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ATOMIC DISPLACEMENTS PRODUCED BY FISSION FRAGMENTS
AND FISSION NEUTRONS IN MATTER

by

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ABSTRACT

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ATOMIC DISPLACEMENTS PRODUCED BY FISSION FRAGMENTS
AND FISSION NEUTRONS IN MATTER

The Passage of Fission Fragments Through Matter

I Introduction

About four-fifths of the energy of the fission process is shared by the two heavy fragments into which the nucleus splits. This energy, of about 80 Mev. per fragment, is transferred to the medium in which the fission takes place in two ways. First, because each fragment is stripped of about half of its electrons during most of its path, it will interact strongly with electrons and thus lose energy through ionizing collisions with other atoms. Secondly, the fragments will lose energy by elastic collisions with atoms as a whole. Each fragment leaves in its wake a cloud of moving electrons and atoms. This cloud will roughly resemble a cylinder whose radius will increase with time on account of the motion of the struck electrons and atoms. We may now investigate two features of the slowing down process. First, as it affects the fragment, i.e. we calculate its range, straggling, etc. And secondly we may work out the motion of the cylinder of moving particles, its effect on the medium, etc.

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II. The Energy Losses of Fission Fragments

A. Methods

We will show in the following that the energy losses, both to electrons and to atoms may be calculated in a simple classical way. It is well known that Quantum Mechanics reduces to classical mechanics provided that the wave length λ of the moving particle is small compared to the extent of the potential, a ; and that the interaction is sufficiently strong. We can see this by the following argument due to E. J. Williams.⁽¹⁾ In order for a classical treatment to apply it must be possible to determine experimentally the calculated orbit of the particle. Let the potential be of the order of V over a distance of the order of a . Then, first, since we are specifying distances of the order of a , the indeterminacy in momentum will be $\hbar/a$ and this must be small compared to the momentum of the particle. Thus we get the condition $\hbar/a \ll Mv$ or $\lambda \ll a$. Secondly the deflexion must be well defined. Here this is determined by the change in momentum in the collision and this is of the order of V/v . The indeterminacy, $\hbar/a$, must be small compared to this in order that the deflexion be well defined. Hence the condition $\hbar/a \ll V/v$ or $Va/\hbar v \gg 1$, expressing the strength of the interaction. For a Coulomb field this becomes $\frac{z Z e^2}{\hbar v} = \frac{z Z}{137\beta} \gg 1$. The conditions for the applicability of classical methods are then: $\frac{\lambda}{a} \ll 1$, $\frac{137\beta}{z Z} \ll 1$.

(1) Rev. Mod. Phys. 17, 217, 1945

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A fission fragment starts out with an energy of about 10^8 e.v. In the table below we give values of λ/a and $\frac{z Z}{137\beta}$ for fission fragments and for electrons of the same velocity. The energy given is that of the fragment.

Energy	electrons		fragments		
	10^6 ev.	10^8 ev.	10^6 ev.	10^8 ev.	100 ev.
λ/a	1.7	.17	10^{-4}	10^{-5}	10^{-2}
$\frac{z Z}{137\beta}$	41	4.1	3760	376	3.76×10^5

Classical methods will certainly work for the collision losses at these energies and for electron losses down to energies of a few Mev. We will see later that losses to electrons cease at about this energy so that the failure of the classical method is of no consequence.

The conditions for the validity of the Born Approximation are $\frac{\lambda}{a} \ll 1$ and $\frac{z Z}{137\beta} \ll 1$ in the case of a Coulomb field. However, it is well known that, for the case of the Coulomb field, the wave method of Born and the classical method give the same scattering formula and it might be thought that it is immaterial which method is used here. This is not so, however, since the field of the nucleus is cut off by (1) the electrons and so is no longer a Coulomb field. Thus Williams shows that, though the probability of scattering through an angle ϑ is

(1) loc. cit.

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formally the same in the two treatments, the field which scatters into ν in the classical case is at a distance r_c , in the wave case at r_w and r_c and r_w are related by the equation: $r_c = \frac{zZ}{137\beta} r_w$. Now the effect of the electrons is to cut off the Rutherford scattering at impact parameters larger than the atomic radius, i.e. small angle scattering. The cut-off takes place at a larger energy (of the struck atom) in the classical case than in the wave case. As Williams shows, ⁽¹⁾ the two cut-off angles are related by $\nu_c = \frac{zZ}{137\beta} \nu_w$. The difference in the two treatments is that the classical one gives higher average and lowest energies of the struck atoms. That this is actually so is shown by comparing the results which are given later in this treatment with those obtained by ⁽²⁾ Seitz using the Born Approximation. For the atoms struck by a fission fragment of 80 Mev. energy, the Born Approximation gives 375 ev. for the average energy and that half of the energy lost is given to atoms whose energy is less than 25 ev. In the same case, the classical method gives 1900 ev. for the average energy, and no recoil atoms with energies less than about 350 ev.

Having shown that classical mechanics is a good approximation to quantum mechanics in this problem, we go on now to calculate the energy losses to electrons and atoms.

(1) loc. cit.

(2) F. Seitz, "Irradiation Physics Notes", page 78 et seq.

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B. Energy Losses to Electrons.

In general, when a particle of energy E , mass M collides elastically with a particle at rest, mass m , the conservation of energy and momentum laws give the equation:

$$\Delta E = 4E \frac{\mu^2}{Mm} \cos^2 \nu \quad - - - - - (1)$$

for the energy, ΔE given to the struck particle. μ is the reduced mass and ν is the angle the recoil makes with the incident direction.

Now if the force between the two particles of charges Z and z is a Coulomb force, then the relation between ν and the impact parameter b , is:

$$\cot \nu = \frac{zZe^2}{v^2 b} \frac{1}{\mu} = \frac{\tilde{E}}{bv^2 u} \frac{1}{\mu} \quad - - - (2)$$

where $\tilde{E} = zZe^2 u$; $u = Z^{1/3}/2a_0$; $a_0 = \hbar^2/me^2 = \text{Bohr radius}$. Using (2) we find the energy taken up by the struck particle to be:

$$\Delta E = 4E \frac{\mu^2}{Mm} \frac{1}{1 + \frac{b^2 v^4 u^2 \mu^2}{\tilde{E}^2}} \quad (3)$$

for an impact parameter $= b$. If there are N particles per cm^3 then the number having impact parameters in db at b is $2\pi N b db$ and so the energy loss per cm. of path is:

$$-\frac{dE}{dx} = 8\pi N E \frac{\mu^2}{Mm} \int_0^{b_{\max}} \frac{b db}{1 + b^2/b_0^2} \quad - - - (4)$$

$$b_0 = \frac{\tilde{E}}{\mu u v^2}$$

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$$\text{or } -\frac{dE}{dx} = 4\pi N E b_o^2 \frac{\mu^2}{Mm} \log \left(1 + \frac{b_{\max}^2}{b_o^2}\right) \quad (4)$$

We apply this equation to the electron losses of a fission fragment. Then m is the electron mass, $z = 1$, $\mu \approx m$ = mass of the electron, $Z = Z^*$ = effective charge of the fission fragment; $N = N_o Z$ where Z is the atomic number of the material. $b_{\max}^2$ is chosen according to Bohr's well known condition, viz, $\frac{b_{\max}}{v} = \frac{\hbar}{E_i}$; $b_{\max} = \hbar v / E_i$ where E_i is the binding energy of the i^{th} electron. We can take into account that for values of b less than about $1/u$, the charge of the fission fragment is unshielded and so is Z_f rather than Z^* . We find then the equation:

$$-\frac{dE}{dx} = \pi \frac{N_o}{u^2} \cdot \frac{\tilde{E}^2}{E} \cdot \frac{M}{m} \cdot \sum_i \left\{ \left(1 - \frac{Z^{*2}}{Z_f^2}\right) \log \left(1 + \frac{1}{u^2 b_o^2}\right) + \frac{Z^{*2}}{Z_f^2} \log \left(1 + \frac{b_{\max}^2}{b_o^2}\right) \right\} \quad (5)$$

Ignoring the correction due to close collisions which is not large and also unity in the second log term we get:

$$-\frac{dE}{dx} = 2 \pi \frac{N_o}{u^2} \cdot \frac{\tilde{E}^2}{E} \cdot \frac{M}{m} \cdot \sum_i \frac{Z^{*2}}{Z_f^2} \log u \frac{\hbar v}{E_i} \frac{mv^2}{E} \quad (6)$$

We next make the substitution $E = \frac{M}{m} \epsilon$ so that ϵ is the energy of an electron moving with the same velocity as the fission fragment and (6) becomes:

$$-\frac{dE}{dx} = \frac{\pi N_o Z^{*2} e^4}{E} \cdot \sum_i \log \frac{\epsilon}{Z^{*2} E_R} \frac{\epsilon^2}{E_i^2} \quad (7)$$

The sum is to be taken over all possible modes of excitation of the bound electrons in the atom where the energies involved are less than ϵ .

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The sum may be written $n(\epsilon) \log \frac{\epsilon}{Z^{1/2} E_R} \frac{\epsilon^2}{E^2}$ where $n(\epsilon)$ is the number of terms and $\bar{E}$ is the geometric mean of the excitation energies which are less than ϵ . For large Z , such as we have here, $n(\epsilon)$ and $\bar{E}$ can probably be estimated fairly closely by the use of the Thomas-Fermi model. $\bar{E}$, for example, could be found by considering the oscillations of an appropriate Fermi gas sphere after the manner of Bloch⁽¹⁾.

Here we will adopt a much simpler procedure and assume that:

(i) $n(\epsilon)$ = no. of electrons in the charge cloud in a Thomas-Fermi atom which are bound with energy less than ϵ and (ii) $E = E_R$, the Rydberg energy. This amounts to saying that all electrons with energies less than

ϵ can be excited and that the geometric mean of the possible excitation energies is E_R .

In the Thomas-Fermi model the potential is $V = Z e u \frac{e^{-x}}{x}$, and the charge density of electrons is $\rho = \frac{u^3}{4\pi} Z e \frac{e^{-x}}{x}$ where $x = ur$ and $u = 0.479 Z^{1/3}/a_0 = 0.905 Z^{1/3} \times 10^8 \text{ cm}^{-1}$. Now the fraction f of charge outside a distance x is:

$$f = \frac{\int_x^\infty \rho(x) d\tau}{Z e} = \int_x^\infty x e^{-x} dx = e^{-x} (x+1) \quad (8)$$

and the value of x at which the binding energy of an electron is ϵ is given by:

$$u Z e^2 \frac{e^{-x}}{x} = \epsilon \quad \text{or} \quad \frac{e^{-x}}{x} = \frac{\epsilon}{u Z e^2} \quad (9)$$

(1) Bloch, Zeits fur Phys. 81, 363, 1933

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It can be shown from (8) and (9) that f varies as $\epsilon^{\frac{1}{2}}$ for the values of ϵ of interest to us.

Then we may write:

$$f = f_0 \sqrt{\frac{\epsilon}{\epsilon_0}} \quad (1) \quad (2) \quad (10)$$

This is the same energy dependence of f as found by Bohr and by Lamb.

Eq. (10) may also be used to estimate the charge of the moving fission fragment for it will have a tendency to lose electrons which are bound with energy less than ϵ . Thus the effective charge is given by:

$$Z^{*2} = f_0^2 Z_f^2 \frac{\epsilon}{\epsilon_0} - - - - - (11)$$

The values of f_0 for U and A_g are: $f_u = 0.471$ and $f_{Ag} = 0.64$ for $\epsilon_0 = 500$ ev. Then $Z^{*2} = 0.41 Z_f^2 \frac{\epsilon}{\epsilon_0}$ and $n(\epsilon) = f_u Z_u \sqrt{\frac{\epsilon}{\epsilon_0}} = 0.471 Z_u \sqrt{\frac{\epsilon}{\epsilon_0}}$.

Experimental evidence⁽³⁾ on the charges of fission fragments gives values of f somewhat lower than the ones we have calculated here. On the basis of the experiments we will choose values of 0.47 for f_{Ag} and 0.35 for f_u . Substituting these into Eq. (7) we obtain:

$$-\frac{dE}{dx} / e = \frac{0.077 \pi N_0 Z_u Z_f^2 e^4}{\epsilon_0^{3/2}} \epsilon^{1/2} \log \frac{\epsilon_0 \epsilon^2}{0.22 Z_f^2 E_R^3} \quad (12)$$

Eq. (12) indicates that, except for the log term, the velocity loss of the particle is constant through out its range. This is found to be so experimentally to a very good approximation⁽¹⁾. Assuming the log

(1) Bohr, Phys. Rev. 59, 270, 1941

(2) Seitz, Irradiation Physics Notes

(3) Perfilov C.R. Acad. Sc. U.R.S.S. 28, 5, 1940

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term is constant we may calculate the range by integrating Eq.(12). We get:

$$R = \frac{2 \epsilon_o^{1/2}}{A \log \frac{\epsilon_o^3}{0.22 Z_f^2 E_R^3}} \quad (13)$$

$$\text{with } A = \frac{m}{M} \frac{0.077 \pi N_o Z_u Z_f^2 e^4}{\epsilon_o^{3/2}} = 2.34 \times 10^4 \text{ (e.v.)}^{1/2} \text{ cm}^{-1}$$

Putting in the value $\epsilon_o = 500 \text{ ev.}$ we find for the range of a typical fragment:

$$R = 4.3 \times 10^{-4} \text{ cm.}$$

C. Energy Losses Due to Collisions with Other Atoms

We will again use the Thomas-Fermi model.

The potential of a neutral atom is:

$$V = \frac{Ze}{r} e^{-ur} \quad (14)$$

The potential energy of a second atom, in the field of the first may be obtained as follows: The charge distribution which produces the potential (14) can be found from Poisson's equation: $\nabla^2 V = -4\pi\rho$. This gives $\rho = -\frac{u^2}{4\pi r} Ze e^{-ur}$. To this distribution we must add a point charge $+Ze$ at the origin. This and the charge density ρ then produce the potential of (14). The potential energy of this charge distribution in the field (14) is then given by (r - distance between centres):

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$$V = \int_{\tau_2} V_1 \rho_2 d\tau_2 = Z^2 e^2 u e^{-ur} \frac{1+ur}{ur} \quad (15)$$

for like atoms and by:

$$V = Z_1 Z_2 e^2 u e^{-ur} \frac{1+ur}{ur} \quad (16)$$

for different atoms if we assume that $u_1 \approx u_2 \approx u$.

The force between two atoms is obtained by differentiating (15):

$$F = - \frac{\partial V}{\partial r} = Z^2 e^2 u^2 e^{-ur} \frac{1+ur+(ur)^2}{(ur)^2} \quad (17)$$

The collision problem for two atoms may now be solved by classical methods. The exact solution cannot be obtained without untabulated functions but we can get a usable approximate solution easily in the following way: if we set $ur = p$, the potential is: $V = Z^2 e^2 u e^{-p} \frac{1+p}{p}$. Now the function of p behaves as e^{-p} for $p \gg 1$, and as $\frac{1+p}{p}$ for $p \ll 1$. We may divide the problem into two parts, for $p < 1$, we write $V = \frac{Z^2 e^2 u}{p}$ and for $p > 1$, $V = Z^2 e^2 u e^{-p}$.

1) $p < 1$. This is the case of Rutherford scattering. Here we have the equation:

$$- \frac{dE}{dx} = \frac{\pi N_0 z^2 Z^2 e^4}{E} \frac{M}{m} \log \left(1 + \frac{4E^2}{z^2 Z^2 e^4} \frac{\mu^2}{M^2} b^2 \right) \quad (18)$$

for the energy loss due to impacts with impact parameters from 0 to b .

z and m are the charge and mass of the struck atom: Z , M , E , are the charge, mass and energy of the moving particle. μ is the reduced mass.

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We will assume that this equation gives the correct energy loss for values of b up to $p = 1$, i.e. $b = \frac{1}{u}$.

ii) $p > 1$. For impact parameters larger than $\frac{1}{u}$ we use the potential $V = Z z e^2 e^{-p}$. In this case the energy loss equation becomes:

$$-\frac{dE}{dx} = \frac{\pi N_0 z^2 Z^2 e^4}{E} \frac{M}{m} (0.22) \quad (19)$$

The Eq. (19) is obtained as follows: the passing particle exerts a force $z Z e^2 u^2 e^{-ur}$ on the particle at rest for a time of the order of $\sqrt{2ur+1}/uv$ and so the energy lost is of the order of $\frac{(\text{force})^2 (\text{time})^2}{(2 \times \text{mass})} = \frac{u^4 z^2 Z^2 e^4 e^{-2ur} (2ur+1)}{2m \cdot u^2 v^2}$ and the energy loss for impact parameters from $1/u$ to ∞ is then

$$-\frac{dE}{dx} = \frac{2\pi N_0 z^2 Z^2 e^4 u^4}{2m u^2 v^2} \int_{1/u}^{\infty} e^{-2ur} (2ur+1) r dr$$

The value of the integral is $0.44/u^2$. Thus we find:

$$-\frac{dE}{dx} = \frac{\pi N_0 z^2 Z^2 e^4}{E} \frac{M}{m} (0.22) \quad (19')$$

Adding (18) and (19') we find for the total energy loss due to collisions:

$$-\frac{dE}{dx} \Big|_c = \frac{\pi N_0 z^2 Z^2 e^4}{E} \cdot \frac{M}{m} \cdot \log \left\{ 1.25 + \frac{5E^2}{z^2 Z^2 e^4 u^2} \frac{M^2}{m^2} \right\} \quad (20)$$

This expression is valid when $E > \tilde{E}$ and so can be used when the energy is high enough so that the nuclei penetrate through most of each others electron clouds. At these energies most of the energy is lost in the close collisions ($p < 1$); the contribution from the distant collisions ($p > 1$) is

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small on account of the exponential falling-off of the force. We will use Eq. (20) for fission fragments and for recoil atoms of sufficiently high energy.

For low energies, $E < \tilde{E}$, there will be little interpenetration of the colliding atoms and so no possibility of the intense Coulomb interaction between the nuclei. Here the collisions will be like those between rigid spheres. We will calculate the energy losses here by an impact parameter method. The energy lost to a struck atom for a given impact parameter b is some function of b depending upon the nature of the interaction. Up to some value of b , b_1 say, we will assume rigid sphere interaction. For $b > b_1$ we will use the exponential interaction. A suitable value of b_1 will be chosen later.

For $b < b_1$ we have for the energy loss per collision:

$$\Delta E = 4E \frac{\mu^2}{Mm} \left(1 - \frac{b^2}{b_1^2}\right)$$

where E is energy of the moving atom, M is the mass of the moving atom, m is the mass of the recoil atom, μ the reduced mass and b_1 the diameter of the atoms. The energy loss per cm. of path then becomes:

$$-\frac{dE}{dx} = 2\pi N_0 \int_0^{b_1} \Delta E b db = 2\pi N_0 E b_1^2 \frac{\mu^2}{Mm} \quad \dots (21)$$

When $b > b_1$ the energy loss is given by an expression like Eq. (19), the interaction being the exponential force. Thus we get:

$$-\frac{dE}{dx} = \frac{\pi N_0 z^2 Z^2 e^4}{2E} \frac{M}{4m} e^{-2ub_1} \left((2ub_1)^2 + 6ub_1 + 3 \right) \quad (22)$$

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We have next to choose a suitable value of b_1 . Our choice here will be the distance of nearest approach of the two nuclei. The reasons for this are that for $b < b_1$ the collision will certainly be a rigid sphere affair, and since, as we will see, this is the main contribution, the end result will not depend much on the interaction chosen for $b > b_1$. b_1 is given by:

$$\frac{E}{2} = z Z e^2 u e^{-ub_1}; \quad ub_1 = \log \frac{\tilde{2E}}{E}; \quad \tilde{E} = z Z e^2 u$$

Putting in this value and adding (21) and (22) we get for the total energy loss:

$$-\frac{dE}{dx} = \frac{\pi N_0}{8u^2} \cdot E \cdot \frac{M}{m} \left\{ \left(\frac{16u^2}{M^2} + 1 \right) \log^2 \frac{\tilde{2E}}{E} + \frac{3}{2} \log \frac{\tilde{2E}}{E} + \frac{3}{4} \right\} \quad (23)$$

For sufficiently low energies, $E < 10^4$ ev. in uranium, for example, we may drop the last two terms in (23) and get:

$$-\frac{dE}{dx} = \frac{\pi N_0}{8u^2} E \frac{M}{m} \left(\frac{16u^2}{M^2} + 1 \right) \log^2 \frac{\tilde{2E}}{E} \quad (24)$$

This may be integrated to give the range, R:

$$R = \int_0^R dx = \frac{8u^2}{\pi N_0} \frac{M}{m} \frac{M^2}{16u^2 + M^2} \frac{1}{\log \frac{\tilde{2E}}{E}} \quad (25)$$

Eq. (25) will be used later to find the ranges of the recoil uranium atoms.

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D. Ratio of Electron Losses to Collision Losses

We have obtained in sections B and C estimates for the rates of energy loss by a fission fragment to electrons and to atoms. These are equations (12) and (20):

$$-\left. \frac{dE}{dx} \right|_e = \frac{0.077 \pi N_o Z_u Z_f^2 e^4}{E_o^{3/2}} E^{1/2} \log \frac{E_o E^2}{0.22 Z_f^2 E_R^3} \quad (12)$$

$$-\left. \frac{dE}{dx} \right|_c = \frac{\pi N_o Z_f^2 e^4 z^2}{E} \log \frac{2}{3} \frac{E}{E} \quad (21)$$

In Eq. (21) we have substituted values for $\frac{M}{m}$ and $\frac{h}{m}$ from the Eq. (20) $M \approx \frac{m}{2}$ and have ignored the losses due to distant collisions ($p > \frac{1}{u}$) since these are small for high energies. Eqs. (12) and (21) are valid in the initial part of the range, from 100 Mev. to about 1 Mev.

We will now calculate an expression for the ratio $\frac{E_e}{E_c}$ where E_e is the energy lost to electrons, E_c the energy lost to atoms. We have seen that the range is given very closely by (12) if we consider the log term to be constant. Then we may find an expression for the range in terms of energy from (12) and use it to calculate E_c from (21). From (12) then, we get:

$$\text{distance travelled} = x = \frac{2}{B} (\sqrt{E_o} - \sqrt{E})$$

$$\text{where } B = A \log \frac{E_o^3}{0.22 Z_f^2 E_R^3} = 1.09 \times 10^5 (\text{ev.})^{1/2} (\text{cm})^{-1}$$

and the energy lost in collisions down to 1 Mev. is:

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$$\begin{aligned}
 E_c &= \int_{E_0}^E dE_c = \int_E^{E_0} \frac{dE}{dx} \bigg|_c dx = \int_E^{E_0} \frac{C}{E} \log \left(\frac{2}{3} \frac{E}{\tilde{E}} \right) \cdot \frac{1}{B} \sqrt{\frac{m}{M}} \frac{dE}{E^{1/2}} \\
 &= \sqrt{\frac{m}{M}} \frac{C}{B} \int_E^{E_0} \frac{-2d(E^{-1/2})}{-2 \log \left(\frac{2}{3} \frac{E}{\tilde{E}} \right)^{1/2}} = \sqrt{\frac{m}{M} \cdot \frac{2}{3\tilde{E}}} \frac{C}{B} \int_x^{x_0} \frac{dx}{\log x} \\
 &= \sqrt{\frac{m}{M} \cdot \frac{2}{3\tilde{E}}} \frac{C}{B} (E_i(\log x_0) - E_i(\log x)); \quad - \quad - \quad - \quad - \quad (22) \\
 x &= \left(\frac{3}{2} \frac{\tilde{E}}{E} \right)^{1/2}
 \end{aligned}$$

In the above, $C = \pi N_0 Z_f^2 e^4 z^2 = 5.83 \times 10^{16} (\text{ev.})^2 (\text{cm})^{-1}$ and $\tilde{E} = z Z e^2 u = 2.59 \times 10^5 \text{ ev} = \text{energy required to approach to a distance of } 1/u.$

From Eq. (22) we find that the energy lost in collisions amounts to

$$E_c = 4.96 \text{ Mev.}$$

as the fragment slows down from 100 Mev. to 1 Mev.

Below 1 Mev. the electron losses will be very small since the equivalent electron energy of the fragment is only about 5 ev.. We will assume that about half the energy is lost to electrons and half to collisions with other atoms. Then we get $E_c = 5.5 \text{ Mev.}$ and $E_e = 94.5 \text{ Mev.}$

III. THE EFFECTS OF FISSION FRAGMENTS ON THE MEDIUM

We have calculated the energy losses and range of fission fragments and saw that about 5½% of the energy was given to other atoms and the rest to electrons. Our next problem is to consider the motion of the struck atoms and electrons. As we shall see the struck atoms constitute a matter current, isotropic and proportional to the number of fissions per unit

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volume. The electrons, on the other hand move out as a high temperature wave, losing energy to the medium. Here we will be interested mainly in the exchange of energy between these electrons and the medium. We consider the motion of the struck atoms first.

A. The Motion of Atoms Struck by Fission Fragments

The fragment leaves behind it a trail of recoil atoms with certain ranges and energies. These recoil atoms may have enough energy to produce secondary recoils and these in turn may produce tertiary recoils. The atoms of the medium are bound in lattice positions with some energy E_d . Following Seitz⁽¹⁾ we will assume that $E_d = 25$ ev. In finding the number of displaced atoms we must ignore all collisions in which the energy exchange is less than E_d .

The energy of a recoil atom of impact parameter b is $\Delta E = 4E \frac{\mu^2}{Mm} \cos^2 \vartheta$. Averaging this over all impact parameters we find

$$\langle E \rangle = \frac{\int_0^{b_1} 2\pi \Delta E b db}{\int_0^{b_1} 2\pi b db} \approx \frac{2M}{m} \frac{E^2}{E} \log \frac{4}{M} \frac{E}{E} - - - (23)$$

where E = energy of fragment, $E = zZe^2u$. This result looks strange since it depends on $\frac{1}{E}$. One must remember, however, that ϑ depends on $\frac{1}{E}$ and it is this dependence that leads to our final result. We should also average $\langle E \rangle$ over the range from the maximum energy to about 1 Mev. This averaging contributes a factor of about 1/2. We have then, for the average energy of a fission recoil the expression:

(1) Irradiation Physics Notes.

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$$\langle E \rangle = \frac{M}{m} \cdot \frac{\tilde{E}^2}{E} \log \frac{\tilde{E}}{M \frac{E}{m}} - - - - (24)$$

For a fragment (Ag) of $Z = 47$, $M = 108$ moving in uranium, $z = 92$, $m = 238$, we have $\tilde{E} = 2.59 \times 10^5$ ev. Then we get for $\langle E \rangle$:

$$\langle E \rangle = \frac{108}{238} \cdot \frac{6.71 \times 10^{10}}{10^8} \log \left(\frac{10^8}{2.59 \times 10^5} \cdot \frac{2}{3} \right) = 1690 \text{ ev.} \quad (25)$$

The lowest energy that a recoil can have may be found by substituting $b = \frac{1}{u}$ in the equation $\Delta E = 4E \frac{z^2}{Mm} \cos^2 \vartheta$. This gives $\Delta E = 390$ ev. and shows that all primary recoils have enough energy to be displaced.

Since all primary recoils get more than 25 ev. of energy we may find the total number of these by dividing $\langle E \rangle$ from (25) into E_c , the total energy lost in collisions.

Assuming that the initial energy of each fragment is 80 Mev., we find
for the total number of primary recoils:

$$N_1 = \frac{160 \times 10^6 \times 0.055}{1690} = 5.2 \times 10^3$$

In section II, part C we developed a collision theory for slow atoms. There we saw that, for $E < 10^4$ ev. in uranium, the energy losses are given by Eq. (24) and the range by Eq. (25). In this region of energy there are no electronic losses so that Eq. (24) represents the total energy loss.

We consider now the production of secondary and tertiary recoils on the basis of the above equations. The energy given to an atom at rest by one moving with an energy E is:

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$$\Delta E = 4E \frac{\mu^2}{Mm} \left(1 - \frac{b^2}{b_1^2} \right)$$

for an impact parameter b . The average energy of those recoils whose energy is greater than E_d is:

$$\langle E \rangle = \frac{\frac{4}{9} (E - E_d) - \frac{E_d}{2} \log \frac{E}{E_d}}{\log \frac{E}{E_d}} \quad - - - \quad (30)$$

In obtaining (30) we have averaged over all impact parameters involved and also over the range of the moving atom assuming that the collision losses depend only on E . (That this is a good approximation can be seen from Eq. (24):

The fraction of the energy lost in encounters where the energy exchange exceeds E_d is $\left(1 - \frac{9}{8} \frac{E_d}{E} \right)$, and if this is averaged over the range from E to E_d , again assuming that $\left. \frac{dE}{dx} \right|_c \sim E$, we find the fraction:

$$1 - \frac{\frac{9}{8} E_d \left(\frac{1}{E_d} - \frac{1}{E} \right)}{\log \frac{E}{E_d}} \quad - - - \quad (31)$$

From Eq. (25) we get the following expression for the range, setting

$m = M$:

$$R = \frac{8}{5} \frac{u^2}{N_0} \frac{1}{\log \frac{2E}{E}} \quad - - - \quad (32)$$

We are now in a position to describe the recoils (primary, secondary, and tertiary) of the fission fragment as to number, average energy, and average range.

We have already seen that there are $N_1 = 5.2 \times 10^3$ primary recoils with average energy $\langle E_1 \rangle = 1690$ ev.

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Their average range is:

$$\langle R_1 \rangle = \frac{8}{5} \cdot \frac{u^2}{\pi N_0} \cdot \frac{1}{\log \frac{2E}{\langle E_1 \rangle}} = 2.8 \times 10^{-7} \text{ cm.}$$

The average energy of the secondary recoils is:

$$\begin{aligned} \langle E_2 \rangle &= \frac{\frac{4}{9} (\langle E_1 \rangle - E_d) - \frac{E_d}{2} \log \frac{\langle E_1 \rangle}{E_d}}{\log \frac{\langle E_1 \rangle}{E_d}} \\ &= 163 \text{ ev.} \end{aligned}$$

and so their average range, $\langle R_2 \rangle$ is:

$$\langle R_2 \rangle = \frac{8}{5} \frac{u^2}{\pi N_0} \frac{1}{\log \frac{2E}{\langle E_2 \rangle}} = 2.03 \times 10^{-7} \text{ cm.}$$

and their number N_2 is

$$N_2 = \left\{ 1 - \frac{\frac{9}{8} E_d \left(\frac{1}{E_d} - \frac{1}{\langle E_1 \rangle} \right)}{\log \frac{\langle E_1 \rangle}{E_d}} \right\} \frac{\langle E_1 \rangle}{\langle E_2 \rangle} N_1 = 3.9 \times 10^4.$$

The average energy of the tertiary recoils is:

$$\langle E_3 \rangle = \frac{\frac{4}{9} (\langle E_2 \rangle - E_d) - \frac{E_d}{2} \log \frac{\langle E_2 \rangle}{E_d}}{\log \frac{\langle E_2 \rangle}{E_d}} = 20$$

Assuming that the fraction of atoms displaced is given by an expression of the form $(1 - e^{-E/E_d})$ we find for the number of tertiary recoils, N_3 :

$$\begin{aligned} N_3 &= \left\{ 1 - \frac{\frac{9}{8} E_d \left(\frac{1}{E_d} - \frac{1}{\langle E_2 \rangle} \right)}{\log \frac{\langle E_2 \rangle}{E_d}} \right\} \frac{\langle E_2 \rangle}{\langle E_3 \rangle} N_2 (1 - e^{-\frac{\langle E_3 \rangle}{E_d}}) \\ &= 6.2 \times 10^4 \end{aligned}$$

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The average range of the tertiary recoils is:

$$\langle R_3 \rangle = \frac{8}{5} \frac{u^2}{\pi N_0} \frac{1}{\log \frac{2E}{\langle E_3 \rangle}} = 1.63 \times 10^{-7} \text{ cm.}$$

In the table below we collect together the numbers, average energies, and ranges of the various recoils produced by a pair of fission fragments:

	No.	Av. E	Av. R.
primary recoils	5.2×10^3	1690	2.8×10^{-7}
secondary recoils	3.93×10^4	163	2.03×10^{-7}
tertiary recoils	6.2×10^4	20	1.63×10^{-7}

The ensemble of moving atoms may be conveniently represented as an isotropic matter current. Thus if the number of fissions per cm^3 per sec. is n , the current of atoms crossing a cm^2 in one second is:

$$J = \frac{n}{3} (N_1 \langle R_1 \rangle + N_2 \langle R_2 \rangle + N_3 \langle R_3 \rangle)$$
$$= 6.5 n \times 10^{-3} \text{ atoms per cm}^2 \text{ per second.}$$

B. The Motion of Electrons Struck by Fission Fragments

This problem is more difficult to treat in any detailed manner than the corresponding one for struck atoms. The reason for this is that neither classical methods nor the Born Approximation apply at the energies involved so that no simple quantum solution can be found. Moreover, when the energies of the struck electrons fall to a few ev., then the problem becomes a many body problem, and consequently more

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difficult to solve. The following discussion is mainly qualitative and attempts to answer two main questions: i) do the electrons produce atomic displacements?, and ii) how does the energy of the struck electrons spread to the rest of the metal?

We calculate first the average energy of the struck electrons. The energy of an electron with impact parameter b is

$$\epsilon = 4 \epsilon_0 \frac{1}{1 + \frac{4b^2 \epsilon_0^2}{Z^{*2} e^4}} \quad - - - (33)$$

where $\epsilon_0 = \frac{m}{M} E$ and E = energy of the fragment. The average overall impact parameters is

$$\bar{\epsilon} = \frac{\int_0^{b_{\max}} 2\pi b \epsilon db}{\int_0^{b_{\max}} 2\pi b db} = \frac{Z^{*2} e^4}{\epsilon_0 b_{\max}^2} \log \left(1 + \frac{4 b_{\max}^2 \epsilon_0^2}{Z^{*2} e^4} \right) \quad - - - (34)$$

Substituting values for Z^{*2} , ϵ_0 , $b_{\max}$, we find

$$\bar{\epsilon} = \frac{5 \times 10^3}{\epsilon_0} \log \left(1 + \frac{\epsilon_0^2}{10^3} \right) \text{ ev.} \quad - - - (35)$$

This gives $\bar{\epsilon} = 63.5$ for $\epsilon_0 = 400$ ev. and $\bar{\epsilon} = 120$ ev. for $\epsilon_0 = 40$ ev. $\bar{\epsilon}$ therefore does not vary wildly over this range of the fragment and may be represented approximately by:

$$\bar{\epsilon} = -\frac{1}{6} \epsilon_0 + 126 \text{ e.v.} \quad - - - (36)$$

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We limit ourselves to the part of the range given above, i.e. 400 to 40 ev. since the classical theory begins breaking down at the lower energies. The neglected part of the range will not make any appreciable difference in the conclusions we will draw. Averaging $\bar{E}$ over this part of the range, we

have then:

$$\langle E \rangle = \frac{\int_{\epsilon_0'}^{\epsilon_0} n(x) \bar{E}(x) dx}{\int_{\epsilon_0'}^{\epsilon_0} n(x) dx} = \frac{\int_{\epsilon_0'}^{\epsilon_0} \frac{dE}{dx} \bigg|_e dx}{\int_{\epsilon_0'}^{\epsilon_0} \frac{1}{\bar{E}(x)} \frac{dE}{dx} \bigg|_e dx} =$$

$$\frac{\int_{\epsilon_0'}^{\epsilon_0} d\epsilon_0}{\int_{\epsilon_0'}^{\epsilon_0} \frac{d\epsilon_0}{\bar{E}}} = \frac{\epsilon_0 - \epsilon_0'}{6 \log \frac{\bar{E}}{\epsilon_0'}} \approx 90 \text{ e.v.} \quad (37)$$

In the above $n(x)$ = no. of struck electrons/cm. Assuming the value of 90 e.v. for the energy of the electrons we now calculate the range. The electrons are scattered by both the lattice atoms and by the free electron gas of the metal. However, the energy losses to the lattice will be negligible in the range from 90 to about 10 e.v. The energy exchange between an atom and the electron is $h\nu^{(1)}$, the maximum value of $h\nu$ being about 10^{-2} e.v. However, the electron can give up large fractions of its energy to the free electrons in the metal. As its energy goes below about 10 e.v., the collisions with the free electrons are restricted by the Fermi statistics so that the scattering by the lattice eventually becomes the dominant mode of energy loss. On the basis of the above reasoning we will adopt

(1) see, for example, Mott and Jones, Theory of Metals and Alloys, Oxford University Press

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the following simplified picture: the electron will lose all of its energy to the free electrons from 90 e.v. to 10 e.v. Below 10 e.v. the losses will be mainly to the lattice atoms.

The energies of the free electron gas are distributed according to the Fermi statistics, at ordinary temperatures. According to quantum mechanics, the number of free states in the momentum range p to $p + dp$ in a volume Ω is $\frac{4\pi p^2 dp \Omega}{h^3}$ and the total number up to a momentum p is $\frac{4\pi}{3} p^3 \cdot \frac{\Omega}{h^3}$. If there are N electrons in the volume, they will occupy the lowest $N/2$ states at absolute zero so that all electrons will lie in a sphere in momentum space of radius p_m where

$$p_m = \left(\frac{3}{4\pi}\right)^{1/3} N_0^{1/3} \pi h; \quad N_0 = \frac{N}{2} \quad - \quad - \quad - \quad (38)$$

This distribution will not change much at ordinary temperatures.

We will now calculate the energy loss using the Rutherford scattering formula and the Fermi statistics. This is given by an expression of the form:

$$-\frac{dE}{dx} \Big|_e = \frac{2\pi N_0 e^4}{E} \int_0^{2p_0} \frac{dp}{p} \quad - \quad - \quad - \quad (39)$$

As usual, we get an infinite result at the lower limit but are saved from this by the Fermi statistics. Only those collisions are possible in which the momentum vectors of the struck electrons fall outside the sphere defined by (38). Obviously, if the momentum exchange is $2p_m$ or more, all collisions are possible. For momentum exchanges between $2p_m$ and 0, however, a certain fraction H , say, are forbidden by the statistics. A

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straightforward, but rather tedious calculation gives the following result for the fraction of collisions that are possible:

$$H_1 = \frac{7x^3 - 12x + 32 - 32(1-x)^{3/2}}{32}; \quad x = \frac{p}{p_m} \quad 0 < p < p_m$$

$$H_2 = \frac{16 + 12x - x^3}{32}; \quad x = \frac{p}{p_m} \quad p_m < p < 2p_m$$

Putting these expressions into (39) we have:

$$-\frac{d\epsilon}{dx}\bigg|_e = \frac{2\pi N_o e^4}{\epsilon} \left\{ \int_0^{p_m} H_1 \frac{dp}{p} + \int_{p_m}^{2p_m} H_2 \frac{dp}{p} + \int_{2p_m}^{2p_o} \frac{dp}{p} \right\} \dots \quad (40)$$

which, on integration, becomes

$$-\frac{d\epsilon}{dx}\bigg|_e = \frac{2\pi N_o e^4}{\epsilon} \left(\log \frac{p_o}{p_m} + 2.55 \right) - \dots - \dots - \dots \quad (41)$$

or, in terms of ϵ_m , the energy corresponding to p_m : $\epsilon_m = \frac{h^2}{2m} \left(\frac{3N_o}{8\pi} \right)^{2/3}$

$$-\frac{d\epsilon}{dx}\bigg|_e = \frac{\pi N_o e^4}{\epsilon} \left(\log \frac{\epsilon}{\epsilon_m} + 5.1 \right) - \dots - \dots - \dots \quad (42)$$

In the above, the values of ϵ_m and p_m have been calculated for uranium assuming that there is one free electron per atom. This gives about 5 e.v. for ϵ_m .

On this model, the range is:

$$R = \frac{1}{\pi N_o e^4} \int_{\epsilon_1}^{\epsilon_2} \frac{\epsilon d\epsilon}{\log \frac{\epsilon}{\epsilon_m} + 5.1} - \dots - \dots - \dots \quad (43)$$

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$$\begin{aligned} &= \frac{1}{\pi N_0 e^4} \left(\frac{\epsilon_m}{164} \right)^2 \left\{ \overline{E_i} \left(2 \log \frac{164 \epsilon_2}{\epsilon_m} \right) - \overline{E_i} \left(2 \log \frac{164 \epsilon_1}{\epsilon_m} \right) \right\} \\ &\approx \frac{1}{\pi N_0 e^4} \left(\frac{\epsilon_m}{164} \right)^2 \left(\frac{164 \epsilon_2}{\epsilon_m} \right)^2 \frac{1}{2 \log \frac{164 \epsilon_2}{\epsilon_m}} \\ &= 1.69 \times 10^{-7} \text{ cm.} \end{aligned}$$

It is probably correct to calculate the total energy losses as we have done using the Rutherford formula and the Fermi Statistics. However, it would not be safe to use this method for such things as the energy distribution of the struck electrons or the average angle of scattering. More detailed quantum mechanical investigation is necessary to estimate these quantities. Quantum methods, of course, also lead to the Rutherford formula. The differences would arise in the cutoff procedure. This would not affect the total energy loss but would certainly change the energy distribution of the recoil electrons. Our model will suffice for the discussion to follow.

The track of the fission fragment is surrounded by a cylindrical cloud of electrons of average energy equal to about 90 e.v. The radius of this cloud is given by $b_{\max} \approx \frac{\kappa v}{E_R} = 2.9 \sqrt{\epsilon_0} \times 10^{-8} \text{ cm.}$, and the average over the range $\epsilon_0 = 400$ to $\epsilon_0 = 40$ is $3.8 \times 10^{-7} \text{ cm.}$ The electrons will move at right angles to the track of the fragment a distance of the order of 10^{-7} cm. given by Eq. (43). By then the average energy has fallen to 10 e.v. and the radius of the cylinder has increased to about $5 \times 10^{-7} \text{ cm.}$ Also, since the primary electrons have lost energy mainly to the free electron gas, there will be a multiplication of

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electrons. Now our model does not allow us to calculate either the energy distribution in or the number of these secondary and possibly tertiary electrons. For our purpose, however, it will be correct enough to assume that all the free electrons in a cylinder of radius 5×10^{-7} cm. have been excited to about 10 e.v. We will see later why this is justified.

We cannot treat this cloud in any detail. The energies of the electrons have become of the order of the energies of the free electron gas in the metal, thus it is a many body problem. Strictly speaking, we must now consider wave functions for the entire piece of uranium including the excited cloud. Such a procedure, however, is not feasible at the present state of quantum mechanics. We will content ourselves here with an approximate heat diffusion method. (1) But first we give a short description of the electrons in a metal.

The electrons in a metal may be classified as follows. First, those that are bound in states so stable that, when the atoms are brought together to form the solid, these states are only slightly broadened and remain bound states. These will be at least all the electrons belonging to the free atom except for those in the outermost or valence shell. As far as the properties of the solid are concerned, they may be considered together with the nucleus as one particle, just as, for example, we consider the nucleus as one particle in spectroscopic problems, though in fact it consists of many protons and neutrons. Secondly, there are those electrons (in the valence shell) which are so loosely bound in the free atom that when the atoms are brought together in the solid, the

(1) Suggested by H. Brooks

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influence of the atomic potentials on one another broadens their states to such an extent that the state of any one electron is not at all one of stationary binding but is rather a free state. In this way the entire metal becomes accessible to these electrons and they may be pictured as having states belonging to the whole metal rather than to any individual atom. (Classically, the picture is that the outer electrons are freed by the lowering of the potential barrier between two nuclei when they are brought together).

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In the theory of solids it is shown how these free electrons account for the electrical and thermal conductivities of metals. It is found that the motion of electrons is restricted only by the irregularities of the lattice caused either by thermal agitation or by the presence of impurities. Our picture will be based on this. We will assume that the electrons are decelerated in their passage through the lattice in the same way that conduction electrons are. This procedure has one obvious defect and it is this. In thermal conduction we know that an equilibrium state is established so that there is a variation of the Fermi distribution of the electrons with temperature gradient. The question is, how is this set up if electron interactions are neglected? We will look a little further into this question later and see just how much it could affect our present considerations.

(1) see, for e.g. Mott and Jones: loc cit.

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Let ψ , ϕ be the temperatures of the electrons and lattice respectively. Then the diffusion equations for ψ and ϕ are:

$$\nabla^2 \psi = H_1 \frac{\partial \psi}{\partial t} + \frac{q}{\rho_1 c_1} (\psi - \phi) \quad (44)$$

$$\nabla^2 \phi = \frac{1}{H_2} \frac{\partial \phi}{\partial t} + \frac{q}{\rho_2 c_2} (\phi - \psi) \quad (45)$$

where $H_1 = \frac{k_1}{\rho_1 c_1}$, $H_2 = \frac{k_2}{\rho_2 c_2}$ are the diffusivities of electrons and lattice, and q is a coupling constant representing the heat flow from electrons to lattice. Initially, the temperature of the electrons is given by:

$$\psi_0 = \frac{10 \text{ e.v.}}{\frac{3}{2} k} = \frac{10}{\frac{3}{2} \times 862} \times 10^4 \sim .8 \times 10^5 \text{ deg.}$$

At this temperature the electron gas is not degenerate and we may use classical statistics to find H_1 . Thus $c_1 = \frac{3}{2} k T N_0$. For k_1 we use the value given by experiment, assuming that all the heat is conducted by electrons. Then $H_1 \approx 4 \text{ cm}^2/\text{sec}$. k_2 is much smaller than k_1 since we are assuming that lattice conduction is negligible compared to electron conduction and so H_2 will be of the order of $0.01 H_1$. We will assume that initially the term $q(\psi - \phi)$ in Eq. (44) is negligible. This will be a good approximation since energy losses to the lattice, as we have seen are small, being of the order of $.02 \text{ e.v.}$ per collision. Then, assuming cylindrical geometry and an δ function for ψ at $t = 0$, the solution of (44) is:

$$\psi = \frac{u}{4\pi^{3/2} \rho_1 c_1 H_1 t} e^{-r^2/4H_1 t} \quad (46)$$

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where u is the total electron heat. This is a gaussian function which spreads with time. To solve (45) we will neglect the term $\nabla^2 \phi$ and use (46). Then the solution is;

$$\phi = \frac{H_2}{H_1} q \frac{u}{4\pi \rho_1 c_1} e^{-H_2 q t} \cdot \int e^{H_2 q t - r^2/4H_1 t} \frac{dt}{t} \dots (47)$$

For small t , the term $e^{-H_2 q t}$ will be nearly unity and ϕ is then given by:

$$\phi_{t \rightarrow 0} \sim -\frac{H_2}{H_1} q \frac{u}{4\pi \rho_1 c_1} \text{Ei} \left(-\frac{r^2}{4H_1 t} \right) \dots (48)$$

Thus for small t and $r^2 \sim 4H_1 t$, the ratio of the temperatures is:

$$\frac{\phi}{\psi} \sim H_2 q t \dots (49)$$

a result which shows that ϕ is much smaller than ψ initially. We can see in a qualitative way that ϕ will not be an appreciable fraction of ψ in the times when ψ has its large initial values. Thus we can conclude that the temperature of the lattice nowhere rises sufficiently high to produce displacements of atoms; this conclusion would hold even with fairly large changes in our choice of constants.

It would be more exact to solve Eq. (44) with the condition that $\psi = \psi_0$ for $r < 5 \times 10^{-7}$ cm and $\psi = 0$ for $r > 5 \times 10^{-7}$ cm at $t = 0$. Then the solution is

$$\psi = \int_0^\infty g(k) J_0(kr) e^{-H_1 k^2 t} k dk$$

with $g(k) = \int_0^{r=5 \times 10^{-7} \text{ cm}} \psi_0 J_0(kr) r dr.$

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However, the conclusions made above would remain the same with this more accurate solution.

We will use the diffusion method no further. Finally we estimate the distance over which the 10 e.v. electrons travel before they come to equilibrium with the electron gas. That is, we will calculate the distance they travel as their energy falls from 10 to about 5 ($= \epsilon_m$) e.v.

In the theory of metallic conduction⁽¹⁾, there is calculated a relaxation time T , which represents the mean time an electron accelerated by a field, will travel before losing all the momentum gained from the field through collisions with lattice atoms. T is related to the mean free path and the conductivity by the equation:

$$l = \frac{Ne^2}{m} p \sigma \quad (50)$$

where p is the momentum of the electron. Using Eq. (50) we find a value for l of about 10^{-6} cm. Thus the energy of the electrons is given up to the lattice in a distance of the order of 10^{-6} cm.

(1) Mott and Jones, Theory of Metals and Alloys, Chap. VII

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Atomic Displacements Produced by Fission Neutrons

The principal mode by which fission neutrons lose energy is elastic scattering. For energies, such as we consider here, 2 Mev. and lower - the scattering in the barycentric system of co-ordinates is spherically symmetric. This is because the wave length, $\lambda = h/p$ of the neutron is somewhat smaller than the dimensions of the nuclear potential well so that only the $l=0$ or s-wave is scattered.

We consider first some features of an elastic collision of a neutron of energy E with an atom of mass A at rest. In the centre of mass system the velocities of neutron and atom are $\frac{Av}{1+A}$ and $\frac{v}{1+A}$. In this system the velocities change only in direction after a collision so that if the atom is deflected through an angle ϑ' in this system, it will be deflected through an angle ϑ , where

$$\vartheta = \frac{\pi}{2} - \frac{\vartheta'}{2} \quad (51)$$

in the laboratory system. This may be seen easily by adding vectorially the velocity of the centre of mass to the velocity of the atom in the centre of mass co-ordinates. From (51) we see that all the recoil atoms go into the forward hemisphere. The velocity of a recoil at angle ϑ in the laboratory system is $u = \frac{2v}{1+A} \cos \vartheta$ and so the energy is

$$\epsilon = \frac{4EA}{(1+A)^2} \cos^2 \vartheta \quad (52)$$

We will assume that the atoms are held in stable positions with an energy $E_d = 25$ e.v. The average value of ϵ , $\bar{\epsilon}$, averaged over collisions in which $\epsilon > E_d$ is given by:

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$$\bar{E} = 4E \frac{A}{(1+A)^2}$$

$$\frac{\int_0^{\vartheta_1} \cos^2 \vartheta \sigma(\vartheta) 2\pi \sin \vartheta d\vartheta}{\int_0^{\vartheta_1} \sigma(\vartheta) 2\pi \sin \vartheta d\vartheta}$$

where ϑ_1 is given by $\cos^2 \vartheta_1 = \frac{E_d}{4E} \frac{(1+A)^2}{A}$. From the definition of the total cross-section we have

$$\sigma(\vartheta) 2\pi \sin \vartheta d\vartheta = \sigma(\vartheta') 2\pi \sin \vartheta' d\vartheta'$$

where the primes refer to centre of mass co-ordinates. Hence

$$\begin{aligned} \bar{E} &= 4E \frac{A}{(1+A)^2} \frac{\int_0^{\vartheta_1'} \sin^2 \frac{\vartheta'}{2} \sigma(\vartheta') 2\pi \sin \vartheta' d\vartheta'}{\int_0^{\vartheta_1'} \sigma(\vartheta') 2\pi \sin \vartheta' d\vartheta'} \\ &= 4E \frac{A}{(1+A)^2} \frac{\int_{x_1}^1 x^3 dx}{\int_1^{x_1} x dx} = 2E \frac{A}{(1+A)^2} \left(1 + \frac{E_d}{4E} \frac{(1+A)^2}{A}\right) \end{aligned} \quad (53)$$

since the cross-section, $\sigma(\vartheta')$ is constant in centre of mass co-ordinates. In the above, $\vartheta_1' = 2 \sin^{-1} \sqrt{\frac{E_d(1+A)^2}{4E A}}$; $x_1 = \sin \frac{\vartheta_1'}{2}$. In terms of the maximum value of E , $E_{\max}$, we have

$$\bar{E} = \frac{1}{2} E_{\max} + \frac{1}{2} E_d \quad (54)$$

The fraction $f = \frac{\text{energy loss to displaced atoms}}{\text{energy loss to all atoms}}$

is obviously given by:

$$f = \frac{\int_0^{\vartheta_1} \cos^2 \vartheta \sin \vartheta d\vartheta}{\int_0^{\pi/2} \cos^2 \vartheta \sin \vartheta d\vartheta} = \frac{\int_{x_1}^1 x^3 dx}{\int_0^1 x^3 dx} = 1 - \frac{E_d}{16E^2} \frac{(1+A)^4}{A^2} \quad (55)$$

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This fraction f falls to 0.99 when the energy of the neutron has fallen to E_1 where

$$E_1 = 10 \frac{E_d}{4} \frac{(1+A)^2}{A} \sim 10^4 \text{ e.v.}$$

Thus the neutron has lost more than 99% of its energy in a part of its range where the losses were almost entirely to atoms that received more than 25 e.v. of energy. We will therefore assume that all the neutron energy goes into displacing atoms.

The energy losses of the struck atoms, in the cases we will be interested in (Fe and U), will be mainly through collisions with other atoms; the losses to electrons will be negligible. The reasons for this will be discussed later but depend mainly on the fact that the velocities of the struck atoms are very low.

Further, the conditions for a classical treatment of the collisions of the recoil with other atoms are that $\chi/a \ll 1$, and $V_a/\chi v \gg 1$. If we use the Fermi Thomas model then, the interaction potential between two atoms is: $V = \tilde{E} e^{-x} \frac{1+x}{x}$ with $\tilde{E} = Z^2 e^2 u$; $x = ur$; $u = 0.905 Z^{1/3} \times 10^8 \text{ cm}^{-1}$.

In the table below we give values of $\tilde{E}$, the corresponding $\chi, \beta = v/c$, $\tilde{E}$, x_1 given by $V_a = \chi v$ and $x_2 = \frac{2\tilde{E}}{E}$ = distance of closest approach in a head on collision, for Fe and U.

Z	A	$\tilde{E}$ e.v.	χ	β	$\tilde{E}$	x_1	x_2	ϵ
26	56	6.9×10^4	2.34×10^{-13}	1.63×10^{-3}	2.6×10^4	10.5	0.66	0.67
92	238	1.7×10^4	2.28×10^{-13}	3.92×10^{-4}	4.9×10^5	14.7	4.30	0.39

Since the extent of the atomic potentials is of the order of 10^{-8} cm ,

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we see that $\chi/a \ll 1$ for both Fe and U. Also, $\frac{v_a}{\chi v} > 1$ for $x < 10.5$ for Fe and $x < 14.7$ for U. This means that classical methods may be applied for encounters in which the impact parameters are less than x_1 . Now the scattering even at distances of order of $x = 4$ or 5 will be negligible since both atoms are neutral so that their potentials fall off exponentially. Thus at distances for which the potential is strong enough to produce appreciable scattering, it is also strong enough to permit a classical calculation. When the interaction is too weak for classical methods, it is also too weak to produce any scattering.

We have yet to show that energy losses to electrons will be negligible. There will be no losses to the bound electrons since the velocity of the atoms is so low. Thus the only electron losses will be to the free conduction electrons. For our purposes, it will suffice to make a very rough calculation which will give us an upper limit for the electron losses.

Let us suppose then that there are N_0 electrons per unit volume and, for simplicity, that they are at rest. Then, in the barycentric system of co-ordinates, the atom will be nearly at rest, on account of its large mass, and the electrons will stream by with a velocity v and be scattered. The velocity v corresponds to an electron energy of the order of 1 e.v. The wave length of such electrons is of the order of 2×10^{-8} cm and from this we see that the scattering is spherically symmetric. The total cross section is given by the well known formula

$$\sigma = 4\pi\chi^2 \sum (2n+1) \sin^2 \eta_n \quad (56)$$

(1) See, for e.g. Mott and Massey, Theory of Atomic Collisions, Oxford University Press.

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and since all but s-scattering is negligible, the maximum value of σ is just $4\pi\lambda^2$. An upper limit to the energy loss, in laboratory co-ordinates, is then given, in order of magnitude, by:

$$-\frac{dE}{dx}\bigg|_e \sim N_0 \sigma \frac{m}{M} E = 4\pi\lambda^2 N_0 \frac{m}{M} E \quad (57)$$

Now the energy of an electron having the same velocity as the atom is given by $\frac{m}{M} E$. Thus (57) becomes:

$$-\frac{dE}{dx}\bigg|_e \sim \pi N_0 \frac{e^4}{E_R} \quad (58)$$

We will see later that the collision loss is given, in order of magnitude, by:

$$-\frac{dE}{dx}\bigg|_c \sim \pi N_0 \frac{e^4}{E_R} \frac{E}{E_R} \quad (59)$$

The ratio $\frac{dE}{dx}\bigg|_c / \frac{dE}{dx}\bigg|_e = \frac{E}{E_R}$ which is of the order of 100 in the region of E where the expression for $\frac{dE}{dx}\bigg|_e$ is valid. This reasoning gives an upper limit to the energy losses to electrons. We may therefore neglect them entirely in the two cases we are discussing.

We calculate next the atomic displacements produced by neutrons in the metal. In particular, we will find expressions for the total number of displacements per 2 Mev. neutron and for the current of displaced atoms produced in a thin slab of metal by a given neutron flux of given energy. The first expression is useful in that it will give directly the total number of displacements (or the matter current) per fission (or per fission/second cm^3). The second expression will give some idea of the distribution of matter current in, for example, the fast neutron shields in a pile.

For the sake of simplicity we will calculate the total number of displacements per neutron in an approximate way. We will find an average primary recoil atom and use it to calculate the number of secondary and tertiary recoils. A more exact calculation, taking into account the distributions in energy of the recoils is possible on the basis of the equations developed here but is long and numerical in nature. In any case our approximate method will not be in error by more than a factor of two.

We will calculate the total number of displacements produced by the neutron as it slows down to 1% of its initial energy. Since, as we have seen, all the energy in this part of the path goes into displacements, and since further as we will show later, most of the energy of the primary recoils goes into secondary and tertiary displacements, this procedure will not affect the final result.

The average energy of the primary recoils in this region will now be found. It is well known that the percent loss of energy per collision of a neutron with an atom remains constant, on the average, throughout the path of the neutron. This loss may be expressed by the parameter ξ where

$$\xi = \left\langle \log \frac{E_i}{E_f} \right\rangle_{\text{av.}} \quad (60)$$

where E_i and E_f are the initial and final energies of the neutron. It may then easily be shown that:

$$\xi = 1 - \frac{A(1 - \frac{1}{A})^2}{2} \log \frac{A+1}{A-1} \approx \frac{2}{A} ; A \gg 1 \quad (61)$$

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where A is the atomic weight of the recoil atom. The energy of the neutron E_n , after n collisions is just:

$$E_n = E e^{-n\xi} \quad (62)$$

and the average energy of the recoil atoms over the region where the neutron energy falls from E to E_n is:

$$\langle \epsilon_1 \rangle = \frac{1}{n} \sum_{i=1}^n \epsilon_i = \frac{E}{n} (1 - e^{-\xi}) \sum_{i=1}^n e^{-(i-1)\xi} = \frac{E}{n} (1 - e^{-n\xi}) \quad (63)$$

where n is given by:

$$n = \frac{1}{\xi} \log \frac{E}{E_n} \quad (64)$$

Since we are taking $E_n = 0.01 E$, this becomes

$$n = \frac{2}{\xi} \sim A \quad (65)$$

using the eq. $\xi \approx 2/A$ for $A > 50$. The average primary recoil energies are then:

$$= \frac{2 \times 10^6}{56} \times 0.865 = 3.10 \times 10^4 \text{ e.v. (Fe)}$$

$$\langle \epsilon'_1 \rangle = \frac{2 \times 10^6}{238} \times 0.865 = 7.26 \times 10^3 \text{ e.v. (U)}$$

The average energy $\langle \epsilon'_1 \rangle$ found above corresponds to the energy whose logarithm is the average of the logarithms of the individual energy losses. Since the number of secondary and tertiary recoils depends on ϵ rather than on $\log \epsilon$ we will actually want the average of ϵ . This is obtained as follows: the energy loss of the neutron is given by:

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$$-\frac{dE}{dx} \approx N_0 \sigma \bar{E} = kE$$

$$\therefore \langle \epsilon_1 \rangle = \frac{\int k' E dx}{\int dx} = \frac{\int k' E \frac{dE}{kE}}{\int \frac{dE}{kE}} =$$

$$\frac{2A}{(1+A)^2} \frac{E - E'}{\log E/E'} = \frac{E}{(1+A)^2} \quad (66)$$

since we are assuming that $E = 0.01 E$.

Eq. (66) gives the values: $\langle \epsilon_1 \rangle = 3.45 \times 10^4$ e.v. for Fe; $= 8.34 \times 10^3$ e.v. for U.

(1)

The collision theory developed elsewhere for slow atoms may now be applied to the primary recoils. The range of a recoil atom of energy E is given by:

$$R = \frac{8}{5} \frac{u^2}{\pi N_0} \frac{1}{\log \frac{2E}{E}} \quad (67)$$

The fraction of energy that it loses in encounters in which the energy exchange exceeds E_d is:

$$f = 1 - \frac{\frac{9}{8} E_d \left(\frac{1}{E_d} - \frac{1}{E} \right)}{\log E/E_d} \quad (68)$$

and the average energy of the recoil atoms of this atom is:

$$\langle \epsilon \rangle = \frac{\frac{4}{9} (E - E_d) - E_d/2 \log E/E_d}{\log E/E_d} \quad (69)$$

Using Eqs. (67), (68), (69), we have calculated the values of N_i , the number of; $\langle \epsilon_i \rangle$ the average energy of; $\langle R_i \rangle$ the average range of the i^{th} generation of recoil atoms.

(1) Passage of Fission Fragments Through Matter, Part II, C

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Element		N_i	$\langle \epsilon_i \rangle$ e.v.	$\langle R_i \rangle$ cm.	f_i	$N_t = \sum N_i$
Fe	primary	58	3.45×10^4	1.1×10^{-6}	1	2.46×10^4
	secondary	800	2.12×10^3	1.4×10^{-7}	0.85	
	tertiary	6400	2.0×10^2	8.2×10^{-8}	0.75	
	quaternary	17300	25	6.0×10^{-8}	0.53	
U	primary	240	8.3×10^3	9.6×10^{-8}	1	2.84×10^3
	secondary	2600	6.2×10^2	4.3×10^{-8}	0.81	
	tertiary	0	--	--	--	
	quaternary	0	--	--	--	

A useful rule of thumb for primary recoils is that for A larger than 50, a neutron makes about A collisions in losing 99% of its energy and that the average energy of the recoils is about $\frac{1}{A}$ x initial neutron energy.

As a second example we will calculate the flux of displaced atoms in a thin slab of material produced by a flux of n neutrons per second per cm^2 and of energy = 1 Mev. We will use Fe as the material. The number of collisions in a thickness dx is:

$$\eta_1 = n N_0 \sigma dx$$

where σ is the neutron elastic cross section and is about 5 barns for Fe. Then this η_1 is just the number of primary displacements and if the neutron energy is 1 Mev., the average energy of the displacements is given by $\langle \epsilon_1 \rangle$, in the foregoing table. The previous methods would

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then give the total number of displacements per primary recoil. Although the primary recoils all move into the forward hemisphere, the secondaries and so forth will not and we may assume that the flux of atoms will be roughly isotropic. Then this flux is given by:

$$j = \frac{1}{3} n N_0 \sigma \left\{ \sum N_i \langle R_i \rangle \right\} dx \text{ atoms/cm}^2 \text{ sec.}$$

For a flux of $n = 1$ Mev. neutrons/cm²/sec we find the following value of j :

$$j = 10^{-5} n dx \text{ atoms/cm}^2 \text{ sec.}$$

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