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STRENGTH OF UNDERGROUND CAVITIES

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March 13, 1972

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SUBJECT

STRENGTH OF UNDERGROUND CAVITIES

NAME

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I. INTRODUCTION

The enclosed set of notes was gathered at the time of review of underground room liners such as U19e, U20c, U20n and U20f.

Because of our continuous interest in underground cavities, soil stresses and displacements in relation to the emplacement hardware, these notes are here entered under the ENN classification for the purpose of general record.

Although this material is not fully edited, it contains the basic information which may be of some interest to the analyst or designer. The reader is assumed to have the minimum knowledge of soil mechanics and the theory of elasticity.

The structural analyst, concerned with underground cavities and stability of rock under the three dimensional stress system, should find the enclosed calculations of rock-bolting of additional interest with particular regard to the concept of yielded zone. A brief comparison of spherical and cylindrical cavities, with respect to the required pressure to cause plastic flow, is therefore included for a quick reference.

Some numerical values of mechanical rock properties, stresses and displacements obtained on full size cavities in mining operations, are given in the report. Rock failure criteria and stability considerations are briefly reviewed with special regard to the Mohr-Coulomb relation between the normal and shear stresses in granular materials.

II. REMARKS ON EXPERIENCE WITH UNDERGROUND OPENINGS

This example of large cavities is based on Technical Report No. 1-83, DASA, 1968 by E.J. Cording

In tuff tangential stresses at the cavity surface were found to exceed the unconfined strength of this material. Grouting and bolting was necessary. Monitoring should include:

- check of bolt loading (plate bending and load cells)
- " of rock fracturing (seismic refraction)
- " of rock displacement (early warning)

Departure from flattening of the load-time curve indicates danger of failure. At that instant, the displacements are usually larger than those predicted on the basis of continuum mechanics.

Continuum theories, however, are not suitable for predicting large discontinuous movements. Continuous displacements include elastic, plastic and creep characteristics. For rock materials with high modulus elastic movements can hardly be restrained by any normal type of rock support system.

Discontinuous displacements are not proportional to stress, and should be restrained as soon as possible. Roof layer is modelled as an uniformly loaded beam with clamped ends, with the fiber stress correlating well with the modulus of rupture.

For excellent quality of rock consisting of bedded shale and limestone, E was found to be 10.4 to 11.0×10^6 psi. As failure approached the modulus decreased from 11×10^6 psi. (Based on Jonathan Mine, Zanesville, Ohio).

Increase in inelastic behavior at a total stress less than one-fourth of the crushing strength was observed in a White Pines mine in Michigan (2200 feet in bedded sandstone, siltstone and shale). Laboratory modulus was 6×10^6 psi to 7×10^6 psi. In situ modulus was 3×10^6 psi to 4×10^6 psi. The displacements could be analyzed with the elastic theory.

In Climax and San Manuel mines the strains observed were of the order of 0.1 to 0.3 percent.

In Star Mine, Burke, Idaho, most of the movement, as the vein was mined, occurred over a period of one to two years. Approximately 67 percent of the rock movement took place within 6 feet of the wall surface. Measurements were taken with strain-gages, tensioned rock bolts, borehole extensometers and transit levels.

In South African Gold Fields rock motion at 5,000 to 8,000 foot depth was found to correlate with elastic theory. Rock bursting was the usual mode of failure.

Massive quartzite (rock) S = 36,000 psi
Stress-strain linear to failure E = 12×10^6 psi

Excavation collapse was often violent due to a brittle character of failure. In such cases stresses are built-up with very little deformation until the rock mass reaches its ultimate strength and fails. Prediction of failure from displacement is therefore difficult. The major disturbance occurs when a three dimensional stress field is changed suddenly into two-dimensional state.

In tunnel work near Niagara rock load on concrete liner was estimated, for a 50 foot diameter, 300 ft beneath the surface in bedded sandstone, limestone and shales.

The rate of wall displacement decreased logarithmically with time along the horizontal chord. At the crown where limestone was, very small vertical movements were detected. The actual rock loads on the lining were much smaller than predicted because of concrete shrinkage. The intact modulus ranged from 1×10^6 psi to 10×10^6 psi. The prediction of vertical displacement was close but that of a horizontal was underestimated by an order of magnitude.

Many large underground openings have been built at depths up to 1450 feet.

In Australia, high natural horizontal stresses were observed in granite with $E = 8 \times 10^6$ psi. The rock bolts used were of 1 inch diameter, 10 to 15 ft long, spaced 4-5 feet apart. Nominal bolt load was 15,000 lbs.

In Rhodesia room construction, concrete linings and the grout were designed for water pressure and rock pressure. Rock bolts were secured during the excavation. The concrete linings were not highly stressed.

In Norad Center, Colorado, rock bolts were designed for static and dynamic loading, set in coarse-grained granite having strength of 10,000 to 20,000 psi. It was emphasized that rock strengthening program should be completed immediately following an excavation.

In a Morrow Point excavation, 78 foot long bolts were placed and drainage provided behind the wall. Predicted elastic displacement at crown was 0.05 inch while the measured value was 0.35 inch.

Large discontinuous displacements near rock surface are difficult to correlate on the basis of a laboratory modulus.

For a depth of 260 feet, the natural vertical stress should be close to 300 psi and horizontal component about 200 to 600 psi.

$$\frac{\nu}{1-\nu} = \frac{300}{300} \quad \therefore \quad \nu = 1-\nu \quad \nu = 0.5$$

$$\frac{\nu}{1-\nu} = \frac{600}{300} = 2 \quad \begin{matrix} \nu = 2-2\nu \\ 3\nu = 2 \end{matrix} \quad \nu = 0.67$$

This does not correlate with our rule.

One of the main problems is to determine the amount of rock-bolt support required to maintain a stable opening. This involves:

- a) defining rock properties
- b) selecting mode of failure around opening
- c) determining the effect of bolting on failure mode
- d) determining restraining load to prevent rock instability.

When building cavities in tuff (which is a very low-strength, relatively unjointed rock), spherical domes and rock-bolts are normally used, up to 32 foot long on 3 foot centers. Bolts are 1-1/8 inch diameter, made of A-431 reinforcing steel, having ultimate strength of 85,000 psi. The tuff is incapable of supporting standard expansion shell anchors. Bearing plates are 8 x 8 x 0.5 inches. Bolt pretension should be 30,000 lbs., after the grout anchor and bearing plate mortar are set. All bolts installed in the tuff are torqued twice, and then checked for tension periodically. 15% of the bolts lose more than half their torque during 2 to 6 months. Based on the University of Illinois experiments on intact tuff specimen, Poisson's ratio varied between 0 and 0.24 for sonic velocities of 8,500 to 9,100 psi.

The stress-strain curve was linear. The average values of 39 samples were:

$$\gamma = 120 \text{ lb./ft}^3; \sigma_u = 1,400 \text{ psi}; E = 0.5 \times 10^6 \text{ psi}; \mu = 0.24; C = 8,750 \text{ ft/sec.}$$

(dilatational at
1,000 psi)

Poisson's ratio and sonic velocity increases with compressive stress.
Dry tuff becomes more brittle.

A measure of rock mass quality is the ratio of field seismic velocity to the laboratory compressional velocity. A ratio of one represents the best quality. Stresses near the surface are about 1.5 to 2.0 times the natural stress away from the opening.

Vertical stresses are equal to the weight of overburden while horizontal are 1/2 the vertical. (In tuff).

$$\frac{\nu}{1-\nu} = \frac{1}{2} \quad 2\nu = 1-\nu \quad \underline{\nu = 0.3}$$

Laboratory tests on quartz monzonite gave:

$$S_u = 26,600 \text{ psi}$$

$$E = 10.4 \times 10^6 \text{ psi (at half compressive strength)}$$

$$\text{Sonic compressional (at zero stress)} = 21,000 \text{ feet/sec.}$$

$$\text{Sonic compressional at (at 5,000 psi)} = 21,600 \text{ fps}$$

$$\text{Unit dry weight} = 167 \text{ lb./ft}^3$$

General tuff behavior is similar to that of a highly stressed, brittle and homogeneous material. In some cases failures were associated with movement and sharp noises, with deflections on the order of a one foot. The bumps and cracks resulted apparently from a rapid release of strain energy.

In situ dilatational velocity in tuff averaged about 7200 ft/sec. In the fractured rock this velocity was 3300 ft/sec. The following elastic relations were used:

$$\left(\frac{v_p}{v_s}\right)^2 = \frac{2(1-\mu_d)}{1-2\mu_d}$$

$$E_d = \left(\frac{\gamma}{g}\right)(v_p)^2 \left[\frac{(1 + \mu_d)(1-2\mu_d)}{1 - \mu_d} \right]; \text{ where } \gamma = 121 \text{ lb./ft}^3 \text{ for tuff}$$

Approx. only $\left(\frac{v_{pf}}{v_{pl}}\right)^2 = \frac{E_{df}}{E_{dl}}$ if $\gamma_f = \gamma_l$
and $\mu_f = \mu_l$

Here

v_{pf} = sonic velocity in the field

v_{pl} = sonic velocity in the lab, $v_{pl} = 8750$ for tuff

v_{sf} = shear velocity in the field

E_{df} = modulus from field velocity = was found to be approx. $2E_t$

E_{dl} = modulus from lab velocity

E_t = modulus from stress/strain $E_t = 0.5 \times 10^6$ psi
for tuff

μ_{df} = Poisson's ratio from field velocity

The success of correlation must depend on:

- a) correct assumption of material properties
- b) correct assumption of boundary conditions
- c) correct assumption of theoretical model

II. YIELD ZONE CRITERIA

For a heavy walled sphere, displacement at any point is
(Faupel, page 247):

$$u = C_1 r + C_2 / r^2$$

where

$$C_1 = \frac{1 - 2\nu}{E(b^3 - a^3)} (P_1 a^3 - P_0 b^3)$$

$$C_2 = \frac{1 + \nu}{2E(b^3 - a^3)} (P_1 - P_0) (a^3 b^3)$$

when

$$P_0 = 0 \text{ and } P_1 = P$$

$$C_1 = \frac{1 - 2\nu}{E(b^3 - a^3)} (Pa^3)$$

$$C_2 = \frac{1 + \nu}{2E(b^3 - a^3)} Pa^3 b^3$$

when

$$b^3 \gg a^3$$

$$C_1 = \frac{(1 - 2\nu)}{Eb^3} \frac{Pa^3}{\rightarrow 0}$$

$$C_2 = \frac{(1 + \nu) Pa^3 b^3}{2Eb^3} = \frac{(1 + \nu) Pa^3}{2E}$$

$$u_s = \frac{C_2}{r^2} = \frac{(1 + \nu) Pa^3}{2E r^2} \quad \text{this is Eq. (1) in the thesis and Cording's report}$$

For a heavy walled cylinder:

$$u = \frac{1 - \nu}{E} \frac{a^2 P_1 - b^2 P_0}{b^2 - a^2} r + \frac{(1 + \nu)}{E} \frac{a^2 b^2 (P_1 - P_0)}{(b^2 - a^2) r}$$

for

$$P_0 = 0$$

$$u = \frac{1 - \nu}{E} \frac{a^2 P_1 r}{b^2 - a^2} + \frac{(1 + \nu) a^2 b^2 P_1}{E(b^2 - a^2) r}$$

when $P_1 = P$ and $b \gg a$

$$b^2 - a^2 \approx b^2$$

and

$$u = \frac{1-\nu}{E} \frac{a^2 P_1 r}{b^2} + \frac{(1+\nu) a^2 P_1}{Er}$$

Since $\left(\frac{a}{b}\right)^2$ is very small by our definition

$$u_c = \frac{(1+\nu) P a^2}{Er}$$

$$u_s / u_c = \frac{(1+\nu) P a^3}{2E r^2} \frac{Er}{(1+\nu) P a^2} = \frac{a}{2r}$$

when

$$\begin{aligned} r = a & \quad u_s / u_c = \frac{1}{2} \\ r \rightarrow \infty & \quad u_s / u_c = 0 \end{aligned}$$

The above equations are the basis of Figure 36, (*) comparing uniform stress-field displacements. It is seen that the static displacements attenuate as $1/r^2$ for the sphere and as $1/r$ for the cylinder.

For an elastic half space, Terzaghi gives the displacement of the surface under uniform pressure q :

$$u = 2a \left[\frac{q (1-\nu^2)}{E} \right]$$

Here a = radius of loaded area

Let yield criterion be represented in an excavation as follows:

$$\tau = c + p \tan \phi = \text{shear stress}$$

$$\phi = \text{angle of internal friction}$$

$$c = \text{cohesion (from Mohr's envelope)}$$

$$p = \text{external hydrostatic pressure}$$

(*) Refers to the text of Cording's paper.

According to Jaeger, the boundary radius between the plastic and elastic zones is given by R ("Elasticity, Fracture and Flow", John Wiley, 1962):

For $t = \tan(45 + \frac{\phi}{2})$
 $2ct = \sigma_{uH} = \text{unconfined compressive strength}$

Jaeger's equation for the boundary radius R , based on Mohr - Coulomb elastic-plastic criterion for a cylinder is:

$$\frac{R}{a} = \left\{ \frac{2 [P(t^2-1) + 2 Ct]}{[P_1(t^2-1) + 2 Ct] (t^2+1)} \right\}^{\frac{1}{t^2-1}}$$

When $P \rightarrow 0$

$$\frac{R}{a} = \left\{ \frac{4 ct}{[P_1(t^2-1) + 2 Ct] (t^2+1)} \right\}^{\frac{1}{t^2-1}}$$

where
 strength of rock)

R = radius of yielded zone. Putting $S_u = 2Ct$ (unconfined

$$\frac{R}{a} = \left\{ \frac{2 S_u}{[P_1(t^2-1) + S_u] (t^2+1)} \right\}^{\frac{1}{t^2-1}}$$

Let

$$\phi = 45 \text{ deg.}$$

$$t = \tan (45 + 22.5) = 2.414$$

$$S_u = 10,000 \text{ psi}$$

$$P_1 = 15 \text{ psi}$$

$$P = 1200 \text{ psi}$$

$$1/(t^2-1) = 1/(5.827-1) = 0.207$$

$$t^2-1 = 4.827$$

$$a = 120 \text{ in.}$$

$$R/a = \left\{ \frac{2[1200 \times 4.827 + 10,000]}{[15 \times 4.827 + 10,000 \times 6.827]} \right\}^{0.207}$$

$$= \left(\frac{31,584.8}{68,342.4} \right)^{0.207} = 0.85 \quad \text{This number indicates no yielded zone for the parameters specified.}$$

$$\text{Let } \phi = 45^\circ \text{ and } S_u = \sigma = 2ct$$

$$\therefore \frac{R}{a} = \left\{ \frac{2[4.227P]}{4.827 P_1 \times 6.827} \right\}^{0.207} = \left(\frac{P}{P_1} \right)^{0.207} \times 0.776$$

$$1.29 \left(\frac{R}{a} \right) = \left(\frac{P}{P_1} \right)^{0.207} \quad \text{Check curve in Figure 38 (Cording's Report)}$$

$$1.29 \times 1.2 = \left(\frac{1}{0.137} \right)^{0.207} \approx 1.509$$

$$1.548 \approx 1.509 \quad \text{O.K. considering reading error for 1.29 at } R/a = 1$$

$$\left(\frac{1}{0.27} \right)^{0.207} = 1.31 \quad \text{O.K.}$$

take

$$\phi = 30 \text{ deg.}$$

$$t = \tan (45 + 15) = 1.7321$$

$$t^2-1 = 2$$

$$t^2+1 = 4$$

$$\frac{R}{a} = \left\{ \frac{2[2400 + 10,000]}{[15 \times 2 + 10,000] 4} \right\}^{1/2}$$

$$= \left(\frac{24,800}{40,120} \right)^{1/2} \approx 0.79$$

If cohesion strength is used instead, for instance

$$\begin{aligned}
 C &= 100 \text{ psi} \\
 \therefore \frac{R}{a} &= \left\{ \frac{2(2400 + 100 \times 3.46)}{[30 + 100 \times 3.46]^4} \right\}^{1/2} \\
 &= \left(\frac{5492}{1500} \right)^{1/2} = \underline{1.92}
 \end{aligned}$$

For $R = a$ the following value of cohesion would be required:

$$2 [2400 + 3.46 C] = [15 \times 2 + 3.46 C]^4$$

$$4800 + 6.92C = 120 + 13.84 C$$

$$C = 4680/6.92 = 678 \text{ psi.}$$

Take for instance

$$\begin{aligned}
 C &= 600 \text{ psi} \\
 \therefore \frac{R}{a} &= \left\{ \frac{2 [2400 + 2075]}{[30 + 2075]^4} \right\}^{1/2} \\
 &= \left(\frac{8950}{8420} \right)^{1/2} = 1.03
 \end{aligned}$$

For zero cohesion and internal pressure Jaeger's equation gives an infinite depth of yielded zone. For $C = 0$ but a finite value of P_i , we get

$$\left(\frac{R}{a} \right) = \left\{ \frac{2 P (t^2 - 1)}{P_1 (t^2 - 1)(1 + t^2)} \right\}^{\frac{1}{t^2 - 1}}$$

$$\left(\frac{R}{a} \right) = \left\{ \frac{2P}{P_1 (1 + t^2)} \right\}^{\frac{1}{t^2 - 1}}$$

at

$$R/a = 1$$

$$2P = P_1 (1 + t^2)$$

$$\left(\frac{P_1}{P}\right) = \frac{2}{1 + t^2}$$

$$t = \tan \left(45 + \frac{60}{2}\right) = 3.732$$

$$\frac{P_1}{P} = \frac{2}{1 + 3.732^2} = 0.134 \quad \text{This agrees with Figure 38.}$$

The depth of yielded zone depends therefore on cohesion and internal pressure, assuming stress criteria. For a material with high cohesion, the effect of internal pressure is very small.

The theory of stresses around circular two dimensional openings under uniaxial stress was developed by Kirsch.

In a particular case of a finite element approximation the tuff properties were given as follows:

$$E = 0.5 \times 10^6 \text{ psi}$$

$$\nu = 0.22$$

$$\sigma_v = 1,000 \text{ natural stresses}$$

$$\sigma_h = 500 \text{ stresses}$$

Incidentally these do not agree with $(\nu/1 - \nu)$ rule.

The maximum elastic stress concentrations, equal to 2.7, were found at the intersection of the dome and the plane face. The major and minor principal stresses are both relatively high. The displacements occurred mostly within the 5 to 10 feet from the surface. The shallow displacement can be corrected by gunite.

The deep-seated discontinuous displacements at depths from 10 to 30 feet were relatively high, and increased with time and stress.

The change in modulus due to confinement did not appear to be significant.

In one example quoted by Cording, rock became unstable breaking 12 rock bolts. Subsequently 200 rock bolts were employed, 48 foot long and on 3 foot centers to stabilize the rock. This was the case of a large plane face. Curvature usually helps prevent opening and movement along extension fractures and joints.

Long delay in providing bolt support allows loosening of the rock.

IV. FRACTURE AND FAILURE OF TUFF

Continuum theory does not allow really for the type of discontinuous displacement, fracturing and loosening of the rock material, with property change in a sudden way. However, the theory may be applied if large movements and failures are prevented with the aid of some structural support. The most difficult problem is to select representative properties, and to evaluate the effect of the support system. The theoretical studies and models provide basically limiting conditions and parametric interpretations.

The support required to prevent the initial failure is usually smaller than that when the rock mass has already failed.

At relatively low loads, rock may have high angle of friction but virtually no cohesion. At high loads, coefficient of friction decreases while cohesion intercept gets higher. This intercept appears to be independent of the amount of displacement. For stable equilibrium the geometry is of utmost importance.

Rock bolts and the curved geometry may provide sufficient restraint when employed together.

Unconfined compression strength may be defined as follows:

$$\sigma_{ult} = 2 C t = 2 C \tan \left(45 + \frac{\phi}{2} \right)$$

A reasonable range of yielded zone in tuff is about 5 to 12 feet, although it can be as high as 30 feet. For the high cohesion materials internal pressure applied by the bolts has practically no effect on the yielded zone.

For the properly supported but fractured material the yielded zone is about 7.5 feet, and so is the observed depth of fractured rock.

Note: For concrete having $\sigma_t = 0.08 \sigma_c$

and $\phi = 45$ deg.

$$t = \tan (45 + 22.5) = 2.414$$

$$\sigma = 2 \sigma_c \times 0.08 \times 2.414 = 0.28 \sigma_c$$

very low

hence $c \neq 0.08 \sigma_c$ and tensile strength can not be easily substituted for cohesion.

if $\phi = 60$ deg.

$$t = \tan 75^\circ = 3.73$$

$$\text{and } \sigma = 0.16 \times 3.73 \sigma_c = 0.60 \sigma_c$$

$$\text{if } \sigma_t = 0.1 \sigma_c$$

$$\text{then } \sigma = 0.2 \times 3.73 \sigma_c = 0.746 \sigma_c.$$

Therefore cohesion for a typical concrete must be very high.

$$\sigma_u = 2 \times C \times 2.414$$

$$c = 0.21 \sigma_u \text{ approx.}$$

In the process of developing yielded zone, stresses are transferred away from the opening into the unfailed deeper rock, thus reducing stress concentration on the surface until a stable condition is reached. As long as the failed rock is properly supported, part of its strength will be

maintained and failure of the opening will not occur. However, the fractured zone must be relatively small and should not extend too deeply.

Load on rock bolts may be estimated on the basis of the depth of the yielded zone; the bolts must have sufficient stiffness to prevent extensive loosening and loss of strength of the rock mass.

The spacing and the maximum depth of fractures, formed around an opening are related to the radius of curvature of surface. On large, plane surfaces the tangential stresses are relatively low but the overstressed zone is deep because of the slow buildup of radial stress with depth.

V. ROCK FAILURE CONSIDERATIONS

Failure of rock-like materials is normally attributed to:

shear, consisting of particles sliding

cleavage, bond breaking and particle separation

pseudo-shear, advocated by Terzaghi, particularly at low confining pressures.

Rate of loading may cause change from ductile shear-type failure to brittle cleavage failure under dynamic conditions.

Velocity of cleavage fracture propagation is higher.

According to Mohr-Coulomb failure criterion, the maximum shear strength, $C + P \tan \phi$, is exceeded along a plane oriented at $[45 - (\phi/2)]$ deg to the direction of the major principal stress. In a circular opening then, under hydrostatic loading, the slip lines intersect the surface at 22 to 45 deg, depending on ϕ . The cohesive or tensile characteristics, of the material appear to be often of greater significance than the frictional characteristics, in case of cleavage mode of failure.

Most failure theories are independent of the size of the opening, although in tuff certain types of failures were only observed in larger cavities.

The stability of larger cavities must be dependent on the size because the weight of the rock to be supported increases with the size of the opening, with no corresponding increase in the strength of the rock surrounding the opening.

The support system must be capable of maintaining the coherency of the rock mass.

In granite, the displacements on the curved surfaces of the dome ranged from about 0.015 to 0.040 inches and very little movement was observed beyond 25 feet from the surface. On plane surfaces the readings were some 0.125 inches and were concentrated within 10 feet from the surface. Displacements on the curved surfaces correlated closely with the predictions based on spherical geometry.

Intact modulus $E = 10 \times 10^6 \text{ psi}$ $\sigma_{ult} = 27,000 \text{ psi}$
Deformation modulus $E = 4 \times 10^6 \text{ psi}$

The displacements were largely proportional to the radius of curvature. The displacements, however, were more concentrated near the surface and dropped off more rapidly with the depth than predicted from theory.

VI. OUTLINE OF ROCK BOLT DESIGN

The bolts provide a confining pressure near the free surface which can increase shearing resistance. The stability can be maintained if the bolts can hold the fractured rock in place. It is desirable to grout the bolts after tensioning to prevent the possible loss of tension, due to the partial failure of the bearing surface.

Bolt length/span ratio $\cong 0.25$

spacing 3 ft

pressure 20 psi. (This last parameter is very

difficult to determine, as it depends on the nature of discontinuities and the magnitude of displacement).

Let

$$R_i = 91"$$

$$R_o = 118"$$

$$R_p = 240"$$

Let γ = specific weight of material surrounding the liner,
assuming to be of the same order of magnitude as
that for concrete

Volume of spherical dome:

$$\frac{1}{2} \times \frac{4}{3} \pi (R_p^3 - R_i^3) = \frac{2\pi}{3} (R_p^3 - R_i^3)$$

Weight of this material:

$$\frac{2\pi\gamma}{3} (R_p^3 - R_i^3)$$

Disribution of Rock Bolts:

$$2\pi R_i = 6.28 \times 91 = 571.48 \text{ in.}$$

$$\text{or } \frac{571.48}{12} = 47.62 \text{ ft}$$

Consider dome area at the yield zone radius:

$$\frac{1}{2} \times 4 \pi R_p^2 = 2 \pi R_p^2$$

As the first approximation take one rock bolt per 9 ft² of surface. Then the number of rock bolts is:

$$n = \frac{2\pi R_p^2}{9 \times 144} \quad \text{on the yield zone}$$

$$\text{or } n = \frac{\pi R_p^2}{72 A_p}$$

Here A_p is expressed in square feet and plastic radius R_p in inches. The corresponding condition on the inner radius R_i is

$$n = \frac{\pi R_i^2}{72 A_i}$$

Hence
$$\frac{\pi R_p^2}{72 A_p} = \frac{\pi R_i^2}{72 A_i}$$

and
$$\left(\frac{A_i}{A_p}\right) = \left(\frac{R_i}{R_p}\right)^2$$

for instance, taking $R_i = 91$

$$R_p = 240$$

$$A_p = 9$$

$$A_i = 9 \left(\frac{91}{240}\right)^2 = 1.29 \text{ ft}^2. \quad \text{Spacing on the inside}$$

$$= (1.29)^{1/2} = 1.14 \text{ ft}$$

By increasing this spacing to 15 in. or 1.25 ft

$$A_i = 1.25^2 = 1.5625 \text{ ft}^2$$

and

$$A_p = A_i \left(\frac{R_p}{R_i} \right)^2 = 1.5625 \left(\frac{240}{91} \right)^2 = 10.87 \text{ ft}^2$$

Spacing on the outside

$$= (10.87)^{1/2} = 3.3 \text{ ft}$$

The corresponding number of rock bolts is:

$$n = \frac{3.14 \times 91^2}{72 \times 1.5625} = 230$$

Again as a crude approximation let the load per bolt is equal to total overburden load due to the weight of $\frac{2\pi\gamma}{3} (R_p^3 - R_i^3)$ divided by n so that

$$F_b = \frac{2\pi\gamma}{3} \frac{(R_p^3 - R_i^3)}{\pi R_i^2} \times 72 A_i$$

$$F_b = 48 \gamma A_i \frac{(R_p^3 - R_i^3)}{R_i^2}$$

let

$$R_p/R_i = k$$

$$R_p = k R_i$$

$$F_b = 48 \gamma A_i \frac{(k^3 R_i^3 - R_i^3)}{R_i^2}$$

$$F_b = 0.028 \gamma R_i A_i (k^3 - 1)$$

Where γ is given in pounds per cubic foot.

Denote nominal bolt diameter by d and its working length equal to the yield zone radius R_p . Let u be the maximum displacement of the curved surface equal to the bolt extension.

$$u = \frac{F_b R_p}{A E}$$

where

$$A = \frac{\pi d^2}{4}$$

$$u = \frac{4 F_b R_p}{\pi d^2 E}$$

The direct tensile stress in the bolt is:

$$S = \frac{F_b}{A}$$

$$u = \frac{S R_p}{E}$$

from which

$$S = uE/R_p$$

for a typical

$$u = 0.1 \text{ in.}$$

$$R_p = 240 \text{ in.}$$

$$E = 30 \times 10^6 \text{ psi}$$

$$S = \frac{0.1 \times 30 \times 10^6}{240} = 12,500 \text{ psi based on displacement criterion.}$$

if now

$$u = \frac{F_b R_p}{A E} = \frac{0.035 \gamma R_i A_i (k^3 - 1) R_p}{d^2 E}$$

and

$$R_p = k R_i$$

$$u = \frac{0.035 \gamma R_i^2 A_i k (k^3 - 1)}{d^2 E}$$

and for $d = 2r$

with $m = R_i/r$

$R_i = mr$

$$u = 0.0088 \gamma A_i m^2 k (k^3 - 1) / E \quad (*)$$

Using Jaeger's equation for k , based on Mohr-Coulomb elastic-plastic criterion, we get:

$$k = \left\{ \frac{2 [P_p (t^2 - 1) + 2Ct]}{[P_i (t^2 - 1) + 2Ct] (t^2 + 1)} \right\}^{\frac{1}{t^2 - 1}} \quad \begin{array}{l} \text{(Cylindrical geometry)} \\ \text{approximately only. Conservative} \end{array}$$

where $t = \tan (45 + \frac{\phi}{2})$

P_i = internal pressure, inside the concrete room

P_p = external pressure, assumed to be equal to the maximum design pressure for the particular concrete liner

ϕ = angle of internal friction.

Note: In the above analysis concrete is lumped together with the surrounding rocks and soil to provide a conservative approach.

Under the actual operating conditions of the liners the term in the denominator $P_i(t^2 - 1)$ will be usually found to be much smaller than the unconfined strength of the surrounding medium, equal to $2Ct$.

$$P_i (t^2 - 1) < < 2Ct$$

Hence, the equation for k may be rearranged as follows:

$$k = \left\{ \frac{2P_p (t^2 - 1)}{2Ct(t^2 + 1)} + \frac{4Ct}{2Ct(t^2 + 1)} \right\}^{\frac{1}{t^2 - 1}}$$

(*) In this formula use γ in lb./ft.³, A_i in ft.² and E in psi to obtain displacement in inches.

$$k = \left\{ \frac{P_p(t^2-1)}{ct(t^2+1)} + \frac{2}{(t^2+1)} \right\} \frac{1}{t^2-1}$$

Note: This equation applies strictly to a cylindrical configuration such as a tunnel.

for

$$\phi = 45^\circ \quad t = 2.414$$

$$P_p = 1100 \text{ psi} \quad c = 250 \text{ psi}$$

$$k = \left(\frac{1100 \times 4.827}{250 \times 2.414 \times 6.827} + 0.292 \right) 0.207$$

$$k = 1.1$$

Numerical Example:

$$\text{Rock bolt dia} \quad 1.0 \text{ in.}$$

$$r = 0.5 \text{ in.}$$

$$R_i = 91 \text{ in.}$$

$$m = 91/0.5 = 182$$

$$\gamma = 120 \text{ lb/ft}^3 = \frac{120}{1728} = 0.069 \text{ lb/in}^3$$

$$k = 240/91 = 2.64$$

$$A_i = 18.75 \text{ in}^2 = 0.13 \text{ ft}^2$$

$$E = 30 \times 10^6 \text{ psi}$$

Then

$$u = 0.0088 \times 120 \times 0.13 \times 182^2 \times 2.64 \times 17.39/30 \times 10^6 = 0.00695 \text{ in.}$$

$$F_b = 0.028 \times 120 \times 91 \times 0.13 \times 17.4 = 690 \text{ lb.}$$

$$S = 0.00695 \times 30 \times 10^6 / 240 = 869 \text{ psi.}$$

$$F_b/u = 869/0.00695 = 125,000 \text{ lb./in.}$$

Spring rate in terms of axial bolt stress:

$$E = \frac{S R_p}{u}$$

$$\begin{aligned} \frac{F_b}{u} &= \frac{\pi R_i}{km^2} \times \frac{S R_p}{u} & R_p &= k R_i \\ &= \frac{\pi R_i}{m^2} \times \frac{S}{u} \times R_i & &= \frac{\pi R_i^2 S}{u m^2} = \frac{\pi S r^2}{u} = \frac{\pi r^2 S}{u} \end{aligned}$$

VII. COMPARISON OF SPHERICAL AND CYLINDRICAL CAVITIES

Basic notation for comparing spherical and cylindrical geometries with respect to the required pressure to cause plastic flow to a boundary defined by radius b .

According to Faupel, pages 423 and 425 give equations for sphere and cylinder respectively

Sphere:

$$P_{bs} = 2\sigma_o \left[\ln \frac{b}{a} + \frac{b^3}{3c^3} \left(\frac{c^3}{b^3} - 1 \right) \right]$$

$$\text{Cylinder: } P_{bc} = \frac{\sigma_o}{\sqrt{3}} \left[1 - \left(\frac{b}{c} \right)^2 + 2 \ln \frac{b}{a} \right]$$

let

$$c/b = 240/118 = 2.036$$

$$b/c = 0.491$$

$$b/a = 118/91 = 1.297$$

$$\ln (1.297) = 0.262$$

$$P_{bs} = 2\sigma_o [0.262 + 0.0395 (7.43)] = 1.11\sigma_o$$

$$P_{bc} = 0.578 \sigma_o [1 + 0.524 - 0.241] = 0.742 \sigma_o$$

$$\therefore \frac{P_{bs}}{P_{bc}} = \frac{1.11}{0.742} = 1.5$$

Hence, for the geometry considered, it takes about 50 percent more pressure for the spherical geometry than for the cylindrical to assure the same depth of plastic penetration.

$$\text{When } b/a \approx 1 \quad \ln b/a = 0$$

and

$$P_{bs} = 2\sigma_o [0.2935] = 0.587 \sigma_o$$

$$P_{bc} = 0.578 \sigma_o (0.759) = 0.439 \sigma_o$$

$$\therefore \frac{P_{bs}}{P_{bc}} = \frac{0.587}{0.439} = 1.337$$

$$\text{When } c/b = 1$$

$$P_{bs} = 2 \sigma_o [0.262] = 0.524 \sigma_o$$

$$P_{bc} = 0.578 \sigma_o [0.524] = 0.306 \sigma_o$$

$$\therefore \frac{P_{bs}}{P_{bc}} = \frac{0.524}{0.306} = 1.713$$

$$\frac{P_{bs}}{P_{bc}} = \frac{3.46 \left[\ln \frac{b}{a} + \frac{b^3}{3c^3} \left(\frac{c^3}{b^3} - 1 \right) \right]}{\left[1 - \left(\frac{b}{c} \right)^2 + 2 \ln \frac{b}{a} \right]}$$

$$\text{let } a/b = 1/k \quad b = ak$$

$$b/c = e \quad c/b = 1/e$$

$$\begin{aligned} \therefore \frac{P_{bs}}{P_{bc}} &= 3.46 \frac{\left[\ln k + \frac{e^3}{3} \left(\frac{1}{e^3} - 1 \right) \right]}{\left[1 - e^2 + 2 \ln k \right]} \\ &= \frac{3.46 \left[\ln k + \frac{(1-e^3)}{3} \right]}{\left[1 + 2 \ln k - e^2 \right]} = \frac{1.15 (1 + 3 \ln k - e^3)}{(1 + 2 \ln k - e^2)} \end{aligned}$$

$$\text{let } \frac{P_{bs}}{P_{bc}} = \frac{1.15 (1 + 3 \ln k - e^2)}{(1 + 2 \ln k - e^2)} = \mathcal{P}$$

$$\frac{a}{b} \times \frac{b}{c} = \left(\frac{a}{c} \right) = \frac{1}{k} \times e = \left(\frac{e}{k} \right)$$

$$\text{at } \frac{a}{c} = 1 \quad e = k$$

$$\text{and } \mathcal{P} = \frac{1.15 (1 + 3 \ln k - k^3)}{(1 + 2 \ln k - k^2)} \quad k = 1$$

If m and k are zero, then $\mathcal{P} = 1.15$ for an infinitely thick cylinder.

$$\frac{e}{k} = 0$$

then

$$k \neq 0$$

$$\frac{b^3}{3C^3} \left(\frac{C^3 - b^3}{b^3} \right) = \frac{C^3 - b^3}{3C^3}$$

$C \gg b$ then $C^3 \gg b^3$ and the above
reduces to $\frac{1}{3}$

so that

$$\mathcal{F} = \frac{3.46 \left[\ln k + \frac{1}{3} \right]}{[1 + 2 \ln k]}$$

when

$$k = 1; \mathcal{F} = 1.15$$

$$k = 5; \mathcal{F} = \frac{3.46 [1.6 + 0.33]}{4.2} = 1.59$$

when

$b/a = k = 1$ there is no plastic boundary

and

$$\mathcal{F} = \frac{1.15 (1 - e^3)}{(1 - e^2)}$$

Second condition for a very thick shell is

$$\frac{a}{c} = \bar{k} = 0$$

and since $k \neq 0$

then

$$e = 0$$

and from the above $\mathcal{F} = 1.15$

NO PLASTIC PENETRATION
VERY THICK WALL

when

$e \rightarrow 1$

$\mathcal{F}$ is indeterminate, so differentiating top and bottom with respect to e , gives:

$$\mathcal{F} = \frac{1.15 (-3e^2)}{-2e} = \frac{1.15 \times 3 \times 1}{2 \times 1} = 1.725$$

$$\mathcal{F} = 1.725$$

no plastic
penetration, very
thin wall

when

$k = 2$ and $e = 0$

$$\mathcal{F} = \frac{1.15 [1 + 0.693 \times 3]}{(1 + 2 \times 0.693)} = \frac{1.15 \times 3.079}{2.386} = 1.484$$

$$k = 1.5 \quad \log_e 1.5 = 0.4055 \quad \mathcal{F} = \frac{1.15 \times 2.2165}{1.8110} = 1.4075$$

$$k = 2.5 \quad \log_e 2.5 = 0.9163 \quad \mathcal{F} = \frac{1.15 \times 3.7489}{2.8326} = 1.5220$$

$$k = 3.0 \quad \log_e 3.0 = 1.0986 \quad \mathcal{F} = \frac{1.15 \times 4.2958}{3.1972} = 1.5450$$

$$k = 4.0 \quad \log_e 4.0 = 1.3863 \quad \mathcal{F} = \frac{1.15 \times 5.1589}{3.7726} = 1.5725$$

$$k = 1.2 \quad \log_e 1.2 = 0.1823 \quad \mathcal{F} = \frac{1.15 \times 1.5470}{1.3646} = 1.3037$$

$$k = 5.0 \quad \log_e 5.0 = 1.6094 \quad \mathcal{F} = \frac{1.15 \times 5.8283}{4.2188} = 1.5887$$

Calculate parameter $t = \tan (45 + \frac{\phi}{2})$

$\phi = 0$	$t = 1$
$\phi = 10^\circ$	$t = \tan 50^\circ = 1.1918$
$\phi = 20^\circ$	$t = \tan 55^\circ = 1.4281$
$\phi = 30^\circ$	$t = \tan 60^\circ = 1.7321$
$\phi = 40^\circ$	$t = \tan 65^\circ = 2.1445$
$\phi = 50^\circ$	$t = \tan 70^\circ = 2.7475$
$\phi = 60^\circ$	$t = \tan 75^\circ = 3.7321$

Calculate parameter "k" for

$$P_p = 1100 \text{ psi}$$

as a function of cohesion C and parameter t.

At $P_p=1100$ and $C=100$ assuming that

$$P_i(t^2-1) \ll 2Ct$$

$$k = \left\{ \frac{P_p(t^2-1)}{Ct(t^2+1)} + \frac{1}{(t^2+1)} \right\} \frac{1}{t^2-1}$$

$$k = \left\{ \frac{1100(t^2-1)}{t(t^2+1)} + \frac{1}{(t^2+1)} \right\} \frac{1}{t^2-1}$$

$$k = \left\{ \frac{11(t^2-1) + t}{t(t^2+1)} \right\} \frac{1}{t^2-1}$$

t	(t^2-1)	$t(t^2+1)$	$\frac{1}{t^2-1}$	$\frac{11(t^2-1) + t}{t(t^2+1)}$	k
1.1918	0.42039	2.8846	2.3787	2.0162	5.300
1.4281	1.03947	4.34067	0.96203	2.96320	2.843
1.7321	2.00017	6.92870	0.49996	3.4254	1.851
2.1445	3.59888	12.0068	0.27786	3.4757	1.414
2.7475	6.54876	23.4877	0.15270	3.1839	1.193
3.7321	12.92857	55.7119	0.07735	2.6195	1.077

SQUARE 10 X 10 IN. 1/4" AS 0807-61

GRAPH-EX-3 R GRAPHIC CONTROLS CORPORATION Buffalo, New York Printed in U.S.A.

z = auxiliary parameter in Jager equation

$$F(t) = \frac{11t^2 + t - 11}{t^3 + t}$$

$$\frac{dF(t)}{dt} = (t^3 + t)(1 + 22t) - (11t^2 + t - 11)(3t^2 + 1) = 0$$

Second approximation:

$$P_p = 1100 \phi; c = 100 \text{ psi}$$

$$\phi = 10^\circ; k = 5.3; \mathcal{S} = 1.59$$

$$P_p = 1100 \times 1.59 = 1749$$

$$k = \left\{ \frac{17.49(t^2 - 1) + t}{t(t^2 + 1)} \right\}^{1/t^2 - 1} \stackrel{\text{EXP } 2.3787}{=} 2.96208 = 13.23$$

$$\phi = 20^\circ \quad k = 2.843 \quad \mathcal{S} = 1.54$$

$$P_p = 1100 \times 1.54$$

Review the method:

If modified Jaeger's equation shows that for a given pressure P_p , the k value is say 5.3 based on cylindrical geometry, and since $\mathcal{S} > 1$, the same pressure for a spherical vessel should produce smaller value of k and not larger. To account for this, substitute $1100/\phi$ for 1100. Based on this assumption make the following calculation:

$$P_p = 1100/1.59 = 691.8, \text{ at } k = 5.3$$

$$\phi = 10^\circ \quad \text{then } k = \left\{ \frac{6.918 \times 0.42039 + 1.1918}{2.8846} \right\}^{2.3787} = 2.308$$

$$\phi = 20^\circ \quad k = \left\{ \frac{7.1428 \times 1.03947 + 1.4281}{4.34067} \right\}^{0.96203} = 1.985$$

$$\phi = 30^\circ \quad \mathcal{S} = 1.47 \quad k = \left\{ \frac{7.4830 \times 2.0017 + 1.7321}{6.9287} \right\}^{0.49996} = 1.552$$

$$\phi = 40^\circ \quad \mathcal{S} = 1.38 \quad k = \left\{ \frac{7.9710 \times 3.59888 + 2.1445}{12.0068} \right\}^{0.27786} = 1.300$$

$$\phi = 50^\circ \quad \mathcal{S} = 1.30 \quad k = \left\{ \frac{8.4615 \times 6.54876 + 2.7475}{23.4877} \right\}^{0.1527} = 1.148$$

Try third approximation:

$$\begin{array}{lcl}
 \phi = 10^\circ & \varphi = 1.51 & k = \left\{ \frac{7.2847 \times 0.42039 + 1.1918}{2.8846} \right\}^{2.3787} = 2.519 \\
 \phi = 20^\circ & \varphi = 1.48 & k = \left\{ \frac{7.4324 \times 1.03947 + 1.4281}{4.34067} \right\}^{0.96203} = 2.049 \\
 \phi = 40^\circ & \varphi = 1.35 & k = \left\{ \frac{8.1481 \times 3.59888 + 2.1445}{12.0068} \right\}^{0.27786} = 1.307
 \end{array}$$

Try fourth approximation:

$$\phi = 10^\circ \quad \varphi = 1.522 \quad k = \left\{ \frac{7.2273 \times 0.42039 + 1.1918}{2.8846} \right\}^{2.3787} = 2.486$$

VIII. EXAMPLE OF ROCK BOLT CALCULATION

Let $A_p = 9 \text{ ft}^2$ (based on mining practice) and $R_i = 91 \text{ in.}$

then $A_i = A_p / k^2$

Let angle of internal friction in tuff be:

$$\phi = 35^\circ$$

then from the "final curve for k",

$$k = 1.45$$

and $A_i = 9 / 1.45^2 = 4.28 \text{ ft}^2 = 616.32 \text{ in}^2$

Hence the number of bolts:

$$n = \frac{3.14 \times 91^2}{72 \times 4.28} = 84$$

$$F_b = 0.028 \times 120 \times 91 \times 4.28 \times 2.05 = 2683 \text{ lb.}$$

For a value of $m = 91/0.5 = 182$, bolt extension is:

$$u = 0.0088 \times 120 \times 4.28 \times 33124 \times 1.45 \times 2.04/30 \times 10^6 = 0.015 \text{ in.}$$

Hence, the approximate bolt length should be 11 feet.

$$R_p = 1.45 \times 91 = 131.95 \text{ in.}$$

$$S = 0.015 \times 30 \times 10^6/131.95 = 3410 \text{ psi.}$$

Let $A_p = 9 \text{ ft}^2$

$$\phi = 25^\circ$$

then $k = 1.82$

$$A_i = 9/1.82^2 = 2.717 \text{ ft}^2 = 391.26 \text{ in.}^2$$

$$n = \frac{3.14 \times 91^2}{72 \times 2.717} = 132$$

$$F_b = 0.028 \times 120 \times 91 \times 2.717 \times 5.028 = 4177 \text{ lb.}$$

$$R_p = 1.82 \times 91 = \underline{165.62 \text{ in.}}$$

$$u = 0.0088 \times 120 \times 2.717 \times 33124 \times 1.82 \times 5.028/30 \times 10^6 = 0.029 \text{ in.}$$

$$S = 0.029 \times 30 \times 10^6/165.62 = \underline{5220 \text{ psi.}}$$

$$R_p = k R_i$$

Depth of the yielding zone

$$R_p - R_i$$

$$Z = R_i (k-1)/12$$

$$\begin{aligned}
 \phi = 10^\circ & \quad Z = 91 \times 1.5/12 = 11.375 \text{ ft} \\
 20^\circ & \quad Z = 91 \times (2.05-1)/12 = 7.963 \text{ ft} \\
 30^\circ & \quad Z = 91 \times (1.55-1)/12 = 4.171 \text{ ft} \\
 40^\circ & \quad Z = 91 \times (1.32-1)/12 = 2.427 \text{ ft} \\
 50^\circ & \quad Z = 91 \times (1.15-1)/12 = 1.138 \text{ ft}
 \end{aligned}$$

$$\text{if } A_i = A_p/k^2$$

$$\text{and } n = \frac{\pi R_i^2}{72 A_i} = \frac{\pi R_i^2 \times k^2}{72 A_p} = \frac{3.14 \times 91^2}{72 \times 9} \cdot k^2 = 40 k^2$$

$$\text{at } k = 1.45 \quad \therefore n = 84$$

$$k = 1.82 \quad \therefore n = 40 \times 1.82^2 = 132$$

$$k = 1.15 \quad n = 40 \times 1.15^2 = 52.9$$

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TID File 4

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