MASTER

**NUMERICAL STUDIES OF MASS TRANSFER AND
ACCRETION IN X-RAY BINARY SYSTEMS**

M. L. Alme

September 25, 1974

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M. L. Alme
(Ph. D. Thesis)

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CONTENTS

Abstract	v
1. Introduction	1
2. Observations of the X-ray Binaries	3
Centaurus X-3	9
Hercules X-1	12
Cygnus X-1	20
SMC X-1	23
3U 0900-40 (Vela X-1)	24
3U 1700-37	25
Cygnus X-3	26
Circinus X-1	27
3. The "Standard" Model for X-ray Sources in Close Binary Systems	30
I. Mass Transfer Mechanisms	31
a. Gravitational Lobe Overflow	32
b. Stellar Winds	34
c. X-ray Induced Stellar Winds	35
II. Accretion onto the Secondary and X-ray Production	37
a. Accretion Disks	37
b. Accretion Shocks	40
c. X-ray Emission from Neutron Stars	43
d. Degenerate Dwarfs	45
4. A Preview of Chapters 5 - 8	47
5. X-ray Emission from a Neutron Star Accreting Material	50
I. Introduction	50

II. Notation and Equations	52
III. Method of Solution	56
IV. Results	56
V. Conclusions	70
Appendix	73
6. X-ray Induced Mass Transfer in the HZ Her/Her X-1	
Binary System	75
I. Model	77
II. Equations	82
a. Equations of Motion	82
b. Opacities	85
III. Method of Solution	89
IV. Non-LTE Test Problem	91
V. Results	95
a. Example Problem	95
b. Mass Loss Rate	102
c. Radial Oscillations of the Atmosphere	115
VI. Conclusions	120
7. Time Variation of Optical Radiation from HZ Herculis	123
8. X-ray Heating of the Infalling Gas Stream	129
I. Introduction	129
II. Geodesic Motion	133
III. Heating with Spherically Symmetric Infall	142
a. Spherically Symmetric Model	152
b. Results	153

IV. Radial Infall with Expansion	157
a. "One and a Half Dimensional" Hydrodynamics	157
b. Results	163
V. An Approximate Treatment of the Gravitational	
Potential in the Binary System	163
a. The Binary Gravitational Field	167
b. Results	167
c. Conclusions	172
9. Summary and Conclusions	177
10. Future Work	179
I. Three Dimensional Hydrodynamics	179
II. Mass Ejection from Supergiants	180
III. Accretion onto a Neutron Star	182
Acknowledgments	183
References	184

ABSTRACT

We numerically investigate several aspects of mass transfer and accretion in X-ray binary systems. We perform computer calculations of (1) the X-ray emission resulting from the accretion of material onto a neutron star; (2) the X-ray heating of the surface of HZ Her; and (3) the X-ray heating of a gas stream leaving HZ Her through the inner Lagrangian point, falling toward Her X-1.

In our accretion calculations we find that for high mass ($> 1.4 M_0$) neutron stars, the emitted spectra differ only slightly from blackbody, particularly at lower luminosities. For intermediate mass neutron stars the spectra differ substantially from blackbody spectra, especially at higher luminosities. The calculated spectra differ greatly from blackbody spectra at all luminosities for low mass neutron stars.

We calculate the attenuation and diffusion of a beam of X-rays incident on the surface of HZ Her. Atmospheric hydrogen is assumed to be in LTE. All other elements are treated non-LTE. Compton scattering and energy exchange are included. These time dependent calculations incorporate hydrodynamics with radiation pressure. Our calculations indicate that (a) The mass transfer rate dM/dt is too small to sustain a self-excited accretion-powered X-ray mechanism unless the photosphere is close to the Roche limit. (b) dM/dt is insensitive to differences in spectral shape between blackbody and exponential fits to the observed

Her X-1 X-ray spectrum. (c) For a fixed energy flux per unit area, dM/dt is a mild function of the angle of incidence for the X-rays. (d) Reducing by 10 the relative abundance of elements with $Z > 2$ decreases dM/dt by 7.3, while increasing by 2 the relative abundance of elements with $Z > 2$ increases dM/dt by 1.8. (e) With reduced abundances for $Z > 2$, a large amplitude pulsation of 3 hour period occurs. We also study the response of HZ Her to time variations in the incident X-ray intensity. Assuming cosmic abundances, the optical modulation resulting from 1.24 s pulsed X-rays is about 18 percent. Time of flight smearing over the surface of the star will not reduce this modulation to the observed values. The optical modulation decreases as the relative abundance of materials with $Z > 2$ is decreased.

In our calculations of the X-ray heating of the gas stream, we assume cold material leaves HZ Her, flowing through the inner Lagrangian point. We then calculate the modifications in the hydrodynamical motion due to the X-ray heating. We are able to calculate strong X-ray heating effects for some choices of boundary conditions, but are unable to determine if, in general, X-ray heating effects dominate the motion of the infalling gas.

1. INTRODUCTION

In 1972 the X-ray pulsar Centaurus X-3 was discovered to be in a close binary system (Schreier et al 1972b). This was immediately followed by the discovery that the only other known X-ray pulsar, Hercules X-1, is also in a close binary system (Tananbaum et al 1972b). No more strong X-ray pulsars of this class have been detected (1), but five more X-ray sources have been observed to be in binary systems. There is a growing feeling that many, perhaps most, discrete X-ray sources are in binary systems. This is due to the apparent success of the accretion powered X-ray source model in explaining the observed characteristics of these X-ray sources. This popular model for the X-ray sources in binary systems assumes that one component of the binary system is a normal stellar object (this member will be termed the primary), and that the other (the secondary) is a compact object (degenerate dwarf, neutron star, or black hole). Material from the normal star is accreted onto the compact object; the gravitational energy released as the material falls onto the compact object is assumed to be released as X-radiation. While most of the details of this model are currently under intensive investigation, its broad outlines have

(1) The Crab Pulsar, NP0532, is the only other object known to produce pulsed X-rays. Analysis of its radio pulsations indicates it is not in a binary system.

won general acceptance. The accretion model can supply both the hard X-ray spectra and high luminosities ($1 \times 10^{36} - 1 \times 10^{38}$ ergs/s) observed from the discrete X-ray sources.

In Chapter 2 we review the observations of the X-ray binaries; in Chapter 3 we describe the "standard" mass transfer and accretion model for X-ray sources in binary systems, highlighting its strengths and weaknesses, and indicating aspects of the model that are being investigated by astrophysicists. Chapters 4-8 contain the original research reported in this thesis. We have performed numerical investigations of some aspects of the mass transfer and accretion. We caution the reader that these investigations will appear highly disjoint, almost unrelated. The unifying thread is that they treat some aspect of the mass transfer or accretion. Chapter 4 is a roadmap to the research presented in the following chapters. It briefly describes the research and discusses the significant conclusions. In Chapter 5 we investigate accretion onto a neutron star. Chapters 6 and 7 examine the effects of X-ray heating of the normal star in the HZ Her/Her X-1 system. In Chapter 8 we assume that Roche lobe overflow initiates the mass transfer, and then look at the effect the X-rays have on the gas falling toward the X-ray source. The principal conclusions of this research are summarized in Chapter 9. In Chapter 10 we describe future work on models for X-ray binaries.

2. OBSERVATIONS OF X-RAY BINARIES

There is good evidence that seven of the X-ray sources are in binary systems. Table 2-1 lists these X-ray sources, and presents certain of their characteristics. Also included in this table is Circinus X-1, which may be in a binary system (see discussion below). Immediately following, we summarize the evidence for the binary nature of each of these sources. We then present a description of the observations of each X-ray source. We caution the reader that the X-ray intensities exhibit variations on timescales from milliseconds to months. Two of these variations are thought to be understood. First, Cen X-3 and Her X-1 emit most of their radiation in very regular pulses spaced about a second apart. While the mechanism for producing the pulsed X-rays is not understood, the pulse period is interpreted as the rotational period of a neutron star. Second, all the proposed X-ray binaries except Cyg X-1 exhibit periodic variations on a timescale ranging from a fraction of a day to a few days. These extremely regular variations are interpreted as either partial or complete occultations of the X-ray source by its orbital companion. In addition to these "understood" variations, many of the sources exhibit extended periods of low intensity. The most notorious example is the quasi-periodic 35 day cycle of Her X-1 (see below). The sources seem to go "off", or emit at much lower intensity levels, for arbitrary periods. This general behavior is not understood. The sources also vary in an apparently

TABLE 2-1*
CHARACTERISTICS OF X-RAY BINARIES

Source	Binary Period (days)	Eclipse Duration (days)	Transition Duration (days)	Short-term Variability	Long-term Variability	Velocity (km/s) X-ray Source	Optical Counterpart
Cen X-3	2.08712± 0.00004	0.488± 0.012	0.035± 0.007	4.842 s pulsations	Extended lows	415.1± 0.4	
Her X-1	1.700165± 0.000002	0.24± 0.1	0.025	1.23782 s pulsations	Extended lows, 34.85 ±1.70 day period	169.2± 0.4	
Cyg X-1	5.5999± 0.0009	none		Quasiperiodic pul- sations as short as a few ms.	Single low energy transition		68.2± 1.7
SMC X-1	3.8927± 0.0010	0.60± 0.04	<0.1	Nonperiodic pulsa- tions on times of minutes	Extended lows		50.± (+) 16.
3U0900-40	8.95± 0.02	1.90± 0.05		Nonperiodic pulsa- tions of times of s	No extended lows		26.± 6.
3U1700-37	3.412± 0.002	1.1± 0.07		Nonperiodic pulsa- tions on times as short as 0.1 s	No extended lows		23.± 5.
Cyg X-3	0.199667± 0.000014	The eclipse is not total		No pulsations on times of s	Possible spectral transitions		
Cir X-1	≠ 12.3, > 15.0 (?) (π)			Sharp variations on times less than 0.1 s			

TABLE 2-1 (continued)
CHARACTERISTICS OF X-RAY BINARIES

Source	Optical Counterpart	Spectral Type	Distance	Peak Luminosity (ergs/s, 2-10 keV)
Cen X-3	unnamed (¶)	O9.5 - B0.5 (§)		
Her X-1	HZ Her	F5 at minimum (§)	6 kpc	1×10^{37}
Cyg X-1	HDE 226868	B0 Ib	2 kpc	3×10^{36} 1×10^{37} -
SMC X-1	Sk 160	B0 Ib	61 kpc	3×10^{38}
3U0900-40	HD 77581	B0.5 Ib	1.3 kpc	4×10^{36}
3U1700-37	HD 153919	O6f (T)	1.7 kpc	3×10^{36}
Cyg X-3	None, but IR source and radio source		10 kpc	6×10^{37}
Cir X-1	None (Δ)			

* This table is based on similar tables in Jones et al (1973b) and Gursky (1973b). Unless otherwise noted, the data are taken from those two sources.

(†) Osmer and Hiltner (1974).

(II) Jones et al (1974), Canizares et al (1974).

(¶) Krzeminski (1973).

(§) Rickard (1974).

(Ξ) Bopp et al (1973).

(T) Bessell et al (1974).

(Δ) Jones et al (1973c), Jones et al (1974).

irregular fashion in seconds or fractions of a second. There are no generally accepted explanations for this behavior, although some feel the millisecond variations from Cyg X-1 are due to orbital motion around a neutron star or black hole.

Cen X-3 and Her X-1 are X-ray pulsars with periods 4.8 s and 1.24 s, respectively. The evidence for the binary nature of these two X-ray sources is conclusive. The binary nature of Cen X-3 was originally established (Schreier et al 1972b) because (a) the X-ray source exhibits periodic (2.08 days) variations in intensity, which are interpreted as the X-ray source being occulted by the optical companion, and (b) the X-ray pulse period varies sinusoidally with the same 2.08 day period. This sinusoidal variation in the pulsation period is interpreted as the Doppler shift as the X-ray source orbits the optical companion. With Cen X-3 as the prototype, the X-ray data from Her X-1 (already known to be an X-ray pulsar) were examined for evidence of periodic X-ray occultation and correlated sinusoidal variation in pulsation period. This examination revealed a binary period of 1.7 days (Tananbaum et al 1972b). Further evidence for the binary nature of Her X-1 came with the discovery of its optical counterpart, HZ Herculis, which exhibits the same 1.7 day variation in its optical light curve (Bahcall and Bahcall 1972). Very recently, the optical counterpart to Cen X-3 has also been identified (Krzeminski 1973), providing more evidence for the binary nature of Cen X-3.

With the exception of Cyg X-1, the remaining sources listed in Table 2-1 undergo periodic X-ray occultation. Three of these sources (SMC X-1, 0900-40, and 1700-37) have been identified with optical

counterparts, each of which exhibits regular variation in optical luminosity with the same period at which the X-ray occultation occurs for the corresponding X-ray source (1). X-ray occultation provides the only evidence of the binary nature of Cyg X-3. An infrared source has been identified with Cyg X-3, but it varies in phase with the X-rays (see below).

The blue supergiant HDE 226868, a spectroscopic binary, is believed to be the optical counterpart to Cyg X-1. This identification is based on a correlated X-ray -- radio transition which occurred during March and April 1971 (see Tananbaum et al 1972a and references cited therein). A weak radio source appeared at the location of HDE 226868, and at the same time, the Cyg X-1 X-ray spectrum became substantially harder. The average X-ray intensity in the 2-6 keV band decreased by a factor of 4, and the average X-ray intensity in the 10-20 keV range increased by a factor of 2. No further X-ray -- radio transitions have been observed, so the identification is based on the single transition of March and April 1971. Searches of the Uhuru data for X-ray occultation have proven negative, but very recently the Copernicus satellite has provided some slight evidence for a partial X-ray occultation with the same period as the orbital period of the spectroscopic binary (see the discussion of Cyg X-1 below).

(1) The reader's attention is drawn to the fact that the most powerful of these stellar X-ray sources, SMC X-1, is in the Small Magellanic Cloud. It is fortunate that this binary system is located in a portion of the SMC that allows observation of the optical counterpart.

Cir X-1 was reported to be an X-ray binary with period 12.3 days (Jones et al 1973c). This was based on X-ray occultation data gathered by the Uhuru satellite. However, the MIT group has reported that their OSO-7 data indicate the X-rays were "on" in the middle of what would have been predicted to be occultation based on the 12.3 day period (Canizares et al 1974).

We now present a brief review of the observational characteristics of each of these X-ray sources.

Centaurus X-3

This was the first X-ray source observed to be pulsing periodically (Giacconi et al 1971), and the first found to be a binary (Schreier et al 1972b). The bulk of the observed X-ray power is in the form of a periodic signal with a period of 4.84239 s. The source is in a binary system: the X-ray emission is eclipsed every 2.08707 days, but more important, the period of the emission is observed to vary sinusoidally with the same 2-day period. This variation in period is interpreted as a Doppler shift of 425 km/s corresponding to the motion of the X-ray source around a central star. The center of the eclipse occurs when the Doppler velocity change is zero, consistent with a binary system. The X-ray variations are illustrated in figure 2-1.

The X-ray emission exhibits other timescales in addition to the 4.8 s pulsations and 2.08 day eclipses. The pulsed portion of the X-ray emission disappears for periods of many days in an apparently erratic fashion. During this time and during the eclipse, continuous X-ray emission is observed whose intensity is about 10 percent of the peak intensity of the pulsed portion. It cannot be determined whether this

Fig. 2-1. X-ray variability of Cen X-3. The upper curve shows the 4.8 second pulsations as they appear in the raw data (each bin is 0.096 s). The fitted curve is the result of a fourier analysis. The middle curve is the average intensity folded modulo the eclipse period of 2 days. The period and phase of the eclipse were obtained from the Doppler velocity variations of the pulsing. The lower curve is the time variability on a scale of months and shows extended period: (e.g., February-April) when the source is weak or absent when it was predicted to be seen. (From Gursky 1973a.)

portion of the radiation is present during the time of the pulsing; however its presence during the eclipse suggests that it may be produced independently of the pulsed portion.

The transition into and out of the eclipse requires about 0.8 hours; spectral variations (deficiency of low energy photons) during this period indicate that the X-ray flux is being absorbed rather than being cut off by a sharp edge; i.e., the source appears to be "setting" into an atmosphere before going behind the central star. Also, the time of the transition varies by an hour or more, indicating that the "atmosphere" is not steady.

Based on the Doppler velocity, the mass function is $15 M_{\odot}$. Hence the minimum mass of the optical counterpart is $15 M_{\odot}$. By considering the eclipse duration and by assuming the optical counterpart is within its Roche lobe, Wilson (1972) and Leach and Ruffini (1973) compute the mass of the secondary to be about $0.2 M_{\odot}$, well within the limits for a neutron star.

Based on the similarity to NP0532, the Crab pulsar, it is generally believed that the X-ray emission from Cen X-3 originates from a rotating, magnetic, neutron star. The 4.8 s period of the pulsations is within the range of permissible rotational periods for a white dwarf; however, the only other X-ray source of this kind that we know of, Her X-1, has a period of 1.24 s which is below the limit of stability for the rotation of normal white dwarfs. The shape of the pulse is not consistent with rotation of a source of radiation on the surface which emits isotropically; the pulse is too sharp. The most natural explanation is that it is produced by the same process as is making the X-rays in NP0532. If this is the case, the radiation is beamed, which

has important consequences regarding the average luminosity and number of sources of this type. While the pulsational mechanism may be similar to that of NP0532, the energy source for the X-rays is likely not. The Crab pulsar is thought to derive its X-ray energy from the rotational energy of the central neutron star (cf Ostriker and Gunn 1969); this interpretation is consistent with the observed slowdown of the pulsar. However the pulsation period of Cen X-3 has been observed to be decreasing (Schreier et al 1972b; Schreier 1973). Since the Cen X-3 neutron star is speeding up, its rotational energy is increasing.

The optical companion for Cen X-3 was not found until very recently (Krzeminski 1973). The source is located right on the galactic plane in the direction of a spiral arm. The large optical obscuration in this region made the search for an optical counterpart difficult. Rickard (1974) has estimated the spectral type of Krzeminski's star to be O9.5 - B0.5 Ib.

Hercules X-1

Her X-1 is the only other pulsating X-ray source observed as yet (Tananbaum et al 1972b). Its essential properties are similar to Cen X-3. The pulse period is 1.24 s, the orbital period is 1.70 days, and the actual duration of the eclipse is about 5 hours. The binary nature of the system is confirmed by the observation of the Doppler shift of the 1.24 s pulse rate and the phase of the eclipse. The primary references on the X-ray observations are Tananbaum et al (1972b) and Giacconi et al (1973).

The short term X-ray variability is displayed in figure 2-2. The pulse shape itself is observed to display a complex structure, which varies on a timescale of hours. In figure 2-2 we see a main pulse and an interpulse. The main pulse has been observed at some times to have a double peaked structure (Doxsey et al 1973; Holt et al 1974). Further, both the magnitude of the interpulse and its phase relative to the main pulse vary (see Holt et al 1974 and references therein).

Like Cen X-3, the X-rays from Her X-1 turn "off" for extended periods. Contrary to Cen X-3 no continuous emission has been observed when the X-ray source is not pulsating. For Her X-1, the "on - off" behavior appears to occur in a quasi-periodic 35 day cycle. The X-rays are "on" for about 11 days and are "off" for about 24 days. In figure 2-3 we show Uhuru data for three of these X-ray "on" periods. The "on" state comes on abruptly in one to two hours; it increases slowly for another 3 days and then declines steadily. The "turn-on" seems to have some orbital phase locking. All observed X-ray "turn-ons" have occurred just after orbital phase 0.2 (at phase 0.0 the X-ray source is directly behind the optical counterpart) or just before phase 0.7. Unlike the hour long transition into and out of eclipse exhibited by Cen X-3, the Her X-1 eclipse transitions are sharp. Further, there is no spectral change as the X-ray source enters or emerges from eclipse. In addition to the 1.7 day eclipses, regular dips in the X-ray intensity are seen. During these dips, which occur once per orbit, the observed X-ray intensity decreases by a factor of 5 to 10. The dips occur before the eclipse, and occur progressively earlier in phase as one approaches the

Fig. 2-2. The counts accumulated in 0.096 second bins from Hercules X-1 during the central 30 seconds of a 100 second pass on 1971 November 6. The heavier curve is a minimum χ^2 fit to the pulsations of a sine function, its first and second harmonics plus a constant, modulated by the triangular response of the collimator. The functional fit is systematically below the peak counting rate partly due to the sharpness of the pulsing and partly due to the minimum χ^2 technique. (From Tananbaum et al 1972b.)

Fig. 2-3. Hercules intensity data (2 - 6 keV) during three "on" states. The vertical lines represent the orbital eclipses, whose positions are determined accurately from the pulsation Doppler analysis. Typical errors appropriate for different groups of data are shown; the statistical error bar is relevant for point-to-point comparisons and the aspect error bar is relevant for day-to-day comparisons. It should be noted that most intensity points below 10 cts/s are upper limits. (From Giacconi et al 1973.)

end of the "on" state (see figure 2-3). The dips and the X-ray "turn-on" exhibit absorption features in that a larger low energy cutoff is observed in the X-ray spectrum.

The X-ray pulse period was observed to decrease by about 4.5 microseconds from January to July 1972. Some time after July 1972 an increase in the pulse period was noted, but the period has since been observed to decrease (Schreier 1973). As in the case of Cen X-3, the decrease in pulsation period is important in that it indicates that if the X-rays are coming from a neutron star, the neutron star is speeding up, not slowing down. Hence the source of the X-ray power is not the rotational energy of the neutron star.

As a final note on the many variations in the X-ray luminosity, there is the possibility (as yet not observed) that the X-ray source turns off for periods of several years. This will be discussed further as we describe the optical variations of HZ Her.

Just as X-ray observations of Her X-1 have proved to be a rich source of data, so too have optical observations of HZ Her provided much data. (For a review of the optical observations see Bahcall, Joss, and Avni 1974, and references therein.) First, the optical light curve varies by about 1.5 magnitudes in phase with the 1.7 day orbital period. The maximum luminosity is achieved when the X-ray star is in front of HZ Her (Bahcall and Bahcall 1972). This excess optical radiation is generally attributed to X-ray heating of, and subsequent reradiation from, either (a) the face of HZ Her, or (b) intervening gas between the two stars. However this variation in the optical light curve persists even when the X-rays are "off". Boynton et al (1973) have found that the optical emission changes substantially during the 35 day X-ray

cycle, but there is still a significant excess optical luminosity in orbital phase with the "off" X-ray source. This has lead some investigators (Avni et al 1973) to postulate that there is an as yet unobserved source of energy from Her X-1 heating HZ Her. This persistent optical variation during the X-ray "off" presents one of the largest puzzles from this amazing system.

Davidson et al (1972) and Groth and Nelson (1972) have found that the optical source sometimes exhibits 1.2⁴ s pulsations at the 0.1 to 0.3 percent level. The definitive observations of these optical pulsations have been made by John Middleditch at UC Berkeley, and from Doppler shift analysis of the data, one finds that sometimes the optical pulsations are coming from HZ Her and sometimes from Her X-1 (Middleditch 1973). These conclusions are supported by observations of Groth (1974).

A search of old photographic records (Jones et al 1973a) has revealed that the optical luminosity from HZ Her has fallen to a low level for several year periods, and then has returned to its present variable level for extended periods. Figure 2-4 indicates this long term behavior. When the star is in the low state, its luminosity is about equal to the optical luminosity observed from the "back side" of HZ Her, when Her X-1 is in complete eclipse (see figure 2-5). The usual explanation for the extended optical lows is that the X-ray heating has ceased, i.e., the X-ray source, for some unknown reason, goes "off" for several year periods.

FIG. 2-4. Long term behavior of HZ Her. All plotted magnitudes occurred at phase $\phi = 0.5 \pm 0.2$, and therefore represent near-maximum brightness. Brightness upper limits are indicated by v's. The cross-hatching indicates the behavior reported by Cherepaschuk et al 1973. (From Jones et al 1973a.)

Fig. 2-5. Light curves of HZ Her. (a) 1945-1948; (b) 1949-1956. All magnitudes were derived from iris photometer measurements made of blue-sensitive plates taken with the 41-cm refractor at Harvard's G. R. Agassiz Station. The one high point in (b) occurred in 1954. Open circles represent repeated points. (From Jones et al 1973a.)

Cygnus X-1

The history of the Cyg X-1 observations has recently been reviewed by Gursky (1973a). Because of the probable association of Cyg X-1 with the massive spectroscopic binary HDE 226868, it is a prime black hole candidate. We will address this point as we discuss the optical observations.

The outstanding observational characteristic of Cyg X-1 is the short timescale variations in the X-ray intensity. The source has been observed by Uhuru to exhibit variation on all timescales down to 0.1 s (the limit of the Uhuru time resolution). Only one other X-ray source, Cir X-1, has been observed by Uhuru to exhibit similar rapid variation (Jones et al 1974). Recently, rocket observations of Cyg X-1 measured variations in the X-ray intensity on timescales of one millisecond (Rothschild et al 1974). In figure 2-6 we present a portion of these observations. These are particularly important observations because millisecond or submillisecond pulses from an X-ray source are evidence for a neutron star or a black hole. There is no model for producing quasi-periodic bursts on a millisecond timescale from degenerate dwarfs, but orbital periods near neutron stars or black holes are of this magnitude. If a "hot spot" formed in the inner regions of an accretion disk, it has been suggested that it might produce a burst of radiation each time it orbited (Pringle and Rees 1972; Shakura and Sunyaev 1973). Since the optical data from HDE 226868 indicate Cyg X-1 is too massive to be a neutron star, these millisecond variations can be regarded as evidence favoring a black hole interpretation for this source.

Fig. 2-6. Eighty milliseconds of exposure to Cyg X-1 containing the peak of the enhancement near 318 s after launch. The count rates are binned every 640 μ s. Bursts with > 11 counts per 1.28 ms are shaded. (From Rothschild et al 1974.)

Examination of data from 35 days of continuous observation of Cyg X-1 by the Uhuru satellite showed no evidence of a 5.6 day variation in the X-ray intensity (Tananbaum et al 1972a). The absence of a 5.6 day eclipse period was confirmed by data from the UC San Diego X-ray telescope onboard the OSO-7 satellite (Baity et al 1973). However, the Copernicus satellite recently observed the source for 6.5 days and found an indication of a 5.6 day period (Sanford et al 1974), with the maximum X-ray intensity occurring at the orbital phase when the unseen companion to HDE 226868 would have been closest to earth (orbital phase 0.5). These Copernicus observations are continuing. Some support for the Copernicus results has been obtained from the MIT X-ray detectors on OSO-7. Li and Clark (1974) report they observed "a transient change in the X-ray spectrum of Cygnus X-1 indicative of an increase in absorption which lasted at least 37 minutes and occurred at a time near phase zero of the secondary of the likely optical counterpart, the spectroscopic binary system HDE 226868." Specifically, they were making 30-40 minute observation runs separated by about one hour gaps. Their data in the 15-45 keV range showed no noticeable variation, but the average counting rates in the 1-6 keV and the 3-6 keV ranges decreased by factors of 2.3 and 1.3, respectively, relative to a local two day average centered on the "absorption" observation. This absorption feature occurred when the secondary of HDE 226868 was farthest from the earth, at orbital phase zero.

Observation of a partial X-ray occultation in phase with the 5.6 day orbital period would provide confirmation of the identification of Cyg X-1 with HDE 226868. This association is currently based on a

single correlated X-ray -- radio transition. A radio source appeared (Braes and Wilev 1971; Hjelming and Wade 1971) within the region of uncertainty of Cyg X-1 (Rappaport et al 1971). The appearance of this radio source coincided to within a few days with a change of intensity and spectrum of the X-ray source (Tananbaum et al 1972a). This provides the evidence for supposing the X-ray and radio emissions are from the same object. Since HDE 226868 lies within the small region of uncertainty of the radio source, it is concluded that this spectroscopic binary is the optical counterpart to Cyg X-1.

HDE 226868 was observed by Webster and Murdin (1972) and Bolton (1972a,b) to be a spectroscopic binary with a period of 5.6 days. It is a B0 supergiant (2) with a probable mass of about $30 M_{\odot}$. The mass function for this system is $0.23 M_{\odot}$ (Brucato and Kristian 1973), i.e.,

$$f(m) = (M_2 \sin i)^3 / (M_2 + M_{HD})^2 = 0.23 M_{\odot}$$

Here M_{HD} is the mass of HDE 226868, and M_2 is the mass of its unseen (X-ray?) companion. Assuming the inclination angle to be 90 degrees, we find the minimum mass for the unseen companion is $6.8 M_{\odot}$. Optical light curves (see Hutchings et al 1973 and references therein) indicate an orbital inclination nearer 30 degrees, which yields a secondary mass of $15.7 M_{\odot}$.

SMC X-1

SMC X-1 was first observed to be a discrete X-ray source by Uhuru (Lelong et al 1971). Further analysis of the Uhuru data indicated the

(2) Walborn (1973) has refined the spectral classification to O9.7.

source underwent periodic intensity changes which were interpreted as eclipses in a binary system (Schreier et al 1972a). The orbital period indicated by the X-ray data is 3.89 days. While no pulsations have been detected from the source, variations on the timescales of hours and minutes have been observed. Also, at non-regular intervals the source exhibits extended periods, not of fixed length, of low intensity. OSO-7 observations (Ulmer et al 1973b) have confirmed the Uhuru results, and taken with the Uhuru results, suggest a total X-ray luminosity in excess of 5×10^{38} ergs/s, well above the Eddington limit for a $2 M_{\odot}$ object.

Webster et al (1972) suggested Sanduleak 160 (Sk 160) as the optical counterpart to SMC X-1. This association was confirmed by Liller (1973) who showed that Sk 160 was an ellipsoidal variable with the same period and phase as SMC X-1. Sk 160 is classified as a B0 I supergiant (Webster et al 1972). Radial velocity measurements (see Osmer and Hiltner 1974 and references therein) show a large scatter, but indicate a small mass ratio. Osmer and Hiltner argue for a secondary mass around $2 M_{\odot}$. If future observations verify this small secondary mass, it will have significant implications in that this will be the first X-ray source observed to be radiating in excess of the Eddington limit.

3U 0900-40 (Vela X-1)

The X-ray source 3U 0900-40 (Vela X-1) was observed by Ulmer et al (1972) to exhibit periodic intensity fluctuations, which were interpreted as X-ray occultations occurring in a binary system with orbital period 8.7 ± 0.2 days. Uhuru observations confirmed the binary nature of the X-ray source and refined the period to 8.95 ± 0.02 days (Foreman et al 1973).

No evidence was found for regular X-ray pulsations, although the Uhuru observations indicated variability on timescales down to tenths of seconds. The source occasionally goes off for times less than half a day, but does not exhibit the extended lows of the systems such as Cen X-3 or Her X-1.

The single lined spectroscopic binary HD 77581, long suggested as an optical counterpart (see Foreman et al 1973 and references therein), was shown to undergo regular variations at optical frequencies with a period of 8.95 days (Hiltner 1973; Vidal et al 1973). The optical identification is then based on the good agreement between the X-ray and optical periods as well as the coincidence of the X-ray minimum with the secondary minimum of the light curve. HD 77581 is classified as a B0.5 Ib star (Morgan, Code, and Whitford 1955), i.e., a blue supergiant. Wickramasinghe et al (1974) suggests the star should be reclassified as B0.5 Ia (a more luminous supergiant), and further suggests that evolutionary calculations indicate a mass of $50 M_{\odot}$. The secondary to primary mass ratio for this system is small, but with a primary mass of $50 M_{\odot}$, the orbital parameters allow one to argue for a secondary mass equal to or exceeding $3 M_{\odot}$. If future observations provide supporting evidence for the $3 M_{\odot}$ secondary mass, this system will become a strong black hole candidate.

3U 1700-37

The X-ray source 3U 1700-37 was first detected by the Uhuru satellite in 1970. Subsequent analysis of the Uhuru data (Jones et al 1973b) indicated the presence of regular X-ray occultations. The orbital period is 3.4 days, with a comparatively long X-ray eclipse of 1.1 days. The eclipse period may be variable, suggesting motion in the

atmosphere of the companion star. The X-ray source has been observed to change by a factor of four in ten minutes; variability on the time-scales of seconds and tenths of seconds has also been observed. However it should be noted that the variations appear less striking than those of Cyg X-1.

Jones et al (1973b) suggested that HD 153919 was a likely optical companion. Jones and Liller (1973a) confirmed this identification with optical observations indicating a phase and period consistent with the X-ray observations. HD 153919 is an O6f star. Bessel et al (1974) argue that its mass may be $65 M_{\odot}$. While the radial velocity data indicate a small mass ratio, the secondary mass could be a $2.7 M_{\odot}$ or greater if the primary mass of $65 M_{\odot}$ were adopted. Subsequent observations may prove this system to be a black hole candidate.

Cygnus X-3

The X-ray intensity from Cyg X-3 was observed by Uhuru (Parsingault et al 1972) and by Copernicus (Sanford and Hawkins 1972) to show smooth, nearly sinusoidal, variations with a period of 0.2 days (4.8 hours). This variation has been confirmed by later OSO-7 observations (Canizares et al 1973) which refined the period to 0.19966 ± 0.00012 days. While the ratio of maximum to minimum intensity in all these observations is about 2 or 3, the OSO-7 observations indicate the average intensity changed by a factor of three from December 1971 to June 1972. They further indicate the shape of the 4.8 hour "pulse" was not the same for all sightings. This shape variation has been confirmed by analysis of other Uhuru observations (Tananbaum 1974). The Uhuru data have been examined for indications of

a change in the 4.8 hour period; this examination provided no positive evidence of a change in the orbital period.

The X-ray spectrum was known to be strongly cutoff at low energies, which is interpreted as an absorption effect. Analysis of the Uhuru data (Parsingalt et al 1972) indicate the low energy cutoff is variable, suggesting the absorption is partially circumstellar.

No optical counterpart for this source has been observed, but Becklin et al (1973) have observed an infrared source which shows the same 4.8 hour variation as the X-rays. Figure 2-7 shows the results of simultaneous infrared observations and Copernicus X-ray observations. The infrared variation is in phase with the X-ray variation. With the good positional information from the infrared data, a search for an optical companion was undertaken. No candidate was found down to a visual magnitude of 23 (Westphal et al 1972).

Also associated with Cyg X-3 is an extraordinarily variable radio source (cf. Gregory et al 1972a; Hjellming and Balick 1972; and Gregory et al 1972b). The radio flux has been observed to increase by two to three orders of magnitude in one to a few days, and then decay in a few days. Six of these radio bursts have been observed thus far (see Marsh et al 1974 and references therein).

Circinus X-1

On the basis of Uhuru observations, Jones et al (1973c) reported that Cir X-1 underwent a regular eclipse with a period of 12.3 days. However, none of the Uhuru data covered a full orbital cycle. Following a report of these results, Canizares et al (1974) studied OSO-7 observations of Cir X-1. They report that they observed the X-ray

Fig. 2-7. The 4.8 h periodicity of Cyg X-3. The X-ray data have been averaged over the 6 min integration times of the infrared data. During the time period shown the infrared flux from a comparison star located within 5 arc min of Cyg X-3 was constant within ± 2 percent. 1973 July 9. (From Becklin et al 1973.)

source to be "on" when the 12.3 day period would predict the source to be in complete eclipse. This seems to rule out the 12.3 day period, but searches are being made for longer periods, especially multiples of the 12.3 day period suggested by the Uhuru data. Current data excludes periods less than 15 days (Jones et al 1974).

It is important to determine whether or not this source is in a binary system. It is the only X-ray source observed by Uhuru to exhibit time variations as sharp as those observed from Cyg X-1. The identification of Cyg X-1 as a black hole will be strengthened if other objects of the same class can be identified. We should note that this source, if it proves to be in a binary system, may have the longest orbital period yet observed in an X-ray binary.

3. THE "STANDARD" MODEL FOR X-RAY SOURCES IN CLOSE BINARY SYSTEMS

The discovery of X-ray sources in close binary systems has focused attention on a class of source models which assume a compact object (the secondary) is in orbit with a comparatively normal stellar object (the primary). X-rays are produced as a result of material accreting onto the secondary. The primary is assumed to be the source of this accreted material.

The secondary is assumed to be a compact object because: (1) the rapid time variations in the observed X-ray intensity indicate a small, high density, emitting region; (2) accretion of material onto the compact object can supply energy at the observed radiation rates; and (3) at the surface of the compact object (the region outside the innermost stable circular orbit in the case of a black hole) this accreted material can produce X-rays very efficiently.

By allowing a variety of compact objects, we obtain a "rich" (in the words of David Pines) class of source models. This richness is enhanced by varying the characteristics within each classification of compact object. For example, one can consider neutron stars with very weak magnetic fields as well as the highly magnetized neutron stars thought to be present in the X-ray pulsars Cen X-3 and Her X-1. Assuming a variety of compact objects allows one to account for a wide range of secondary masses as well as permitting large flexibility in accommodating evolutionary scenarios.

I. Mass Transfer Mechanisms

If the compact object is a neutron star or a black hole, the gravitational potential energy released per unit mass of accreted material will be about $0.1 c^2$. The infalling material is likely to be primarily hydrogen; if so, fusion beneath the surface of the neutron star (see Rosenbluth et al 1973) will add about $0.71 c^2$ per unit mass. In the case of accretion onto a degenerate dwarf, fusion may be the primary energy source, with gravitational potential energy contributing about 10 percent of the radiated energy. Hence, neutron stars and black holes require accretion rates in the range $10^{16} - 10^{18}$ g/s ($10^{-10} - 10^{-8} M_{\odot}/\text{yr}$) to provide the observed X-ray luminosities of $10^{36} - 10^{38}$ ergs/s; a degenerate dwarf model would require accretion rates an order of magnitude larger (two orders of magnitude if gravitational energy powers the X-ray intensity). These are very moderate mass transfer rates when compared with the rates thought to be achievable in close binary systems; there is reason to believe that only a small fraction of the material leaving the primary is accreted onto the secondary, and that the remaining material from the primary escapes from the system.

Roche lobe overflow is frequently mentioned as the mechanism by which the primary supplies material to the secondary. However, Davidson

(1) By co-rotation we mean the primary is rotating with the same period as the binary orbital period.

and Ostriker (1973) noted that the assumption of co-rotation (1) may not be valid in some, or all, of the X-ray binaries. They suggested that the Roche lobe geometry be replaced with a tidal lobe geometry (cf. Jeans 1928). Davidson and Ostriker also suggested two alternate mass transfer mechanisms: (1) If the primary is a sufficiently massive O or B supergiant, it is likely to have a strong stellar wind, and some of this mass outflow may be accreted by the secondary. (2) If the secondary has a sufficiently high X-ray luminosity, the X-ray heating of the surface of the primary may drive off sufficient material to power the X-ray source. In both of these cases only a small fraction of the material leaving the primary would be captured by the secondary.

a. Gravitational Lobe Overflow

For purposes of understanding the mass transfer, there is no essential difference between Roche lobe overflow, tidal lobe overflow, or ... lobe overflow. The surface of the primary should roughly follow a surface of constant potential (gravitational plus rotational). There is a maximum critical equipotential surface which encloses the primary, but not the secondary. If the primary attempts to expand beyond this critical equipotential, material will spill over into the lobe surrounding the secondary. We will adopt the term gravitational lobe to describe this critical equipotential surface, regardless of the form of the primary rotational potential.

The rate at which the primary loses mass due to gravitational lobe overflow is presumably determined by internal conditions; during the

later stages of central hydrogen burning (case "A"), or after exhausting its hydrogen core (case "B"), the primary tends to expand, but finds itself limited in size by the gravitational lobe. The mass transfer then proceeds with timescales characteristic of the later stages of stellar evolution. Mass transfer rates from 10^{-8} to $10^{-3} M_{\odot}/\text{yr}$ can be achieved by suitably choosing system parameters. This concept of mass exchange in binaries due to gravitational lobe overflow is fundamental to current ideas about the evolution of normal stellar objects in binary systems. An extensive review of the evolution of mass-exchange binary systems has been given by Paczynski (1971). There the reader will find several computed examples of binary evolution with details of the inferred mass transfer rates.

The general feeling is that gravitational lobe overflow can easily supply enough material to power the X-ray source; of more concern is what happens when material is supplied at a rate substantially exceeding the rate at which it can be accreted onto the secondary.

Material will be ejected from the primary in the vicinity of the inner Lagrangian point with an initial velocity comparable to the thermal velocity on the surface of the primary, a much smaller velocity than the orbital velocity of the secondary. Hence this ejected material will have a net angular momentum with respect to the secondary. The angular momentum will likely prevent the material from falling directly onto the secondary, instead resulting in the formation of an accretion disk around the secondary (Prendergast and Burbidge 1968). The inner regions of this accretion disk have been the subject of intensive

investigation (cf Pringle and Rees 1972; Shakura and Sunyaev 1973; Novikov and Thorne 1973); several detailed models have been constructed for accretion disks around neutron stars and black holes. However little work has been done on modeling the outer regions of these accretion disks. (See the discussions in Chapters 8 and 10.)

b. Stellar Winds

Observations of OB stars (late O or early B) indicate mass loss rates on the order of $10^{-6} M_{\odot}/\text{yr}$ (Morton 1967, 1969; Carruthers 1968), with the ejected material achieving velocities of 1000 - 1500 km/s. These experimental results are in good agreement with recent calculations of Castor et al (1974). Since five of the identified optical counterparts to X-ray binaries are OB supergiants, it does not seem unreasonable to attribute strong winds of this type to these stars. The secondary would then have to capture about 0.1 percent of the material being driven off the primary.

As Davidson and Ostriker (1973) have pointed out, the wind velocities exceed the orbital velocities, resulting in a supersonic wind flowing by the secondary. This should produce an accretion shock behind the secondary (Hoyle and Lyttleton 1939; Bondi and Hoyle 1944), with all the material in a cylinder whose radius is roughly $R_a = 2GM_X/V^2$ being captured by the secondary. If we take $V = 10^8$ cm/s and $M_X = 1 M_{\odot}$ then $R_a = 2.6 \times 10^{10}$ cm. Since radii for the OB counterparts are $1 - 2 \times 10^{12}$ cm, the secondary capture radius subtends $10^{-5} - 10^{-4}$ of the solid angle of the primary, implying accretion rates of $10^{-11} - 10^{-10} M_{\odot}/\text{yr}$. While these would be too small to power the X-ray sources at the

observed luminosities, the capture efficiency is a strong function of the wind velocity (fourth power), and a slight adjustment of the wind parameters can raise the accretion rate by two or three orders of magnitude. In the absence of any realistic model calculations for this accretion process, it may be reasonable to expect this accretion mechanism to work in some binary systems where the wind is characterized by a relatively larger mass loss rate and a relatively smaller velocity.

We have noted that the accretion efficiency for a supersonic wind may be marginal. There are two ways the X-rays from the secondary may reduce the efficiency, perhaps making this wind accretion model untenable. First, the X-ray heating of the gas will produce hydrodynamical motion opposing accretion. The radiation pressure of the X-rays will also oppose accretion. Second, X-rays will likely strip the ions responsible for the acceleration of the wind. The acceleration, most of which takes place near the surface of the primary, is due to line radiation pressure (Castor et al 1974). Hence the wind may tend to turn off on the side of the primary facing the secondary. Another possibility is that these two mechanisms act together to regulate the accretion rate.

c. X-ray Induced Stellar Winds

Davidson and Ostriker (1973) suggested that X-ray heating of the surface of the primary and the subsequent evaporation of material from this surface is a possible mass loss mechanism for some of the primaries in X-ray binaries. Her X-1 and SMC X-1 seem to be the best candidates

for this mechanism: the X-ray luminosity exceeds the optical (2) by two orders of magnitude in the Hercules system while the optical and X-ray luminosities are comparable for SMC X-1. In the other four X-ray binaries with identified optical counterparts the optical luminosity exceeds the X-ray luminosity by two to three orders of magnitude, and the X-ray heating effects will presumably be small.

Arons (1973) and Basko and Sunyaev (1973) examined this X-ray induced mass transfer mechanism for HZ Her/Her X-1. Both investigations concluded that this mechanism could work in the Hercules system. However, we have investigated this mechanism in greater detail (see Chapter 6) and conclude that it probably does not operate for HZ Her/Her X-1, even if one assumes all the material ejected from HZ Her is captured by the secondary. Further support for this conclusion comes from the fact that recent calculations by Alme and Wilson (see Chapter 10) and by McCray (1974) indicate the X-ray source will be able to capture only a small fraction of the ejected material.

While the X-rays may not be able to drive enough material from the surface of HZ Her if it is well inside the gravitational lobe, the X-ray heating can still produce substantial distention of the atmosphere. If the primary fills or nearly fills its gravitational lobe, an increase in X-ray intensity may cause it to expand, resulting in gravitational lobe

(2) Here we are using optical loosely. We mean the total luminosity of the optical counterpart-visible, UV, etc.

overflow. It is likely that the X-ray heating plays a significant role in the mass transfer for the Hercules system.

II. Accretion onto the Secondary and X-ray Production

We now turn our attention to models for accretion onto the secondary and mechanisms for converting the gravitational (or nuclear) energy of the infalling material to X-rays. In the previous section we described how gravitational lobe overflow could lead to the formation of a disk rotating around the secondary. We also noted that stellar winds may result in supersonic flow by the secondary, with material being captured from a tail shock behind the secondary. These are the most frequently mentioned mechanisms for accretion onto the secondary, which stems in large part from their being limiting cases. The accretion disk represents the limit in which the orbital velocity dominates the hydrodynamical and thermal velocities, while the standing accretion shock in the supersonic wind represents the limit of small thermal and orbital velocities in comparison with the wind velocity.

a. Accretion Disks

If the mass transferred from the primary has so much angular momentum that it cannot fall directly onto the secondary (and if the X-rays do not heat the infalling material excessively - see Chapter 8), it seems likely that the matter will form a differentially rotating disk around the secondary, with the material following approximately

Keplerian orbits. If viscosity in the disk can produce a significant outward flux of angular momentum, there will be a corresponding inward flux of material. The energy dissipated by viscosity will heat the material in the disk, resulting in radiation from the disk. When Prendergast and Burbidge (1968) originally developed the disk model, they were attempting to understand if the inner regions of a disk around a white dwarf would become hot enough to produce X-rays. (They were not able to address the difficult questions of how the disk connects to the secondary and what happens at the surface of the secondary.) Pringle and Rees (1972) have examined models for accretion disks around neutron stars and black holes; similar models for disks around black holes have been considered by Shakura and Sunyaev (1973). Both these investigations concluded that disks around black holes could radiate most of their power as X-rays. Pringle and Rees further concluded that if a disk surrounded a neutron star with a negligible magnetic field, the disk could extend down to the surface of the neutron star, with the disk and the surface of the neutron star producing comparable amounts of X-ray power. When the central object is a black hole, the disk is expected to extend in to the region of the innermost stable circular orbit. The binding energy of this orbit is $0.06 mc^2$ for a non-rotating black hole, and $0.42 mc^2$ (cf Novikov and Thorne 1973) for material co-rotating with a maximal Kerr hole. Hence disks around black holes are expected to produce about 10^{20} ergs of X-ray energy for each gram of material swallowed by the hole. The innermost orbit of a disk

extending in to the surface of a neutron star will have a binding energy equal to half the gravitational potential at the surface of the star. Assuming a surface potential of 10^{20} ergs/g, we find the disk radiating about 5×10^{19} ergs for each gram accreted, while the other 5×10^{19} ergs/g is released as X-rays when the material is stopped on the surface of the neutron star.

If an accretion disk surrounds a highly magnetized neutron star, Pringle and Rees suggested the magnetic field would disrupt the disk at a relatively large radius, with the material flowing down magnetic field lines onto the neutron star. Then most of the radiation would be emitted from the neutron star. Lamb, Pethick, and Pines (1973) have considered accretion onto highly magnetized neutron stars in much more detail, and their results have confirmed the suggestions of Pringle and Rees. We will return to this point below in the section on X-rays from neutron stars.

A large viscosity must be assumed in order to make the accretion disk models work. Atomic and ionic viscosities fail by orders of magnitude. All investigators to date have had to invoke either a turbulent viscosity or a magnetic viscosity. The standard practice is to take the viscous stress to be αP where α is a dimensionless parameter less than or equal to unity, and P is the total pressure. Recently it has been shown (Lightman and Eardley 1973; Lightman 1974 a,b) that such a viscous stress produces instabilities in the inner regions of the disks (where radiation pressure dominates gas pressure). It remains to be seen if more detailed viscosity models show similar instabilities.

b. Accretion Shocks

If the material leaves the primary with velocities comparable to or exceeding the orbital velocity of the secondary, some of the material will be able to flow "behind" the secondary. If a substantial fraction of the material leaving the primary achieves this counter-rotational motion, it is not obvious that an accretion disk will form. Instead, a shock may form near the secondary with some of the material being accreted after passing through the shock.

This shock accretion has been studied extensively assuming supersonic flow past a gravitating body. The study began with work by Hoyle and Lyttleton (1939) and Bondi and Hoyle (1944); it has continued through the present time (cf Salpeter 1964; Hunt 1971; Ruderman and Spiegel 1971; Lyttleton 1972; Davidson and Ostriker 1973). The most detailed numerical simulations have been those of Hunt (1971) and LeBlanc and Wilson (1973) (3).

If the flow is thoroughly supersonic (and axi-symmetric), the picture that emerges is that the material, as it passes the massive object, is deflected toward the axis, and forms a tail shock (see figure 3-1). Some of the material loses enough energy in passing through the shock to fall toward the central object, forming an "accretion column" along the axis of symmetry. This accretion process sweeps out a

(3) These two investigations also considered subsonic flow.

Fig. 3-1. A schematic view of the flow. The dotted line is the axis of accretion. The heavy line is the standing shock front; the thin lines are (laminar) flow lines. (From Salpeter 1964.)

cylinder of radius R_a (the "accretion radius"), where R_a is approximately

$$R_a = 2GM/V^2.$$

Here V is the initial upstream velocity. The accretion rate is then

$$dM/dt = \pi R_a^2 \rho V$$

or

$$dM/dt = 4\pi(GM)^2 \rho / V^3$$

where ρ is the initial upstream density.

As the Mach number of the flow is reduced toward unity, the shock structure changes. Hunt (1971) considered Mach numbers of 1.4 and 2.4, finding in both cases that a bow shock formed. The accretion rate is roughly as calculated before with V being replaced by $V(1+V/M)$ where M is the Mach number. This is in general agreement with an earlier conjecture by Bondi (1952).

There are two reasons for being very cautious in applying these results to X-ray binaries. First, X-ray heating effects and/or radiative cooling of the shocked material may substantially alter the calculated accretion rates. Strong X-ray heating might result in an isothermal shock with a different structure, and presumably a different accretion rate. Another possibility is that some regions of the flow may be shielded from X-rays, while others are not, resulting in a far more complex flow pattern than previously modeled. The second reason for caution is that orbital velocities are not small in comparison with even the largest observed wind velocities, therefore destroying the axial symmetry and reintroducing angular momentum considerations. Shakura and Sunyaev (1973) suggest that even if an accretion shock is

formed around a black hole, the angular momentum of the captured material is likely to be non-zero, resulting in the formation of a disk inside the shock. This suggestion may be motivated in part by the fact that the accretion disk is the only mechanism proposed to date that is capable of "dissipating" the angular momentum, thereby allowing material to fall onto the central object. The centripetal support of angular momentum seems to pose a formidable problem. The specific angular momentum of material in Keplerian orbit is $(GMr)^{1/2}$, so material in circular orbit at 4×10^{10} cm must get rid of 99.5 percent of its angular momentum in order to be in Keplerian orbit at the surface of a 10 km neutron star. However, it is our feeling that angular momentum is given too much consideration. The three dimensional calculations of Alme and Wilson (see Chapter 10) indicate that inertial effects of material "blow-off" caused by X-ray heating dominate the hydrodynamical motion near the X-ray source, making the angular momentum "dissipation" easy, or even irrelevant. Finally, magnetic fields in the accreted matter have been absent from our discussions. While the quantity of the discussion is commensurate with our knowledge of what effect these fields will have, it is easy to imagine small initial fields undergoing large amplification during accretion, producing large modifications both in the radiation mechanisms and the flow.

c. X-ray Emission from Neutron Stars

Zel'dovich and Shakura (1969) have studied the emission of X-ray resulting when accreting material strikes the surface of a neutron star.

The material loses its kinetic energy, typically 100 MeV per nucleon, in a few grams per square centimeter due to Coulomb collisions. For reasonable accretion rates, the heating electrons radiate most of their energy in the X-ray range. We have continued this study (see Chapter 5), and find that by varying the accretion rate and the surface potential, a large variety of X-ray spectra can be produced, ranging from a few keV blackbody spectra to 50 or 60 MeV bremsstrahlung spectra. While these studies have neglected (1) the changes in the radiation mechanisms that result if strong ($\sim 10^{12}$ gauss) magnetic fields are present at the surface, and (2) the effect of X-ray heating on the infalling material, they make the essential point that if material strikes the surface of a neutron star with large kinetic energy, X-rays are produced very efficiently. The heated atmosphere is an excellent insulator, and all the kinetic energy of infall is converted to outgoing radiation.

The X-ray pulse shapes of Cen X-3 and Her X-1 apparently cannot be explained by an isotropically radiating "hot spot" on the surface of a neutron star -- a beamed source is required. By analogy with the radio pulsars, it is assumed that the beaming is due to strong magnetic fields on the neutron star. Calculations of radiation rates and slowing down times indicate radio pulsars have fields $\sim 10^{12}$ gauss (cf Ostriker and Gunn 1969). This canonical value of 10^{12} gauss applies to radio pulsars, but this is the value that model makers take for the surface fields in

the X-ray pulsars (4). The reader is referred to the article by Lamb et al (1973) for a detailed description of some of the effects that may arise from the presence of a large magnetic field, but a major point is that, assuming a dipole magnetic field for the neutron star, the ram pressure (ρv^2) of the infalling gas will be offset by the magnetic pressure at a large radius ($\sim 10^8$ cm for a surface field of 10^{12} gauss), thereby inhibiting fluid flow across the field lines and channeling the flow down the field lines onto the magnetic poles of the neutron star. This radius at which the magnetic pressure is equal to the ram pressure is called the Alfen radius, and presumably this radius marks the inner boundary of any accretion disk.

d. Degenerate Dwarfs

The role played by degenerate dwarfs in the X-ray binaries has not been defined. There is a real lack of mechanism for converting gravitational or nuclear energy into X-rays. While infalling material can strike the surface with a few hundred keV per nucleon kinetic energy, it does not seem likely that this will maintain keV temperatures on the surface of the dwarf; recall that in the case of neutron star

(4) While isotropically radiating hot spots are not in favor, fields two or three orders of magnitude smaller than 10^{12} gauss can, at the surface of the star, produce magnetic pressures comparable to the ram pressure of the infalling material, and therefore could have some effect on the flow, perhaps causing beaming.

surfaces, 100 MeV per nucleon infall energies only produce several keV surface temperatures. A luminosity of 10^{38} ergs/s (about the Eddington limit for a $1 M_{\odot}$ object) taken with a radius of 2×10^8 cm only implies an effective surface temperature of a tenth of a keV. There have been various suggestions for burning hydrogen in bursts very near the surface, but the very difficult calculations required to test this hypothesis remain to be carried out.

While there is a lack of an X-ray mechanism, degenerate dwarfs do have some attractive features. (An excellent summary of the arguments for degenerate dwarfs is given by Lamb and Van Horn (1973), but their discussion falters when it comes to the point of an X-ray production mechanism.) From an evolutionary viewpoint, it is easier to understand the presence of dwarfs in close binary systems than it is to understand the presence of neutron stars or black holes. The general conception is that it is hard to form a neutron star or a black hole in a manner sufficiently non-violent to avoid disruption of the binary system. This objection may vanish as current calculations of the late stages of stellar evolution progress, and as a better picture of neutron star formation is developed. Recent calculations by James R. Wilson (unpublished) indicate there may be a wide range of violence (or non-violence) associated with neutron star formation.

4. A PREVIEW OF CHAPTERS 5 - 8

Chapter 5 discusses computer calculations of the X-ray emission resulting from the accretion of material onto a neutron star. We assume a spherically symmetric flow of cold hydrogen gas, and treat the deceleration of the infalling gas at the surface of the neutron star, modeling the X-ray emission resulting when the kinetic energy of infall is converted to thermal energy at the surface. Although the kinetic energy of infall is ~ 100 MeV per nucleon, the surface temperature of the star is a few keV, and virtually all the radiation emitted from the surface of the star is in the form of X-rays. By varying the surface potential and the infall rate we are able to produce a wide variety of spectra ranging from a few keV blackbody to 40 - 50 keV bremsstrahlung. These calculations neglect surface magnetic fields on the neutron star as well as the interaction of the outgoing radiation with the infalling gas.

Davidson and Ostriker (1973) suggested that X-ray heating of the surface of HZ Her may induce a stellar wind, thereby providing a mass transfer mechanism other than gravitational lobe overflow for this system. We test this suggestion by calculating the attenuation and diffusion of a beam of X-rays incident on the surface of HZ Her, and by calculating the resulting hydrodynamical motion. In Chapter 6 we report the results of an extensive parameter study in which we vary the

incident X-ray intensity, the incident spectrum, the atmospheric composition, and the position of the atmosphere. We find that the X-ray induced wind is too small to power the X-ray source. We conclude that gravitational lobe overflow is the most likely mass transfer mechanism for the Hercules system.

Chapter 7 reports calculations of the optical response of HZ Her to variations in the incident X-ray intensity. We calculate the response due to a sudden "turn-off" in the X-rays as well as the response to 1.24 s pulsations. In the "turn-off" calculations we find that the long term (>5 seconds) decay is in general agreement with estimates reported in the literature, but that there is a much more rapid decrease in the optical radiation rate during the first second after "turn-off" than other investigators had predicted. Since the X-rays are pulsed at 1.24 s, this short term behavior is more important for modeling the response of the atmosphere to the X-ray pulses. While we use these results to draw some conclusions about the HZ Her/Her X-1 system, the major significance of these optical response calculations is as input for other investigators attempting to model this system.

It is our feeling that gravitational lobe overflow is the mass transfer mechanism in the Hercules system. In Chapter 8 we assume a cold stream of material flows through the inner Lagrangian point, and then calculate the X-ray heating effects as the material falls toward Her X-1. By suitably adjusting the boundary parameters, we can find strong heating effects, but these calculations do not give a clear answer as to whether the X-ray heating effects dominate the hydrodynamical

motion of the infalling material. The results of Chapter 8 are being used as boundary conditions for three dimensional hydrodynamical calculations now in progress.

5. X-RAY EMISSION FROM A NEUTRON STAR ACCRETING MATERIAL

I. Introduction

In this chapter we describe computer calculations of the X-ray emission resulting from the accretion of material onto a neutron star. Our calculational model assumes a steady-state, spherically symmetric, infall of hydrogen gas from radial infinity in a Schwarzschild metric. This gas falls onto a nonrotating neutron star where we calculate the deceleration of the gas, treating it as a beam of charged particles passing through the neutron star atmosphere. We consider only Coulomb collisions in calculating this deceleration since they dominate over nuclear collisions at the energies of interest (~ 100 MeV per nucleon). The neutron star atmosphere, also assumed to be hydrogen, is heated by the infalling hydrogen gas and radiates. The atmosphere is treated as a hydrodynamical fluid characterized by temperature, density, and radial velocity. Its equation of state is taken to be that of an ideal gas. The transport of the radiation field in the atmosphere, described by its energy density per unit frequency, is treated in a diffusion approximation. An important limitation of these calculations is that we do not allow the radiation field to interact with the infalling hydrogen gas - we consider neither X-ray heating of this gas nor momentum transfer to the gas from the outgoing radiation.

An initial temperature and density distribution is assumed for the atmosphere, which is then allowed to evolve dynamically until a steady-state is achieved. The resulting atmosphere is sufficiently thin that we may ignore the gravitational acceleration of the infalling gas in the atmosphere. We also ignore all time derivatives in treating the transport of the infalling gas through the atmosphere.

As we have noted, the calculated atmosphere is very thin in comparison with the radius of the neutron star. The atmosphere is also very much less massive. Hence we may ignore the self-gravitational effects of the atmosphere and set up on the surface of the neutron star a static proper reference system (coordinate lengths and times equal to proper lengths and times)(Bardeen, Press, and Teukolsky 1972). The laws of physics in this proper reference frame are those of special relativity augmented by the gravitational acceleration

$$g = \frac{GM}{R^2(1 - 2GM/c^2R)^{1/2}} .$$

Further, since the internal energy density of the gas is small compared with the rest mass energy density and since the fluid velocities are small compared to c , we may use a Newtonian description for the hydrodynamics. However, the infalling hydrogen gas has velocities approaching c , so it must be treated special-relativistically. The reader is referred to the Appendix for a description of a distant observer's interpretation of the results.

These calculations can be regarded as a more detailed study of the accretion calculations performed by Zel'dovich and Shakura (1969).

As we have previously noted, a limitation of our calculations is that we ignore the interaction of the outgoing radiation with the infalling gas. A more serious limitation is that our calculations assume a spherically symmetric geometry. However, they can be interpreted per unit surface area and thus can be applied locally in any situation where the infall is primarily in the radial direction.

II. Notation and Equations

R is the value of the radial coordinate at the surface of the neutron star (Schwarzschild curvature coordinates), and M is the star's gravitational mass. All other quantities occurring in the equations are the quantities as measured in the static proper reference system set up in the atmosphere. In particular, dt is the differential proper time and dr is the differential proper length in the radial direction. (Here t and r are not the Schwarzschild curvature coordinates - see Appendix.)

a) Equation of Motion

The equation of motion is taken as

$$\rho \frac{dv}{dt} = - \frac{\partial P}{\partial r} - \frac{GM\rho}{R^2(1 - 2GM/Rc^2)^{1/2}} + \frac{dM/dt}{4\pi R^2} \frac{\partial}{\partial r} \left(\frac{v}{[1 - (v/c)^2]^{1/2}} \right) + \frac{1}{c} \int_0^\infty \rho (v'_a + v'_s) F_v dv,$$

where ρ is the density, V is the radial fluid velocity, P is the pressure, dM/dt is the accretion rate, v is the velocity of the infalling hydrogen gas at the top of the atmosphere, ν is the photon energy, K'_a is the absorption opacity corrected for induced emission, K_s is the Compton scattering opacity, and F_ν is the radiation flux. The gravitational constant is G , and the velocity of light is c . The $(1 - 2GM/Rc^2)^{-1/2}$ factor is included in the gravitational force to facilitate a relativistic interpretation of the results (see Appendix).

b) Radiation Diffusion

The radiation field is described by the equation

$$\frac{dF_\nu}{dt} + \frac{\partial}{\partial r} (F_\nu) = c\rho K'_a (B_\nu - E_\nu) + \frac{F_\nu}{\rho} \frac{\partial \rho}{\partial t} - \frac{V}{c} \rho (K'_a + K_s) F_\nu + \frac{\nu \rho}{m_e c} \frac{\partial}{\partial \nu} [T(\nu \frac{\partial F_\nu}{\partial \nu} - 3E_\nu) + \nu F_\nu (1 + \frac{h}{15\pi \nu^3})]$$

Here E_ν is the radiation energy density per unit photon energy, B_ν is the blackbody radiation energy density per unit photon energy, and T is the electron temperature. The total blackbody energy density is given by aT^4 . The change in photon energy resulting from Compton collisions is treated in the Fokker-Planck approximation (Chang and Cooper 1970). The radiation flux is taken as

$$F_\nu = \frac{\lambda_{\text{total}} c (\partial F_\nu / \partial r)}{3 + \lambda_{\text{total}} |\partial F_\nu / E_\nu \partial r| [1 + 3 \exp(-\lambda_{\text{total}} |\partial F_\nu / F_\nu \partial r| / 2)]}$$

where $\lambda_{\text{total}} = (\rho K'_a + \rho K_s)^{-1}$. The denominator of the diffusion coefficient has been selected so that, for optically thin systems, the radiation flux will be approximately correct. This form of the

diffusion coefficient has been tested against exact solutions on slabs of one to five mean free paths in thickness, and found to have less than 10 percent error in flux and energy density.

c) Material Temperature

The material temperature is determined from

$$\begin{aligned} \rho c_v \frac{\partial T}{\partial t} = & c \int_0^\infty \rho K'_a (E_\nu - B_\nu) d\nu + \frac{dM/dt}{4\pi R^2} \frac{\partial}{\partial r} \left\{ \frac{c^2}{[1 - (v/c)^2]^{1/2}} \right\} \\ & + \frac{\partial}{\partial r} (K_{\text{electron}} \frac{\partial T}{\partial r}) + \frac{P}{\rho} \frac{\partial \rho}{\partial t} \\ & + \frac{\rho}{m_e c} \int_0^\infty K_B \left(\pi \left(\nu \frac{\partial E_\nu}{\partial \nu} - 3E_\nu \right) + \nu E_\nu \left(1 + \frac{\pi^2 E_\nu}{15 a \nu^3} \right) \right) d\nu. \end{aligned}$$

Here c_v is the material specific heat, K_{electron} is the electron thermal conductivity, and P is the material pressure. When the temperature gradient is superadiabatic, K_{electron} is modified to include convective heat transfer in the mixing length approximation.

d) Opacity

The absorption opacity is taken as

$$K_a = \frac{2.787^3 \rho}{\nu_A^2 \pi^{1/2}} g^{ff},$$

where g^{ff} is a tabular fit to the free-free Gaunt factors given by Karzas and Latter (1961), and ν and π are given in keV. K'_a is then

$$K'_a = K_a (1 - \exp[-\nu/\pi]).$$

e) Deceleration of Infalling Gas

The rate of energy loss for a proton traveling in a plasma is (Shkarofsky, Johnston, and Bachynski 1966)

$$\frac{dE_p}{dt} = - [\phi(x_e) - x_e(1 + m_e/m_p)\phi'(x_e)] \frac{4\pi n_e e^4}{m_e v} \ln \Lambda_e,$$

where we have neglected slowing down as a result of ions in the plasma. Subscript p denotes the infalling protons and subscript e denotes the electrons in the plasma. The number density is n_e , the electric charge is e , the proton velocity is v , the Coulomb logarithm is Λ_e , the error function is

$$\phi(x) = 2\pi^{-1/2} \int_0^x \exp(-z^2) dz,$$

and $x_e = (m_e v^2 / 2kT_e)^{1/2}$. We take Λ_e to be the larger of $2m_e v^2 / (\gamma e^2 \omega_p)$ (Kihara and Aono 1963) and $3\pi^{3/2} / (e^2 \omega_p m_e^{1/2})$ (Spitzer 1962). The plasma frequency is ω_p , and Euler's constant is $\ln \gamma$. When $v/2c$ or $v_{th}(\text{electrons})/2c$ is greater than the fine-structure constant, α , we also use a quantum-mechanical correction for Λ_e (Spitzer 1962), i.e., $\Lambda_e^{QM} = \Lambda_e 2\alpha c/v$. We approximate $f(x_e) = \phi(x_e) - x_e(1 + m_e/m_p)\phi'(x_e)$ as

$$f(x_e) = \frac{2x_e(2x_e^2/3 - m_e/m_p)/\pi^{1/2}}{1 + 2x_e(2x_e^2/3 - m_e/m_p)/\pi^{1/2}}.$$

Then, for each gram of infalling material, we have

$$\frac{d}{dt} \frac{c^2}{[1 - (v/c)^2]^{1/2}} = - f(x_e) \frac{4\pi n_e e^4}{v m_e m_p} \ln \Lambda_e.$$

III. Method of Solution

The equation of motion, the radiation diffusion equation, and the infalling-gas deceleration equation are approximated by finite-difference equations. The atmosphere is divided into approximately 50 spatial zones. The radiation field is described by 20 frequency groups ranging from 0.5 to 150 keV. The numerical treatment used to calculate the Compton energy exchange is the scheme developed by Chang and Cooper (1970).

As a starting condition for the computer code we assume a radiation field and temperature and density distributions in the atmosphere. A rate of mass infall is specified. Then a dynamic problem is run and the atmosphere is allowed to readjust until a steady state is achieved.

IV. Results

Calculations were performed for the five neutron star configurations given in table 5-1. Rates of mass infall were selected to provide luminosities ranging from 10^{35} to 10^{38} ergs/s. The calculated X-ray spectra for these five configurations are presented in figures 5-1-5.

A crucial parameter in determining the departure of the radiated spectrum from a blackbody spectrum is the total amount of material per unit area required to decelerate the incoming proton beam. Since the stopping distance is a strong function of the infall velocity, the lower surface potentials in the smaller-mass neutron stars result in larger departures from a blackbody spectrum. Typical

TABLE 5-1
Neutron Star Configurations

Mass (M_0)	Radius (km)	$\frac{E_{\text{infall}} - Mc^2}{Mc^2}$	Critical Luminosity (ergs/s)
2.4	10	0.86	3.0 (38)
1.4	12	0.24	1.8 (38)
0.86	10	0.16	1.1 (38)
0.5	13	0.062	6.3 (37)
0.3	20	0.023	3.8 (37)

Fig. 5-1. X-ray spectra for $2.4 M_{\odot}$ star as seen by distant observer. Labels indicate luminosity in ergs/s. Solid curves are calculated spectra. Dashed curves are blackbody spectra with the same luminosities.

Fig. 5-2. X-ray spectra for $1.4 M_{\odot}$ star as seen by distant observer. Labels indicate luminosity in ergs/s. Solid curves are calculated spectra. Dashed curves are blackbody spectra with luminosities of 1.3×10^{36} and 1.3×10^{38} ergs/s.

Fig. 5-3. X-ray spectra for $0.86 M_{\odot}$ star as seen by distant observer. Labels indicate luminosity in ergs/s. The dashed curve is a blackbody spectrum with luminosity 1.5×10^{36} ergs/s.

Fig. 5-4. X-ray spectra for $0.5 M_{\odot}$ star as seen by distant observer.
Labels indicate luminosity in ergs/s.

Fig. 5-5. γ ray spectra for $0.3 M_{\odot}$ star as seen by distant observer.
Labels indicate luminosity in ergs/s.

stopping distances are 100 g/cm^2 for the $2.4 M_{\odot}$ star, 18 g/cm^2 for the $1.4 M_{\odot}$ star, 8 g/cm^2 for the $0.86 M_{\odot}$ star, $2-4 \text{ g/cm}^2$ for the $0.5 M_{\odot}$ star, and $2-3 \text{ g/cm}^2$ for the $0.3 M_{\odot}$ star. For a given star there is a weak variation in stopping distance with infall rate. Figures 5-6 and 5-7 illustrate physical conditions in the atmosphere for the $1.4 M_{\odot}$ star.

Zel'dovich and Shakura (1969) have suggested that plasma instabilities may play a significant role in decelerating the infalling protons. Following this suggestion, we assume that Landau damping suppresses the two-stream instability between protons of the infalling gas and the atmospheric electrons when the electron thermal velocity is greater than the infalling gas velocity. We require the stopping power to be such as to raise the local electron thermal velocity to at least equal to the local infalling-gas velocity. Incorporating this instability model, we repeated our calculation for the $1.4 M_{\odot}$ neutron star with luminosity 2×10^{37} ergs/s. The stopping distance was reduced from 18 gm/cm^2 to 3 g/cm^2 . Modifications of the spectrum and physical conditions in the atmosphere are illustrated in figures 5-8 through 5-10.

We are unsure of the existence of strong instabilities. According to Spitzer (1962) the growth rate for the most unstable wave is

$$\omega_p \frac{3^{1/2}}{2^{4/3}} \left(\frac{n_b m_e}{n_e m_b} \right)^{1/3} .$$

The Landau damping rate takes the form (Spitzer 1962)

$$-\omega_p v_{th}(\text{electrons}) / (v_b 3^{1/2}) .$$

Fig. 5-6. Temperature T versus atmospheric depth d for $1.4 M_{\odot}$ star. The luminosity associated with each temperature profile is given in ergs/s.

Fig. 5-7. Energy flux and number flux vs atmospheric depth for $1.4 M_{\odot}$ star. The solid curves are for luminosity 4.1×10^{36} ergs/s (local luminosity 6×10^{36} ergs/s). The dashed curves are for luminosity 4.1×10^{37} ergs/s (local luminosity 6×10^{37} ergs/s).

Fig. 5-8. X-ray spectra for $1.4 M_{\odot}$ star with luminosity 1.3×10^{37} ergs/s as seen by distant observer. Curve A illustrates the spectrum when no instability model is used. Curve B indicates the spectrum after introduction of a plasma instability model. The dashed curve is a blackbody of luminosity 1.3×10^{37} ergs/s.

Fig. 5-9. Temperature T vs atmospheric depth d for a $1.4 M_{\odot}$ star with luminosity 1.3×10^{37} ergs/s. Curve A illustrates the temperature profile when no instability model is used. Curve B indicates the temperature profile after introduction of a plasma instability model.

Fig. 5-10. Energy flux and number flux vs atmospheric depth d for a $1.4 M_{\odot}$ star with luminosity 1.3×10^{37} ergs/s (local luminosity 2×10^{37} ergs/s). The solid curves illustrate the energy and number fluxes when no instability model is used. The dashed curves indicate the energy and number fluxes after introduction of a plasma instability model.

We have examined the calculations that include only collisional stopping and find that

$$\frac{v_{th}(\text{electrons})}{v_b} \gg \left(\frac{n_{b,e}}{n_{e,b}} \right)^{1/3},$$

except in the first few hundredths of a gram per square centimeter of the atmosphere. For example, in the $1.4 M_{\odot}$ star with luminosity 2×10^{37} ergs/s, the smallest value for $v_{th}(\text{electrons})/v_b$ is 0.15, while 99 percent of the energy is deposited in a region such that

$$(n_{b,e}/n_{e,b})^{1/3} < 0.004.$$

Hence Landau damping dominates over the two-stream instability at the shorter wavelengths. However, as Weaver and Chapline (1974) have pointed out, at longer wavelengths the reverse will be true. With increasing wavelength the Landau damping rate decreases much more rapidly than the two-stream growth rate decreases; thus there is reason to believe that instabilities may be present at longer wavelengths.

For purposes of calculating the emergent X-ray spectrum, the most important property of a strong plasma instability would be the electron thermal velocity at which the instability is saturated. The velocity distribution is complex and a numerical simulation would be required in order to determine this saturation velocity. One has a version of the "bump-on-tail" instability, with the incoming protons being the bump on the atmospheric electron velocity distribution. Recent investigations (see Papadopoulos 1973 and references therein, and Papadopoulos et al 1974) indicate that this instability may saturate

without "flattening" the velocity distribution as predicted by quasi-linear theory - compare the curves of figure 5-11. Since the electrons then have a lower velocity, the X-ray spectrum is softer. We hope to perform numerical simulations of the problem in the near future.

V. Conclusions

The temperature at the top of the atmosphere, characteristically tens of kiloelectron volts, is primarily determined by an energy balance between heating by the incoming beam and Compton cooling. The temperature at the bottom of the atmosphere, of the order of 1 keV, is set by the requirement that there be no net energy flow into the star. The atmospheric depth at which the transition from kiloelectron volts to tens of kiloelectron volts takes place is the primary parameter determining the emergent spectrum. If the bulk of the energy is deposited several Compton mean free paths into the atmosphere, the radiation must diffuse out. The photons will tend toward a Planckian distribution as a result of the scatterings. If too few photons are being emitted, the Planckian distribution will be dilute.

If the deposition takes place in only one or two Compton mean free paths, as in the $0.5 M_{\odot}$ and $0.3 M_{\odot}$ stars, the emitted photons stream out. As they suffer only one or two scatterings, there is only slight modification of the bremsstrahlung emission spectrum.

Our model neglects any deceleration of the infalling gas as a result of radiation pressure. This approximation obviously breaks down

Fig. 5-11a

Fig. 5-11b

Fig. 5-11. Figure 5-11a illustrates the final form of the electron distribution function predicted by quasi-linear theory, Figure 5-11b indicates the final form of the electron distribution function predicted by the parametric oscillating two-stream instability. (From Papadopoulos et al 1974.)

as the luminosity approaches the Eddington limit, $L_E = 4\pi cGM/K_g$. We also neglect any modification of the spectrum as a result of scattering in the cold infalling gas. For this assumption to have some validity, each photon should encounter less than one mean free path of material as it travels out through the infalling gas. This is a more stringent requirement than the Eddington limit; it results in $L < (v_{gas}/2c)L_E$. Because of this limitation we cannot use our model to calculate accretion onto white dwarfs.

While we have refrained from proposing specific models for some of the observed X-ray sources, we feel it is significant that this accretion model, with all its limitations, can produce a wide variety of emission spectra. Further, this can be done considering only Coulomb collisions as a stopping mechanism. Plasma instabilities need not be invoked, although they will produce even harder spectra. Although Cygnus X-1 is generally thought to be a black hole, there is a suggestive similarity in the high-energy tail observed from Cygnus X-1 and the high energy spectra we calculate for the lower mass neutron stars.

APPENDIX

Since we neglect the self-gravitation of the atmosphere, the gravitational field in the atmosphere, as well as exterior to the atmosphere, is the Schwarzschild gravitational field:

$$ds^2 = - \left(1 - \frac{2GM}{c^2 \hat{r}}\right) d\hat{t}^2 + \frac{d\hat{r}^2}{\left(1 - 2GM/c^2 \hat{r}\right)} + \hat{r}^2(d\hat{\theta}^2 + \sin^2 \hat{\theta} d\hat{\phi}^2),$$

where $\hat{t}$, $\hat{r}$, $\hat{\theta}$, and $\hat{\phi}$ are the Schwarzschild curvature coordinates.

Differential proper time dt and differential proper radial length dr in our static proper reference system are

$$dt = (1 - 2GM/c^2 R)^{1/2} d\hat{t}$$

and

$$dr = (1 - 2GM/c^2 R)^{-1/2} d\hat{r}$$

where R is the value of $\hat{r}$ at the surface of the neutron star.

In the atmosphere the relativistic hydrostatic equation is:

$$\frac{\partial P}{\partial \hat{r}} + \frac{GM}{R^2(1 - 2GM/c^2 R)} \left(\rho + \frac{\rho \epsilon}{c^2} + \frac{P}{c^2} \right) = 0,$$

where ρ is now the proper rest-mass density, and ϵ is the thermal energy per unit rest mass. If we neglect the thermal energy density and pressure in comparison with the rest-mass energy density, the hydrostatic equation becomes

$$\frac{\partial P}{\partial r} + \frac{\rho GM}{R^2(1 - 2GM/c^2 R)^{1/2}} = 0.$$

Material freely falling from radial infinity has, as measured in the static proper reference system on the surface of the star, radial velocity

$$\frac{dr}{dt} = \left(\frac{2GM}{c^2 R}\right)^{1/2}$$

and total energy per unit rest mass of $c^2(1 - 2GM/c^2 R)^{-1/2}$.

The accretion rate, luminosity, and photon energy used in the calculations are the quantities measured by a distant observer as follows:

$$\frac{dM}{dt} = (1 - 2GM/c^2 R)^{1/2} \frac{dM}{dt} ,$$

$$L_{\text{distant}} = (1 - 2GM/c^2 R) L_{\text{local}} ,$$

and

$$v_{\text{distant}} = (1 - 2GM/c^2 R)^{1/2} v_{\text{local}} .$$

6. X-RAY INDUCED MASS TRANSFER IN THE

HZ HER/HER X-1 BINARY SYSTEM

Davidson and Ostriker (1973) noted that X-ray heating of the surface of the primary and the subsequent evaporation of material from this surface is a possible mass loss mechanism for HZ Her. This chapter reports the results of investigating this suggestion. Our major conclusion is that the X-ray induced mass transfer mechanism does not work for HZ Her/Her X-1.

Arons (1973) and Basko and Sunyaev (1973) examined Davidson and Ostriker's X-ray induced mass transfer mechanism for the Hercules system. Both investigations concluded that self-excited mass flow is possible for this system. We have constructed a more detailed model for the X-ray heating of the surface of HZ Her, and have performed computer calculations of the X-ray induced mass flow. Contrary to the conclusion reached by Arons and by Basko and Sunyaev, we find that the X-ray induced mass flow is too small to power the X-ray source unless the photosphere of HZ Her is very near the Roche lobe. We reach this conclusion without taking into account the capture efficiency of the secondary, i.e., we find the X-rays do not drive off enough material from HZ Her to power Her X-1 even if all the mass lost from HZ Her is captured by Her X-1. Recent calculations by Alme and Wilson (see Chapter 10) and by McCray (1974) indicate that the X-ray source

will be able to capture only a small fraction of the ejected material.

Our time dependent calculations include (a) Newtonian hydrodynamics with radiation pressure, (b) a frequency dependent treatment of radiation transport and Compton energy exchange, and (c) non-LTE opacities. The gravitational effects of both members of the binary system are included in the hydrodynamics. The frequency dependent radiation field is treated in two parts. The first part is the beam of X-rays incident on HZ Her which has not been absorbed or Compton scattered. The second part then includes all the rest of the radiation field, i.e., the thermal radiation and the X-rays which have been Compton scattered or emitted via radiative recombination. The incident X-ray beam is assumed to be monodirectional and is transported through the atmosphere suffering exponential attenuation due to absorption and Compton scattering. The balance of the radiation field is treated in a diffusion approximation. The sources for this diffusion radiation field are all photons emitted due to bound-free or free-free transitions and photons from the incident beam which have Compton scattered. Line transitions are neglected. The Compton energy exchange for both radiation fields is treated in the Fokker-Planck approximation. The scattering is assumed to be isotropic and there is no energy-angle correlation for the scattered radiation. The opacities are computed assuming the hydrogen is in local thermodynamic equilibrium (LTE). All other elements are treated non-LTE. Auger transitions are also included.

I. Model

An important limitation on our calculations is that we are restricted to either a spherically symmetric geometry or a plane parallel geometry. We choose to work in a spherically symmetric geometry which has its origin in the center of the star being irradiated (HZ Her). In this spherically symmetric geometry the radial dependence we assume for the gravitational potential and the incident X-ray beam is the spatial variation along the line of centers from HZ Her to Her X-1. In figure 6-1 we indicate the gravitational potential in the region of the inner Lagrangian point. The masses we have assumed for each component of the binary system are $2 M_{\odot}$ for HZ Her and $1 M_{\odot}$ for Her X-1. The separation is $8 R_{\odot}$ (1). We assume the gravitational potential is the Roche

(1) These masses were chosen after examining the parameters given by Forman et al (1972), Leach and Ruffini (1973), and Bahcall et al (1974). We have chosen the parameters which will give the self-excited accretion powered X-ray mechanism the best opportunity to operate. For a smaller neutron star mass, HZ Her will subtend a larger fraction of the X-ray source solid angle, increasing the X-ray induced mass transfer rate. If the neutron star is reduced from $1 M_{\odot}$ to $0.38 M_{\odot}$ ($M_{\text{HZ}} = 1.6 M_{\odot}$), the Roche lobe of HZ Her subtends 7.1 percent of the X-ray source solid angle, compared with 4.9 percent for $1 M_{\odot}$. This increase of 1.45 is not enough to offset the 0.38 decrease in X-ray production efficiency due to the reduced gravitational potential at the surface of the neutron star. We have neglected the increase in radius of the neutron star as the mass decreases; this will further reduce the surface potential. We are assuming the X-ray induced mass transfer rate is primarily determined by the total X-ray flux intercepted by the primary (see Section V).

Fig. 6-1. Roche equipotential lines in the plane of the orbit. Contours are spaced 5 eV apart. Origin of the X-Y coordinate system is at the HZ Her center of mass. The straight lines are rays to the center of HZ Her.

potential, and take the form appropriate for the line of sight from the center of HZ Her to Her X-1, i.e.,

$$\phi(r) = -G^4_{\text{HZ}} \left\{ \frac{1}{r} + \frac{q}{a-r} + \frac{(1+q)(r-qa/(1+q))^2}{2a^3} \right\} \quad (1)$$

Here r is the distance from the center of HZ Her, G is the gravitational constant, M_{HZ} is the mass of HZ Her, a is the separation between the two stars, and q is the mass ratio of the system. In our spherically symmetric geometry, the saddle in the gravitational potential at the inner Lagrangian point becomes a lip. We treat only the outer 25 g/cm^2 of the irradiated primary (8 Compton scattering mean free paths). This is a compromise between computing costs and providing sufficient material to attenuate the incident X-rays. We choose a radius for the inner boundary of the problem. Across this inner boundary we assume there is (a) an outward directed flux of thermal radiation equal to 10^{35} ergs/s to represent the intrinsic luminosity of the star, and (b) a flux of material equal to the rate at which material is being lost at the outer boundary of the problem. X-rays are assumed to be reflected by the inner boundary. This does not cause a buildup of X-rays at the boundary; the radiation energy density at all X-ray frequencies decreases smoothly as we approach the inner boundary. Note the decrease in heating rate per gram in figure 6-5. We take the outer boundary of the problem to be at a radius well outside the lip in the gravitational potential, but inside $r = a$. At this outer boundary we specify the incident X-rays by total flux and spectral shape. The incident X-ray flux per unit

area is $L/(4\pi[a - r]^2)$ where L is the X-ray luminosity from Her X-1. This X-ray flux is subject to a $1/(a - r)^2$ geometric attenuation as it passes through our spherically symmetric problem geometry. The spectral shape is assumed to be either a blackbody with temperature T_g or an exponential $e^{-\nu/\bar{\nu}}$ (energy per unit frequency). We also experimented with a low energy cutoff - see Section V. For the exponential spectrum $\bar{\nu}$ is simply a constant which describes the exponential fall-off of the spectrum. $h\bar{\nu}$ is not the temperature for a thermal bremsstrahlung spectrum. We set the X-ray luminosity and compute the resulting mass loss rate. To have self-excited flow, this mass loss rate would have to be equal to or greater than the accretion rate necessary to power the X-ray source. We do not require a self-consistent X-ray luminosity and mass loss rate.

The hydrodynamical timescale (pressure scale height/sound speed) in the photosphere is $\sim 10^3$ seconds. The radiation-material coupling timescale (material specific energy/X-ray heating rate) is $\sim 0.1 - 1$ seconds. Hence for the steady state calculations of mass loss the 1.24 s X-ray pulsation is unimportant, and we do not include it. For further discussion of the radiation-material coupling timescale see Section V.

The composition of the atmosphere is taken from the cosmic abundances given by Allen (1963) except for iron, where we have assumed the abundance given by Dalgarno and McCray (1972). We have lumped the C, N, O, and Ne abundances as O, the silicon group elements as Si, and the iron group elements as Fe. The resulting abundances

are (number fraction relative to hydrogen) $N_{\text{He}}/N_{\text{H}} = 1.4 \times 10^{-1}$,
 $N_{\text{O}}/N_{\text{H}} = 1.3 \times 10^{-3}$, $N_{\text{Si}}/N_{\text{H}} = 8.0 \times 10^{-5}$, and $N_{\text{Fe}}/N_{\text{H}} = 3.0 \times 10^{-5}$.
 In some problems we vary the abundance of all elements with atomic
 number $Z > 2$.

To summarize our steady state calculations, the parameters we
 vary from problem to problem are (a) the inner radius of the outer
 25 g/cm^2 of the atmosphere (the radius of the photosphere is then
 a result of our calculations), (b) the luminosity of the X-ray source,
 (c) the spectral shape of the incident X-rays, and (d) the abundance
 of elements with $Z > 2$. We have also performed calculations in which
 we include time variation of the incident X-rays (see Chapter 7).

II. Equations

a) Equations of Motion

The mass conservation equation is

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho v) = 0 \quad (2)$$

where ρ is the material density and v is the radial velocity of the fluid.

The equation of motion is taken to be

$$\frac{\partial (\rho v)}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho v^2) + \rho \frac{\partial \phi}{\partial r} + \frac{\partial P}{\partial r} - \frac{1}{c} \int_0^\infty \frac{1}{\lambda_{\text{total}}} F_{\nu}^{\text{total}} d\nu = 0 \quad (3)$$

where ϕ is the gravitational potential, P is the material pressure, c is light velocity, λ_{total} is the photon mean free path for absorption and Compton scattering, and F_{ν}^{total} is the total flux of radiation energy, including both the flux for the diffusion field and the incident beam of X-rays.

The attenuation of the incident beam of X-rays is determined from

$$\frac{\partial E_{\nu}^b}{\partial t} - \frac{c}{(a-r)^2} \frac{\partial}{\partial r} ((a-r)^2 E_{\nu}^b) = -c\rho(K_S + \sum_{Z>1} K_Z) E_{\nu}^b \quad (4)$$

Here E_{ν}^b is the energy density per unit frequency in the incident beam and K_Z is the bound-free opacity for the species with atomic number Z .

The diffusion radiation field is described by

$$\begin{aligned}
 \frac{\partial E_\nu}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 F_\nu) = & \quad c\rho K'_a(B_\nu - E_\nu) - c\rho \sum_{Z>1} K_Z E_\nu \\
 & + c\rho \sum_{Z>1} A_Z \delta(\nu - \nu_Z) \int_{\nu_Z}^{\infty} K_Z \nu_Z (E_\nu + E_\nu^b) d\nu' / \nu' \\
 & + \frac{\nu_0 K_s}{m_e c} \frac{\partial}{\partial \nu} \left\{ kT \left(\nu \frac{\partial E_\nu}{\partial \nu} - 3E_\nu \right) + h\nu E_\nu \left(1 + \frac{c\pi^4 F_\nu}{60\sigma \nu^3} \right) \right\} \\
 & + c\rho K_s (1 - g(\nu)) E_\nu^b
 \end{aligned} \tag{5}$$

E_ν is the energy density per unit frequency and F_ν is the flux of radiation energy per unit frequency for the diffusion field. B_ν is the Planck function, K'_a is the opacity for bound-free transitions for hydrogen and free-free transitions for all species. It is corrected for induced emission. $\delta(\nu - \nu_Z)$ is the Dirac delta function, ν_Z is the frequency associated with the K-shell ionization potential for species Z, m_e is the electron mass, T is the material temperature, and σ is the Steffan-Boltzmann constant. $g(\nu)$ is the fraction of the photon energy that is lost as a result of Compton scattering. $c\rho K_s E_\nu^b$ is the Compton scattering rate for the incident beam, $g(\nu)c\rho K_s E_\nu^b$ is the electron heating rate, and $(1 - g(\nu))c\rho K_s E_\nu^b$ is the source rate for the diffusion field. Note that while we remove the correct amount of energy from the radiation field as a result of the first scattering,

we do not shift the photon frequency. The frequency shift is included for photons in the diffusion field. For an exponential spectrum incident we take $g(\nu)$ as

$$g(\nu) = \frac{h\nu - kT\nu / \bar{\nu} - 3kT}{m_e c^2} \quad (6)$$

In A_Z we incorporate our model for the Auger effect, thermal broadening of K-shell recombination radiation, and radiative recombination following collisional ionization. We take A_Z to be

$$A_Z = (1 - f_Z^A) \frac{\bar{\nu}(T, Z)}{\nu_Z} (1 - R_Z^C)$$

where f_Z^A is the fraction of electron transitions to the K-shell for species Z that are Auger transitions, $\bar{\nu}(T, Z)$ is the average energy of a photon emitted as a result of radiative recombination to the K-shell, and R_Z^C is the ratio of collisional ionization to photoionization for K-shell electrons of species Z. Radiative recombination completely dominates ($> 10^6$) collisional recombination at the temperatures and densities we are considering; collisional recombination is not included in the energy budget.

The radiation flux F_ν is computed as

$$F_\nu = - \frac{\lambda_{\text{total}} c \partial E_\nu / \partial r}{3 + \lambda_{\text{total}} |\partial \ln E_\nu / \partial r| \{1 + 3 \exp(-\frac{1}{2} \lambda_{\text{total}} |\partial \ln E_\nu / \partial r|)\}} \quad (7)$$

The denominator of the diffusion coefficient has been selected so that, for optically thin systems, the radiation flux will be approximately

correct. This form of the diffusion coefficient has been tested against exact solutions on slabs of one to five mean free paths in thickness, and found to have less than 10 percent error in flux and energy density.

The material temperature is determined from

$$\frac{\partial}{\partial t}(\rho C_v T) + \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 \rho C_v T v) + \frac{P}{r^2} \frac{\partial}{\partial r}(r^2 v) = \rho C_v \frac{\partial T}{\partial t} \Big|_{\text{rad}}$$

$$\begin{aligned} \rho C_v \frac{\partial T}{\partial t} \Big|_{\text{rad}} = & c \rho \int_0^\infty K'_a (E_v - B_v) dv + c \rho \sum_{Z \geq 1} \int_0^\infty K_Z (v - A_Z v_Z) (E_v + E_v^b) dv / v \\ & + \frac{\rho}{m_e c} \int_0^\infty K_S \left\{ kT \left(v \frac{\partial E_v}{\partial v} - 3 E_v \right) + h v E_v \left(1 + \frac{c \pi^4 E_v}{60 \sigma v^3} \right) \right\} dv \quad (8) \\ & + c \rho \int_0^\infty K_S g(v) E_v^b dv \quad . \end{aligned}$$

Here C_v is the specific heat, which we take as the perfect gas value appropriate to complete ionization.

b) Opacities

We assume the hydrogen is in LTE. The state of ionization is determined using the average ion, modified Stromgren model (Chandrasekhar 1957). For all other elements, we consider only the K-shell electrons. These K-shell populations are computed with a non-LTE

model which neglects any screening of the nucleus by the electrons; both K-shell electrons are assumed to have ionization potential $Z^2 h\nu_H$ where Z is the nuclear charge and $h\nu_H$ is the ionization potential of the hydrogen K-shell. Let $N(Z, Z')$ be the number density of ions with nuclear charge Z and ionic charge Z' . The ionization equilibrium is then given by

$$\frac{N(Z, Z-1)}{N(Z, Z-2)} = \frac{2 \int_0^\infty c(E_\nu + E_\nu^b) \sigma_Z d\nu / h\nu + N_e C_{Z, Z-2}^i}{N_e (\alpha_{Z, Z-1} + C_{Z, Z-1}^r)} \quad (9)$$

$$\frac{N(Z, Z)}{N(Z, Z-1)} = \frac{\int_0^\infty c(E_\nu + E_\nu^b) \sigma_Z d\nu / h\nu + N_e C_{Z, Z-1}^i}{N_e (\alpha_{Z, Z} + C_{Z, Z}^r)}$$

where σ_Z is the bound-free cross section per K-shell electron for species Z , N_e is the electron number density, $\alpha_{Z, Z'}$ is the radiative recombination rate, $C_{Z, Z'}^i$ is the collisional ionization rate, and $C_{Z, Z'}^r$ is the collisional recombination rate. $\alpha_{Z, Z'}$ is taken to be

$$\alpha_{Z, Z} = 2.07 \times 10^{-11} Z^2 T^{-1/2} (0.4288 + \frac{1}{2} \ln \beta + 0.46988 \beta^{-1/3}),$$

$$\text{where } \beta = \frac{Z^2 I_H}{kT} \quad (10)$$

and T is given in $^{\circ}\text{K}$. This recombination coefficient is based on fits

to hydrogenic recombination rates by Tarter (1971). The recombination coefficient includes only recombination to the K-shell. The expression was formed by subtracting the fit to the recombination rate summed over all levels above the ground state from the fit to the recombination rate summed over all levels. We include only recombination directly to the K-shell because we wish to consistently neglect all line transitions in our calculations. We justify this neglect by the fact that our problems are optically thick in all lines involving transitions to the K-shell (see Section IV for further discussion). The collisional ionization rate, $C_{Z,Z}^i$, was taken from Cox and Tucker (1969) with their "focusing factor" set to unity. $C_{Z,Z}^r$ was then set from detailed balancing.

1) Bound-free

The cross section for ionization of an electron in a state with principal quantum number n is taken as

$$\sigma_Z^{bf} = \frac{64\pi^4 m_e e^{10} Z^4}{3^{3/2} c h n^5} \frac{g^{bf}}{v^3} \quad (11)$$

where g^{bf} is the bound-free gaunt factor given by Karzas and Latter (1961).

ii) Free-free

The free-free cross section used is

$$\sigma^{ff} = \frac{16\pi^4 Z^2 e^6 N_e}{3^{3/2} (2\pi m_e)^{3/2} (kT)^{1/2}} \frac{g^{ff}}{v^3} \quad (12)$$

where g^{ff} is the free-free gaunt factor of Karzas and Latter.

iii) Frequency Averaging

The opacity used in the radiation transport calculations is an inverse average over the frequency group of interest. For the j^{th} frequency group we have

$$K_a(\nu^j) = \frac{\nu^{j+1/2} - \nu^{j-1/2}}{\int_{\nu^{j-1/2}}^{\nu^{j+1/2}} K_a^{-1} d\nu} \quad (13)$$

iv) Auger Transitions

We do not explicitly compute the L-shell occupation. We estimate the L-shell occupation from our calculated K-shell occupations by using the relation of K and L-shell occupations in a 22 keV exponential spectrum computed by C. Bruce Tarter (private communication). Any time a K-shell photoionization occurs when there are two or more electrons in the L-shell, we assume an Auger transition takes place; we neglect fluorescence (Weisheit 1974).

III. Method of Solution

Our system of coupled partial differential equations (Section II) is approximated by a set of finite-difference equations. Extensive use is made of the operator splitting technique (Richtmyer and Morton 1967). Because we have material flowing through our problem boundaries, we have chosen to use Eulerian difference equations for the hydrodynamics. The atmosphere of the primary is divided into about 50 spatial zones, with a smoothly varying zone size (Δr is restricted to vary from zone to zone by no more than 10 percent in regions where physical quantities are varying rapidly). The finest zoning in a given problem is concentrated in the photospheric region. The zone size here is at most $1/3$ the local pressure scale height (see figure 6-4). This zoning procedure allows us to represent the photospheric region reasonably well but does not allow us to resolve the temperature-density discontinuity that occurs for certain incident spectra (see Section V).

The radiation field is described by 50 frequency groups ranging from 1 eV to 140 keV. The numerical treatment used to calculate the Compton energy exchange for the diffusion radiation field is the scheme developed by Chang and Cooper (1970). In solving the beam attenuation equation we neglect the time derivative $\partial E_V^b / \partial t$ (no retardation).

As a starting condition for the computer code we assume a radiation field and temperature and density distributions in the atmosphere of the primary. The X-ray luminosity and spectral shape are specified. As noted in Section I, the 1.24 s variation in the incident

X-ray flux is neglected in calculating the mass flow. A dynamic problem is then run allowing the atmosphere to readjust until a steady state is achieved. Since the hydrodynamical timescale is about four orders of magnitude larger than the radiation timescale in the photosphere we have found it necessary to run the hydrodynamics on the hydrodynamical timescale while the radiation transport is run on the radiation timescale. In regions of weak radiation-material coupling the differential heat capacity was decreased sufficiently so that a steady state would be approached in a reasonable number of problem cycles. This procedure can be regarded as a technique for implicit radiation transport. The time dependent calculations of the decay and the 1.24 s modulation of the optical output reported in Chapter 7 were performed with no modification of the heat capacity.

IV. Non-LTE Test Problem

The beam of X-rays incident on the surface of the primary has an energy density, because of geometric attenuation, equal to that of a blackbody radiation field with temperature one to a few electron volts. The cooling rate per gram is proportional to density for cooling processes other than Compton. The heating rate per gram is density independent. Material of sufficiently low density will simply come to Compton equilibrium with the incident X-ray beam with a temperature equal to a quarter of the energy averaged photon energy. Material with a sufficiently high density will cool strongly enough to convert the incident X-rays to a blackbody radiation field with the same energy density. This variation of temperature with material density in a dilute X-ray field is well illustrated by the calculations of Tarter, Tucker, and Salpeter (1969) (hereafter referred to as TTS).

To test the heating and cooling rates resulting from our non-LTE model, we performed the following test problem: First we took the radiation field just above the photosphere from a typical calculation (see the following section for a complete description of this calculation) and computed the steady state temperature as a function of density for material heated by this radiation field. The radiation field included both the incident beam of X-rays and the diffusion field reflected or re-emitted from the primary. The irradiated samples of material were optically thin ($\rho \Delta r < 10^{-3} \text{ g/cm}^2$) so there was little self-absorption of radiation emitted by the material samples. This attempts to simulate the "optically thin" computational

procedure of TTS. Figure 6-2 presents the material temperature as a function of density (curve A). The incident beam was taken to be an exponential with $h\nu = 22$ keV. In our second test problem we computed the steady state temperature for material heated only by the incident beam (curve B of figure 6-2). The diffusion radiation field is substantially softer than the incident beam; hence the temperature is lower in the Compton dominated region when the diffusion radiation field is included. We also present in figure 6-2 temperatures computed by C. Bruce Tarter (curve T) as a result of heating by the incident beam only. This calculation was performed with an updated version of the code reported in TTS (see Tarter and McKee 1973).

Our results are in good agreement with Tarter's, comparing curves B and T, despite our relatively crude non-LTE model. For the temperature and density regime of these problems, the most important differences between our code and Tarter's are (a) we do not include collisionally excited lines, and (b) we do not allow radiative recombination to levels with $n > 1$ since the eventual transition to the $n = 1$ level would be a line transition. We feel it is a reasonable approximation to neglect both of these processes because they involve line transitions to the K-shell. Any photon emitted will be reabsorbed immediately, and will have no net effect on the K-shell population. For example, the $1s-2p$ transition in oxygen will have an opacity at the line center that is $10^6 - 10^7$ larger than the opacity at the K edge of oxygen. In the example problem described in the following section, the temperature increases from 10 eV to 30 eV in $\sim 10^{-2}$ mean

Fig. 6-2. Steady state temperatures as a function of density. Curve A is the temperature resulting from the total radiation field at the photosphere from model 14 (see Section Va). Curve B is the temperature resulting from only the 22 keV exponential spectrum. Curve T was generated by C. Bruce Tarter using the 22 keV exponential spectrum and is to be compared with curve B.

free paths (measured at the oxygen K edge). This implies $10^4 - 10^5$
mean free paths for the 1s-2p transition.

V. Results

a) Example Problem

We have selected one of our problems (model 14) (2) to illustrate in detail what happens to the surface of the primary as it is irradiated by the X-ray beam. The inner boundary of the 25 g/cm^2 was placed 32 eV (3) from the lip in the gravitational potential. The resulting photospheric radius was 23 eV from the lip. The X-ray luminosity was set to 1.3×10^{37} ergs/s. Its spectral shape was specified to be a 22 keV exponential. The abundances for elements with $Z > 2$ were set to our cosmic abundances.

Figure 6-3a illustrates the incident spectrum and the reflected X-ray spectrum. 30 percent of the radiation returning from the primary is above 100 eV. The thermal spectrum emitted by the primary is presented in figure 6-3b. In figure 6-4 we give the temperature, density, velocity, and material pressure distributions in the atmosphere. Also indicated in figure 6-4 is the spatial zone size for the

(2) Model numbers refer to the model numbers assigned in table 6-1.

(3) We define the gravitational potential in eV to be the product of Φ (eq. 1) and the average mass per particle of completely ionized material: $[m_p(1 + \bar{Z})/\bar{A}]$. It is normalized to zero at the lip in the potential. The radius is quoted in eV to facilitate a comparison with the material temperature. Table 6-1 specifies all positions in both eV and cm.

Fig. 6-3a. The incident (dashed) and reflected (solid) radiation spectra for model 14. Figure 6-3a gives the spectra from 0.1 to 100 keV, and figure 6-3b gives the reflected spectrum from 1 to 100 eV. Both incident and reflected spectra integrated from 0.001 to 100 keV are normalized to 1 erg/s. In figure 6-3a large bumps are seen from the recombination at the oxygen and silicon K edges.

Fig. 6-3b. The reflected (solid) radiation spectrum for model 14.

Figure 6-3b gives the reflected spectrum from 1 to 100 eV, while figure 6-3a gives the spectrum from 0.1 to 100 keV. Both incident and reflected spectra integrated from 0.001 to 100 keV are normalized to 1 erg/s. Figure 6-3b includes a 1.6 eV blackbody spectrum of arbitrary normalization for comparison with the thermal spectrum.

Fig. 6-4. Temperature, density, pressure, and velocity distributions for model 14. The ordinate range for density is 10^{-13} to 10^{-9} g/cm^3 , the range of pressure is 10 to 10^5 ergs/cm^3 , and the velocity range is 10^4 to 10^8 cm/s .

the minimum temperature zone. In figure 6-5 we illustrate the X-ray heating rates due to Compton energy exchange and bound-free absorption by the various elements in the atmosphere. For species with atomic number Z the quantity presented is

$$c \int_{\nu_Z}^{\infty} K_Z (\nu - A_Z \nu_Z) (E_{\nu} + E_{\nu}^b) d\nu / \nu$$

Also illustrated in figure 6-5 is the hydrodynamical cooling rate. This includes the advective cooling $(\nu \partial [C_V T] / \partial r)$ as well as the decompressional $(P \dot{V})$ cooling. These two rates are comparable in magnitude. At density 10^{-13} g/cm³, hydrodynamical cooling has limited the temperature to ~ 150 eV (figure 6-4), while the radiation field would allow a temperature ~ 150 eV (figure 6-2). Compton energy exchange accounted for 29 percent of the total X-ray heating, absorption in helium accounted for 14 percent, absorption in oxygen 27 percent, silicon 11 percent, and iron 19 percent. Figure 6-6 presents the total X-ray heating rate integrated to a depth D in the atmosphere. The mass loss rate for this problem is 3×10^{-16} g/(s-sterad).

For this problem the ρ - T curve for material in radiative equilibrium with the photospheric radiation field (curve A of figure 6-2) is single valued in pressure for temperatures below 25 eV. With radiative equilibrium ρT is an increasing function of density. This results in a continuous temperature transition outside the photosphere. As the pressure decreases, the material follows the radiative equilibrium ρ - T curve as long as hydrodynamical effects are not im-

Fig. 6-5. X-ray heating rates versus atmospheric depth D . Curve C is the Compton heating rate. Other X-ray heating rates for individual elements are labeled by the element symbols. Unlabeled curve is the total X-ray heating. Curve Hydro is the cooling rate due to hydrodynamical expansion and advection. The dip in the He curve at $D = 0.03$ is due to the interplay of the decreasing X-ray intensity with depth and the increasing K-shell occupation.

Fig. 6-6. Total X-ray heating rate integrated to atmospheric depth
D. Curve is normalized to 1 erg/c.

portant. As the hydrodynamical effects become important, the material follows a ρ -T curve which drops away from the radiative equilibrium ρ -T curve (lower T for a given ρ). This drop begins about 20 μ . Changing the incident spectra or the composition can result in radiative equilibrium ρ -T curves which are not single valued in pressure for the low temperature region. Our calculations then produce sharp jumps in temperature and density outside the photosphere. These sharp jumps can also occur for higher luminosities (10^{38} ergs/s) or low angles of incidence (see Section Vb) where there is significant attenuation of the incident X-rays in the transitional region itself. Recall that the ρ -T curves in figure 6-2 are computed in the "optically thin" approximation which assumes no attenuation.

b) Mass Loss Rate

We have examined the mass loss rate as a function of (a) the incident X-ray spectrum, (b) the incident X-ray luminosity, (c) the position of the photosphere relative to the lip in the gravitational potential, (d) the relative abundance of elements with $Z > 2$, and (e) the angle at which the beam of X-rays is incident on the surface of the primary. Table 6-1 summarizes problem parameters from the various mass loss calculations we have performed. Models 13 through 24 were the most carefully executed calculations. As a group they had the finest spatial zoning and were followed the furthest in time. Within the context of our physical model, we regard the mass loss values for these calculations to be uncertain by 10 percent. For the

TABLE 6-1*

SUMMARY OF PROBLEMS

MODEL	L_X (ergs s ⁻¹)	R_0 (10 ¹¹ cm)	R_{ph} (10 ¹¹ cm)	Φ_{ph} (eV)	g_{ph} (cm s ⁻²)	$f_{2>2}$	$\dot{M}$ (g s ⁻¹ sr ⁻¹)	ϵ (g erg ⁻¹)	T_e (keV)	ρ_e (g cm ⁻³)	v_e (cm s ⁻¹)	REFLECTED X-RAY ENERGY† (fraction of energy > hv)		
												hv > 0.10 keV	hv > 1.2 keV	hv > 11 keV
1.....	1.3 (38)	2.93	†	†	†	1.0	3.2 (19)	1.1 (-18)	†	†	†	.48	.39	.29
2.....	1.3 (37)	2.93	†	†	†	1.0	8.0 (18)	4.2 (-18)	†	†	†	.50	.39	.29
3.....	1.3 (38)	2.98	3.19	0.1	9.6 (11)	1.0	3.0 (18)	1.9 (-19)	.16	3.5 (-12)	8.4 (6)	.47	.38	.29
4.....	1.3 (37)	2.90	3.13	0.4	3.1 (2)	1.0	2.0 (17)	1.4 (-19)	.10	3.2 (-13)	6.0 (6)	.44	.35	.28
5.....	1.3 (36)	2.90	3.08	1.1	4.8 (2)	1.0	1.4 (16)	1.1 (-19)	.04	2.8 (-14)	5.0 (6)	.37	.31	.26
6.....	1.3 (38)	2.86	3.09	1.0	4.6 (2)	1.0	2.5 (18)	1.7 (-19)	.30	1.8 (-12)	1.3 (7)	.47	.37	.29
7.....	1.3 (37)	2.86	3.04	2.1	6.6 (2)	1.0	1.5 (17)	1.2 (-19)	.17	1.3 (-13)	1.1 (7)	.43	.34	.27
8.....	1.3 (36)	2.86	3.00	3.2	7.8 (2)	1.0	7.6 (15)	6.1 (-20)	.05	1.1 (-14)	6.5 (6)	.36	.31	.26
9.....	1.3 (38)	2.86	†	†	†	0.1	2.0 (18)	1.2 (-19)	†	†	†	.67	.59	.41
10.....	1.3 (37)	2.86	3.12	0.5	3.8 (2)	0.1	1.1 (16)	8.6 (-21)	.16	1.5 (-14)	7.0 (6)	.59	.52	.38
11.....	1.3 (38)	2.72	2.86	9.5	1.3 (3)	1.0	1.4 (18)	1.3 (-19)	.38	7.0 (-13)	2.0 (7)	.45	.36	.29
12.....	1.3 (37)	2.72	2.83	11	1.4 (3)	1.0	1.1 (17)	1.1 (-19)	.50	1.1 (-13)	9.0 (6)	.42	.33	.27
13.....	1.3 (38)	2.58	2.69	22	1.9 (3)	1.0	5.6 (17)	6.5 (-20)	.90	1.6 (-13)	3.3 (7)	.33	.31	.27
14.....	1.3 (37)	2.58	2.67	23	2.0 (3)	1.0	8.0 (16)	9.6 (-20)	.21	4.4 (-14)	1.7 (7)	.30	.29	.26
15.....	1.3 (36)	2.58	2.65	25	2.1 (3)	1.0	9.6 (15)	1.2 (-19)	.05	1.0 (-14)	9.5 (6)	.26	.26	.24
16.....	1.3 (38)	2.58	2.71	20	1.9 (3)	0.1	2.4 (17)	2.8 (-19)	1.0	8.3 (-14)	2.8 (7)	.59	.52	.38
17.....	1.3 (37)	2.58	2.74	18	1.8 (3)	0.1	1.4 (17)	1.7 (-19)	.23	7.6 (-14)	1.8 (7)	.25	.24	.22
18.....	1.3 (37)	2.58	2.67	23	2.0 (3)	2.0	1.4 (17)	1.7 (-19)	.23	7.6 (-14)	1.8 (7)	.25	.24	.22
19.....	1.3 (37)	2.58	2.67	23	2.0 (3)	0.1	1.1 (16)	1.4 (-20)	.23	5.6 (-15)	1.9 (7)	.46	.45	.37
20.....	1.3 (37)	2.58	2.66	24	2.1 (3)	1.0	1.8 (17)	2.1 (-19)	.49	6.3 (-14)	2.7 (7)	.41	.40	.34
21.....	1.3 (37)	2.58	2.64	26	2.1 (3)	1.0	3.2 (17)	3.8 (-19)	1.0	8.0 (-14)	3.8 (7)	.48	.47	.39
21b.....	1.3 (36)	2.58	2.64	26	2.1 (3)	1.0	5.3 (16)	6.3 (-19)	.22	2.9 (-14)	1.8 (7)	.40	.39	.33
22.....	1.3 (37)	2.58	2.67	23	2.0 (3)	1.0	8.0 (16)	9.6 (-20)	.23	4.3 (-14)	1.8 (7)	.31	.31	.27
23.....	1.3 (37)	2.58	2.68	23	2.0 (3)	1.0	6.4 (16)	7.8 (-20)	.26	2.9 (-14)	2.1 (7)	.32	.32	.29
24.....	1.3 (37)	2.58	2.68	23	2.0 (3)	1.0	6.0 (16)	7.3 (-20)	.25	2.9 (-14)	2.0 (7)	.33	.33	.30
25.....	1.3 (38)	2.34	2.41	53	3.2 (3)	1.0	5.8 (17)	1.0 (-19)	.85	1.7 (-13)	3.3 (7)	.44	.35	.28
26.....	1.3 (37)	2.34	2.40	55	3.2 (3)	1.0	5.8 (16)	1.0 (-19)	.28	1.9 (-14)	1.9 (7)	.41	.28	.25

* L_X is the Her X-1 luminosity. R_0 is the radius for the inner boundary of the 25 g cm⁻² atmosphere. R_{ph} is the radius of the photosphere, g_{ph} is the photospheric gravity, $f_{2>2}$ is the abundance for elements with $Z > 2$ (relative to our cosmic abundances). $\dot{M}$ is the mass loss rate, ϵ is the ratio of mass flux to incident X-ray at the surface of HZ Her, and T_e , ρ_e , and v_e are the temperature, density, and velocity at the tip in the gravitational potential.

† For fixed abundances, variations from problem to problem are primarily due to our using different numerical models for the Auger effect. Our best model was used in models 13-24. Some of the variation is due to frequency dependence of the geometric attenuation.

‡ In these problems the photosphere extended over the lip in the gravitational potential

§ All models except these were run with the X-ray angle of incidence, μ_{ph} (see text), set to 1.0. For model 19, $\mu_{ph} = 0.3$. For models 20 and 21, $\mu_{ph} = 0.1$.

|| All models except these assumed a 22-keV exponential spectrum incident. Model 22 assumed a 22-keV exponential spectrum with a 1-keV low-energy cutoff. Model 23 assumed a 22-keV exponential spectrum with a 2-keV cutoff. Model 24 incorporated a 5-keV blackbody spectrum.

remaining models we feel the uncertainty is about 30 percent. This is because of poorer zoning at the thermal transition, and because the problems were not followed as far in time.

In most of our calculations we have assumed the incident spectrum is a 22 keV exponential (4). We have selected the exponential spectrum over a blackbody spectrum because the Uhuru data for Her X-1 indicate the spectrum is flat (constant energy per unit frequency) in the 2 - 20 keV range (Giacconi et al 1973). To examine the effect of spectral shape on mass loss rate, we repeated the example problem (model 14) reported in the first part of this section with the 22 keV exponential replaced with (a) a 22 keV exponential spectrum with a low energy cutoff at 1 keV (model 22), (b) a 22 keV exponential spectrum with a 2 keV cutoff (model 23), and (c) a 5 keV blackbody spectrum (model 24).

Introducing the 1 keV cutoff produced no noticeable change in mass loss rate. However, the 2 keV cutoff resulted in a 20 percent decrease in mass loss rate, and the 5 keV blackbody produced a 25 percent decrease in mass loss rate (see table 6-1). The problems were much different in character. The problem with no low energy cutoff and the problem with the 1 keV cutoff both relaxed quickly to a steady state (< 5 percent variation in physical quantities over times $\sim 10^4$ seconds). The problems with the 2 keV cutoff and the 5 keV blackbody spectrum exhibited persistent fluctuations at the thermal

(4) This is a fit to preliminary OSO-7 spectra provided to us by M. Ulmer. These spectra have since appeared in Ulmer et al (1973a).

transition (5). The period of these approximately sinusoidal fluctuations is $\sim 10^4$ seconds. The mass loss rate varied by as much as 15 percent from the average mass loss rate for the 2 keV cutoff problem and by 40 percent for the 5 keV blackbody problem. The 5 keV calculation was followed for one day (9 oscillations) and no damping of the oscillations was observed. We can only follow a few oscillations because of computer time limitations.

The difference in these four calculations can be traced to their different ρ -T curves (see Section IV) for material in radiative equilibrium with the incident X-rays. In figure 6-7 we present the steady state temperature as a function of density for material heated by the radiation field just above the photosphere in these calculations. These ρ -T calculations include both the incident beam of X-rays and the diffusion field reflected or re-emitted from the primary. For the 22 keV exponential with no low energy cutoff the pressure decreases with decreasing density for temperatures below 25 eV. For all other incident spectra considered, the pressure initially decreases with density but then begins increasing at a low temperature. For the 2 keV cutoff and the 5 keV blackbody, this increase begins ~ 3 eV. A sharp decrease (increase) in the density (temperature) must be present in the atmosphere calculations to maintain a decreasing pressure with increasing radius. Since we are limited to a spatial zone size $\sim 10^8$ cm, we cannot resolve this region of sharp transition. With our

(5) See Section Vc for a description of the calculation (model 16) which exhibited the largest scale, as well as the most repeatable, fluctuations at the thermal transition.

Fig. 6-7. Temperature versus density in radiative equilibrium using the radiation field at the photosphere for several input spectra. Curve A is for model 14, curve B is for model 22, curve C is for model 23, and curve D is for model 24.

computational method, we see a smoothly decreasing density from zone to zone until we reach the transition. Then the density drops one or two orders of magnitude across one zone boundary, with a corresponding increase in temperature. The density then decreases smoothly again. In the finely zoned problems, a typical pressure drop across this transition is 15 percent. Our computational procedure is to assume an initial configuration for the atmosphere, usually from another of our models, and run a dynamical problem until the atmosphere relaxes to a steady state. In those problems where the sharp transition in density is present, we observe oscillations in the temperature, density, and pressure at the transition during this dynamical phase of the calculations. The pressure varies above and below the value we would expect from examining the ρ - T curves, i.e., the pressure at the high density knee in figure 6-7.

We cannot be sure whether the oscillations we observe in these calculations are due to our physical model or to our numerical methods; it is our feeling that the physical model is the primary cause. The numerical feature of our calculations that we are suspicious of is the hydrodynamical boundary condition we adopt at the inner radius of the 25 g/cm^2 . We assume this inner problem radius is a rigid boundary. Hence sound waves are reflected. The period of the oscillations is $\sim 10^4$ seconds. The hydrodynamical timescale (pressure scale height/thermal velocity) is $\sim 10^4$ seconds in the high temperature region outside the photosphere, and $\sim 10^3$ seconds in the photosphere. $\sim 10^4$ seconds is also the time required for a sound signal to propagate

from the photosphere to the inner problem boundary and back, so the period of the oscillations does not allow us to discriminate between periodic readjustments in the high temperature region or initial perturbations traveling back and forth from the photosphere to the inner problem boundary. The reason we do not believe it is the latter is that in almost all problems, many of which start from configurations quite far from equilibrium, a steady state is achieved very quickly, typically in 2 or 3 periods. Further support for the physical origin of the persistent oscillations is that we note much more damping in the more coarsely zoned problems. The oscillations are the most persistent and have the greatest amplitude in those problems with the finest spatial zoning.

Even if the oscillations are primarily due to initial conditions in the atmosphere, we would expect these calculations to have some application to the HZ Her/Her X-1 system. They should be indicative of the way the atmosphere readjusts to changes in the incident X-ray luminosity. In particular, if a thermal jump is present above the HZ Her photosphere, readjustment to a new steady state might take one to a few days. During this readjustment we might see variations in the optical light on a timescale of 10^4 seconds.

In figure 6-8 we present the mass loss rate as a function of photospheric position (gravitational potential) and incident X-ray luminosity. These results were obtained assuming a 22 keV exponential spectrum and our cosmic abundances. After the photosphere is 5 to 10

Fig. 6-8. Mass loss rate versus gravitational potential of photosphere. Labels indicate Her X-1 luminosity. See table 6-1 for incident luminosity 1.3×10^{36} ergs/s.

eV inside the lip the mass loss rate varies slowly with increasing gravitational potential and surface gravity. Some of this variation is due to the increased geometric attenuation of the incident X-ray beam. In table 6-1 we see that ϵ , the ratio of mass flux to the incident flux, exhibits less variation than the mass loss rate with increasing potential.

When the photosphere is very near the lip in the potential, the X-rays can induce significant mass flow. However, these calculations would apply only to a very small fraction of the surface area of the primary. Perhaps the most important aspect of these calculations with the photosphere near the lip is that they suggest that the X-rays could cause substantial modification of the mass flow through the inner Lagrangian point if the primary filled its Roche lobe and if the inner Lagrangian point were not shielded from the X-rays. Of course this is the most likely point of the primary to be shielded if an accretion disk is present. Even in the absence of an accretion disk the mass stream through the inner Lagrangian point can provide almost complete shielding. Another potentially important result of these calculations is that for a fixed inner problem radius, increasing the incident luminosity distends the atmosphere, pushing the photosphere closer to, or over (compare models 9 and 10 in table 6-1), the lip in the potential. If the primary is very near its Roche lobe, there is the suggestion that an increase in the X-ray luminosity could cause the primary to fill or overfill its Roche lobe. Decreases in luminosity would have the opposite effect.

To test the sensitivity of mass loss rate to the choice of composition, we repeated our example problem with the abundances for elements with $Z > 2$ set to (a) twice our cosmic abundances (model 17) and (b) one tenth our cosmic abundances (model 18). Doubling the abundances increased the mass loss rate from 8.0×10^{16} to 1.4×10^{17} g/(s-sterad). Decreasing the abundances to 10 percent of cosmic reduced the mass loss rate to 1.1×10^{16} g/(s-sterad). These changes are due to changes in the density, temperature, and pressure at the thermal transition. Increasing the abundances increases the heating and increases the density at the high density knee of the ρ -T curve for radiative equilibrium. Decreasing the abundances has the opposite effect.

Thus far all calculations have been appropriate for the line of sight from the center of HZ Her to Her X-1. Other regions on the surface of the primary see a reduced flux of X-rays because of increased distance to Her X-1 and because the surface is no longer perpendicular to the incident beam. The oblique incidence also causes the beam to be attenuated in fewer g/cm² of the atmosphere (measured radially). To simulate this increased attenuation, we replaced $\partial/\partial r$ in our beam attenuation equation (eq. 4) with $(1/\mu_{PS})(\partial/\partial r)$. Here $\mu_{PS} = |\cos \theta|$ where θ is the angle at which the beam is incident on the photosphere. θ is measured from the normal to the surface. In the ionization equilibrium equation (eq. 9), E_v^b was replaced with E_v^b/μ_{PS} . We then repeated our example problem with μ_{PS} set to 0.3 (model 19) and 0.1 (model 20). For $\mu_{PS} = 0.1$, we also performed a

calculation with 1.3×10^{36} ergs/s incident (model 21). The gravitational potential was not modified for this series of calculations with variable μ_{PS} . Table 6-2 summarizes these calculations. Note that we have not changed our problem geometry; constant source luminosity implies constant energy flux per unit area on the primary. The energy density in the incident beam has been increased by $1/\mu_{PS}$. In the actual system, a fixed source luminosity implies constant incident beam energy density on the surface of the primary and incident energy flux decreasing as μ_{PS} . This is neglecting the nontrivial differences in geometric attenuation over the surface of the primary.

There is a significant increase in mass loss rate with decreasing μ_{PS} . Varying μ_{PS} from 1.0 to 0.3 doubles the mass loss rate. The mass loss rate doubles again as μ_{PS} decreases from 0.3 to 0.1. This increase results from the increase in radiation energy density at the thermal transition. The radiative equilibrium ρ - T curves scale with the ratio of the radiation energy density to the material density. Doubling the radiation energy density and the material density results in the same equilibrium temperature. If just the incident beam were driving the mass loss mechanism, and if there were no attenuation, the density at the thermal transition would vary as $1/\mu_{PS}$. Neglecting differences in radiation pressure and hydrodynamical cooling, the mass loss rate would vary as $1/\mu_{PS}$ also. The calculated mass loss exhibits less variation because the energy density of the diffusion field is varying only slightly with μ_{PS} and because of attenuation in and outside the thermal transition region. While the energy density of the diffusion field is not changing substantially at the

TABLE 6-2*

MASS LOSS FOR VARIABLE ANGLE OF INCIDENCE

Model [†]	L_X (ergs/s)	v_{PS}	R_{PS} (10^{11} cm)	g_{PS} (cm/s ²)	$\dot{M}$ (g/[s-sterad])	ϵ (g/erg)
14	1.3 (37)	1.0	2.67	2.0 (3)	8.0 (16)	1.9 (-20)
19	1.3 (37)	0.3	2.66	2.3 (3)	1.8 (17)	2.1 (-19)
20	1.3 (37)	0.1	2.64	2.1 (3)	3.2 (17)	3.8 (-19)
15	1.3 (36)	1.0	2.65	2.1 (3)	9.6 (15)	1.2 (-19)
21	1.3 (36)	0.1	2.64	2.1 (3)	5.3 (16)	6.3 (-19)

* v_{PS} is defined in the text. All other parameters are the same as in table 6-1. The incident spectrum in all problems was a 22 keV exponential.

[†] Model numbers from table 6-1.

thermal transition, its spectral shape is. In table 6-1 note the increase in the reflected X-ray energy as μ_{PS} is decreased.

To estimate the mass loss over the region of the primary being heated by the X-rays, the variation in the gravitational potential must be considered along with the variation in the incident flux and in μ_{PS} . The surface gravity varies from zero to 4×10^3 cm/s^2 over the portion of the primary Roche lobe exposed to direct X-ray heating. On this similarly exposed portion of the 25 eV isopotential, the surface gravity varies from 2×10^3 to 4×10^3 cm/s^2 . We have explored this range of surface gravity with our mass loss calculations. For surface gravities $> 10^3$ cm/s^2 , we find that ϵ , the ratio of mass flux to incident energy flux, varies weakly with surface gravity. We also find that ϵ varies weakly with the incident luminosity over the range $10^{36} - 10^{38}$ ergs/s. To make an estimate of the mass loss rate over the primary, we neglect the variation of ϵ with surface gravity and incident luminosity. We then find that the variation in μ_{PS} over the heated region should increase the mass loss rate by 50 to 70 percent. Specifically, if $A(\mu)$ is the mass loss rate as a function of μ , and $f(\mu)$ is the fraction of the X-ray heating that occurs for $\mu_{PS} > \mu$, then

$$\left[\int_0^1 A(\mu) df(\mu)/d\mu d\mu \right] / A(1) \approx 1.5 - 1.7 .$$

This average over μ has little dependence on the form assumed for $A(\mu)$ in the range $0 < \mu < 0.1$. 90 percent of the incident X-rays

strike the photosphere with $v_{ps} > 0.33$. While we have neglected the variation of ϵ with surface gravity over the range $1 \times 10^3 < g < 4 \times 10^3$ cm/s², it should be noted that the overall enhancement of the mass loss rate due to variable v_{ps} is comparable to the variation in ϵ over this range in surface gravity. We conclude that the ϵ we calculate along the line of centers is representative of ϵ averaged over the whole region of the primary subject to X-ray heating. Hence the mass loss rate is determined primarily by the total X-ray flux intercepted by the primary. We conclude that $\dot{M}/L_X$ is close to 10^{-19} g/erg.

c) Radial Oscillations of the Atmosphere

In Section Vb we note that for certain incident spectra we observe persistent oscillations in the physical variables at the thermal transition. The model in which these oscillations were the most pronounced was model 16. The incident luminosity for this model was 1.3×10^{38} ergs/s and the abundances for $Z > 2$ were set to 10 percent of cosmic. In other model calculations where we observed oscillations at the thermal transition, there was very little radial motion ($< 5 \times 10^8$ cm). Further, the mass loss rate never dropped more than 40 percent below the average mass loss rate. In model 16, the radial excursion was 4×10^9 cm, and the minimum mass loss rate was about an order of magnitude below the average mass loss rate. We have followed about 10 of these oscillations, and they seem very repeatable. The radial excursion of the photosphere encompasses 10

radial mesh points. The optical radiation rate varies by 20 percent during this oscillation. Figure 6-9 shows the photospheric radius, mass loss rate, and radiation rate for 1 and 2.6 eV as a function of time. These quantities were taken from the last three cycles of the calculation.

We attribute these oscillations primarily to the nature of the ρ -T curve for material in equilibrium with the radiation field just above the photosphere. In figure 6-10 we present the equilibrium temperature as a function of density for material heated by this radiation field (curve B). Reducing the abundances for $Z > 2$ shifts the high density knee to a much lower density, and makes the knee much sharper. The ρ -T curve for cosmic abundances (curve A) is included for comparison. Curve A is similar to curve A of figure 6-2 but the density has been shifted by an order of magnitude because of the increase in luminosity.

We observe another interesting hydrodynamical phenomenon when we take a steady state configuration with high incident luminosity and low photospheric gravity, and compute the evolution after reducing the incident luminosity by 10 or 100. The mass loss rate first decreases but then there is a sharp increase followed by a decay to the range appropriate for the reduced incident luminosity. As the most extreme example of this, we reduced the incident X-ray luminosity to 1.3×10^{36} ergs/s for model 3 ($L_X = 1.3 \times 10^{38}$ ergs/s, $\phi_{PS} = 0.01$ eV). After decreasing for 10^4 seconds, the mass loss rate increased to a maximum

Fig. 6-9. Photospheric radius, mass loss rate, and radiation rates for 1.0 and 2.6 eV photons as a function of time. Units for $\dot{M}$ and F_v are arbitrary. Quantities are from model 16 which exhibits persistent radial oscillations (see Section Vc).

Fig. 6-10. Curve B is the steady state temperature as a function of density for material subject to total photospheric radiation field of model 16 (see Section Vc) with reduced abundances for $Z > 2$. Curve A is for normal cosmic abundances (same as curve A of figure 6-2 but with the density shifted by an order of magnitude).

of 7.0×10^{19} g/(s-sterad) at 1.5×10^4 seconds. It then decayed to the steady state value appropriate to $L_x = 1.3 \times 10^{36}$ ergs/s after 5×10^4 seconds. The total mass lost during this pulse was 2×10^{23} g/sterad, which, if we assume a solid angle of 0.5 steradians, would be enough to power the X-ray source at the Eddington limit for one to two days.

VI. Conclusions

Our calculations indicate the X-ray induced mass loss rate is too small to sustain an accretion powered X-ray mechanism unless the photosphere of HZ Her is very near the Roche limit. For each erg incident on the surface of the primary, we transfer $\sim 10^{-19}$ grams of material (ϵ in table 6-1). Each gram transferred produces $\sim 0.1 c^2$ ergs as it falls onto Her X-1. $\sim 4 - 5$ percent (7) of the emitted X-rays will strike HZ Her. Hence, $2 - 3 \times 10^{-19}$ g/erg is the minimum possible value for ϵ for which the self-excited flow can operate. This range for ϵ assumes Her X-1 is 100 percent efficient in capturing the material driven off the primary. The capture efficiency will be substantially less than this because the X-rays will continue to heat the material, which has already reached a temperature of several hundred eV at the lip (8), as it falls toward Her X-1.

We did not achieve this self sufficient range for ϵ until the photosphere was very near or over the lip in the potential. Even for these calculations it is not obvious that ϵ is increasing rapidly

(7) For the system parameters we have chosen, the Roche lobe of the primary subtends ~ 4.9 percent of the solid angle of Her X-1, while the 25 eV isopotential surface of the primary subtends ~ 4.3 percent of the solid angle. If we take $\sin i = 1$, the $.24^d$ duration (Giacconi et al 1973) of the X-ray eclipse in the 1.7^d orbital period implies the primary subtends ~ 4.8 percent of the solid angle of the X-ray source.

(8) The potential of the Roche lobe, as measured by an observer at infinite radius, is ~ 0.6 keV.

enough to compensate for the fact that it applies to a smaller and smaller fraction of the heated area. ~90 percent of the X-ray exposed portion of the primary Roche lobe is at a surface gravity $> 1 \times 10^3$ cm/s² and from table 6-1 we see that surface gravities above 10^3 have efficiencies near 10^{-19} g/erg.

We would like to comment on two major points of difference between our calculations and those of Basko and Sunyaev (1973). First, they calculated values for ϵ more than an order of magnitude greater than those we have calculated, and therefore reached the opposite conclusion as to whether the X-ray powered mass transfer mechanism could operate in the HZ Her/Her X-1 system. Second, they found that introducing a 1 keV cutoff in the incident spectrum reduced the mass loss rate by 50 percent. We calculated no decrease in mass loss rate as a result of a 1 keV cutoff, and a 20 percent decrease as a result of introducing a 2 keV cutoff.

Referring to tabel I of Basko and Sunyaev, we find that for their HZS(20) model $\epsilon \approx 6 \times 10^{-18}$ g/erg and for HZ(20) $\epsilon \approx 3 \times 10^{-18}$ g/erg. These much larger values for ϵ are due to their computing a higher density for the thermal transition. Referring to their figure 4, we see that the pressure at the thermal transition is $\sim 10^3$ ergs/cm³. Assuming a temperature of 1.4×10^4 °K, an ideal gas equation of state, and cosmic abundances, this implies a density $\sim 6 \times 10^{-10}$ g/cm³ just interior to the transition. We infer that with their non-LTE model, the high density knee in the radiative equilibrium p-T curve occurs at a density more than an order of magnitude

larger than the density at which the knee occurs in our calculations. We presume they are calculating a larger heating rate than we are. Three differences in our non-LTE model that could produce less heating are: (a) Our oxygen opacity is effectively smaller since it is falling off as ν^{-3} rather than $\nu^{-2.4}$ as Basko and Sunyaev assumed. (b) We assume a smaller radiative recombination rate since we only allow recombination to the K-shell. This reduces K-shell population. (c) We re-emit a K edge photon unless an Auger transition takes place, so the net energy transferred to the material is $h\nu - h\nu_I$. Our peak heating then occurs at $\nu = (3/2)\nu_I$, and is down by factor six from the heating rate that one would calculate at ν_I without the re-emitted K edge photon. This is a nontrivial difference since ~ 2 percent of the incident energy is in the 0.4 keV interval immediately above the oxygen K edge.

Introducing a 1 keV cutoff in our calculations has virtually no effect because photons below 1 keV do little heating until they get near the bottom of the transition. The helium is almost completely ionized and does not make a substantial contribution to the heating rate for temperatures greater than 4 or 5 eV.

Radial oscillations persisted in several of the calculations. They were most pronounced in model 16 which assumed an incident luminosity of 1.3×10^{38} ergs/s and had reduced abundances for $Z > 2$. The timescale for these oscillations varied from 10^4 to 2×10^4 seconds. There was substantial modulation of the optical radiation rate during these oscillations.

7. TIME VARIATION OF OPTICAL RADIATION FROM HZ HERCULIS

We have studied the optical response of the atmosphere of HZ Her to variations in the incident X-ray luminosity. First we took the steady state configurations from our example problem (model 14 of Chapter 6), and, at time $t = 0$, reduced the incident X-ray luminosity from 1.3×10^{37} to 1.3×10^{36} ergs/s. We then computed the time decay of the optical radiation rates. In figure 7-1 we present the time decay of the radiation rates for photon energies from 2.0 to 3.3 eV and for the 25 keV radiation reflected from the atmosphere. Note the exponential decay time is very short over the first second and then approaches a value ~ 30 seconds at large times.

Next we subjected several different steady state configurations from our mass loss problems of Chapter 6 to an incident X-ray luminosity with the same time averaged value, but varying as $\sin^4(\pi t/1.24)$. Although this is a much smoother time variation than the X-ray pulse shape we observe from Her X-1, it is far more peaked in time than the optical response we calculate. In all cases the variation in the optical radiation rate is very smooth during the 1.24^s cycle, and, neglecting a phase lag, is well described by the function $a + b \sin(\pi t/1.24)$. Hence we conclude $\sin^4$ is an adequate simulation of the X-ray pulse shape.

Recall that we assume the ionization equilibrium (eq. 9 of Chapter 6) adjusts immediately to any change in the radiation energy density, electron number density, or temperature. Also recall that we neglect retardation (photon time of flight) in computing the attenuation of

Fig. 7-1. Response of frequency dependent radiation flux from the primary in model 14 (of Chapter 6) after reduction (at $t = 0$) of incident X-ray luminosity from 1.3×10^{37} to 1.3×10^{36} ergs/s. Labels indicate photon energy in eV. Each radiation flux is normalized to its value at $t = 0$.

the incident beam. These are still reasonable assumptions to use in the computation of the response of the atmosphere to the 1.24^S variation in the incident X-ray flux. The material outside the sharp transition in temperature has virtually no effect in these calculations of the optical modulation. One is effectively just computing the response of the atmosphere from the thermal transition on inward. Light travel times and recombination times in this region are substantially less than the 1.24^S X-ray pulsation period. In the example problem of Section Va, Chapter 6, ~ 0.5 percent of the X-ray heating takes place outside $R = 2.673 \times 10^{11}$ cm ($D = 1.8 \times 10^{-2}$ g/cm²); ~ 20 percent of the X-ray heating takes place between this radius and $R = 2.654 \times 10^{11}$ cm ($D = 2.1 \times 10^{-11}$ g/cm²). Light transit time across this interval is 0.06 seconds. In the same example problem, the oxygen recombination time is ~ 10 percent of the pulse period at 8 eV, and is decreasing as $T^{3/2}$ (neglecting the change in pressure over the transitional region). Finally, the material energy content of the region in which the oxygen recombination time varies from 0.1 to 3 seconds is about 1 percent of the energy in each 1.24^S pulse of X-rays. Regardless of what this region does between X-ray pulses, it takes only a small fraction of the X-ray energy to effect any ionizational equilibrium.

We repeated our pulsational calculations for this example problem replacing our assumption of instant ionization equilibrium with the assumption that the ionization equilibrium was that of the steady state calculation. This decreased the 1 - 3.8 eV modulation from 19 percent

to 18 percent.

In table 7-1 we list the computed modulation (1) for photon energies from 1 to 3.3 eV as well as the modulation of the energy summed from 1 to 3.8 eV. Time of flight differences across the photosphere will decrease the optical modulation by a factor of approximately $cP/2\pi R \approx 0.02$. Here P is the pulsation period. The observed modulation (Middleitch 1973) is at most 0.3 percent and is usually less ($\sim 0.15\%$).

Within the context of our model, our calculations suggest that optical modulation should be present at the 0.3 - 0.4 percent level at all times that the pulsed X-rays are striking the surface of HZ Her (assuming cosmic abundances). Modifying the abundances and/or the spectral shape can reduce the modulation to the 0.1 percent level, but this would still be within the observational limits. Our primary reason for believing that the X-ray driven wind is not the basic mass transfer mechanism in the HZ Her/Her X-1 system is that we calculate too low a mass transfer rate. We take the absence of persistent optical modulation to be further support for this conclusion. We feel a realistic model for this system must be different from the model we studied in Chapter 6.

(1) We define the modulation for frequency ν to be $(F_{\nu}^{\max} - F_{\nu}^{\min}) / (F_{\nu}^{\max} + F_{\nu}^{\min})$, where $F_{\nu}^{\max}$ is the maximum radiation rate during the 1.24^s cycle, and $F_{\nu}^{\min}$ is the minimum radiation rate.

TABLE 7-1*
MODULATION OF OPTICAL RADIATION

Model [†]	L_X (ergs/s)	ξ_{PS} (cm/s ²)	$f_{\gamma>2}$	Modulation (percent)					
				$h\nu = 1.0$ eV	1.6 eV	2.0 eV	2.6 eV	3.3 eV	1-3.0 eV
7	1.3 (37)	6.6 (2)	1.0	18.	15.	12.	11.	23.	17.
10	1.3 (37)	3.6 (2)	0.1	3.0	2.3	1.8	1.7	5.0	2.9
13	1.3 (38)	1.9 (3)	1.0	25.	20.	18.	17.	18.	18.
14	1.3 (37)	2.0 (3)	1.0	21.	23.	21.	18.	19.	19.
14 [¶]	1.3 (37)	2.0 (3)	1.0	9.2	10.	9.5	8.5	8.5	8.9
15	1.3 (36)	2.1 (3)	1.0	13.	17.	16.	15.	15.	15.
16	1.3 (38)	1.9 (3)	0.1	8.7	8.5	8.0	8.0	10.	9.0
16	1.3 (38)	1.8 (3)	0.1	9.5	8.0	7.0	7.5	8.0	7.5
19	1.3 (37)	2.1 (3)	1.0	26.	28.	25.	23.	24.	25.
20	1.3 (37)	2.1 (3)	1.0	38.	39.	34.	29.	31.	33.
23	1.3 (37)	2.0 (3)	1.0	15.	17.	15.	14.	15.	15.
24	1.2 (37)	2.0 (3)	1.0	9.4	10.	9.4	8.3	8.2	8.7

*All parameters are the same as in table 6-1.

[†]Model numbers from table 6-1.

[¶]For this calculation half the incident energy was pulsed and half was steady state.

It is important to note that our optical modulation calculations can be applied to other models of the HZ Her/Her X-1 system. If one assumes that HZ Her fills or nearly fills its Roche lobe at all times, and that Roche lobe overflow is the mass transfer mechanism, then the surface gravity over that portion of HZ Her that could be exposed to direct irradiation from Her X-1, if there were no intervening material, ranges from zero to $4 \times 10^3 \text{ cm/s}^2$. This is the range of surface gravities we have considered in our mass transfer calculations, and in our optical modulation calculations. Increasing the surface gravity from 2×10^3 to $4 \times 10^3 \text{ cm/s}^2$ decreases scale heights. This increases slightly the amplitude of the optical response to the 1.24^{s} X-ray pulsation. Material around the X-ray source, say in an accretion disk, will provide partial or total shielding of the surface of HZ Her. But if the line of sight from Her X-1 to any region on the surface of HZ Her is relatively free of material ($< 0.1 \text{ g/cm}^2$), then our calculations of the optical modulation should apply for the portion of the energy that is pulsed and that is directly incident on the surface. If we assume half the energy is pulsed and half the energy is steady state, the modulation is half the modulation calculated when all the radiation is pulsed (compare the two calculations for model 14 in table 7-1). The material free line of sight could be a transient phase during the 35 day X-ray cycle.

8. X-RAY HEATING OF THE INFALLING GAS STREAM

I. Introduction

In Chapter 6 we concluded that for the HZ Her/Her X-1 system, the X-rays incident on the surface of the primary could not drive off enough material to power the X-ray source. If this conclusion is correct, either a stellar wind or gravitational lobe overflow must provide the mass transfer for this system. There is no reason to attribute a strong stellar wind to the primary. Its mass is about $2 M_{\odot}$ and the star seems normal except for the reflection effect due to X-ray heating. This normal character is confirmed by optical observations during the extended periods (several years) of "X-ray off" when the optical light curve exhibits only the "2 θ " variation of the orbital period (cf Whelan 1973), and shows no X-ray heating effects. Normal $2 M_{\odot}$ stars do not exhibit large winds. Since HZ Her appears to be a normal star, we assume that it does not have a strong wind. We conclude that the mass transfer mechanism most likely to operate in the HZ Her/Her X-1 system is gravitational lobe overflow.

At this point we have a stream of gas flowing through the inner Lagrangian point toward the secondary. What happens to this material as it falls toward the secondary? Conventional wisdom has the cold material entering an accretion disk around the compact X-ray object.

This accretion disk in turn transports the material onto the neutron star (Pringle and Rees, 1972; Shakura and Sunyaev, 1973; Novikov and Thorne, 1973). These authors expect an accretion disk to form because the infalling material has high angular momentum relative to the compact object. They assert that this will cause the material to enter approximately circular orbits about the compact object, forming a differentially rotating disk. They then go on to compute the general properties and structure of these disks. The rate at which these disks can transport material inward is determined by the rate at which they can transport angular momentum outward. In order to transport enough material inward to power the X-ray source, one needs an anomalously large viscosity. Disk models constructed thus far invoke either a turbulent viscosity or a magnetic viscosity. These ad hoc viscosities are likely to be the weakest part of the disk models. Another weak point is that the disk models do not address the question of how material enters the accretion disk. They do not model the interaction of the infalling gas stream with the outer edge of the disk, and they do not look at the effect of the X-ray heating on the infalling gas stream.

As a starting point for this investigation, we assume that cold material does flow through the inner Lagrangian point. Our goal is to determine how the X-ray heating affects the mass flow. If the X-ray

heating can produce high material temperatures (1) at large distances from the secondary, hydrodynamical motions with large velocities will result, causing the gas stream to expand as it falls toward the secondary. There are two interesting possibilities. First, if the hydrodynamical velocities are less than but still comparable to the infall velocities, the infalling gas stream could have a large cross sectional area when it reaches the outer radius of the accretion disk, extending far out of the orbital plane. This creates a potential problem for the disk models because they assume the disk is rather flat ($z/r \approx .05 - .10$). The outer edge of the disk may be able to interact effectively with only a small fraction of the infalling stream. Second, if the hydrodynamical velocities exceed the infall velocities, then the material may be able to flow on both sides of the secondary. If the hydrodynamical velocities are sufficiently large there is the exciting possibility that we may be able to replace the accretion disk model for HZ Her/Her X-1 with a model where the accretion is taking place out of

(1) Without X-ray heating, observational evidence indicates the infalling material will form a disk around the secondary. There are numerous examples of gas streams and disks observed in close binary systems (non-X-ray binary systems). In the absence of an X-ray luminosity we would agree with the disk proponents that the high relative angular momentum of the infalling material would result in the formation of a disk around the secondary. The crucial question would then be whether there is a sufficiently large viscosity in the system so that enough angular momentum can be transported outward to provide an adequate inward mass flow.

a "stellar wind" produced between the primary and secondary. This model may be able to avoid the viscosity problems of the accretion disk. We should caution that this alternate model may have some serious problems of its own. Shakura and Sunyaev (1973) argue that with an accretion shock standing around the secondary, the material accreted will have a net angular momentum with respect to the secondary, and will form a disk inside the accretion shock. Their examination of this issue does not go beyond the simple assertion that if the secondary is accreting out a wind with an angular momentum gradient, the accreted material will have a net angular momentum. In the absence of detailed calculations, we do not find this a persuasive argument. The X-rays can provide tremendous heating, with the resulting internal energy of the gas well above the gravitational binding energy. If some of the material, after passing through the shock, still has too much angular momentum to fall onto the secondary, the material pressure might push it out of the system. If the X-ray heating dominates the gas temperature, the shock structure will be very peculiar. At sufficiently low densities, the X-rays want an isothermal temperature distribution because Compton heating and cooling is the dominant energy exchange mechanism. However, at intermediate densities the X-rays adjust the temperature to produce a nearly constant pressure. Recall in Chapter 6 that the radiative equilibrium ρ - T curve followed a roughly constant pressure contour from ~ 1 keV down to ~ 2 eV. Preliminary results from three dimensional hydrodynamical calculations, with very crude X-ray transport, suggest the shock structure may be far different than can be understood in the context of simple fluid dynamics. (Here we are referring to calculations being performed by Alme and Wilson - see Chapter 10.)

In Section II of this chapter we will digress from our examination of the X-ray heating and study geodesic motion of test particles in the gravitational field of the binary system. The aim is to provide insight into the motion of a cold gas stream, and to indicate the modifications in the gas motion that the X-ray heating can produce. In Section III we examine the X-ray heating that results assuming radial infall in a spherically symmetric geometry. In Section IV we allow expansion of the infalling gas stream during infall. Finally, in Section V we generalize the results of Section IV to incorporate an approximate treatment of the binary gravitational potential.

II. Geodesic Motion

Assume that the stars revolve around one another in circular orbits. Use coordinates x, y, z such that the z axis passes through the center of mass of the binary system, normal to the orbital plane, and the primary lies on the negative x axis. In this rotating coordinate system, the equation of motion for a test particle (restricted three-body problem) is

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} + 2\vec{\omega} \times \vec{u} = \nabla \phi$$

where

$$\phi = -\frac{GM_p}{r_p} - \frac{GM_s}{r_s} + 1/2 \omega^2 (x^2 + y^2) \quad .$$

Here M_p is the primary mass, M_s is the secondary mass, r_p is the distance to the center of the primary, and r_s is the distance to the center of the secondary. ω is the rotational velocity of the binary system. If we restrict the motion to the orbital plane, the equation of motion becomes

$$\frac{du}{dt} = 2\omega v - \frac{GM_p(x - x_p)}{r_p^3} - \frac{GM_s(x - x_s)}{r_s^3} + \omega^2 x$$

$$\frac{dv}{dt} = -2\omega u - \frac{GM_p y}{r_p^3} - \frac{GM_s y}{r_s^3} + \omega^2 y$$

where $u = dx/dt$, $v = dy/dt$, and x_p and x_s are the x coordinates of the primary and secondary, respectively.

To determine the trajectory of material leaving the surface of the primary, we must know its initial position and velocity. The surface of the primary should approximately follow a surface of constant potential (gravitational plus rotational). There is a maximum critical equipotential surface which encloses the primary, but not the secondary. If the primary attempts to expand beyond this critical equipotential, material will spill over onto the secondary. This is the phenomenon we have termed gravitational lobe overflow. The shape of the critical equipotential depends not only on the masses of the primary and secondary, but also on the rotational velocity of the primary. If the primary is in synchronous rotation, the Roche potential (see Chapter III of Kopal, 1959, for a definitive discussion) will be the relevant potential. This potential should be a reasonable approximation for the

outer portions of HZ Her since this binary system seems to be very old, and tidal forces have had a long time to operate. The Roche potential may be a poor approximation for the systems with massive supergiant primaries since these supergiants are very young stars, and tidal forces have not had time to enforce co-rotation in the outer layers of the primary (2). If the rotational period of the primary is small compared with the orbital period, the tidal potential (cf Jeans 1928) will apply. We note that when the mass ratio is near unity, as is the case for HZ Her/Her X-1, the position of the saddle in the critical equipotential is little affected by the rotational potential. As we are primarily interested in the Hercules system, we assume the Roche equipotential describes the surface of the primary. We assume the velocity of the material coming through the saddle is comparable to the thermal velocity at the surface of the star.

Figure 8-1 indicates the motion of three test particles leaving the inner Lagrangian point, each with a kinetic energy of 1 eV (3). To

(2) Little is known about how much angular momentum is transferred during evolution in close binaries. It is possible that selection effects operate during the mass exchange phases of binary evolution, in turn making the angular momentum distribution in the outer layers of massive stars in close binary systems much different from the angular momentum distribution observed in the outer layers of isolated massive stars.

(3) This is the kinetic energy for a particle with mass equal to the average mass per particle of fully ionized material, assuming cosmic abundances.

Fig. 8-1a.

Fig. 8-1a. Geodesics followed by three test particles leaving the saddle with a kinetic energy of 1 eV. Initial velocity of the upper and lower trajectories is 60° from line of centers. Labels on curves are 10^2 seconds apart.

Fig. 8-1b.

Fig. 8-1b. Geodesics followed by five test particles leaving the saddle with a kinetic energy of 1 eV. Initial velocity of the upper and lower trajectories is 60° from line of centers. Labels on curves are 10^2 seconds apart.

simulate the velocity distribution of gas streaming through the saddle, one of the particles is directed along the line of centers, while the other two are directed 60 degrees away from the line of centers. The variation in initial velocity produces no essential differences in the motion. In the absence of X-ray heating, pressure effects will cause negligible expansion. The material can cool very effectively by radiative mechanisms, and any compressional heating can easily be dissipated. The material falls toward the secondary until it interacts with material already in orbit. Neglecting the question of angular momentum transport, the infalling material will then go into approximately Keplerian orbit about the secondary. We emphasize that the interaction distances for the infalling stream are very small. At density 10^{-12} g/cm³, assuming a collisional cross section of 10^{-16} cm², the mean free path for a particle is about 10^5 cm. Typical dimensions in the binary system are 10^{11} cm.

To have accretion out of a wind, an appreciable fraction of the infalling material must go into counter-rotation, flowing on the "back side" of the star. What temperatures, at what radii, are required to produce this counter-rotation? To crudely simulate hydrodynamical effects, we follow one test particle along a chosen geodesic. At an arbitrarily specified position, we "explode" the test particle, emitting new test particles with an isotropic velocity distribution in the frame of the original test particle. We then follow this new family of geodesics. In figure 8-2a-c, we indicate the motion resulting when a

Fig. 8-2a.

Fig. 8-2. Motion resulting from 100 eV "bursts." Each group of test particles was emitted with an isotropic velocity distribution in the frame comoving with the central geodesic of figure 8-1. Labels are spaced 10^2 seconds apart.

Fig. 8-2b.

Fig. 8-2. Motion resulting from 100 eV "bursts." Each group of test particles was emitted with an isotropic velocity distribution in the frame comoving with the central geodesic of figure 8-1. Labels are spaced 10^2 seconds apart.

Fig. 8-2c.

Fig. 8-2. Motion resulting from 100 eV "bursts." Each group of test particles was emitted with an isotropic velocity distribution in the frame comoving with the central geodesic of figure 8-1. Labels are spaced 10^2 seconds apart.

group of particles is emitted at various positions along the central geodesic (initial velocity along the line of centers) of figure 8-1. The kinetic energy of each emitted particle is 100 eV, measured in the frame co-moving along the original geodesic. In figures 8-3a-c, the kinetic energy per particle in each burst is 200 eV, while in figures 8-4a-c, the kinetic energy is 400 eV. We see that where the "heating" occurs at large radii, several of the particles pass behind the secondary. When the "heating" takes place closer to the secondary, some of the particles are ejected from the system, but none go behind the secondary. As a final example, we have followed the motion resulting from 5 keV bursts (figures 8-5a-c). 5 keV is the maximum temperature the X-rays from Her X-1 can produce, assuming a 20 keV exponential spectrum. While these geodesic calculations are no substitute for hydrodynamical calculations, they make it clear that if the X-rays are going to turn the gas stream through the inner Lagrangian point into a wind, the X-ray heating must occur at a large radius.

III. Heating with Spherically Symmetric Infall

The geodesic calculations of the previous section indicate that if the X-rays can heat the infalling material to a temperature of several hundred eV at a distance $\sim 10^{11}$ cm from the X-ray source, a substantial fraction of the infalling material may be able to flow behind the secondary. In this section and the following two sections we calculate the X-ray heating in a sequence of models, a sequence which attempts to develop progressively more realistic models for the binary system

Fig. 8-3a.

Fig. 8-3. Motion resulting from 200 eV "bursts." Each group of test particles was emitted with an isotropic velocity distribution in the frame comoving with the central geodesic of figure 8-1. Labels are spaced 10^2 seconds apart.

Fig. 8-3b.

Fig. 8-3. Motion resulting from 200 eV "bursts." Each group of test particles was emitted with an isotropic velocity distribution in the frame comoving with the central geodesic of figure 8-1. Labels are spaced 10^2 seconds apart.

Fig. 8-3c.

Fig. 8-3. Motion resulting from 200 eV "bursts." Each group of test particles was emitted with an isotropic velocity distribution in the frame comoving with the central geodesic of figure 8-1. Labels are spaced 10^2 seconds apart.

Fig. 8-4a.

Fig. 8-4. Motion resulting from 400 eV "bursts." Each group of test particles was emitted with an isotropic velocity distribution in the frame comoving with the central geodesic of figure 8-1. Labels are spaced 10^2 seconds apart.

Fig. 8-4b.

Fig. 8-4. Motion resulting from 400 eV "bursts." Each group of test particles was emitted with an isotropic velocity distribution in the frame comoving with the central geodesic of figure 8-1. Labels are spaced 10^2 seconds apart.

Fig. 8-4c.

Fig. 8-4. Motion resulting from 400 eV "bursts." Each group of test particles was emitted with an isotropic velocity distribution in the frame comoving with the central geodesic of figure 8-1. Labels are spaced 10^2 seconds apart.

Fig. 8-5a.

Fig. 8-5. Motion resulting from 5 keV "bursts." Each group of test particles was emitted with an isotropic velocity distribution in the frame comoving with the central geodesic of figure 8-1. Labels are spaced 10^2 seconds apart.

Fig. 8-5b.

Fig. 8-5. Motion resulting from 5 keV "bursts." Each group of test particles was emitted with an isotropic velocity distribution in the frame comoving with the central geodesic of figure 8-1. Labels are spaced 10^2 seconds apart.

Fig. 8-5c.

Fig. 8-5. Motion resulting from 5 keV "bursts." Each group of test particles was emitted with an isotropic velocity distribution in the frame comoving with the central geodesic of figure 8-1. Labels are spaced 10^2 seconds apart.

geometry.

a. Spherically Symmetric Model

Our initial model assumes a spherically symmetric geometry with its origin at the center of the secondary. In this spherically symmetric geometry the radial dependence we assume for the gravitational potential and the X-ray flux is the spatial variation along the line of sight from the center of the secondary to the center of the primary. We use the same equations of motion, the same radiation transport model, and the same non-LTE model that we used in the X-ray induced mass transfer calculations of Chapter 6. The positions of the primary and secondary have been exchanged, and the incident beam has been placed at the center of the problem rather than at the outer boundary.

The outer boundary of the problem is placed at the lip in the one dimensional gravitational potential. At this boundary we specify the temperature, density, and velocity of the infalling material. The temperature is set to a value representative of the X-ray heated surface of the primary, and the velocity is set equal to the thermal velocity associated with this temperature. We set the density by specifying

- (1) a mass flow rate through a "throat" at the inner Lagrangian point, and
- (2) a cross-sectional area for the throat. The density is calculated as

$$\rho = \frac{dM/dt}{\pi R_{\text{throat}}^2 v} \quad .$$

The throat area is πR_{throat}^2 , where R_{throat} is the perpendicular distance from the line of centers to the gravitational isopotential which lies C_V outside the Roche equipotential. R_{throat} is measured in the plane which passes through the inner Lagrangian point, and is perpendicular to the line of centers.

Since the distance of closest approach to the secondary by the 1 eV geodesics is a few times 10^{10} cm, we take 4×10^{10} cm as the inner boundary of the problem. At this inner boundary we assume there is no material pressure gradient, and allow material to flow across the boundary. These assumptions are not appropriate if an optically thick accretion disk surrounds the X-ray source, shielding the infalling gas stream, and providing a pressure gradient. However, it is our hope that as we progress through our model sequence we will calculate progressively more X-ray heating, finally achieving a situation where the X-ray heating can produce an optically thin wind flowing by the X-ray source. In this final situation our internal boundary conditions would be appropriate.

b. Results

We have run two problems with this spherically symmetric model. In the first problem (test 1) we adopted straightforward estimates for the parameters of the HZ Her/Her X-1 system. We took the mass of the primary as $2 M_{\odot}$, the mass of the secondary as $1 M_{\odot}$, and the separation as 6×10^{11} cm. The surface temperature of the primary was assumed to be 1 eV, with the resulting throat radius 1.85×10^{10} cm. The X-ray

luminosity was set at 1.3×10^{37} ergs/s, and the mass flow rate was set to a value consistent with the X-ray luminosity, i.e., the X-ray source would have to capture all the material streaming through the inner Lagrangian point in order to radiate at 1.3×10^{37} ergs/s (assuming an X-ray production efficiency of 10% of the infalling rest mass). Element abundances were assumed to be our cosmic abundances (see Chapter 6).

The throat subtends just over one tenth of one percent of the solid angle of the X-ray source. In the 4π of our spherically symmetric problem geometry we have a total mass infall rate of about a thousand times that required to power the X-ray source. We are asking if the X-rays can penetrate through this infalling material and heat the material at radii $\sim 10^{11}$ cm.

In this first problem, the calculated X-ray heating was negligible. The maximum temperature calculated was 4 eV, in comparison with a maximum allowable temperature of 15 eV. This maximum allowable temperature assumes (a) no attenuation of the incident X-rays, and (b) negligible hydrodynamical heating and cooling. In figure 8-6, we present the computed temperature and the maximum allowable temperature as a function of radius. In figure 8-7, we show the density, velocity, and column depth as a function of radius. We conclude that for this set of problem parameters, the infalling stream is well shielded, and the X-ray heating effects are small.

The radius of the throat was doubled for the second problem (test 2), reducing the boundary density by a factor of four. All other

Fig. 8-6. Temperature versus radius for test 1. Solid curve is computed temperature. Dashed curve is maximum allowable temperature ("optically thin" approximation).

Fig. 8-7. Velocity, density, and column depth versus radius for test 1. Ordinate range for density is 10^{-11} to 10^{-9} g/cm^3 . The range for column depth is 10^{-1} to 10 g/cm^2 .

problem parameters were the same as for test 1. The maximum temperature computed was 80 eV. In figure 8-8 we plot the computed temperatures and maximum allowable temperatures versus radius. Figure 8-9 presents the density, velocity, and column depth versus radius. While none of these temperatures are interestingly large, the large change in temperature associated with the four-fold decrease in density indicates we may be near a region where lower densities and less shielding will produce interesting temperatures.

IV. Radial Infall with Expansion

In the previous section we saw that to heat the infalling material at large radii, the density in the infalling stream would have to be reduced. This decreases the self shielding of the infalling stream, and increases the ratio of the X-ray energy density to the material density, allowing higher material temperatures. In our spherically symmetric model we were effectively calculating the behavior in a cone which had the throat as its base and the origin as its vertex. By confining the infalling gas to the cone, we neglected the motion out the sides of the cone that the pressure would cause. This sideward motion would have little effect of the radial velocity, but it would reduce the density of the infalling gas, permitting more heating. The following model attempts to simulate this sideward motion.

a. "One and a Half Dimensional" Hydrodynamics

We establish a spherical polar coordinate system with its origin at

Fig. 8-8. Temperature versus radius for test 2. Solid curve is computed temperature. Dashed curve is maximum allowable temperature ("optically thin" approximation).

Fig. 8-9. Velocity, density, and column depth versus radius for test 2. Ordinate range for density is 3×10^{-12} to 10^{-9} g/cm^3 . The range for column depth is 3×10^{-2} to 10 g/cm^2 .

the center of the secondary and its $\theta = 0$ axis along the line of centers. We maintain our spatial zoning in the radial direction. At each radial mesh point we assume there is one spatial zone. Each of these spatial zones, assumed to be axisymmetric, has a θ extent, say $\theta(r)$. At the beginning of the problem, each spatial zone has the same θ , as in figure 8-10a. As the problem evolves, θ is allowed to change for each zone. When a steady state is achieved, the zoning may be as in figure 8-10b. A θ velocity is computed at each zone using a difference equation approximation to the standard $d\theta/dt$ equation of hydrodynamics:

$$\frac{\partial \dot{\theta}}{\partial t} = - \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r \dot{\theta}) + \dot{\theta} \frac{\partial v_r}{\partial r} - \frac{1}{\rho r^2} \frac{\partial}{\partial \theta} (1/2 \rho r^2 \dot{\theta}^2 + P) .$$

To recapitulate, we treat two dimensional hydrodynamics in an axisymmetric geometry. We adopt spherical polar coordinates r and θ , with the origin at the center of the secondary, and the $\theta = 0$ axis directed toward the primary. In our numerical treatment we utilize a mesh with a reasonable number of zones in the radial direction, but with only one zone in the θ direction. The numerical treatment is a mixed Eulerian-Lagrangian method. A problem cycle consists of a cycle of Lagrangian hydrodynamics, followed by a remapping of the mesh onto a new mesh with the same radial coordinates as at the beginning of the hydro cycle (see figure 8-11). Momentum transfer due to the radiation flow is included in the computation of the θ acceleration.

The diffusion radiation field is allowed to leak out the sides of

Fig. 8-10. Evolution of r, θ mesh during calculation. Figure 8-10a illustrates the mesh at the beginning of the calculation, and figure 8-10b indicates the mesh after a steady state has been reached.

Fig. 8-11. Evolution of mesh during one problem cycle. Figure 8-11a indicates the mesh at the beginning of the hydrodynamical cycle, figure 8-11b indicates the mesh at the end of one cycle of Lagrangian hydrodynamics, and figure 8-11c illustrates the problem mesh remapped onto the original radial mesh.

the mesh. This flow is described by a diffusion equation with a "flux-limited" diffusion coefficient. This diffusion coefficient limits the radiation flux to the product of light velocity and the radiation energy density.

b. Results

With the "one and a half dimensional" code we ran one problem (test 3) where we assumed the same boundary conditions as in test 1. Allowing the sideward motion produced a dramatic change in the X-ray heating. In figure 8-12 we present the computed temperatures and the maximum allowable temperatures. In figure 8-13 we plot the density, velocity, and column depth versus radius. The final r, θ configuration is presented in figure 8-14. For comparison we also plot in figure 8-14 the cone to which the material was confined in test 1.

V. An approximate Treatment of the Gravitational Potential in the Binary System

In the previous section we saw that by allowing θ motion of the material, we can reduce the density of the infalling material, dramatically increasing the X-ray heating effects. In this section, we add an approximate treatment of the rotation that the infalling material undergoes in the binary system gravitational potential. We wish to see if the centripetal effects will hold the material away from the X-ray source enough to allow more heating and expansion, without an off-setting increase in the self shielding.

Fig. 8-12. Temperature versus radius for test 3. Solid curve is computed temperature. Dashed curve is maximum allowable temperature ("optically thin" approximation).

Fig. 8-13. Velocity, density, and column depth versus radius for test 3. Ordinate range for density is 6×10^{-13} to 10^{-10} g/cm³. The range for column depth is 6×10^{-3} to 1 g/cm².

Fig. 8-14. Final r, θ configuration for test 3. Also indicated is the "cone" of test 1.

a. The Binary Gravitational Field

We maintain our two dimensional r, θ geometry. In our equations of motion, we replace the radial gravitational acceleration with the radial acceleration along the central 1 eV geodesic of figure 8-1, i.e., we add the centripetal and coriolis acceleration to the gravitational acceleration. In figure 8-15 we compare the gravitational acceleration along the line of centers with the radial acceleration experienced by the central test particle of figure 8-1. While we changed only the gravitational acceleration in our calculations, we now interpret the θ expansion as expansion relative to material following the "zero-energy" geodesic of figure 8-1.

b. Results

We assumed the same boundary conditions as in test 1 and ran a problem (test 4) with the modified gravitational potential. In figure 8-16 we present the computed temperatures and the maximum allowable temperatures. In figure 8-17 we plot the density, velocity, and column depth. Figure 8-18 shows the final r, θ configuration for this calculation. We also present in figure 8-18 the r, θ configuration from test 3.

Including the modified gravitational acceleration resulted in more expansion in the θ direction. This increased expansion did not result from lower densities and higher temperatures. The temperatures (densities) were slightly lower (higher) than in test 3, but the material fell in with substantially smaller radial velocities, resulting

Fig. 8-15. Radial acceleration versus radius for central geodesic of figure 8-1 (dotted curve). Solid curve is gravitational acceleration along the line of centers.

Fig. 8-16. Temperature versus radius for test 4. Solid curve is computed temperature. Dashed curve is maximum allowable temperature ("optically thin" approximation).

Fig. 8-17. Velocity, density, and column depth versus radius for test

4. Ordinate range for density is 10^{-12} to 10^{-9} g/cm^3 .

The range for column depth is 10^{-3} to 1 g/cm^2 .

Fig. 8-18. Final r, θ configuration for test 4 (solid curve). Dashed curve is r, θ configuration from test 3.

in more θ motion. In test 4 the material had more time to heat, and more nearly achieved the maximum allowable temperature distribution.

As previously noted, we now interpret the θ motion as motion with respect to the central geodesic of figure 8-1. In figure 8-19 we indicate the fluid motion in the binary geometry.

The final test problem (test 5) assumed the same boundary conditions as test 1 except the mass infall rate was increased by a factor of 5, increasing the boundary density by a factor of 5. This resulted in much less X-ray heating and much less θ expansion. In figure 8-20 we indicate the final temperature distribution for test 5 and in figure 8-21 we indicate the final r, θ configuration for test 5. For comparison the final r, θ configuration from test 4 is also presented in figure 8-21.

c. Conclusions

It is clear that the X-ray heating of the infalling gas stream can result in large modifications of the hydrodynamical motion, but it is also clear that the heating is very sensitive to the boundary conditions adopted. If we take our most straightforward estimates of the parameters for HZ Her/Her X-1, we get substantial heating, although not enough to produce what is obviously a wind. In calculations we have not reported here, we found that by modifying the boundary parameters (e.g. increasing the throat area or increasing the boundary velocity) we could

Fig. 8-19. θ motion superimposed on central geodesic of figure 8-1.

Fig. 8-20. Computed temperature versus radius for test 5.

Fig. 8-21. Final r, θ configuration for test 5 (solid curve). Dashed curve is r, θ configuration from test 4.

"surround" (4) the X-ray source at large radii ($\sim 10^{11}$ cm). The X-ray heating effects in these calculations were so strong that we were unable to achieve a steady state.

However, in the wind scenario, we will not be able to achieve the 100 percent capture efficiency assumed in test 4 and the $\theta = \pi$ calculations. In test 5 we increased the mass flow enough to allow for a 20 percent capture efficiency, but found that we lost the strong X-ray heating effects. By adjusting the boundary parameters we could restore the strong heating effects, but the basic fact is that our model has not given a clear answer as to whether X-ray heating effects dominate the hydrodynamical motion, or whether they have no significant effect. Rather than continue with this rather limited model, it was felt that a more realistic problem geometry would be a more fruitful area for investigation. Hence we are using the results from this "one and a half" dimensional hydrodynamics as boundary conditions for a three dimensional hydrodynamical code (see Chapter 10).

(4) By "surrounding" the source we mean θ becomes equal to π at some radius. Since θ cannot decrease with decreasing radius, once the stream closes on itself, it remains closed from that point inward.

9. SUMMARY AND CONCLUSIONS

In Chapter 5 we present calculations of the X-ray emission resulting from the accretion of material onto a non-magnetized, non-rotating neutron star. The major significance of these calculations is that they demonstrate that it is possible to produce extremely non-equilibrium X-ray spectra with a very simple accretion model. Some investigators have attempted to argue against accretion as the power supply for the X-ray sources because (1) many of the observed sources are characterized by very hard, non-blackbody spectra, and (2) accretion should just heat the surface, producing thermal blackbody spectra of one to a few keV. Our accretion calculations clearly show the second part of this argument is incorrect.

We calculate the heating of the surface of HZ Her and find the X-ray induced stellar wind is too weak to power the Her X-1 source. Chapter 6 reports the results of an extensive parameter survey, a survey which strongly suggests HZ Her fills, or very nearly fills, its gravitational lobe.

Chapter 7 reports calculations of the optical response of HZ Her to variations in incident X-ray intensity. We find that the short term (< 1 second) response is substantially larger than has been estimated by other investigators. Hence we would predict observable optical modulation with period 1.24 s at all times when the X-ray pulses are

directly incident on the surface of HZ Her. 1.24 s optical modulation is observed only during certain phases of the 1.7 day orbital period, and then it is often observed with the Her X-1 Doppler shift (Middleditch 1973). The optical modulation with a Doppler shift appropriate for the surface of HZ Her (Middleditch 1973, Groth 1974) may be the result of a transient phase where the line of sight from Her X-1 to some regions of HZ Her is relatively free of material. The recent observations of Cocke et al (1973) limiting the optical modulation to 0.01 percent early in the X-ray "off" phase of the 35 day X-ray modulation cycle would, taken with our results, suggest that when we do not see the X-ray pulsations, HZ Her is also not subject to pulsed X-rays.

In Chapter 8 we assume Roche lobe overflow is the mass transfer mechanism in the Hercules system, and estimate the modifications in the hydrodynamical motion due to X-ray heating effects. While we are able to calculate strong heating effects for some choices of boundary conditions, these calculations are disappointing in that they do not indicate whether X-ray heating effects dominate the motion. The major importance of these calculations is that they are being used to provide boundary conditions for three dimensional hydrodynamical calculations currently in progress.

10. FUTURE WORK

Chapters 4 through 8 of this thesis report our investigations of a few aspects of the accretion model. In this chapter we discuss other studies of the mass transfer and accretion that we have undertaken, or hope to undertake.

I. Three Dimensional Hydrodynamics

We have begun three dimensional hydrodynamical calculations of the fluid flow near the X-ray source (1). These time dependent calculations include the treatment of the X-ray transport. The "one and a half dimensional" results of Chapter 8 are being used as boundary conditions for these 3-D calculations. The 1+1/2-D mesh extends from 2.5×10^{11} cm to 4×10^{10} cm, while the 3-D mesh extends from 7.5×10^{10} cm to 5×10^9 cm. We pick up the density and velocity at 7.5×10^{10} cm from the 1+1/2-D calculations and use these to define a gas stream coming in the

(1) The code being used for these 3-D calculations is a modified version of a 3-D hydro code written by James R. Wilson to numerically test the stability of rapidly rotating stars (unpublished work). We have incorporated (a) the boundary conditions describing the incident gas stream, (b) our X-ray transport model, and (c) a viscosity based on the radial gradient of the ϕ velocity.

"side" of the 3-D mesh. In the initial phases of these calculations we have assumed X-ray transport in the "optically thin" approximation, and have further assumed the material is always in equilibrium with the local radiation field. While hydrodynamical heating and cooling have not been included in these first calculations, we hope to incorporate time dependent heating and cooling from the radiation field as well as from the hydrodynamics.

In the first 3-D problem we took the results of test 4 (Chapter 8) as boundary conditions. The 3-D code quickly achieved a steady state in which 0.1 - 1.0 percent of the material was being captured in the central region, and the rest of the material was being ejected from the problem mesh with large velocities. For the second 3-D problem the results of test 5 (Chapter 8) were used as boundary conditions. The 3-D steady state solution for this problem consists of a "finger like" gas stream wrapping about 180 degrees around the X-ray source. The narrow tip of this finger touches the central zone in the problem mesh, with about 20 percent of the material entering the problem in the beam being captured in the central region. The remaining 80 percent is ejected out the ends and sides of the mesh. Since the mass flow rate in the beam is five times that required to power the X-ray source, this second 3-D problem has a consistent X-ray luminosity and accretion rate.

II. Mass Ejection from Supergiants

In Chapter 3 we noted that theory and observation indicate mass loss rates as high as a few times $10^{-6} M_{\odot}/\text{yr}$ occur in some massive supergiants. We also noted that the driving mechanism for the wind is

assumed to be line radiation pressure, and that in a X-ray binary there is the possibility that the X-rays turn off the wind by stripping the ions responsible for the radiation acceleration. Because of this possibility of X-ray regulation of the wind, and because of the low capture efficiency that may result with a supersonic wind flowing by the secondary, we have begun investigating the mass transfer that may result from the gravitational field of the secondary. 30 to 60 $M_{\odot}$ supergiants can have intrinsic luminosities as high as a quarter to a half of the Eddington limit. If a supergiant is in a binary system, radiation pressure from absorption and Compton scattering will prevent it from filling its gravitational lobe. For example, if we consider a 30 $M_{\odot}$ star radiating at a third of its Eddington luminosity, the Compton scattering will, by itself, effectively reduce the primary mass from 30 $M_{\odot}$ to 20 $M_{\odot}$. The Compton modified critical equipotential surface will then be well inside the gravitational lobe. We are then attempting to understand if the supergiant can achieve a configuration such that there is a preferential mass loss on the side toward the secondary.

For this investigation we are using a modified version of the code described in Chapter 6, with all the X-ray transport and non-LTE physics removed from the code. We assume an initial configuration for the primary such that the photosphere is well inside the Roche lobe, and such that the mass loss rate is very small. Since we do not include lines, we can find a photospheric position such that the mass loss rate is $\ll 10^{-8} M_{\odot}/(\text{sterad-yr})$. The radius of the photosphere is then increased until a large mass loss rate per steradian is achieved.

These calculations are giving surprising results in that density

instabilities seem to set in just as the mass loss rate begins to become appreciable. A density inversion develops in the photospheric region, and then propagates out to the lip in the gravitational potential. The force driving these density waves seems to be radiation pressure. What appears to be happening is that material clumps, causing the opacity to increase with the increasing density. Radiation pressure then drives the density wave over the potential lip. If these instabilities turn out to be physical and not numerical, we may have a mass transfer mechanism that is unique to binary systems. In a single star, a wave propagating away from the star will reach a density sufficiently low so that it loses its radiation support. However in the binary system, the wave may reach the lip before the r^2 divergence drops the density enough to lose radiation support.

III. Accretion onto a Neutron Star

In our discussion of the possible presence of two-stream instabilities at the surface of a neutron star (Chapter 5) we indicated our interest in performing numerical simulations of the two-stream instability between the infalling protons and the atmospheric electrons. A different aspect of the neutron star accretion that we can now treat is the interaction of the outgoing radiation with the infalling gas. The non-LTE model developed for the X-ray heating calculations of Chapter 6 is also appropriate for treating the interaction of the outgoing X-rays with this infalling material.

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