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Inverse Bremsstrahlung Absorption Coefficient and
Initiation of Laser-Supported Detonation Waves in Air

Alan M. Winslow

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Inverse Bremsstrahlung Absorption Coefficient and Initiation of Laser-Supported Detonation Waves in Air

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We have modified the inverse bremsstrahlung absorption coefficient in LASNEX¹ for application to laser effects calculations at temperatures below 10 eV. These modifications are described below and the initiation condition for a laser-supported detonation (LSD) wave is crudely estimated for air in ^{the} vicinity of a solid target.

1. Ionic and atomic inverse bremsstrahlung absorption coefficients

For electron-ion bremsstrahlung, the Maxwell-averaged inverse bremsstrahlung absorption coefficient is given by²

$$K = \frac{4}{3} \frac{e^6}{m^2 c^2 h^2 \nu^3} \sqrt{\frac{2\pi}{3mkT}} n_e \langle n_i Z_i^2 \rangle \left(1 - e^{-\frac{h\nu}{kT}} \right) \bar{g}_{ff}$$

$$= 2.61 \times 10^{-41} \left(\frac{Ry}{h\nu} \right)^3 \left(\frac{Ry}{kT} \right)^{1/2} n_p^2 \bar{Z} \cdot \bar{Z}^2 \left(1 - e^{-\frac{h\nu}{kT}} \right) \bar{g}_{ff} \quad (\text{cm}^{-1})$$

(1)

where ν is the laser radiation frequency, T the electron temperature, n_e the electron density, n_i the density of ions of charge Z_i , n_p the total density of heavy particles, $\bar{g}_{ff}$ the Maxwell-averaged Gaunt factor, $\bar{Z} = n_e/n_p$, $\bar{Z}^2 = \langle n_i Z_i^2 \rangle / n_p$, and the remaining quantities are the usual atomic constants.

Maxwell-averaged hydrogenic Gaunt factors have been calculated by Karzas and Latter³ and are shown in Fig. 1 as functions of the parameters

$$y^2 = \frac{Ry \bar{Z}_+^2}{kT}$$

$$u = \frac{h\nu}{kT}$$

where $Ry = 13.59 \text{ eV}$ and $\bar{Z}_+^2$ is given by

$$\bar{Z}_+^2 = \langle n_i Z_i^2 \rangle / n_p^+$$

the average being taken over the ionic species. Here n_p^+ is the total number density of ions excluding neutral atoms.

When $\gamma^2 \leq .01$ the Karzas and Latter Gaunt factors reduce to the Born approximation:

$$\begin{aligned}\bar{g}_{ffB} &= \frac{\sqrt{3}}{\pi} e^{u/2} K_0(u/2) \\ &\approx \frac{\sqrt{3}}{\pi} e^{u/2} [\ln(4/u) - .577] \quad u \lesssim 1 \\ &\approx \sqrt{3/\pi u} \quad u \gtrsim 2\end{aligned}$$

However Figure 2 shows that, for air, $\gamma^2 \gtrsim 4$ for temperatures up to 100 eV, so that the Born approximation is in error by nearly a factor of 2. Therefore we have used the Karzas and Latter results fitted with the expression⁴

$$\bar{g}_{ff} = 1.08(1 + ay^b) \quad (2)$$

where

$$a = .25 - .07716x$$

$$b = 1.198/a^{.27}$$

$$x = \log_{10}(10\gamma^2)$$

$$y = \log_{10}(10/u)$$

for $u \leq 10$, $\gamma^2 \leq 100$. We also require $u < \gamma^2$, implying $\lambda > .09\mu$. For $\gamma^2 > 100$ we set $\bar{g}_{ff} = 1.08$. This formula is compared with the Karzas and Latter results in Table I. The brackets show, for each value of γ^2 , the domain of values of u that is of interest for wavelengths between 1μ and 10μ and temperatures below 1 Kev. The accuracy for the indicated domains of u and γ^2 is seen to be approximately 5%.

In order to calculate γ^2 for the above fit, we need Z_+^2 . This quantity may be obtained by modifying our Saha equation subroutine⁵ which provides values of $\bar{Z}$, $\bar{Z}^2$, etc. as functions of temperature, density and chemical composition. Alternatively, we may proceed as follows. If only single

and double ionization have occurred ($\overline{Z^2} \lesssim 2.5$ for air) it may be shown that

$$\overline{Z_+^2} = \frac{\overline{Z^2}}{\overline{Z} - \frac{1}{2}(\overline{Z^2} - \overline{Z})} \quad (\overline{Z^2} < 2.5) \quad (3)$$

and $\overline{Z_+^2} \rightarrow 1$ as $T \rightarrow 0$ since $\overline{Z^2} \rightarrow \overline{Z}$. When this assumption is no longer valid,

$$\overline{Z_+^2} = \overline{Z^2} \quad (\overline{Z^2} > 2.5) \quad (4)$$

since there is no longer any significant number of neutral atoms. Equations (3) and (4) are accurate to 1% (see Fig. 3 for air at normal density).

In Fig. 4 we compare our fit to the Karzas and Latter results with the Born approximation result for $\lambda = 10.6\mu$ in normal density air for $.025 \leq T \leq 100$ eV. The difference approaches a factor of two for $T < .1$ eV.

For the electron-atom bremsstrahlung absorption coefficient, we add $n_a \langle Z^2 g \rangle$ to $\langle n_i Z_i^2 \rangle \bar{g}_{ff}$ in Eq. (1), where n_a is the density of neutral atoms. J. Green⁶ has calculated $\langle Z^2 g \rangle$ for atomic oxygen. To estimate the potential surrounding an oxygen atom, he uses the zero temperature Thomas-Fermi solution confined to a volume which is the atomic volume of liquid oxygen, based on the assumption that, as long as the atom is not ionized, its internal state, and consequently its radius, will not change much with temperature.

Green's results show an increase of $\langle Z^2 g \rangle$ with temperature, which can be attributed to the fact that higher energy electrons penetrate through the centrifugal barrier more deeply into the atom. Our fit to his results, good to about 5%, is

$$\langle Z^2_g \rangle = \begin{cases} .081T^{7/3} & (T \leq .5 \text{ eV}) \\ .0749T - .0214 & (T \geq .5 \text{ eV}) \end{cases}$$

(T in eV).

We have applied this formula to absorption by air. Since one would not expect it to be particularly accurate for nitrogen, we compare it with the classical absorption coefficient for weak ionization, given approximately by⁶

$$K_c = \frac{\omega_p^2}{\omega^2} \frac{\nu_c}{c}$$

where ω_p is the plasma frequency = $4\pi e^2 n_e / m$, $\omega = 2\pi\nu$, and ν_c is the collision frequency for electron-atom scattering, which we may approximate by

$$\nu_c = n_a \langle \sigma v \rangle_{ea} \sim n_a \langle \sigma \rangle_{ea} \langle v \rangle$$

For $\langle v \rangle$ we use the mean thermal electron velocity given by

$$\bar{v} = \left(\frac{8kT}{\pi m} \right)^{1/2} = 6.69 \times 10^7 T^{1/2} \text{ cm/sec}$$

(T in eV). Hence we have

$$K_c = \frac{4\pi e^2}{mch^2} \sqrt{\frac{8kT}{\pi m}} n_e n_a \langle \sigma \rangle \quad (\text{cm}^{-1})$$

The electron-ion bremsstrahlung coefficient K_i is compared to the electron-atom coefficients K_a and K_c with $\sigma = 1 \times 10^{-15} \text{ cm}^2$ (estimated from Ref. 7) in Table II. We see that, for normal density air, $K_a > K_i$ for

$T \sim 1.2$ eV. We also observe that $K_a \sim K_c$ within a factor of four over the entire temperature range, which is reasonably good agreement in view of the approximations made.

The effective absorption coefficient is given by $K/\sqrt{\epsilon}$ where ϵ is the dielectric constant, given approximately by

$$\epsilon = 1 - \frac{\omega_p^2}{\omega^2} = 1 - \frac{n_e}{n_c}$$

where n_c is the critical electron density = $9.91 \times 10^{18} \text{ cm}^{-3}$ for $\lambda = 10.6\mu$. We have given values of $\sqrt{\epsilon}$ in Table II. We see that $\sqrt{\epsilon} \sim 1$ for $T \sim 1$ eV and $\sqrt{\epsilon} = 0$ for $T \sim 1.6$ eV, when $n_e = n_c$.

2. Heating of low-temperature air by laser absorption

We may use the above expressions for K_i and K_a (or K_c) to estimate the heating rate of air by laser absorption, leading to the initiation of an LSD wave in the vicinity of a solid target such as aluminum. Initially the air, at $T = .025$ eV, is essentially nonabsorbing, and the laser energy is deposited in the target vaporizing a thin layer. The expansion of this vapor compresses a thin layer of adjacent air by approximately a factor of ten ($\gamma_{\text{air}} \sim 1.2$) and heats it to a few thousand degrees ($T \sim .1-.2$ eV), producing weak ionization. The subsequent heating of these electrons by laser absorption can be crudely estimated if we neglect hydrodynamic expansion and energy exchange with the heavy particles. Then the initial rate of increase of T_e is given simply by

$$\frac{\partial T_e}{\partial t} = \frac{T_e}{\tau_{\text{abs}}} \quad (5)$$

We derive τ_{abs} as follows. Neglecting losses, we can write for the heating rate of the electrons in a layer of thickness Δx

$$\frac{\partial \epsilon_e}{\partial t} = \frac{WK}{\rho} \left(\frac{1 - e^{-K\Delta x}}{K\Delta x} \right) \sim \frac{WK}{\rho} \quad (6)$$

if $K\Delta x \ll 1$, where W is the laser beam intensity and ϵ_e is the specific energy of the electrons. Treating the electrons as an ideal gas, we have

$$\epsilon_e = \frac{n_e}{\rho} \left(\frac{3}{2} kT_e + I \right) \sim \frac{n_e I}{\rho} \quad (7)$$

where I is the average ionization potential ($= 14.4$ eV for air) and $kT_e \ll I$. Combining (5), (6), and (7) we have

$$\tau_{\text{abs}} = \frac{IT_e}{WK} \left(\frac{\partial n_e}{\partial T_e} \right) \quad (8)$$

In order to calculate n_e we make use of the Saha equation in the LASNEX version⁴, which neglects processes such as molecular dissociation and electron attachment which are dominant at low temperatures. A complete equilibrium calculation for air, taking into account all important molecular species, has been done by Gilmore.⁸ Comparison of these results with our Saha equation shows that the latter underestimates n_e by a factor of 100 at $T_e = .2$ eV, although the error decreases as T_e increases and falls below 10% for $T_e \gtrsim .8$ eV.

Nevertheless we observe from (8) that $\tau_{\text{abs}} \sim \frac{1}{n_e} \frac{\partial n_e}{\partial T_e}$, and we find that the Saha values for this ratio agree with Gilmore's results to better than a factor of two. Thus we can obtain acceptable estimates of τ_{abs} in spite of the error in n_e .

The Saha equation for $\bar{Z} \equiv n_e/n_p$ is, at low temperatures,

$$\bar{Z}^2 = \text{const } T_e^{3/2} e^{-I/T}$$

so that

$$\frac{\partial \bar{Z}}{\partial T} \sim \frac{\bar{Z}}{2} \frac{I}{T^2}$$

and (8) becomes

$$\tau_{\text{abs}} = \frac{n_e}{2WK} \frac{I^2}{T_e} = 8.95 \times 10^3 \left(\frac{\rho/\rho_0}{W/W_0} \right) \frac{\bar{Z}}{KT_e} \quad \text{sh} \quad (T_e \text{ in eV})$$

where $\rho_0 = .00129 \text{ g/cm}^3$ and $W_0 = 10^7 \text{ watts/cm}^2$. Since $\bar{Z} \sim \rho^{-1/2}$, $K \sim \rho^{3/2}$, we see that $\tau_{\text{abs}} \sim \rho^{-1}$. We can fit τ_{abs} reasonably well by

$$\tau_{\text{abs}} = 50 \left(\frac{\rho_0}{\rho} \right) \left(\frac{W_0}{W} \right) \left(\frac{T_0}{T} \right)^2 \quad \text{sh} \quad (9)$$

where $T_0 = .2 \text{ eV}$.

Values of τ_{abs} when $W = W_0$ are shown in Fig. 5 for $\rho = \rho_0$ and $\rho = 10\rho_0$, the latter corresponding to the density of shocked air. Since $\tau_{\text{abs}} \sim T^{-2}$ from (9), we can integrate (5) and obtain

$$t_{1 \rightarrow 2} = \frac{\tau_1}{2} \left[1 - \left(\frac{T_1}{T_2} \right)^2 \right] \approx \frac{\tau_1}{2} \quad \text{if } T_2 \gtrsim 3T_1$$

for the time to heat the electrons from temperature T_1 to approximately $3T_1$, where $\tau_1 = \tau_{\text{abs}}(T_1)$. If the laser intensity is pulsed in time, with a characteristic pulse time t_p , ignition of an LSD wave will not occur unless the electrons initially produced by the target vapor are sufficiently hot so that $\frac{\tau_{\text{abs}}}{2} \ll t_p$.

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Table I

Comparison of Eq. (2) with Karzas and Latter $\bar{g}_{ff}$

	γ^2/u	.003	.01	.03	.1	.3	.5	1	3	10
Ref. 3:	.1	3.54	2.94	2.42	1.92	1.53	1.37	1.17	.87	.59
Eq. (2):		3.50	2.91	2.43	1.98	1.64	1.51	1.35	1.17	1.08
	1	[3.22 3.19]	[2.65 2.63]	2.19	1.78 1.79	1.50 1.50	1.40 1.39	1.29 1.27	1.15 1.13	.97 1.08
	3	[2.99 3.03]	[2.44 2.48]	2.02 2.06	1.65 1.69	1.42 1.43	1.34 1.33	1.26 1.23	1.18 1.12	1.09 1.08
	10	2.72 2.85	[2.21 2.31]	1.82 1.91	1.51 1.57	1.32 1.35	1.26 1.27	[1.20 1.18]	1.15 1.10	1.13 1.08
	100	2.23 2.76	1.80 2.03	1.51 1.60	1.29 1.31	1.17 1.13	1.13 1.13	[1.10 1.10]	1.08 1.08	1.08 1.08

Table II

Inverse bremsstrahlung absorption coefficients for normal
density air at $\lambda = 10.6\mu$

T(eV)	$\bar{Z}^*$	$K_i(\text{cm}^{-1})$	$K_a(\text{cm}^{-1})$	$K_c(\text{cm}^{-1})$	$\sqrt{\epsilon}$
.05	1.01×10^{-59}	1.92×10^{-112}	1.33×10^{-57}	1.61×10^{-57}	1
.1	4.49×10^{-30}	2.10×10^{-53}	1.62×10^{-27}	1.01×10^{-27}	1
.2	3.93×10^{-15}	7.70×10^{-24}	3.24×10^{-12}	1.25×10^{-12}	1
.3	4.45×10^{-10}	6.20×10^{-14}	5.63×10^{-7}	1.74×10^{-7}	1
.4	1.65×10^{-7}	6.09×10^{-9}	2.78×10^{-4}	7.46×10^{-5}	1
.5	6.11×10^{-6}	6.36×10^{-6}	.0127	.00308	1
.7	4.13×10^{-4}	.0193	1.04	.246	.999
1	.0107	8.34	27.4	7.53	.971
1.2	.0386	87.3	94.8	29.0	.889
1.5	.138	844	291	104	.499
2.0	.439	5.97×10^3	558	248	0

*From subroutine described in Ref. (5).

Figure 1

Maxwell-averaged Gaunt factor vs u
(from Ref. 3)

Figure 2

γ^2 and u vs temperature

Figure 3
 $\bar{Z}^2$ vs temperature for normal density air

Figure 4

Comparison of Born approximation and Karzas and Latter
calculations of $\bar{\sigma}_{ff}$ vs temperature for air

Figure 5

Inverse bremsstrahlung absorption time for air vs temperature

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