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**RADIATION GAGE IMAGING
FOR NONDESTRUCTIVE TESTING APPLICATIONS**

Donald W. Sweeney

September 5, 1974

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Contents

Abstract	1
Introduction	1
Basic Equations	2
Use of Discrete Data	6
Required Sampling Rate	7
Effect of Finite Aperture Size	9
Experimental System	11
Scanning System	11
Radiation System	13
Recording System	13
Experimental Procedure and Results	17
Discussion	16
References	18

RADIATION GAGE IMAGING FOR NONDESTRUCTIVE TESTING APPLICATIONS

Abstract

A radiation gage is used to reconstruct cross-sectional images of the linear absorptivity distribution of complex objects for nondestructive testing applications. The objects must be scanned in many different directions, the amount of data needed depending on the resolution required in the reconstructed image. For typical objects scanned in

nondestructive testing, attainable resolution is limited by the finite size of the detector aperture. Two Fourier-transform reconstruction techniques are described in detail. One of the techniques, direct inversion, is used to reduce data from a scanning radiation gage. The resolution of the resulting reconstructed images approached the resolution expected.

Introduction

When electromagnetic radiation passes through an absorbing material the radiation is attenuated by an amount proportional to the absorptivity of the material and the geometric length of the path followed by the radiation through the material. If the geometric length of the path is known, average absorptivity along the path can be determined. Since the absorptivity of a material is a function of its density, it is possible to detect undesirable density variations in the material nondestructively by measuring radiative attenuation. Attenuation measurements using either gamma- or x-ray radiation are well-known nondestructive testing techniques,^{1,2} and radiation gages, often called DXT gages, are commercially available for this purpose.

This report describes a procedure for using radiation gage measurements when

the geometric length of the radiation path through an object is not known and when absorptivity varies along that path. The procedure results in a cross-sectional image of the scanned object. Deriving this image from the radiation gage data is referred to as image reconstruction.

The basic components of a radiation gage are shown in Fig. 1. A source is placed behind a collimating aperture so that only a thin beam of radiation can propagate through the target object. The radiation passes directly through the object and then on through the detector aperture until it strikes the detector. The two apertures (collimating and detector) determine the position and shape of the beam. More importantly, these apertures limit the effects of scatter. Once a photon is scattered, it is unlikely to pass through the detector aperture.

Fig. 1. Basic radiation gage components.

This means only the radiation that passes directly through the object is measured.

When a radiation gage is used to detect density variations, the ratio of transmitted radiation intensity at the detector I to radiation intensity without the object

present I_0 often is expressed in the simplified form

$$I/I_0 = \exp(-\mu t), \quad (1)$$

where μ is the linear absorptivity coefficient and t is the thickness of the target material. Equation (1) implicitly assumes constant absorptivity along the radiation path and does not include the effect of finite aperture size. For objects composed of several different materials in complex geometry, a more precise description of the attenuation process is necessary.

The basic integral equations that describe the attenuation process in complex objects and a procedure for inverting the resulting integral equation are presented in the next two sections. The result of inverting this integral equation is a cross-sectional image of the linear absorptivity coefficient of the object. Following the mathematical derivations, the remainder of the report describes a scanning radiation gage and presents examples of the imaging process.

Radiation gage imaging is very similar to imaging processes in radio astronomy, electron microscopy, optical interferometry, and plasma physics. Relevant papers in these fields are referenced.

Basic Equations

For simplicity, the effects of finite beam size are not considered in this section, but are discussed separately in the next section. Also, to conform with notation used in related literature, the symbol f is used to denote linear

absorptivity, rather than the more familiar μ .

Consider a single ray of a radiation beam passing through an heterogeneous material with an absorption coefficient that is a function of the spatial coordinates

x , y , and z . The differential attenuation of the beam as it travels a distance ds through a region with absorptivity $f(x, y, z)$ is

$$\frac{dI_i}{I_i} = -f(x, y, z) ds_i, \quad (2)$$

where the subscript i denotes the i th ray. Total attenuation can be determined by integrating Eq. (2) to obtain

$$(I/I_0)_i = \exp \left[\int_{s_i} -f(x, y, z) ds_i \right], \quad (3)$$

where the line integral is evaluated along the ray path s_i . Taking the natural log of Eq. (3) results in

$$-\ln(I/I_0)_i = \int_{s_i} f(x, y, z) ds_i. \quad (4)$$

Again, to conform with existing literature, we define

$$g_i = -\ln(I/I_0)_i,$$

where g_i is the pathlength of the i th ray. Substituting this definition into Eq. (4) results in

$$g_i = \int_{s_i} f(x, y, z) ds_i. \quad (5)$$

The objective of image reconstruction is to determine $f(x, y, z)$, given g_i values for a large collection of rays.

For simplicity, a two-dimensional distribution $f(x, y)$ can be considered without loss of generality, since any arbitrary plane within a three-dimensional distribution can be studied separately. It is convenient then to rewrite Eq. (5)

$$\int_S f(x, y) ds = g(\rho, \theta), \quad (6)$$

where ds is the differential element of the geometric ray path and $g(\rho, \theta)$ is the pathlength of the ray specified by coordinates (ρ, θ) , defined in Fig. 2. All possible rays are included in the range $-90^\circ < \theta \leq 90^\circ$.

Fig. 2. Notation for Fourier analysis of pathlength data. In spatial plane, ray is specified by parameters ρ and θ . In frequency plane, pathlength data for all rays with viewing angle θ determine the transform values along a radial line through the origin with angle $\theta + \pi/2$.

The solution of Eq. (6) was considered first in radio astronomy. Bracewell,³ Bracewell and Riddle,⁴ and Smerd and Wild⁵ presented detailed discussions on the Eq. (6) solution, using Fourier analysis. Later, DeRosier and Klug⁶ independently developed the same relationships for electron microscopy. Work in this area has been summarized by Klug.⁷ Crowther et al.⁸ have proposed several numerical techniques for image reconstruction schemes. Rowley⁹ also developed the Fourier relationships for use in three-dimensional interferometry. A good summary of a variety of inversion schemes is given by Gordon.¹⁰

Basically, inversion schemes can be divided into two groups: series expansion techniques and Fourier transform methods. All series expansion techniques use the expansion

$$\tilde{f}(x, y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} a_{mn} H_{mn}(x, y) \quad (7)$$

to represent $f(x, y)$. In this expansion $H_{mn}(x, y)$ are the generating functions and a_{mn} are their coefficients. Since a finite number of terms are used in the expansion, the series $\tilde{f}(x, y)$ is an approximation of $f(x, y)$.

Introducing the series expansion, Eq. (7), into the basic integral equation, Eq. (6), and interchanging the order of integration and summation yields

$$\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} a_{mn} \int_S H_{mn}(x, y) ds = g(\rho, \theta). \quad (8)$$

The objective is to select a_{mn} coefficients that will satisfy Eq. (8).

Often, determining a_{mn} coefficients reduces to solving a system of simultaneous equations. Since the work reported here did not use series reconstruction techniques (for reasons discussed later) and since these techniques already are well documented in the literature, they are not discussed here. A good guide to this literature is presented in Ref. 10.

Basic Fourier transform relationships can be obtained easily. Consider a single view of the field $f(x, y)$ normal to the y axis (i.e., $\theta = 0$). The pathlength data for this view are $g(\rho, 0)$, making Eq. (6)

$$g(\rho, 0) = g(y, 0) = \int_{-\infty}^{+\infty} f(x, y) dx, \quad (9)$$

where integration limits have been extended to $\pm\infty$. Taking the one-dimensional Fourier transform of both sides of Eq. (9), with respect to y , results in

$$\begin{aligned} \mathcal{F}_y\{g(y, 0)\} &= \int_{-\infty}^{+\infty} \exp(-2\pi i f_y y) dy \int_{-\infty}^{+\infty} f(x, y) dx \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) \\ &\quad \times \exp\{-i2\pi(f_x x + f_y y)\} dx dy \Big|_{f_x=0}. \quad (10) \end{aligned}$$

Fourier transform notation is shown in Fig. 2.

The last expression in Eq. (10) indicates that the Fourier transform of pathlength data for a view normal to the y axis is equal to the two-dimensional Fourier transform of $f(x, y)$ evaluated along the f_y axis. This relationship, generalized to include all θ values between $\pm 90^\circ$, is

$$F_p[g(\rho, \theta)] = F(R, \beta) - F(R, \theta + \pi/2) \\ \int_{-\infty}^{+\infty} g(\rho, \theta) \\ \times \exp(-i2\pi R\rho) d\rho \quad (11)$$

Equation (11) shows that pathlength data for each direction of view can be used to generate a single radial line through the origin in the Fourier transform plane of $f(x, y)$. The Fourier transform then can be filled one line at a time until the entire frequency plane is determined. Then, $f(x, y)$ can be determined by taking the inverse transform of $F(f_x, f_y)$:

$$f(x, y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(f_x, f_y) \\ \times \exp[i2\pi(f_x x + f_y y)] df_x df_y. \quad (12)$$

In practice, data can be collected for only a finite number of views and positions. Therefore, Fourier transform relationships must be modified to accept discrete data. To accomplish this, Eqs. (11) and (12) can be recast in terms of discrete Fourier transforms; however, certain format problems arise in treating the frequency plane. (In this report, image reconstruction techniques employing Eqs. (11) and (12) are referred to as Fourier synthesis.)

The direct inversion of Eq. (6) can be obtained by formally inverting Eq. (11) to express $f(x, y)$ as a functional of $g(\rho, \theta)$. This technique, when implemented numerically, avoids the format problems associated with Fourier synthesis. The direct relationship between $f(x, y)$ and

pathlength data has been developed independently by Berry and Gibbs,¹¹ Alwang et al.,¹² and Juninger and van Haeringer.¹³ This relationship can be obtained quite easily by converting Eq. (12) to polar coordinates, introducing an identity, and using a few basic properties of the Fourier transform. Both spatial and frequency planes in Fig. 2 can be represented in polar coordinates

$$x = r \cos \alpha \\ y = r \sin \alpha \\ f_x = R \cos \beta = -R \sin \theta \\ f_y = R \sin \beta = R \cos \theta. \quad (13)$$

When Eq. (13) is substituted into Eq. (12), the result is

$$f_p(r, \alpha) = \int_{-\pi/2}^{+\pi/2} d\theta \int_{-\infty}^{+\infty} |R| F_p(R, \theta) \\ \times \exp[i2\pi Rr \sin(\alpha - \theta)] dR, \quad (14)$$

where Hermitian symmetry properties have been used and the subscript p indicates that F and f have been recast in polar coordinates. Equation (14) can be rewritten by introducing the identity

$$|R| \equiv R \operatorname{Sgn}(R) = (-i2\pi R) [1\pi \operatorname{Sgn}(R)] 2\pi^{-2},$$

where

$$\operatorname{Sgn}(R) \equiv \begin{cases} +1 & \text{if } R > 0 \\ 0 & \text{if } R = 0 \\ -1 & \text{if } R < 0. \end{cases}$$

If we let $z = r \sin(\alpha - \theta)$, Eq. (14) becomes

$$f_p(r, \alpha) = \frac{1}{2\pi^2} \int_{-\pi/2}^{+\pi/2} d\theta \int_{-\infty}^{+\infty} dR$$

$$\times (-i2\pi R)[i\pi \text{Sgn}(R)]F_p(R, \theta) \\ \times \exp(i2\pi Rz). \quad (15)$$

Using Eq. (11) and a few fundamental Fourier transform relationships, Eq. (15) can be rewritten

$$f_p(r, \alpha) = \frac{1}{2\pi^2} \int_{-\pi/2}^{+\pi/2} d\theta \left[\frac{\partial g(\rho, \theta)}{\partial z} \right] = \frac{1}{z}, \quad (16)$$

where the asterisk denotes the convolution integral. Expanding the convolution and noting that $z = r \sin(\alpha - \theta)$, the final result is

$$f_p(r, \alpha) = \frac{1}{2\pi^2} \int_{-\pi/2}^{+\pi/2} d\theta \int_{-\infty}^{+\infty} \\ \times \frac{\partial g(\rho, \theta)/\partial \rho}{r \sin(\alpha - \theta) - \rho} d\rho. \quad (17)$$

Equation (17) is a direct expression of absorptivity distribution in terms of radiation pathlength.

USE OF DISCRETE DATA

When discrete data are used, the Fourier transforms defined in Eqs. (11) and (12) must be recast in terms of discrete Fourier transforms. The discrete Fourier transform of a two-dimensional function $f(x, y)$ is (cf. Ref. 14)

$$F\left(\frac{m}{L_x}, \frac{n}{L_y}\right) = \frac{1}{4B_x B_y} \sum_{l=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} \\ \times f\left(\frac{l}{2B_x}, \frac{k}{2B_y}\right) \\ \times \exp\left[-i2\pi \frac{ml}{2B_x L_x} + \frac{nk}{2B_y L_y}\right], \quad (18)$$

and the discrete inverse Fourier transform is

$$f\left(\frac{l}{2B_x}, \frac{k}{2B_y}\right) = \frac{1}{L_x L_y} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \\ \times F\left(\frac{m}{L_x}, \frac{n}{L_y}\right) \\ \times \exp\left[-i2\pi \left(\frac{ml}{2B_x L_x} + \frac{nk}{2B_y L_y}\right)\right], \quad (19)$$

where L_x and L_y are the spatial limits of the function $f(x, y)$ and B_x and B_y are its bandwidths. Values of the function $f(x, y)$ located between the sampled locations $x = l/2B_x$ and $y = k/2B_y$ are determined in terms of the series

$$f(x, y) = \sum_{l=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} f\left(\frac{l}{2B_x}, \frac{k}{2B_y}\right) \\ \times \text{sinc}\left[2B_x \left(x - \frac{l}{2B_x}\right)\right] \\ \times \text{sinc}\left[2B_y \left(y - \frac{k}{2B_y}\right)\right], \quad (20)$$

where $\text{sinc } x \equiv \sin \pi x / \pi x$. A similar expansion can be used in the frequency plane.

One approach to reconstructing the Fourier transform of the refractive index field is to use a discrete, truncated form of Eq. (11). As pathlength data are taken for various angular orientations, the transform plane is determined at sampled locations along radial lines extending from the origin. Figure 3 shows several representative positions at which the transform plane can be determined. The two-dimensional inverse Fourier transform, Eq. (19), and the interpolation series, Eq. (20), can be evaluated to

determine $f(x, y)$. Before the inverse transformation can be used, however, values in the frequency plane must be specified for the rectangular array of sample points required by Eq. (19).

This reformatting of the frequency plane can be accomplished in several ways. One method is to apply concepts developed by J. L. Harris¹⁵ in super-resolution. Although mathematically rigorous, this technique normally requires solving a large set of simultaneous algebraic equations. Another method is to interpolate between values on the known radial lines to determine values at specified grid points. Interpolation may be used if pathlength data are available for a sufficiently large number of angular orientations (discussed later).

A procedure based on the direct inversion formula, Eq. (17), avoids frequency plane formatting problems. Care must be exercised, however, in numerically evaluating the integral over ρ in Eq. (17). It is convenient to use the direct solution in the form of Eq. (16). Using the convolution theorem and a few basic properties of Fourier transforms, it can be shown that

$$\begin{aligned} \int_{-\infty}^{\infty} \frac{[\partial g(\rho, \theta)] \partial \rho}{r \sin(\alpha - \theta) - \rho} \\ = \frac{\partial g(z, \cdot)}{\partial z} \cdot \frac{1}{z} \Big|_{z=r \sin(\alpha-\theta)} \\ - 2\pi^2 \mathcal{F}_f^{-1} \left\{ \left| \mathcal{F}_z [g(z, \theta)] \right\} \Big|_{z=r \sin(\alpha-\theta)} \right. \end{aligned} \quad (21)$$

For a fixed θ , the value of the integral over ρ in Eq. (11) can be calculated by

taking the Fourier transform of the pathlength data, multiplying it by a linear factor, and then inverse-transforming. This operation determines the value of the integral for all values of r . The procedure is repeated once for each direction of view (i.e., for each discrete θ value).

This direct inversion technique is particularly attractive for discrete data in that the fast Fourier transform algorithm developed by Cooley and Tukey¹⁶ enables the inversion process to be completed quite rapidly and with a minimum of computer storage.

REQUIRED SAMPLING RATE

When using discrete data (i.e., sampled $g(\rho, \theta)$ values), it is important to determine the required $g(\rho, \theta)$ sampling frequency for both variables ρ and θ . It is well known from sampling theory¹⁷ that the Fourier transform of a continuous function can be determined exactly from sampled values collected at a finite rate (Nyquist rate) equal to twice the bandwidth of the function. Hence, if Eq. (11) is used to reconstruct the Fourier transform along a radial line in the frequency plane, the spacing between pathlength samples (as a function of ρ) for that viewing angle should not exceed $1/2B$, where B is the effective bandwidth of the field in the corresponding θ direction. If this condition is not satisfied, the transform is undersampled and aliasing will ultimately lead to errors in the reconstructed image. In practice, some errors of this type are unavoidable, since distributions that are spatially limited cannot also be band limited. An effective bandwidth can be defined, however, if the Fourier components

of samples can be compared with the space-bandwidth product (SBWP) of the reconstructed image ($4B_x B_y L_x L_y$), which is a measure of the total number of degrees of freedom in the image. Since the number of degrees of freedom is invariant under Fourier transformation (if Hermitian symmetry is noted), the SBWP of the series expansion for the frequency plane is also $4B_x B_y L_x L_y$. And, since

each independent pathlength measurement should provide a new degree of freedom in the reconstruction, the suggested sampling scheme yields nearly twice as many optical pathlength measurements as are theoretically needed. Thus, if Eq. (11) or Eq. (17) (the direct inversion technique) is used as presented here, it will have a data redundancy of about 2.

Effect of Finite Aperture Size

Since a radiation gage has a finite aperture size, the observed attenuation distribution $g(\rho, \theta)$ is smoother than the true distribution. The greater the size of the aperture the greater the smoothing effect. This effect of aperture size is important because it determines the maximum obtainable resolution in the final reconstructed image. In addition, aperture size determines the rate at which the function $g(\rho, \theta)$ must be sampled. The smoothing effect can be at least partially removed to improve resolution in the reconstructed image.

A clear description of the smoothing effects of finite aperture size is provided by Bracewell and Roberts¹⁹ in connection with aerial smoothing in radio astronomy. Here, only the basic equations and some of the unique aspects of smoothing in radiation gaging are discussed.

The relationship between the measured distribution $g(\rho, \theta)$ and the true distribution that would be measured if aperture were infinitely small $\bar{g}(\rho, \theta)$ is expressed as

$$g(\rho, \theta) = \int_{-\infty}^{+\infty} \bar{g}(\xi, \theta) a(\rho - \xi) d\xi, \quad (23)$$

where $a(\xi)$ is the aperture reception pattern. In linear systems analysis, $a(\xi)$ would be referred to as the impulse response. In Eq. (23), θ should be considered a parameter rather than a variable. That is, Eq. (23) is valid for each discrete value of θ .

The function $a(\xi)$ is the signal obtained when an infinitely thin slit is passed across the face of the aperture. This function can be calculated analytically or can be obtained experimentally by passing a step wedge in front of the aperture and differentiating the signal.

Using radiation gage response to a slit input to define the aperture reception pattern involves an approximation. The aperture of a typical radiation gage is circular. This means the signal received by the detector includes an integration (or smoothing) along a direction normal to the x-y plane (i.e., in the z-axis direction, see Fig. 2). However, this derivation was previously limited to consider only slices of objects in the x-y plane.

By defining the aperture reception pattern as the response to a slit input, we have included the effects of the circular

aperture but neglected the smoothing effects of the aperture in the z-direction. Therefore, the following discussion applies directly only to objects that do not exhibit rapid absorptivity variations in the z-direction. For objects which do not meet this requirement, the analysis must be extended to include the third dimension. In this case, the aperture reception pattern would be defined in terms of the system response to a point source.

The relationship between $g(\rho, \theta)$ and $\bar{g}(\rho, \theta)$ can be seen by comparing the Fourier spectrums of $g(\rho, \theta)$ and $\bar{g}(\rho, \theta)$. Taking the Fourier transform of both sides of Eq. (23) results in

$$G(s, \theta) = \bar{G}(s, \theta)A(s), \quad (24)$$

where capital letters denote the Fourier transform and s is the independent variable in the frequency plane. Equation (24) indicates that the measured Fourier components equal the Fourier components of the true distribution multiplied by the Fourier components of the detector reception pattern. Equation (24) also suggests a method for restoring the received signal by multiplying both sides of Eq. (24) by $1/A(s)$. This is a valid procedure for regions where $A(s)$ is not zero.¹⁹

As discussed in the "Required Sampling Rate" section, the function $g(\rho, \theta)$ must be sampled as a function of ρ at a rate equal to or greater than twice the maximum frequency of $g(\rho, \theta)$. If the transform $A(s)$ is effectively zero beyond a certain frequency, it controls the maximum frequency that can be present in $g(\rho, \theta)$. Thus, the maximum sampling rate re-

quired is determined by finite aperture size and is equal to twice the effective maximum frequency transmitted by the aperture.

Equation (24) shows that the maximum frequency present in $g(\rho, \theta)$ determines the number of views that must be used. Therefore, aperture size determines the number of views required.

Aperture size also controls resolution of the reconstructed image. Using the Rayleigh criterion as a measure of resolution, resolution is defined as the separation that would have to exist between two slits in the object field so that the response of the detector, as it scans across the slits, can "just resolve" the two separate slits. The term "just resolve" is often defined as requiring a 19% dip in the detector signal with respect to the maximum signal when the detector is centered between the two slits. For a symmetric aperture reception pattern, resolution is equal to the root of the following equation:

$$\frac{a(\xi/2)}{a(0)} - \left(\frac{1 - .19}{2} \right) = 0, \quad (25)$$

where $a(0)$ is the value of the reception pattern at the origin.

For example, consider a radiation gauge with a circular aperture of radius r . Let the object to be scanned by the gauge be bounded in the $x - y$ plane by a circle of radius R . Now, to simplify the equations, assume that the radiation entering the detector aperture is uniform in intensity when no object is present. In this case,

$$a(\xi) = \begin{cases} [1 - (\xi/r)^2]^{1/2} & \text{for } \xi/r < 1 \\ 0 & \text{otherwise.} \end{cases} \quad (26)$$

The Fourier transform of Eq. (26) is

$$A(s) = \mathcal{F}\{a(\xi)\} = r \frac{J_1(2\pi rs)}{2rs}, \quad (27)$$

where J_1 is the first order Bessel function. Figure 4 shows sketches of $a(\xi)$ and $A(s)$. Clearly, $A(s)$ has no sharp cutoff. If the first zero of $A(s)$ is taken as the effective maximum frequency, the maximum frequency present in $g(\rho, \theta)$ is

$$s_{\max} = 3.83/2\pi r, \quad (28)$$

where 3.83 is the first zero of J_1 . The required spacing between samples of $g(\rho, \theta)$, as a function of ρ , is

$$\Delta \rho = \frac{1}{2s_{\max}} = \frac{2\pi r}{(2)(3.83)}, \quad (29)$$

which is slightly less than the radius r . The angular spacing between views $\Delta \theta$ is determined using Eq. (22):

$$\Delta \theta = \tan^{-1}\left(\frac{1}{BL}\right) = \frac{2\pi r}{(3.83)(2)(R)}. \quad (30)$$

Resolution d is obtained using Eqs. (25) and (26):

Fig. 4. Aperture reception pattern $a(\xi)$ for circular aperture of radius r and its Fourier transform $A(s)$.

$$d = 2r \left[1 - \left(\frac{.81}{2} \right)^2 \right]^{1/2} \approx 1.8r. \quad (31)$$

Experimental System

The radiation gage used in this study is shown in Fig. 5. Originally designed and built by a private contractor to Lawrence Livermore Laboratory specifications, the gage basically consists of three separate systems: a scanning system, a radiation measurement system, and a data recording system. These systems are described briefly in the following subsections. In these descriptions, numbers in parentheses refer to labeled components in Fig. 5.

SCANNING SYSTEM

The radiation gage has a fixed source and detector. An $x-y$ translation table (2) laterally positions the object to be scanned, and a rotation stage (8) does the angular positioning. Referring to Fig. 5, moving the translation stage (2) in the x -track direction changes the ρ variable (defined in Fig. 2), and turning the rotation stage (8) corresponds to changing the θ variable (also defined in

Fig. 2). Translation in the y-direction was not used.

Both translation table (2) and rotation stage (8) are driven by stepping motors (14, 3). The digital control system for these motors produces the following scan sequence. First, the table translates a programmable distance in the x-direction. Then, there is about a 3-sec pause to allow vibrations of the radiation gage structure to subside. One second before the end of the pause, a command pulse initiates the radiation measurement system. After the pause, the translation table again moves in the x-direction and the scan-pause process is repeated. After a predetermined number of pauses,

Fig. 5. Radiation gage imaging system: 1. Source containment vessel, 2. Translation table, 3. Rotary table stepping motor, 4. Source aperture, 5. Object to be scanned, 6. Detector, 7. Preamplifier, 8. Rotary table, 9. Cooling water circulation lines, 10. Interferometer retroreflector, 11. X-direction lead screw drive, 12. Y-direction lead screw drive, 13. Interferometer laser, 14. X-direction stepping motor.

the rotation stage is rotated and the direction of the translation table is reversed. Scan length, number of pauses per scan, and rotation between scans all are programmable variables. After these initial parameters are entered into the system, the entire scan process is automatic.

To avoid errors introduced by backlash in the lead screw (11) moving the translation table, the x-position of the translation table was measured with an interferometer. The interferometer laser (13) and its retroreflector (10) are shown in Fig. 5. The interferometer can measure the x-position to within 0.01 mm.

RADIATION SYSTEM

The radiation source in these experiments was Iridium 192, with a strength of about 40 curies. A number of photon energies are emitted per disintegration of Iridium 192. However, only three photon energies were of significant abundance in the radiographic applications discussed here. For these primary photon energies, 61% of the photons were at 0.31 MeV; 27% were at 0.47 MeV; and 12% were at 0.60 MeV. The source, located inside a lead containment vessel (1), is exposed by opening a shutter on the container. Mounted on the front of this shutter is the source aperture, a tungsten cylinder (4) 49 mm in diameter and 60 mm long with a circular hole 3.1 mm in diameter drilled along the axis of the cylinder.

The detector aperture, located about 400 mm from the source aperture, consists of a 2.5-mm diameter hole in a 30-mm-thick uranium cylinder.

The detector is a scintillating crystal followed by a photodiode (6). The temperature of the crystal is controlled by circulating constant-temperature water through the frame of the detector. Current output from the photodiode is amplified in a preamplifier (7) and sent to a vibrating reed electrometer, which produces output voltage in proportion to the input current. The output voltage is integrated for 1 sec and then digitized using an integrating digital voltmeter. Output integration is initiated by a trigger command from the scan system.

RECORDING SYSTEM

Digital position data from the interferometer and voltage data from the integrating digital voltmeter are recorded on magnetic tape with an incremental digital tape recorder. After the scan process is complete, the data on magnetic tape are transferred to a CDC-7600 computer disk for reduction.

Experimental Procedure and Results

Several different objects were scanned in this study, and the resulting radiation gage data were processed to reconstruct a cross-sectional image of the objects. Following is the typical procedure followed for each object:

1. Scan parameters were calculated, using the techniques discussed in the previous sections. The total number of pauses for each direction of view was determined so that there were a few stations at which the object was completely out of the beam. This allowed

- a new normalization factor to be obtained for each direction of view.
2. The object was scanned, using the parameters calculated in step 1. Voltage data from the digital voltmeter and position measurements from the interferometer were recorded on magnetic tape.
3. Magnetic tape data were transferred to CDC-7600 computer disk. Voltages $V(\rho, \theta)$ for each scan position were reduced by computer, using the relationship

$$\frac{I(\rho, \theta)}{I_0} = \frac{V_0 - V(\rho, \theta)}{V_0}, \quad (32)$$

where V_0 was the maximum voltage and corresponds to the voltage when the object is out of the beam. Input for the reconstruction algorithms, then, was

$$\begin{aligned} g(\rho, \theta) &= -\ln \left[\frac{I(\rho, \theta)}{I_0} \right] \\ &= -\ln \left[\frac{V_0 - V(\rho, \theta)}{V_0} \right]. \end{aligned} \quad (33)$$

4. Using $g(\rho, \theta)$ values as input for the reconstruction algorithm, the computer calculated the reconstructed image by the direct inversion technique.
5. The calculated image was plotted by computer graphics. The display technique consisted of dividing the $x - y$ plane into 60 resolution elements in both x - and y -directions. The absorptivity coefficient was calculated at the center of each of these 3600 square resolution elements and the entire resolution element was assigned this value. Each resolution element was represented on photographic film as a square array of four points. By controlling the intensity of the four points, it was possible to obtain a total of eight grey levels. Darker regions of the image corresponded to higher linear absorptivity coefficients. This was not the optimum display method for this application, primarily because of its limited grey-scale range. However, more suitable display techniques were not available at the time of the study.

The reconstructed images of some of the objects scanned are shown in Figs. 6 through 10. The scan parameters for each of the images are given in the figure captions.

Figure 6 shows the reconstructed image of three aluminum posts: one circular 25 mm in diameter, another square with 25-mm sides, and the third hexagonal with 25-mm between opposite faces. The shapes and relative positions of the three posts are clearly evident in the image. The dark regions around the perimeter of each post are overshoots, a characteristic of images reconstructed using Fourier transform techniques. This overshoot effect appears only at sharp edges and is known as Gibb's phenomena.

Figure 7 shows the reconstructed image of the handle of a two-cell plastic flashlight. The low-density plastic case around the batteries is clearly visible. The high-density battery case and its core also can be seen. The material between the battery case and the core is

Fig. 6. Reconstructed image of three aluminum posts. Spacing between samples 1.25 mm (function of ρ). Ninety equally-spaced views used between 0 and 180°.

Fig. 7. Reconstructed image of two-cell, plastic flashlight. Spacing between samples 1.25 mm (function of ρ). Ninety equally-spaced views used between 0 and 180°.

Fig. 9. Reconstructed image of stapler. Spacing between samples 1.25 mm (function of ρ). Ninety equally-spaced views used between 0 and 180°.

Fig. 8. Reconstructed image of speaker in telephone headset. Spacing between samples 1.25 mm (function of ρ). Ninety equally-spaced views used between 0 and 180°.

Fig. 10. Reconstructed image of magnifying glass. Spacing between samples 1.25 mm (function of ρ). Ninety equally-spaced views used between 0 and 180°.

of such low absorptivity that it is effectively zero for this coarse grey-level display. The small rectangular dark region between the battery and the plastic case is the copper conduction strap.

Figure 8 shows the reconstructed image of the speaker in a telephone headset. The plastic case and part of the speaker assembly are visible. The thin membranes in the center of the speaker

are not visible. Although these membranes have a high absorptivity, the image reconstruction process pictures the membranes of low absorptivity because they are thinner than the resolution limit of the system. In this case, absorptivity is lowered to effectively zero for this display technique. This misrepresentation of the membranes is an example of the smoothing effect of the finite aperture.

Figure 9 shows the image of a cross-sectional slice of a paper stapler. The metal base, plastic handle, and body of

the staples are clearly visible. The two circular structures within the body are the springs that hold the staples in position. The skew appearance of the stapler handle is an artifact of the display device.

Figure 10 is the reconstructed image of a magnifying glass inside its plastic case. The dark area in the figure is the glass lens, the light area is the case. The image clearly shows the shape of the lens. It would be difficult to determine lens shape from a conventional radiographic shadowgraph.

Discussion

Two factors to consider in radiation gage imaging are the amount of data needed and the time required to collect this information. Using the radiation gage described here, roughly 10,000 data points and about 6 hr are needed to obtain a 1-mm-resolution image of an object bounded in the $x-y$ plane by a 100-mm-diameter circle. The time required to collect the data does not present a major problem, since scanners could be designed to complete the entire scan in only a few minutes.

The amount of data required, however, does present limitations. The series reconstruction techniques in particular require very large computer memories. In this regard, only the Fourier synthesis and direct inversion techniques are readily applicable, particularly if images are to be reconstructed by mini-computer. Although Fourier techniques are as accurate as series techniques, they are not as flexible. For example, Fourier techniques require that data be uniformly

spaced and collected over a 180° angle of view; series techniques do not have this requirement.

Image reconstruction could also be accomplished by making a number of conventional radiographic shadowgraphs from a number of different viewing angles. The film could then be digitized to provide the necessary input data. While theoretically possible, this technique would be difficult to implement because of the effects of scatter and the limited dynamic range of radiographic films.

No attempt was made in this study to image test objects of known absorptivity distributions, primarily because the radioactive isotope used (Iridium 192) emitted three significant photon energies. For a given material, radiation in these three energy levels have different linear absorptivity coefficients. This introduces a nonlinearity in the attenuation process that is not included in the image reconstruction procedure. Since cobalt and cesium are readily available sources that

emit photons at nearly a single energy level, it was felt that precise testing of the imaging system should be done with either of these sources. The results presented in the previous section indicate, however, that the multiple photon energy levels did not significantly affect the experiments.

Recently, there has been growing interest in radiation scanners for medical applications, particularly scanners for *imaging cross-sectional slices* of the brain to detect tumors. Although image reconstruction problems are similar in both medical and industrial radiation scanning, there are a few factors unique to industrial, nondestructive testing applications. Objects scanned in non-destructive testing generally are stationary and not subject to radiation damage. Therefore, high-energy-photon-emitting sources and long scan times are permissible. Also, resolution in non-destructive testing typically is quite low; resolution requirements on the order of 0.1 to 0.5 mm are not uncommon. Typical required resolution for medical scanning, however, is a few millimeters. Finally, industrial detectors must have a larger *dynamic range*, since the contrast of

objects to be imaged in nondestructive testing generally is greater than in medical applications.

This study demonstrated the feasibility of using *radiation gage imaging for non-destructive testing*. With a fast scanning system coupled directly to a mini-computer, it should be possible to produce cross-sectional images of scanned objects within a few minutes and with a resolution in the submillimeter range; but such a system would be expensive. Constructing a fast scanner system, or even using the slow scanner described here, to produce images is justified only if a particular application demands such a sophisticated analysis. Merely to locate a defect, radiographic shadowgraphs taken from several different directions can be used; radiation gaging would produce an image of the defect. There are applications, however, in which a shadowgraph is too complicated to interpret. Consider, for example, the problem of finding a defect in a bundle of several hundred tubes from a nuclear reactor cooling system. In this situation, radiation gage scanning would be an extremely valuable nondestructive testing method. This study has demonstrated the feasibility of such a technique.

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