1 \frac{1.}{[E_6 \rho_i^P + E_7 \bar{\rho}^P + E_8 \rho_o^C]} \quad (58)$$

Let

$$D_1 = 1 - .0867 (M_s + M_D + M_U)$$

$$G_1 = -.0556 M_s D_1 \quad (59)$$

$$G_2 = -.0523 (M_D + M_U) D_1 \quad (60)$$

The calculation of the feedback due to hydrogen in the Beryllium and aluminum components in the reflector and the lateral support structure is:

$$M_B = A_B \Delta X \sum \rho_i \quad (61)$$

$$M_A = A_A \Delta X \sum \rho_i$$

$$M_{LS} = A_{LS} \Delta X \sum \rho_i$$

$$A'_4 = -M_R (.0126 - M_R (.028)) + M_A (.319 + M_A (.117)) + M_{LS} (.577 - M_{LS} (.0761)) \quad (62)$$

$$A_4 = A'_4 \left[\frac{1.}{E_9 \rho_i^P + E_{10} \rho_o^P + E_{11} (\rho_i^P)^2 + E_{12} (\rho_o^P)^2} \right] \quad (63)$$

$$G_4 = M_A (-.0257 + M_A (.0149)) \quad (64)$$

The core neutronic temperature feedback reactivity calculation is:

$$A_5 = [1. + .106 (A_1 + A_2 + A_3)] / (1 + E_{13} W_{ss}) \quad (65)$$

If the average core temperature $\bar{\theta}_c$ is less than or equal to 532.8°R

Set

$$G_5 = 0$$

if greater than 532.8°R

Set

$$\text{DEL} = \bar{\theta}_c - 532.8$$

then

$$G_5 = \text{DEL} (-2.02 \cdot 10^{-5} + \text{DEL} (4.58 \cdot 10^{-9})) \quad (66)$$

The core radial thermal expansion calculation on feedback reactivity is:

$$\text{DER} = 8.04 \cdot 10^{-5} \text{DEL}$$

$$A_6 = -7.44 \quad (67)$$

$$G_6 = .277 \text{DER}$$

The calculation of the beryllium temperature effect on the feedback is

if the average beryllium temperature $\bar{\theta}_B$ is less than 532.8°R set

$$A_7 = 0 \quad (68)$$

$$G_7 = 0$$

if greater than 532.8°R set

$$\text{DEL} = \bar{\theta}_B - 532.8$$

$$A_7 = 1.$$

$$G_7 = \text{DEL} (0.730 \cdot 10^{-4} - \text{DEL} (7.28 \cdot 10^{-8})) \quad (69)$$

The aluminum cylinder support structure radial displacement effect on reactivity feedback is calculated as follows:

$$\text{DEL} = -3.74 \cdot 10^{-5} (P_i - P_o) + 2.20 \cdot 10^{-4} (\bar{\theta}_{AL} - 532.8) \quad (70)$$

$$A_8 = -1.87$$

$$G_8 = -.23 * \text{DEL}$$

The fractional change in drum worth F_θ is

$$F_\theta = (1 - G_1) (1 - G_2) (1 - G_4) (1 - G_5) (1 - G_6) (1 - G_7) (1 - G_8) - 1 \quad (71)$$

In the predictor the following calculations are made:

$$F_1 = E_1 \rho_i^P + E_2 \rho_o^P \quad (72)$$

$$F_2 = E_3 \rho_i^P + E_4 \bar{\rho}^P + E_5 \rho_o^P \quad (73)$$

$$F_3 = E_6 \rho_i^P + E_7 \bar{\rho}^P + E_8 \rho_o^P \quad (74)$$

$$F_4 = E_9 \rho_i^P + E_{10} \rho_o^P + E_{11} \left(\rho_i^P \right)^2 + E_{12} \left(\rho_o^P \right)^2 \quad (75)$$

if $\bar{\theta}_c^P < 532.8$

$$F_5 = -.145 \left[\ln_e \left(\frac{\bar{\theta}_c^P}{532.8} \right) \right] \left(1 + E_{13} W_{ss}^P \right) \quad (76)$$

if $\bar{\theta}_c^P \geq 532.8$

$$\text{DEL} = \bar{\theta}_c^P - 532.8$$

$$F_5 = \text{DEL} (-2.95 \cdot 10^{-4} + \text{DEL} (.446 \cdot 10^{-7})) [1 + E_{13} W_{ss}^P] \quad (77)$$

$$F_6 = 8.04 \cdot 10^{-5} (\bar{\theta}_c^P - 532.8) \quad (78)$$

$$\text{DEL} = \bar{\theta}_B^P - 532.8$$

$$F_7 = \text{DEL} (-5.70 \cdot 10^{-5} - \text{DEL} (1.27 \cdot 10^{-7})) \quad (79)$$

$$F_8 = -3.74 \cdot 10^{-5} (P_i - P_o) + 2.20 \cdot 10^{-4} (\bar{\theta}_{AL}^P - 532.8) \quad (80)$$

Then in the predictor the total feedback may be calculated (Equation 54).

3.1.3 Data Base Requirements

Standard 80 column computer input cards shall be used.
Data may be entered in columns 2 to 80.

For arrays, only a single value may be entered if the array name is subscripted. e.g. $A(1) = 2.0$, is o.k.; $A(1) = 2.0, 3.0$, is illegal.

If the array name is given without being subscripted, data for the whole array must be given. e.g. DIMENSION A(5) enter data into the array A as follows: $A = 1.0, 2.0, 3.0, 4.0, 5.0$, is o.k.; $A = 5 * 0.0$ is illegal because the * ends the namelist read. $A = 1.0, 2.0, 3.0$, is also illegal since the array isn't filled.

The program shall require two sets of pump-data tables consisting of pump efficiency vs speed/flow ratio ($N/WDOT$, rpm-sec/lbm), speed (N , rpm), and saturated-vapor head/speed-squared ratio (HSV/N^2 , but/lb-rpm**2) and of pump pressure-rise/flow-squared ratio ($DELP/WDOT^2$, psai-sec**2/in.**2-lbm**2) vs $N/WDOT$, N , and HSV/N^2 . Since the tables are identical in format, the input format will be described in terms of an arbitrary function F (representing pump efficiency or $DELP/WDOT^2$) vs $N/WDOT$, N , and HSV/N^2 .

Any point in the table can be described as a point in a $M(1) \times M(2) \times M(3)$ matrix, where $M(1)$ is the number of HSV/N^2 points in the Table, $M(2)$ is the number of N points, and $M(3)$ is the number of $N/WDOT$ points. Let $F(A,B,C)$ represent the value of F corresponding to the a^{th} value of HSV/N^2 , the b^{th} value of N , and the c^{th} value of $N/WDOT$. Then any value of F in the matrix can be located with the use of the proper subscripts.

The input format follows:

```
* Table = M(1), M(2), M(3),  
HSV/N2(1),....., HSV/N2 [M(1)],  
N(1),....., N[M(2)],  
N/WDOT(1),....., N/WDOT [M(3)]  
F(1,1,1),....., F(1,2,3), F(2,1,1),.....,F[2,1,M(3)],  
.....F[M(1),M(2),M(3)]  
  
* Insert proper name for table
```

Briefly stated, the values of F are input in ascending order of their subscripts where, in the case of F(A,B,C), A is of higher order than B and B is of higher order than C. It is further required that the values of HSV/N2, N, and N/WDOT be entered in ascending order. The program performs a linear triple interpolation to find the value of F corresponding to given values of HSV/N2, N, and N/WDOT.

The values of M(1), M(2), and M(3) must not be less than 1. The sum, $M(1) \times M(2) \times M(3) + M(1) + M(2) + M(3) + 3$, must not exceed 300 for the efficiency table, or 200 for the work table.

END is the last card entered on each namelist data deck. The * ends the reading of a namelist data deck.

3.1.4 Human Performance

If printing is off line, a summary for steady-state or transient time point shall be displayed to the computer operator if desired. Restart information will be preserved for intervals of 10 minutes or less.

3.2 Interfaces

The CDM shall interface with the more detailed component thermodynamic programs; current interfacing is manual not requiring a specific format. However, all data shall be on tape in order to provide for future automatic interfacing.

The CDM shall interface with the SPASTIC program and plot programs.

3.2.1 Interface Requirements

The CDM shall be programmed for analysis on a Sigma 5 computer for Phases I and II. With minimum modifications, it shall be capable with the IBM 360/75 and CDC 6600. Phase III shall use TBD equipment.

The CDM shall be programmed for a Sigma 5 computer with the following hardware:

- (a) Model Sigma 5 containing 32K words core memory. Each word is made up of 32 bits.
- (b) Model 7020 teletype (ASR) with paper tape reader and punch and controller; operates at 10 characters/second.
- (c) Model 7204 Rapid Access Data Storage System with three million byte capacity.
- (d) Model 7120 card reader 400 cards/minute.
- (e) Model 7361/7362 Magnetic tape transport and controller for 7-track tape. Tape is written in the forward direction in 200, 556, or 800 bits per inch.
- (f) Potter Line Printer, 300 lines per minute.

The Sigma 5 has a 850 nanosecond memory cycle time. It is a word-oriented memory (32 bits) addressable and alterable by byte (8 bites),

half word, word and double word. It includes indirect addressing with or without indexing. Priority interrupts up to 16 levels are included.

3.2.1.1 Interface Block Diagrams

The output from the CDM shall be used as an input to the SPASTIC program. The SPASTIC program requires a tape with all inlet and outlet conditions for each part.

3.2.1.2 Detailed Interface Definition

The program shall be in American Standard FORTRAN IV-H. The CDM shall be able to use either assembly language or American Standard FORTRAN-IV versions of the hydrogen properties. In addition, the CDM program shall be able to use helium or nitrogen fluid properties in place of hydrogen properties.

3.2.2 Government-Furnished Property List

The CDM shall use the hydrogen properties computer programs developed by the National Bureau of Standards. The hydrogen properties shall be those described in NBS Report 9288, August 18, 1967, with the latest version of the computerized properties being used.

3.3 Design Requirements

3.3.1 Program Organization

The CDM shall be organized with two basic parts in the main body: a predictor and a corrector. The predictor subroutine shall in itself be a complete simplified program with each major engine component represented

by a simplified pressure drop, enthalpy rise, and heat capacitance equations. The engine flow balances shall be performed within the predictor, along with the engine controller calculations and the turbopump calculations.

The corrector subroutine shall be used to update the predictor to improve its accuracy. The corrector shall contain the detailed heat transfer and energy balance relationships, and includes a capability to use multi-mode representations for each engine part.

Program Documentation

3.3.2 Documentation shall be kept current and each production code shall be backed by information containing at least:

- a. Complete check case run.
- b. Required change authorization signatures.
- c. Reasons for changes.
- d. Date of changes
- e. Storage on at least two devices to prevent loss of program such as cards, tape and/or disk.
- f. Storage of standard data deck for documentation.

TABLE I
INPUT DATA

PARTS AND FLOW PATH

PARTS	Maximum of 25 parts available. One to four character name may be used. Three cards must always be used to make up the PARTS data deck. The name of the first part must be entered in columns 1-4 of the first card. Four columns must be skipped and the second name placed in columns 9-12 and so on for a maximum of 10 part names in cards 1 and 2. The third card will only use the first 40 columns for a maximum of 5 part names. The parts may be entered in any order, but the part name location must correspond to the array subscript of the part data.
MPARTS	The logic flow path of the main parts and valves of the engine system. A maximum of 40 is available, although the program is set up for only 25 parts, 5 valves and 1 turbine at the present time. Four cards must always be used to make up the MPARTS data deck. The names would be entered the same as for PARTS on the cards, but must be in the order of the flow path for the main parts.
XPARTS	This shows the parallel flow branches of the main engine flow path defined in MPARTS. Ten cards must be entered to make up the XPARTS data deck. Each card representing a possible bypass flow. Ten entries may be made on each card in the same manner as the PARTS card. The first entry must be the name

in MPARTS where the bypass flow will be extracted. The last name entered on the card will be a name in MPARTS where the bypass flow returns. This leaves 8 locations available for entering part and valve names of the bypass circuit.

The following names are used to define a special part, the valves and the turbine.

1. REFL - reflector
2. TURB - turbine
3. TPCV - turbine power control valve
4. TPBV - turbine power blocking valve
5. OLCV - open loop control valve
6. PDV - pump discharge valve
7. STMV - structural support control valve

NAMelist (single valve)

ABY	area turbine bypass valve (TPBV)
AOLCV	area open loop control valve (OLCV)
AVN	area turbine power control valve (TPCV)
DRATE	Isp derating factor (multiply Isp from table by this factor)
DTROT	diameter at nozzle throat
DTURBL	turbine blade diameter
ETAMIN	minimum pump efficiency
GATEEG	turbine power control valve position. If this valve is zero, the program will use the curve input as GP
HGO	internal heat from last time point
HPOUT	enthalpy out of the pump. This is on initial valve used to obtain pump outlet density
KBEAR	loss coefficient for turbopump bearing coolant flow
KVMIN	minimum loss coefficient for TPCV
MPR	pressure ratio adjusting factor (turbine)
MTIME	minimum delta time
MUPT	main turbine and pump inertia factor
NOZEFF	main nozzle efficiency factor in choke flow calculations
NTIME	number of time points

NTURP	initial turbine speed other than time zero
NWORK	minimum turbine speed
PCORO	core outlet pressure (corrector value)
PMINO	minimum nozzle back pressure. If zero, program will use input back pressure curve PAMB.
PPUMOP	pressure out of the pump. This is an initial value to obtain pump outlet density and is also iterated on for constant enthalpy across pump.
PTANK	tank pressure. If zero, program will use table of pressures input at PINLET.
PTROT	nozzle throat pressure (corrector value)
PUMOG	pump outlet pressure guess at time zero. If zero, program will use tank pressure as initial pump discharge guess at zero time.
RNTROT	recovery factor at nozzle throat
STATE	steady-stage flag. If value not zero, program will use steady-state equations for this number of time points.
TIMEI	initial starting time
TIMER	when time equals the value in timer, new data will be read in and the run continued.

TIMEX	delta time. Computing time interval. If negative value entered, the program will use the variable time grid scale entered in table TIME.
TTANK	tank enthalpy. If zero, program will use table of enthalpy input at TINLET.
TVL	flag. Equal zero when TPCV in bypass line. Non zero when TPCV in line with turbine.
TURSPD	initial turbine speed at time zero
WNOZ	nozzle flow rate (corrector value)
WPMAX	maximum pump flow rate
WPUMP	initial pump flow rate guess other than time zero
WPUMPG	initial pump flow rate guess at time zero
WORK	minimum speed/flow
OPTION	flag to tell the type option to run. e.g. standard, $P_c T_c$, controllers, etc.
TANKM	mass of propellant in tank at initial time point. Used if tank heating option desired.
ARATIO	nozzle expansion ratio (used in I_{sp} calculation)

TCOROR	difference in temperature between nozzle probe and core outlet temperature (corrector value, used when demand T_c at the probe)
HCORO	core outlet enthalpy (corrector value)
HCHOK	nozzle throat enthalpy (corrector value)

NAMelist (array values)

AFBAR	average flow area of a part (25 parts max)
ALH	difference in enthalpy between the inlet and outlet of a part (corrector value) (25 parts max)
ALP	difference in pressure between the inlet and outlet of a part times a density divided by part flow rate squared. Density is a function of inlet pressure and average inlet and outlet enthalpy. (Corrector value) (25 parts max)
BARSIZ	length of a part (25 parts max)
CAP01	table of part material heat capacity vs material temperature (25 parts max) (30 storage cells max for each part)
CAP02	
.	
.	
.	
.	
CAP25	
DELHN2	table of HSVP, speed, Q/speed and suction head/speed squared (200 storage cells max)
EP	table of pump bearing torque influence coefficient vs pump Q/speed. (30 storage cells max)
FBAR	integral of fraction of internal heat for each part (25 parts max)

FEE1 average part material heat capacity divided
by the sum of the material heat capacity of
each node divided by the number of nodes.

$$\left(\frac{\frac{C_R}{\text{nodes}}}{\frac{\Sigma CR}{\text{nodes}}} \right) \quad (\text{corrector value}) \quad (25 \text{ parts max})$$

FEE2 average part fluid temperature divided by the
sum of the part inlet and outlet fluid
temperature. $\frac{T_{\text{avg}}}{T_{\text{in}} + T_{\text{cut}}}$

GP TPCV gate position vs time. Used if GATEFG
is zero.(30 storage cells max)

FPBAR average wetted perimeter of a part.
(25 parts max)

HFBAR average film coefficient of a part
(corrector value) (25 parts max)

HG internal heat generation rate vs time.
(30 storage cells max)

KBP table of PDV valve K/A^2 vs time.
(30 storage cells max)

KSS table of SSCV valve K/A^2 vs position.
(30 storage cells max)

KV table of TPCV valve K vs position
(30 storage cells max)

NT	table of turbine speed vs time (used with speed controller option) (30 storage cells max)
OLC	table of OLCV valve K vs time (30 storage cells max)
PAMB	main nozzle back pressure vs time. Used if $PMIN\phi$ is zero. (30 storage cells max)
PC	table of demand chamber pressure vs time (30 storage cells max)
PINLET	table of tank pressure vs time. Used if PTANK is zero. (30 storage cells max)
PRATIO	table of speed/ $\sqrt{T}$, speed, turbine pressure ratio and $\frac{\text{flow} * \sqrt{T}}{\text{MPR} * \text{pressure}}$ (200 storage cells max)
PUMP	table HSVP, speed, Q/speed and pump efficiency (200 cells max)
QFLUD3	average part fluid temperature of previous time point (25 parts max)
QMETL3	average part material temperature of previous time point (25 parts max)
QMETLY	conduction term added in if TTK(N) is flagged and WALL(n) is zero. (corrector value)
QTANK	run tank heat input vs time. PTANK, TTANK and TANKM must be non zero. (30 storage cells max) density of part material. If zero, material calculations for a part eliminated. (25 parts max)

SSCV	table SSCV position vs time (30 storage cells max)
TBYK	table of TPBV valve K vs time (30 storage cells max)
TC	table of demand chamber temperature vs time (30 cells max)
TIMES	variable delta time table with start time, delta time, time of change, new delta time, time of change, etc. Table used if negative value entered in TIMEX (11 storage cells max)
TINLET	table of tank enthalpy vs time. Used if TTANK is zero (30 storage cells max)
TNODE	to be used as a flag for a demand temperature other than chamber temperature. [Not to be used at the present time. (25 parts max)]
TTK	at the present time used as a flag when non zero to add in the radial conduction term QMETL4 to a part. WALL(n) is also used. (25 parts max)
TUREFF	table of speed/ $\sqrt{T}$, speed, tip velocity/gas velocity, and turbine efficiency (200 cells max)
WALL	thickness of pipe wall. At present time used only as flag. When equal to zero and TTK non zero radial conduction term QMETALY used. (25 parts max)

WDIL table of propellant tank pressurization flow
vs time. Bleed flow schedule is associated
to a tap off part that doesn't have a return
to the main engine.

WDOTX Used only with timer input. Contains values
entered in WX at initial time point. These
values are used as flags with bypass flows.
(10 bypass loops max)

WRATEO Flow rate through part during previous time
point. (25 parts max)

WX initial flow rate guess through bypass part.
If less than 0, but greater -1, value will be used
as a fraction of pump flow. If equal or less
-1 but greater -100, value used as percent of the
main flow before bypass tap off. If equal to
-100 all flow put through bypass loop.
(10 bypass loops max)

AEROJET NUCLEAR SYSTEMS COMPANY
SACRAMENTO, CALIFORNIA

COMMON DIGITAL MODEL CHECK CASE

DATE 06:41 NOV 17, 71

CHAMBER PRESSURE = 450.

CHAMBER TEMPERATURE = 4254.

THRUST - - - - - 75819. LBF

SPECIFIC IMPULSE - - - - - 826. LBF-SEC/LBM

REACTOR POWER - - - - - 1530. MEGAWATTS

DRUM POSITION - - - - - 68. DEGREES
STRUCTURAL SUPPORT CONTROL VALVE POSITION - - - - - 22. DEGREES
PUMP DISCHARGE CONTROL VALVE POSITION - - - - - 0. DEGREES
TURBINE BYPASS CONTROL VALVE POSITION - - - - - 39. DEGREES
TURBINE BYPASS FRACTION - - - - - .085

TURBOPUMP SHAFT SPEED - - - - - 24245. RPM
PUMP EFFICIENCY - - - - - 67. PERCENT
TURBINE EFFICIENCY - - - - - 86. PERCENT
NET POSITIVE SUCTION PRESSURE - - - - - 7.1 PSIA
AMBIENT PRESSURE - - - - - .1 PSIA

	STATE NO.	FLUX LBM/SEC	PRESSURE PSIA	TEMPERATURE DEG R	ENTHALPY BTU/LBM
STATE POINT 1					
PROPELLANT TANK OUTLET	1.0	92.4	21.8	36.4	-109.6
PUMP INLET	4.0	46.2	21.8	36.4	-109.6
PUMP OUTLET	5.0	45.9	1403.	55.3	-25.7
STRUCTURAL SUPPORT FEED LINE INLET	9.3	7.1	1377.	55.6	-25.7
SSCV INLET	14.0*	3.5	1376.	55.6	-25.7
SSCV OUTLET	15.0*	3.5	1162.	57.5	-25.7
STEM INLET		7.1	1160.	78.0	43.5
STEM OUTLET	19.1	7.1	1118.	846.0	2893.
BYPASS LINE OUTLET		3.2	1106.	62.9	-11.0
SUPPORT PLATE INLET		7.1	1118.	846.0	2893.
SUPPORT PLATE OUTLET	23.2	7.1	1106.	796.2	2718.
POCV INLET	11.0	42.4	1374.	55.6	-25.7
POCV OUTLET	12.0	42.4	1368.	55.6	-25.7
STRUCTURAL SUPPORT BYPASS INLET	16.1	3.2	1350.	55.8	-25.7
NOZZLE TUBE INLET	20.0	81.7	1330.	56.0	-25.7
REFLECTOR INLET	22.0*	81.7	1159.	186.9	459.1
REFLECTOR ALUMINUM OUTLET		22.8	1106.	264.7	762.0
REFLECTOR BERYLLIUM OUTLET		51.6	1106.	253.2	714.9
PRESSURE VESSEL ANNULUS OUTLET	23.0*	7.3	1106.	197.5	500.7
PERIPHERAL SHIELD INLET	23.3	91.9	1106.	283.2	838.5
SHIELD OUTLET	25.0	66.7	1094.	283.6	839.9
SHIELD BYPASS OUTLET	24.0*	25.2	1094.	283.5	839.5
DOME EXIT LINE INLET	26.0*	91.9	1094.	283.6	839.8
DOME EXIT LINE OUTLET	26.1	91.9	1082.	283.5	839.8
TURBINE INLET	32.0	42.0	1057.	283.5	839.8
TURBINE OUTLET	33.0	42.0	718.	260.8	747.5
TURBINE DISCHARGE BLOCK VALVE OUTLET	35.0	84.6	718.	259.6	742.7
BCV INLET	28.0	3.9	1081.	283.5	839.8
BCV OUTLET	29.0	3.9	724.	283.4	839.8
STAGE PRESSURIZATION	36.0	.6	704.	261.8	750.9
DOME INLET LINE INLET	29.2	91.8	704.	261.8	750.9
CENTRAL SHIELD INLET	37.0*	91.8	674.	262.1	750.9
CORE SUPPORT PLATE INLET	37.2	91.8	634.	263.9	756.4
CORE INLET	37.3	91.8	622.	267.0	769.0
FUEL CHANNEL OUTLET		89.2	450.	4340.	16490.
LATERAL SUPPORT OUTLET		2.6	450.	417.8	1366.
NOZZLE CHAMBER	38.0*	91.8	450.	4252.	16054.

* DENOTES NSS INTERFACE POINTS

TABLE II

TABLE III
PREDICTOR FLOW LOGIC

1. Assume a flow split ($K = \dot{W}_{by} / \dot{W}_{tot}$) for each bypass removal point.
2. Assume an inlet flow rate and/or pressure and start through the system calculating pressures and temperatures.
3. Where a bypass leaves the main flow path, apply the flow split to the inlet flow to get the bypass and main flows. Calculate conditions in the bypass leg first.
4. Where a bypass leg rejoins the main leg - calculate a new flow split and average pressure as follows:

$$F^1 = \frac{F \sqrt{(P_1 - P_2) / (P_1 - P_{2a})}}{1 - F + F \sqrt{(P_1 - P_2) / (P_1 - P_{2a})}}$$

$$P_2^1 = P_1 - (P_1 - P_2) \left(\frac{1 - F^1}{1 - F} \right)^2$$

Where:

P_1 = Pressure where bypass leg leaves main path

P_2 = Pressure in main leg where bypass leg rejoins it

P_{2a} = Pressure in bypass leg where it rejoins the main leg

F = Flow Fraction = $\dot{W}_{by} / \dot{W}_{tot}$

5. Calculate a mixed mean enthalpy from

$$H^1 = \frac{\dot{W}_{by} \cdot H_{by} + \dot{W}_m \cdot H_m}{\dot{W}_m + \dot{W}_{by}}$$

6. Continue calculations using P_2^1 and H^1 as inlet conditions to the next part.
7. Go completely through the engine system this way - do not go back and iterate on flow fractions unless there is a negative pressure - then use the following procedure:
 - a. If the negative pressure is encountered in the main flow path, go back to the last bypass branching point and increase the bypass flow - continue calculations from that point.
 - b. If the negative pressure is found in a bypass leg, go back to the branching point for this leg and decrease the bypass leg flow - continue calculations from that point.
 - c. If a negative pressure is found in both the main and the bypass paths, reduce pump flow and start pressure drop calculations over from the pump discharge.
8. Next pass - use all the new flow fractions.
9. Stop when every $(F^1 - F)$ is within tolerance and flow is closed on choke flow in the nozzle.

TABLE IV

EQUATION DERIVATIONS

P_1

Given P_1, T_1 (inlet to bypass)

P_2, T_2 (outlet of main path at bypass
rejoining point)

P_2, P_{2a}

P_{2a}, T_{2a} (outlet of bypass)

$$K_m = \frac{P_1 - P_2}{\dot{w}_m^2}, \quad K_{By} = \frac{P_1 - P_{2a}}{\dot{w}_{By}^2}, \quad w_m + w_{By} = w_{Tot} = w_m^1 + w_{By}^1$$

$$\frac{(P_1 - P_2) \dot{w}_m^2}{\dot{w}_m^2} = \frac{(P_1 - P_{2a}) \dot{w}_{By}^1^2}{\dot{w}_{By}^2}$$

$$\frac{P_1 - P_2}{P_1 - P_{2a}} = \frac{\dot{w}_{By}^1^2}{\dot{w}_{By}^2} \cdot \frac{\dot{w}_m^2}{\dot{w}_m^2} = \frac{\dot{w}_m^2}{\dot{w}_{By}^2} \cdot \frac{\dot{w}_{By}^1^2}{\dot{w}_m^1^2}$$

Call $R = (w_{By}/w_m)$

$$\frac{P_1 - P_2}{P_1 - P_{2a}} = \frac{R^1}{R} \quad R^{12} = \frac{P_1 - P_2}{P_1 - P_{2a}} R^2$$

$$R^1 = R \sqrt{\frac{P_1 - P_2}{P_1 - P_{2a}}}$$

Now, call $F = w_{By}/w_{Tot}$ (flow fraction)

$$\text{Then,} \quad F = \frac{R}{1 + R} \quad \text{or} \quad R = \frac{F}{1 - F}$$

Substituting:

$$F^1 = \frac{R \sqrt{P_1 - P_2/P_1 - P_{2a}}}{1 + R \sqrt{P_1 - P_2/P_1 - P_{2a}}}$$

$$F^1 = \frac{F \sqrt{P_1 - P_2/P_1 - P_{2a}}}{1 - F + F \sqrt{P_1 - P_2/P_1 - P_{2a}}}$$

Solving for the mix point correct pressure:

$$K = \frac{P_1 - P_2}{\dot{w}_m^2} \quad \text{so} \quad P_2^1 = P_1 - K \dot{w}_m^2$$

$$\text{or} \quad P_2^1 = P_1 - \frac{P_1 - P_2}{\dot{w}_m^2} \dot{w}_m^2$$

but since $\dot{w}_m = (1 - F) \dot{w}_{\text{Tot}}$

$$P_2 = P_1 - (P_1 - P_2) \frac{1 - F^1}{1 - F}$$

NERVA ENGINE STEADY STATE OPERATIONAL CONSTRAINT MAP

Figure 1

NERVA ENGINE OPERATIONAL CONSTRAINT MAP

FLOW SCHEMATIC
COMMON DIGITAL MODEL

MODELING A SYSTEM BY ARRANGING PROGRAM SUBROUTINES

BASIC INTERACTION BETWEEN PREDICTOR AND
CORRECTOR ROUTINES

FIGURE 4