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TRANSPORT CALCULATIONS FOR D-T BURNING TOKAMAK REACTORS

by

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For presentation at:

American Nuclear Society's First Topical Meeting
on the Technology of Controlled Nuclear Fusion,
San Diego, California, April 16-18, 1974

MASTER

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ARGONNE NATIONAL LABORATORY, ARGONNE, ILLINOIS

March 15, 1974

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Work performed under the auspices of the U. S. Atomic Energy Commission.

Transport Calculations for D-T Burning Tokamak Reactors

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One-dimensional particle and energy balance equations are solved for a 50-50 D-T burning Tokamak operating in the reactor regime. Steady-state solutions which satisfy MHD equilibrium and stability criteria are obtained for banana, plateau, and classical transport coefficients by suitable enhancement of the coefficients. The effects of trapped particle instabilities and impurities are investigated.

Unresolved questions about the physical phenomena which will determine particle and energy transport in the reactor regime constitute one of the major uncertainties in predictions of the performance of future fusion power reactors. The purpose of this paper is to examine the effect of various assumptions about the particle and energy transport mechanisms upon the predicted performance of hypothetical Tokamak power reactors with a 50-50 D-T mixture.

The analysis is carried out for a one-dimensional model of an axisymmetric Tokamak in which particle loss is by radial diffusion and energy loss is by radial diffusion and heat conduction. A two-fluid MHD (magnetohydrodynamic) model is used to determine steady-state solutions.

First, the magnitude of the transport enhancement or diminution which is required in order to obtain solutions that satisfy MHD equilibrium and stability criteria is determined for neoclassical transport theory, assuming that this transport enhancement or diminution can be modeled as a multiplicative factor upon the transport coefficients. Transport coefficients corresponding to three different regimes—trapped particle or banana, plateau, and classical—are considered, although neoclassical theory predicts that the plasma would be in the banana regime. Next, a possible mechanism for transport enhancement—trapped particle instabilities—is considered. Transport coefficients based upon both trapped ion and trapped electron instabilities are modeled in the calculations. Finally, the effect of impurities is investigated.

The principal results of the study are: (1) neoclassical banana regime transport coefficients must be enhanced by two to three orders of magnitude in order to obtain a plausible solution—i.e. one which satisfies the MHD stability and equilibrium criteria; (2) trapped particle instabilities produce transport coefficients which must be diminished by one to two orders of magnitude in order to obtain a plausible solution; (3) plausible solutions can be obtained without transport enhancement if impurities are present to a sufficient extent; and (4) the relative importance of the different components of the particle and energy balances is quite different for the three different ways of obtaining a plausible solution.

Two significant conclusions are suggested by these results. Trapped particle instabilities enhance the transport of particles and energy to such an extent that it is doubtful that adequate energy confinement can be obtained in their presence, if current theoretical models are valid. The mechanism by which energy confinement spoiling is achieved relative to neoclassical theory (i.e. multiplicative enhancement of transport coefficients, accounting for trapped particle instabilities, impurities) has an important impact upon the characteristics of the solution.

TRANSPORT EQUATIONS

The classical particle and energy transport equations [1] are modified to include an external source, S , of ions (e.g. neutral beam or pellet injection) and the destruction of particles and production of energy due to the fusion event. In radial geometry, the equations are of the form:

$$\frac{1}{r} \frac{\partial}{\partial r} (r n V_r) = S - \frac{1}{2} \overline{\sigma v} n^2, \quad (1)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{3}{2} n T_x V_r \right) + \frac{1}{r} \frac{\partial}{\partial r} (r q_x) + n T_x \frac{1}{r} \frac{\partial}{\partial r} (r V_r) = Q_x, \quad x = i, e, \quad (2)$$

where

$$Q_i = Q_{\alpha i} - Q_{ie}, \quad (3)$$

$$Q_e = Q_{\alpha e} + Q_{ie} + Q_{\Omega} - Q_{\text{Brem}} - Q_{\text{cyc}}, \quad (4)$$

with

$$Q_{\alpha i} = 1/4 \overline{\sigma v} n^2 (U_{\alpha i} - 3T_i) \quad (\alpha\text{-heating of ions}) \quad (5)$$

$$Q_{\alpha e} = 1/4 \overline{\sigma v} n^2 (U_{\alpha e} - 3T_e) \quad (\alpha\text{-heating of electrons}), \quad (6)$$

$$Q_{ie} = 1.2 \times 10^{-18} \frac{n^2 (T_i - T_e)}{T_e^{3/2}} \quad (\text{ion-electron Coulomb scattering}), \quad (7)$$

$$Q_{\Omega} = \eta j_z^2 \quad (\text{ohmic heating}), \quad (8)$$

$$Q_{\text{Brem}} = 3 \times 10^{-21} n^2 T_e^{1/2} \quad (\text{bremsstrahlung}), \quad (9)$$

$$Q_{\text{cyc}} = 3.75 \times 10^2 B_z^{5/2} n^{1/2} \left[T_e^{-11/4} + 0.0049 T_e^{15/4} \right] \quad (\text{cyclotron radiation}). \quad (10)$$

Here, n is the ion density (a single ion species with mass of 2.5 to simulate the 50-50 D-T mixture), T is the temperature of the (assumed) Maxwellian distribution, $\overline{\sigma v}$ is the Maxwellian-average fusion cross section, B_z is the toroidal magnetic field, j_z is the toroidal current density which satisfies

$$\mu j_z = \frac{1}{r} \frac{\partial}{\partial r} (r B_{\theta}), \quad (11)$$

where B_θ is the poloidal magnetic field which satisfies

$$\frac{\partial}{\partial r} \left[\frac{n}{1 - 1.95(r/R)^{1/2}} \frac{1}{r} \frac{\partial}{\partial r} (r B_\theta) \right] = 0. \quad (12)$$

The resistivity is given by

$$\eta = \frac{3.3 \times 10^{-8}}{T_e^{3/2}}, \quad (13)$$

and μ is the permeability.

The heat conduction is assumed to be described by

$$q_\chi = -K_\chi \frac{\partial T_\chi}{\partial r}, \quad (14)$$

and the radial drift velocity is assumed [2] to be given by

$$V_r = -D \left[\frac{1}{n} \frac{\partial n}{\partial r} + \frac{1.5}{T_i + T_e} \left(\frac{\partial T_i}{\partial r} + \frac{\partial T_e}{\partial r} \right) \right]. \quad (15)$$

The 3.52-MeV alpha particle is assumed to distribute its energy between the ions ($U_{\alpha i}$) and electrons ($U_{\alpha e}$) at the radius of the fusion event, according to a simple model [3]. The alpha particles are not otherwise treated.

Symmetry boundary conditions at the magnetic axis ($r = 0$) and vanishing boundary conditions at the plasma radius ($r = a$) are used for Eqs. (1) and (2). Boundary conditions for Eq. (12) are $B_\theta(0) = 0$ and $B_\theta(a) = \mu J_z / 2\pi a$, where J_z is the toroidal current.

The equations are solved numerically. All quantities are in the MKS system, except T , which is in keV.

TRANSPORT COEFFICIENTS

Neoclassical theory [2] predicts three different regimes characterized by different transport mechanisms, according to the relationship between the electron-ion collision frequency (ν_e) and the frequency ν_1 below which particle trapping is likely, where

$$\nu_e = \frac{n e^4 \ln \Lambda}{45.3 \sqrt{\pi} \epsilon_0^2 m_e^{1/2} T_e^{3/2}}, \quad (16)$$

and

$$\nu_1 = \frac{(r/R)^{3/2} \sqrt{Z T_e / m_e}}{r B_z / B_\theta}. \quad (17)$$

Here, m is the particle mass, $\ln \Lambda = 20$, ϵ_0 is the permittivity, and R is the major radius. The transport coefficients are modeled after Ref. 4.

Banana Regime $v_e < v_i$

$$D^B = 1.1 \left(1 + T_i/T_e \right) \rho_{e,\theta}^2 v_e (r/R)^{1/2}, \quad (18)$$

$$K_i^B = 0.68 n_{i,\theta}^2 v_i (r/R)^{1/2}, \quad (19)$$

$$K_e^B = 1.81 n_{e,\theta}^2 v_e (r/R)^{1/2}. \quad (20)$$

Plateau Regime $v_i < v_e < (R/r)^{3/2} v_i$

$$D^P = \sqrt{\pi/2} \left(1 + T_i/T_e \right) \frac{m_e^{1/2} T_e^{3/2}}{e^2 B_z B_\theta} (r/R^2), \quad (21)$$

$$K_i^P = 3\sqrt{\pi/2} \frac{n_i m_i^{1/2} T_i^{3/2}}{e^2 B_z B_\theta} (r/R^2), \quad (22)$$

$$K_e^P = 15/4 \sqrt{\pi/2} \frac{n_e m_e^{1/2} T_e^{3/2}}{e^2 B_z B_\theta} (r/R^2). \quad (23)$$

Classical Regime $(R/r)^{3/2} v_i < v_e$

$$D^C = \rho_{e,z}^2 v_e (1 + q^2), \quad (24)$$

$$K_i^C = n_{i,z}^2 v_i (1 + 1.6q^2), \quad (25)$$

$$K_e^C = 2.33 n_{e,z}^2 v_e (1 + 1.6q^2). \quad (26)$$

In these definitions, e is the electron charge,

$$\rho_{x,y} \equiv \frac{\sqrt{2m T_x}}{e B_y}, \quad (\text{Larmor radius}), \quad (27)$$

$$v_i \equiv \frac{n e^4 \ln \Lambda}{25.8 \sqrt{\pi} \epsilon_0^2 m_i^{1/2} T_i^{3/2}}, \quad (\text{ion-ion collision frequency}), \quad (28)$$

and

$$q \equiv \frac{r B_z}{R B_\theta}, \quad (\text{safety factor}). \quad (29)$$

Using a reactor model with $R = 7.5$ m, $a = 1.5$ m, $S = 10^{19}$ m⁻³-sec⁻¹, and $B_z = 200$ kG, a toroidal current $J_z = 7.7$ mA leads to $q(a) = 3.9$, which is more

than adequate to satisfy the macroscopic stability criterion $q(a) > 2.5$. Solutions of Eqs. (1), (2), and (12) which are consistent with the MHD equilibrium condition

$$\bar{\beta}_\theta = \frac{n_{\max} k (T_{e \max} + T_{i \max})}{B_\theta^2 / 2\mu} \leq \frac{R}{a} = 5 \quad (30)$$

are obtained by varying a constant, γ , which multiplies the transport coefficients D , K_i , and K_e . Solutions are obtained for banana, plateau and classical coefficients, although $v_e < v_i$. Salient features of these solutions are given in Table I, where P_n is the lineal neutron power and f_b is the fractional burnup of the D-T ions. In each case, the minimum value of γ is determined by the MHD equilibrium condition and the maximum value of γ is limited by numerical difficulties which are thought to signify a thermally unstable regime [5].

For similar $n_{\max}$ and $T_{\max}$, the radial particle distribution is flatter for the plateau and classical coefficients than for the banana coefficients. Also, the cross-over temperature at which $T_i > T_e$ is somewhat greater for the plateau and classical coefficients than for the banana coefficients.

Most importantly, the values of γ which are required in order to obtain plausible solutions imply that substantial transport enhancement is required when the banana and classical coefficients are used.

Because $v_e < v_i$, neoclassical theory predicts that the banana coefficients are appropriate. However, this theory and the resulting predictions of Table I are based upon an idealized model of the plasma.

TRAPPED PARTICLE INSTABILITIES

One means by which particle and energy transport may be enhanced is the microscopic turbulence associated with the trapped ion and trapped electron instabilities [6]. When the collision frequency is in the range $\nu_0 < \nu_e < \nu_i$, where

$$\nu_0 = \frac{T_e}{eB_z r^2}, \quad (31)$$

the transport coefficients resulting from the trapped particle instabilities can be represented by a scaling of the plateau coefficients:

Trapped Ion Instability

$$D^{TI} = (T_i/T_e)^2 (R/a) (\bar{\nu}_i/\nu_e) D^P, \quad (32)$$

$$K_e^{TI} = K_i^{TI} = n D^{TI}; \quad (33)$$

Trapped Electron Instability

$$D^{TE} = (R/a) \frac{\bar{\nu}_i \nu / \bar{\nu}_0^2}{1 + \nu_e^2 / \bar{\nu}_0^2} D^P, \quad (34)$$

$$K_e^{TE} = (R/a)^2 \frac{\bar{v}_1 v_e / \bar{v}_0^2}{1 + v_e^2 / \bar{v}_0^2} K_e^P \quad (35)$$

The trapped electron instability presumably does not alter the ion heat conductivity, which continues to be given by K_i^B . The overbar on the collision frequencies in Eqs. (32)-(35) indicates that the defining expressions are to be evaluated with $r = a$ and $B_0 = B_0(a)$.

The reactor model discussed previously, but now with the injected ion source attenuated exponentially from $r = a$ inward with attenuation coefficient a^{-1} , was used to obtain three new sets of solutions corresponding to the banana, trapped ion and trapped electron coefficients. Salient features of these solutions are shown in Table II; R , L_i and L_e represent the fractional energy losses by radiation, ion heat leakage (conduction, transport and work) and electron heat leakage, respectively.

The most striking point about these solutions is that, while the banana transport coefficients must be enhanced by a factor of approximately 200 in order to obtain a plausible solution, the trapped particle instabilities enhance the transport coefficients too much.

Other features of the solutions are quite different for the three sets of transport coefficients. The energy loss hierarchies are: (1) $L_i > R > L_e$ for banana coefficients; (2) $L_i \approx L_e > R$ for trapped ion coefficients; and (3) $L_e \gg L_i \approx R$ for trapped electron coefficients. The crossover temperature at which $T_i > T_e$ is lower for the trapped particle coefficients than for the banana coefficients. Solutions can be obtained at lower fractional burnup with the trapped ion coefficients than with the other two sets of coefficients, which may imply a greater range of thermal stability for the former than for the latter.

IMPURITIES

The presence of impurities, which heretofore have been ignored, can also spoil the energy confinement and thereby provide another means for obtaining a plausible solution when the banana coefficients are used. In order to account for the effect of alpha particles and other impurities with mass m_I and atomic number z_I , it is convenient to define factors

$$\begin{aligned} R_\alpha &\equiv 1 + \frac{n_\alpha}{n} + \frac{n_I}{n}, & R_z &\equiv 1 + 2 \frac{n_\alpha}{n} + z_I \frac{n_I}{n}, \\ R_{z2} &\equiv 1 + 4 \frac{n_\alpha}{n} + z_I^2 \frac{n_I}{n}, & R_{z2m} &\equiv 1 + 4 \frac{n_\alpha m}{n m_\alpha} + z_I^2 \frac{n_I m}{n m_I}, \\ R_m &\equiv 1 + \frac{n_\alpha m_\alpha}{n m} + \frac{n_I m_I}{n m}, & \alpha &\equiv R_{z2m} R_z / R_\alpha^2, \\ \epsilon &\equiv R_{z2} / R_n, & \xi &\equiv R_{z2} R_z / R_n^2, \end{aligned} \quad (36)$$

in terms of which the previous transport coefficients and balance equations can be modified

$$\bar{D}^B = \epsilon D^B, \quad \bar{K}_1^B = \left(R_m/R_n\right)^{1/2} \left(R_z/R_n\right)^2 K_1^B, \quad \bar{K}_e^B = \left(R_z/R_n\right) \bar{K}_e^B. \quad (37)$$

$$\bar{Q}_{\alpha X} = Q_{\alpha X}/R_n^2, \quad \bar{Q}_{\text{Brem}} = \epsilon Q_{\text{Brem}}, \quad \bar{Q}_{ie} = \alpha Q_{ie}, \quad (38)$$

$$\bar{\eta} = \left(R_{z2}/R_z\right) \eta. \quad (39)$$

In addition, the first and third terms in Eq. (2) for T_e must be multiplied by the factor R_z/R_n which is the ratio of the electron and ion densities.

Accounting for the presence of alpha particles in this manner, but still using a multiplicative factor, γ , to enhance the banana transport coefficients, reduces the fractional burnup and temperatures. The ion temperature is reduced by a greater amount than is the electron temperature, and the reduction in both is greater at lower temperatures. When $n_\alpha/n = 1\%$, f_b is reduced by approximately 1%; when $n_\alpha/n = 10\%$, f_b is reduced by approximately 5%.

Using the banana transport coefficients with $\gamma = 1$ and spoiling the energy confinement by introducing an oxygen impurity, in addition to a 10% alpha particle concentration, leads to solutions which violate the MHD equilibrium requirement. However, satisfactory solutions can be obtained by reducing the injected ion source strength, as shown in Table III. For the impurity-spoiled solutions, the energy loss is essentially entirely by radiation and the particle confinement is extremely good, as reflected by the high fractional burnup.

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TABLE I

Comparison of Banana, Plateau, and Classical Transport Coefficients

	Maximum Values			P_n (MW/m)	$\bar{\delta}_\theta$	f_b
	n ($10^{20}/m^3$)	T_e (keV)	T_i (keV)			
<u>Banana Coefficients</u>						
$\gamma = 222$	0.99	33.6	31.8	10.2	2.4	0.12
200	1.02	37.0	39.8	13.2	3.0	0.16
150	1.03	42.6	62.5	18.8	4.1	0.24
130	1.08	44.7	70.9	23.6	5.3	0.26
100	1.14	46.4	86.2	24.6	5.8	0.32
80	1.18	50.0	97.4	26.0	6.6	0.34
<u>Plateau Coefficients</u>						
$\gamma = 1.5$	1.52	29.2	23.5	15.4	3.0	0.20
1.2	1.36	33.1	25.4	16.8	3.1	0.22
1.0	1.34	35.3	28.2	17.8	3.2	0.24
0.5	1.29	40.0	38.8	25.0	3.9	0.32
0.2	1.31	47.8	60.8	35.6	5.4	0.46
0.1	1.40	49.3	77.7	45.4	6.8	0.58
<u>Classical Coefficients</u>						
$\gamma = 1500$	1.47	38.6	33.0	15.4	4.0	0.20
1450	1.43	44.7	47.5	21.0	5.0	0.28
1400	1.35	49.7	80.7	39.6	6.7	0.50
1200	1.38	51.4	95.9	43.0	7.7	0.56
1000	1.49	53.2	103.3	44.8	8.8	0.58

TABLE II
Comparison of Banana, Trapped Ion Instability, and
Trapped Electron Instability Transport Coefficients

	Maximum Values			P_n (MW/m)	δ_θ	Particle and Energy Balance			
	n ($10^{20}/m^3$)	T_e (keV)	T_i (keV)			f_b	R	L_i	L_e
<u>Banana Coefficients</u>									
$\gamma = 150$	1.04	46.1	70.1	19.8	4.6	0.25	0.40	0.43	0.17
200	1.00	44.0	54.7	16.8	3.7	0.21	0.38	0.47	0.15
222	1.00	42.5	49.5	15.2	3.5	0.20	0.35	0.47	0.18
250	0.97	37.2	35.7	12.2	2.7	0.16	0.29	0.50	0.21
<u>Trapped Ion Instability Coefficients</u>									
$\gamma = 0.0005$	0.65	33.3	44.0	7.6	1.9	0.10	0.31	0.37	0.32
0.001	0.61	29.7	36.6	6.0	1.5	0.08	0.27	0.36	0.37
0.005	0.60	23.4	23.6	3.4	1.1	0.05	0.22	0.39	0.39
0.010	0.64	19.9	19.0	2.8	1.0	0.04	0.20	0.42	0.38
0.050	0.81	10.1	9.8	1.4	0.6	0.02	0.15	0.37	0.48
<u>Trapped Electron Instability Coefficients</u>									
$\gamma = 0.005$	1.07 ^a	30.4	63.1 ^a	13.9		0.18	0.14	0.16	0.70
0.010	0.91 ^a	27.6	52.6 ^a	13.4		0.17	0.12	0.14	0.74
0.100	1.47 ^a	14.2	15.9 ^a	9.0		0.12	0.09	0.15	0.76

^aMaximum value occurs off magnetic axis.

TABLE III

Effect of Oxygen Impurity (Banana Transport Coefficients)

	Maximum Values			P_n (MW/m)	θ_θ	f_b
	n ($10^{20}/m^3$)	T_e (keV)	T_1 (keV)			
No. Impurities, $S = 10^{19} \text{ cm}^{-3}\text{-sec}^{-1}$						
$\gamma = 150$	1.04	46.1	70.1	19.8	4.6	0.25
200	1.00	44.0	54.7	16.8	3.7	0.21
222	1.00	42.5	49.5	15.2	3.5	0.20
250	0.97	37.2	35.7	12.2	2.7	0.16
Oxygen Impurities, $n_o/n = 0.10, S = 10^{19} \text{ cm}^{-3}\text{-sec}^{-1}$						
$n_I/n = 0.130$	1.96	57.3	91.8	50.6	14.6	0.65
0.140	1.97	55.1	86.5	49.5	14.2	0.63
0.147	1.92	46.1	64.4	42.7	11.1	0.55
0.150	2.46	28.6	31.2	37.0	8.0	0.47
0.165	2.50	27.7	30.3	36.3	8.1	0.46
Oxygen Impurities, $n_o/n = 0.10, S = 10^{18} \text{ cm}^{-3}\text{-sec}^{-1}$						
$n_I/n = 0.120$	0.73	38.1	54.6	5.5	3.4	0.71
0.130	0.71	35.9	50.0	5.3	3.1	0.68
0.140	0.75	35.4	45.7	4.9	3.2	0.62
0.155	0.99	27.0	30.1	4.7	3.1	0.60