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MASTER

YANKEE ATOMIC ELECTRIC COMPANY
RESEARCH AND DEVELOPMENT PROGRAM

THE EFFECT OF LOCAL BOILING ON PRESSURE DROP
AND FLOW DISTRIBUTION IN THE YANKEE REACTOR CORE

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WESTINGHOUSE ELECTRIC CORPORATION
ATOMIC POWER DEPARTMENT

PITTSBURGH, 30

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Yankee Atomic Electric Company
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The Effect of Local Boiling on Pressure Drop
and Flow Distribution in the Yankee Reactor Core

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For The Yankee Atomic Electric Company
Under Research and Development Subcontract
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TABLE OF CONTENTS

	<u>Page No.</u>
LIST OF FIGURES.	4
I. ABSTRACT.	6
II. INTRODUCTION	7
III. SUMMARY.	8
IV. NOMENCLATURE AND ABBREVIATIONS	10
V. HYDRAULIC AND THERMAL CHARACTERISTICS.	14
VI. SINGLE PHASE FRICTION FACTOR.	14
A. Quadratic Resistance Law.	14
B. Equivalent Diameter and Reynolds Number Correlation	14
C. Infinite versus Finite Array	16
D. Single Phase Friction Factor Correlations.	17
VII. THE MECHANISM OF LOCAL BOILING.	17
VIII. THE LOCAL BOILING FRICTION FACTOR.	18
IX. LOCAL BOILING PRESSURE DROP CORRELATIONS	22
X. TWO PHASE PRESSURE DROP.	25
A. Introduction.	25
B. Martinelli Correlation	25
XI. GRAPHICAL METHOD FOR DETERMINING THE EXTENT OF LOCAL BOILING IN A REACTOR CORE.	27
A. Heat Transfer and Surface Temperature Correlations	27
B. Flux Distribution and Local Boiling Geometry	28
C. Engineering Hot Channel Factor Considerations. . . .	28
D. Graphical Procedure.	37
E. Radial and Axial Limits of Local Boiling	37

TABLE OF CONTENTS (cont'd)

	<u>Page No.</u>
XII. ANALYTICAL METHOD FOR DETERMINING THE EXTENT OF A CORE IN LOCAL BOILING.	38
A. Introduction.	38
B. Development of Expressions and Equations	39
C. Analytical Step-by-Step Procedure.	40
XIII. DETERMINATION OF THE LOCAL BOILING FRICTION FACTOR	42
A. By Means of the Two-Phase Pressure Drop Correlations	42
B. Calculation of the Local Boiling Friction Factor from the Heat Transfer Momentum Analogy.	42
XIV. FLOW REDISTRIBUTION CAUSED BY LOCAL BOILING.	45
A. Geometrical Considerations.	45
B. Flow Analysis Assumptions.	46
C. Pressure Drop Analysis	46
D. The Effect of Fuel Rod Supports (i.e. Ferrules and Straps) on Flow Redistribution.	53
E. Magnitude of Flow Redistribution	53
XV. BIBLIOGRAPHY.	56
XVI. ACKNOWLEDGEMENT.	58

LIST OF FIGURES

<u>Figure No.</u>		<u>Page No.</u>
1	Schematic of the Yankee Reactor Core	12
2	Full Size Strap Design Fuel Assembly Model	13
3	Values of Related Hydraulic and Thermal Parameters	15
4	Enthalpy versus Distance from Core Entrance (Second Parameter Study)	20
5	Enthalpy versus Distance from Core Entrance (First Parameter Study)	21
6	Martinelli Two-Phase Pressure Drop Correlation for the Zero to 0.05 Quality Region	26
7	Ratio of the Local Boiling Friction Factor to the Single Phase Friction Factor	29
8	Surface Temperature Along the Center Axis (Radius- Zero Feet)	30
9	Surface Temperature at a Radius of 0.5 Feet	31
10	Surface Temperature at a Radius of One Foot	32
11	Surface Temperature at a Radius of 1.2 Feet	33
12	Surface Temperature at a Radius of 1.24 Feet	34
13	Surface Temperature at a Radius of Zero Feet (First Parameter Study)	35
14	Surface Temperature at a Radius of 1.16 Feet (First Parameter Study)	36
15	Maximum Volumetric Limits of Local Boiling (First Parameter Study)	41
16	Local Boiling Friction Factor Based on Momentum-Heat Transfer Analogy (Second Parameter Study)	44
17	Pressure Gradient in an Average Channel (First Parameter Study)	47
18	Pressure Gradient in an Average Channel (Second Parameter Study)	48
19	Mass Flow Rate in the Hot Channel (First Parameter Study)	50

LIST OF FIGURES (cont'd)

<u>Figure No.</u>		<u>Page No.</u>
20	Mass Flow Rate in the Hot Channel (Second Parameter Study)	51
21	Water Temperature vs Distance from Core Inlet (First Parameter Study)	52

I. ABSTRACT

Where local boiling can occur in a semi-open heterogeneous reactor core consisting of parallel flow channels, differences in the terms comprising the total pressure drop in the local boiling regions and in parallel adjacent regions cause flow redistribution in the reactor. This report covers the study of the effect of local boiling on the friction factor and momentum components of the total pressure drop.

The local-boiling to single-phase friction factor ratio has a maximum value of 1.42 when a linear relationship is assumed between the enthalpy of the coolant and the friction factor ratio. Analytical and graphical methods for determining the region of the core in local boiling are developed. For a cylindrical reactor core similar to the Yankee reactor core the maximum possible region of the core in which local boiling should occur would be an ellipsoid whose surface is the locus of points of local boiling.

In the Yankee core the maximum possible volume in local boiling is 10.4 cubic feet and 14.4 cubic feet in the first and second parameter studies respectively. These volumes are 4.5% and 5.5% of the total volume of the core.

The nature and mechanism of local boiling is discussed and its relation to the single phase and true two-phase pressure drop is reviewed.

Flow redistribution out of the hot channel due to local boiling is 17.6% and 23.8% for the first and second parameter studies respectively.

The first parameter study is for a 392 megawatt reactor and the second parameter study is for a 482 megawatt reactor.

II. Introduction

As power demands on water cooled heterogeneous reactors increase, the condition that local boiling be permitted during steady state operation has become a design criterion. The verification of the existence of favorable corrosion and crud deposition characteristics during local boiling, and the favorable interpretation of hot channel factors and burnout concepts will be instrumental in establishing the aforementioned design criterion. Recent impetus in local boiling studies has been motivated by a desire to understand the cause of the large increase in heat transfer and pressure drop accompanying the local boiling phenomena.

Local boiling is a thermodynamic state which occurs when a fluid below its saturation temperature contacts a surface hot enough to cause boiling at the interface. In forced circulation the bubbles which form on the heated surface collapse on the surface or in the main body of the fluid. No net steam is generated in local boiling although the vapor bubbles may exist for some distance from where they are generated.¹

Since it is postulated that local boiling may be common in future water cooled reactors, a comprehensive understanding of the effects of this mode of heat transfer on reactor core design is mandatory. In addition, the method used to determine the isothermal, or non-boiling pressure drop is reviewed because the local boiling friction factor is usually expressed as, f_{LB}/f_{iso} , a function of the isothermal friction factor. In other words the prediction of the friction factor during local boiling is directly related to the value of the friction factor during the single phase flow.

Two parameter studies are analyzed in detail. The first parameter study is for a 392 megawatt reactor and the second parameter study is for a 482 megawatt reactor. Local boiling during steady state operation is a design criterion for the Yankee reactor.

III. Summary

Experimental and theoretical local boiling investigations at 2000 psia for flow parallel to rods are meager. Additional information is required before the effects of local boiling can be accurately extrapolated from one set of parameters to another with confidence. Intensified research in the field of bubble dynamics has yet to uncover the true nature of the local boiling mechanism. A discovery of this nature will provide a procedure for a concerted attack on the solution to associated problems such as obtaining expressions for the local boiling pressure drop and the heat transfer coefficient in terms of system parameters that can be measured directly.

The range of values found in the literature for the local boiling friction factor at 2000 psia shows considerable spread. In fact, the values reported are 10% to 100% greater than the single phase friction factor. This large spread indicates a difference in what is considered the local boiling friction factor and the method by which it is measured. For example, where the local boiling friction factor is obtained by measuring the pressure drop across a conduit and subtracting the elevation pressure drop, and the momentum pressure drop for the change in liquid density, the value obtained is high. Photographic studies also indicate that at a given heat flux the effect of the bubbles should be more pronounced as the sub-cooling decreases. The conclusion is that a comparison among the local boiling friction factors found in the literature is only valid if the value of heat flux and sub-cooling are equal, even if the absolute pressure is the same.

Where the possible volume of the core in local boiling is small and a transverse degree of freedom exists for flow, the apparent increase in pressure drop caused by local boiling results in flow redistribution. The pressure drop across the core should be determined on the basis of an average channel.

Since flow out of the hot channel can occur at any longitudinal point in an open lattice, the effect of flow distribution is more important when the possibility of burnout is being considered.

An attempt is presently being made at the Westinghouse Atomic Power Department in utilizing the well known single phase friction factor to determine the local boiling friction factor. In local boiling the

bubbles formed have a smaller diameter than bubbles formed in bulk boiling at similar conditions of heat flux and surface conditions.

Spherical bubbles cover the maximum amount of surface when the bubble is a semi-sphere. Furthermore, if bubble generation can be considered constant, the roughness might be calculated by multiplying the bubble radius by the fraction of the surface covered by the bubbles. Where the bubble radius is large compared with the distance between bubbles, the bubbles can be assumed to control the roughness. The equivalent diameter is then calculated as for single phase flow and the friction factor is obtained from the usual friction factor versus Reynolds number graph. The Reynolds number uses the coolant velocity and the equivalent diameter of the conduit.

It cannot be over-emphasized that the straps used to support the fuel rods greatly influence the flow redistribution. In designing the fuel assembly careful consideration should be given to the type of support used, the pressure drop caused by the straps, and the location of the support. If local boiling is possible the support should be placed in the local boiling region to minimize flow redistribution in the hot channel. This is especially true where the greater fraction of the pressure drop across the support is form loss (i.e. pressure drop caused by an expansion or a contraction).

Flow redistribution in either parameter study is not great enough to cause burnout.

The second parameter study shows greater possible local boiling and flow redistribution.

Flux distribution is assumed to be a cosine function longitudinally and a Bessel function radially. The effects of control rods on the flux distribution is not considered.

The analytical and graphical methods for determining the volume of the core in local boiling give almost identical results.

IV. Nomenclature and Abbreviations

A, C, E, F, H, K, M, N, S,

c, n. constants

D_e = Equivalent Diameter

f = Fanning friction factor

G = Mass flow rate $\frac{lb}{hr-ft^2}$

g = Local acceleration of gravity (32.2 ft/sec²)

g_c = Conversion factor between mass and force units

h = Heat transfer coefficient Btu/hr-ft²-°F

J_0 = Zero order Bessel function

K = Thermal conductivity Btu/hr-ft-°F

L = Actual length of a fuel rod

L'' = Extrapolated length of a fuel rod

N_u = Nusselt number $(\frac{hD}{K})$

P = Pressure

Re = Reynolds Number $(\frac{Dv\rho}{\mu})$

r = Radius (ft)

R' = Extrapolated radius (ft)

T = Temperature °F

Pr = Prandtl number

K_{form} = Form loss coefficient

X = Distance Measured from entrance to the core

V = Volume (ft³)

ΔT_{sat} = Temperature difference between saturation temperature and actual fuel rod temperature (°F)

ΔT_{max} = Maximum water temperature rise (°F)

Nomenclature and Abbreviations. (cont'd).

- λ = Four times the Fanning friction factor
 ϕ = Heat flux (Btu/hr ft²)
 ρ = Density (lbs/ft³)

Subscripts

- B = Water conditions
B₀ = Water inlet conditions
iso = Isothermal
LB = Local Boiling
Re = Reynolds number
s = Surface
sat = Saturation
TP = Two Phase
 θ = Film conditions
max = Maximum

SECTION A-A

SECOND PARAMETER STUDY

A = 76.46"

B = 38.23"

FIRST PARAMETER STUDY

A = 76.5"

B = 38.25"

SCHEMATIC OF THE YANKEE REACTOR CORE

FIGURE 1

288 012

Figure 2
Full Size Strap Design
Fuel Assembly Model

V. Hydraulic and Thermal Characteristics

The Yankee reactor core is essentially an open lattice which consists of 76 fuel assemblies and 24 control rods. Fig. 1 shows that a form fitting baffle encloses the core. The fuel assemblies are supported between an upper and a lower support plate. From Fig. 2 it can be seen that fuel rods are supported and held together in a bundle by means of straps or ferrules and the fuel assembly handling sockets.

Water flows up through the lower support plate then parallel to the fuel assemblies and control rods and finally through the upper support plate and out of the core.

In an open lattice, similar to the one being analyzed, the resistance to transverse flow is low and the water has a transverse degree of freedom to flow. Resistance to cross flow is offered by the control rods and the restriction between the fuel rods. Since the fuel assemblies and control rods are in parallel and the resistance to cross flow is very low, the condition of equal pressure drop is imposed on the core at any elevation.

Two sets of parameters are studied to determine the effect of core geometry and temperature on the isothermal and local boiling pressure drop. Fig. 3 lists the parameters for the two studies.

VI. Single Phase Friction Factor

A. Quadratic Resistance Law

The general solution to the determination of turbulent resistance in a conduit has been achieved mainly by means of semi-empirical equations based on experimental results.² This approach has been necessary because a complete mathematical solution has not been found. Experimental data indicate that turbulent resistance is proportional to the square of the average velocity and the density of the fluid. This relation is commonly called the quadratic resistance equation and is known as the Fanning friction factor. Whereas the Fanning friction factor appears in most of the semi-empirical equations which define the proportionality factor, a value equal to four times the Fanning factor is used extensively in charts and nomographs on which the friction factor is plotted as a function of the Reynolds number.² Throughout this report the symbol λ (lambda) will be used to represent the value of the friction factor.

B. Equivalent Diameter and Reynold Number Correlations

The Reynolds number, which is actually the ratio between the inertia and viscous forces acting on an element of fluid, is the accepted

	PARAMETER STUDIES	
	First	Second
Core Inlet Temp. °F	499	495
Average Core Outlet Temp. °F	532	536
Hot Channel Exit Water Temp. (No Flow Redistribution)	603	620
Maximum Water Temp. Rise (Hot Channel) °F	104	125
Maximum Film Drop Temp., °F	106	99.2
Maximum Flux Btu/hr-ft ²	450,000	435,000
System Pressure (PSIA)	2,000	2,000
Roughness, (ε) inches Rms.		
Control Rod	0.000125	0.000125
Core Baffle	0.000125	0.000125
Fuel Rod	0.000063	0.000063
Average Roughness	0.0000651	0.0000656
Core Equivalent Diameter (Ft, De)	0.0287	0.0253
Equivalent Roughness (ε/D)	0.000189	0.000216
Friction Factor (λ)	0.0164	0.0168
Fuel Rod Diameter (Inches)	0.337	0.305
Lattice Spacing (Inches)	0.425	0.380
Total Number of Rods	23,218	28,842
Perimeter (Feet)		
Fuel Rods	2,050	2,300
Baffle	25.5	25.5
Control Rod	70	70
Cross Section Area of Core (ft ²)	15.4	15.2
Active Rod Length (ft)	7.5	8.5

Fig. 3

Values of Hydraulic & Thermal Parameters

criterion for the general nature of fluid flow. When the Reynolds number is calculated for a non-circular geometrical shape similar to the Yankee core, the question arises as to what value should be used for the characteristic length dimension. The equivalent diameter, which is four times the flow area divided by the wetted perimeter, is the correlation parameter for turbulent flow in circular conduits. However, the equivalent diameter has no real theoretical basis and its generality for widely different geometrical shapes was, until recently, questionable. The current view is that for all practical purposes the equivalent diameter is a satisfactory correlation parameter for turbulent flow through non-circular ducts, annuli, and parallel flow systems.³ Although the friction factor has been established as a unique function of the Reynolds number for turbulent flow through pipes, the corresponding relationship for non-circular ducts has not been completely established. Indications are that the phenomenon of momentum transfer in non-circular ducts is more complex than in round pipes since it has been shown experimentally that secondary flow occurs in the corners of non-circular ducts.³

Observe that the equivalent diameter concept assumes that the pressure drop is proportional to the wetted perimeter per unit flow cross sectional area. If we assume that the friction factor is independent of geometry, then the Reynolds number based on the hydraulic equivalent diameter is a sufficient criterion for friction drop calculations, especially in the highly turbulent range.

C. Infinite versus Finite Array

For the purpose of obtaining an equivalent diameter the Yankee reactor core is considered to be a finite array. The hydraulic diameter based on a finite array includes the perimeter of the core baffle and the perimeter of the control rods; whereas, the hydraulic diameter based on an infinite array includes only the perimeter of a unit cell. Since the perimeter of the fuel rods is large compared with the sum of the control rods and core baffle perimeters, there is little difference between the finite and infinite array equivalent diameters. The use of a finite array results in a slightly smaller equivalent diameter and a correspondingly increased friction factor. Examination of Fig. 3 will reveal that the fuel tube roughness has a lower value than the roughness of the control rods and core baffle. The procedure of using a weighted roughness is justified because the fuel rod perimeters are a large fraction of the total perimeter.

D. Single Phase Friction Factor Correlations

Although some experimental data⁴ report values of the isothermal or single phase friction factor as high as 65% greater than the friction factor calculated from the appropriate friction equation, an equal number of data and theoretical investigations⁵ support the direct use of the Moody chart or the Colebrook equation for obtaining a friction factor for water flowing parallel to fuel rods. Dressler and Taylor⁶ calculated friction factors for flow parallel to fuel rods by integrating the velocity distribution across a channel. Their results show that the friction factor is a function of the rod-spacing to rod diameter. The friction factor also increases at a decreasing rate as the spacing between the rods increases. For the fuel rod-spacing to rod-diameter ratios considered in the present analysis, the Dressler-Taylor friction factor is somewhat lower than the corresponding factor for flow inside of smooth tubes.

The isothermal or non-boiling single phase friction factor for the analysis considered is obtained in the usual way by calculating the equivalent roughness and Reynolds number. For both parameter groups considered the Reynolds number is in the range $De/2\epsilon/Re\sqrt{f} \leq 0.005$, and the Colebrook equation applies.⁷ In the above range the friction factor is a function of the equivalent roughness and Reynolds number.

An average velocity is assumed at each cross section of the reactor for all flow analyses included in this report. The only variation in velocity profile accounted for is that due to differences in bulk water temperatures. For exact analysis a factor could be applied to the kinetic energy term in the Bernoulli equation to correct for the additional effect of reactor core geometry on fluid velocity distribution. For turbulent flow in pipes the kinetic energy factor ranges from 1.01 to 1.1.⁸ However, for the Yankee core the correction factor is not known.

VII. The Mechanism of Local Boiling

The mechanism of local boiling pressure drop is inherently complicated by the simultaneous change of pressure due to three different causes: the friction, momentum, and static pressure effects.

One generally accepted view is that the mechanism of stirring is responsible for the increase in momentum and heat transfer observed during local boiling. Stirring can be described as a mechanism whereby the bubbles that are constantly growing and collapsing create a hydrodynamic turbulence in the quiescent region of liquid adjacent to the fuel rod surface in excess of that normally found in simple forced convection.

Stirring would be expected to affect momentum and heat transfer similarly. However, an alternate view is that the mechanism is one in which a cold liquid is brought in contact with the fuel rod surface when the bubbles collapse. The latter mechanism resembles conduction and the exact relation between momentum and heat transfer is not easily discernible. Finally, if the large increase in heat transfer is caused strictly by mass transfer an analogous relationship between heat transfer and fluid friction would exist.

It has also been observed that at high fluid flow rates and low degrees of local boiling the bubbles formed collapse on the heated surface thereby producing an undulating film.⁹ In this region the bubbles can be thought of as an increased surface roughness. Thus, at high fluid flow rates and low degrees of local boiling the bubbles do not appreciably disturb the normal liquid convection mechanism whereas at low flow rates and high degree of local boiling (i.e. a large number of bubbles) bubble mechanisms become the dominant factor and the actual fluid velocity has a small effect on the pressure drop.

The above discussion would indicate that the mechanism of local boiling can be explained by dividing the region of local boiling into two sub-regions: at high sub-cooling the prime mechanism causing the stirring at the wall is the formation and collapse of the bubbles while they are attached to the surface, but at low sub-cooling the prime mechanism is the inrush of liquid behind the rapidly moving bubbles as they retreat from the surface.

Whereas in pure two-phase flow the degree is expressed by the quantity of steam present or the quality, the concept of degree of local boiling is not commonly used. However, some advantage would be gained by expressing the degree of local boiling as a function of heat flux and/or sub-cooling. A degree of sub-cooling based on the bubble concentration loses significance because of the difficulty of expressing this quantity in terms of system parameters that are easily measured.

A better understanding of bubble hydrodynamics and the mechanisms by which the increase in momentum is brought about during local boiling would provide a more complete insight to the parameters affecting the local boiling pressure drop.

VIII. The Local Boiling Friction Factor

Considerable experimental work¹⁰ has been done on bulk boiling friction factors, however, corresponding data, for local boiling for geometrical shapes similar to the Yankee and at 2000 psia are not available.

There appears to be little justification in classifying local boiling friction factors separately from nucleate boiling friction factors since

the bubble mechanism at the heated surface has not been shown to be different. The main reason for this somewhat artificial sub-division of local boiling is the difficulty encountered in determining the void fraction in local boiling. The implication that void fractions exist in local boiling is contrary to the original definition; however, recent work at Battelle¹¹ indicates that there is little agreement between theoretical and experimental quality determinations in the low quality range, especially in the vicinity of zero quality. This disagreement indicates that in the region of zero quality, vapor is generated in the sub-cooled liquid. Additional experimental data show that an existing void fraction can remain over a large portion of the local boiling range. In the early stages of local boiling the flow pattern is probably a separated one and the detection of a small fraction in this type configuration is difficult.²⁸

When accurate methods are available for measuring void fractions in local boiling, the local boiling friction factor might be placed on the same basis as bulk boiling friction factors.

The increase in the friction factor caused by local boiling is conventionally expressed as a ratio of the local boiling friction factor to the isothermal friction factor. Since for relatively small temperature differences there is little difference between the isothermal and heating friction factors, the symbol λ_{LB}/λ in this report means the ratio between the local boiling friction factor and the friction factor calculated by the usual forced convection theory at the existing temperature of the coolant. Observe that under conditions of constant mass flow through a conduit the friction factor decreases as the water is heated. Examination of the Reynolds number expression shows that the Reynolds number is inversely proportional to the friction factor; the overall effect is to decrease the friction factor.

The phenomenon of local boiling is thought to increase the pressure drop because: (a) the vapor bubbles formed at the heated surface impose a drag force on the fluid flowing past the surface, (b) the formation of vapor at the surface of the tube causes the local value of the kinematic viscosity at the wall to increase signifying that the viscous forces of the fluid are becoming more pronounced at the wall, (c) of an increase in momentum.

No established analytical equation, or for that matter no empirical equation, has been developed that predicts the local boiling pressure drop over a wide range of the previously mentioned variables. Any generally applicable equation should express the local boiling friction factor as a function of the properties of the fluid and characteristics of the conduit. For instance if the local boiling friction factor is expressed as a function of a modified Reynolds number based on the bubble formation rate and size, the bubble rate and size should be expressible as a function of heat flux, surface tension, etc. Some success along this line has been achieved by Rohsenow, Foster and Zuber¹² in obtaining bulk boiling heat transfer equations. (Figures 4 and 5 give the values of enthalpy versus distance along the length of the core for the two parameter studies).

Figure 4

Enthalpy vs. Distance from Core Entrance
(Second Parameter Study)

Figure 5

Enthalpy vs Distance from Core Entrance
(First Parameter Study)

IX. Local Boiling Pressure Drop Correlations

A review of existing correlations indicates that the local boiling pressure drop increases with increasing heat flux, decreasing sub-cooling, and decreasing absolute pressure. The local boiling pressure drop also decreases with increasing mass flow rate. No correlation was found to include the effect of geometry or surface roughness. It appears, however, that the rougher the surface the lower the temperature of the surface during boiling. This implies that a longer length of the conduit is in local boiling for a rough surface than for a smooth one assuming all other parameters are equal. From a bubble dynamics standpoint the surface tension should also be a parameter.

Even in the absence of experimental local boiling pressure drop data, application of the momentum-heat transfer analogy would predict an increase in the pressure drop since it has been conclusively demonstrated that heat transfer rates are higher in local boiling than for single phase flow under the same conditions.

J. B. Reynolds¹³ reported that the local boiling pressure drop can be expressed within 20% of the experimental value by the following equation.¹³

$$P_{LB} = \left(\frac{dp}{dL} \right)_o \frac{G' D C_p (\Delta t_{sub})_o}{4 a q''} \sinh \frac{4 a q'' L}{G' D C_p (\Delta t_{sub})_o} \quad (1)$$

where

P_{LB} = Local boiling pressure drop

$\left(\frac{dP}{dL} \right)_o$ = Isothermal liquid pressure gradient at average bulk water temperature

q'' = Heat flux = q/A_s Btu/hr-ft²

a = Parameter related to heat flux $a = 4.6 \times 10^{-6} q'' + 1.2$

D = Inside tube diameter (ft)

$(\Delta t_{sub})_o$ = Sub-cooling at point where local boiling starts °F

C_p = Specific heat of water Btu/lb

L = Distance from the point where local boiling starts to any point in question (along the length of local boiling) up to the point where net boiling starts $(\Delta t_{sub}) = 0$

G = Mass velocity lbs/hr-ft²

Although this equation cannot be used at 2000 psia it represents one of the first local boiling pressure equations published. Test conditions were:

Uniform heat flux: (130,000 to 304,000 Btu/hr/ft²)
 Mass Velocity: 343 to 652 lbs/sec-ft²
 Pressure: 45 to 100 psia
 Horizontal stainless steel tube: (3/8" I.D. x 1/2" O.D. x 6' long)

The Bettis thermal and hydraulic sub-committee reported at the Heat Transfer Conference of 1956⁴ the following equation for local boiling pressure drop at 2000 psia.⁴

For local boiling beginning at bulk temperature below 560°F

$$f = f_{iso} \text{ for } T_B \leq 560^\circ\text{F}$$

$$f = f_{iso} \left[1 + \frac{T_B - 560}{126.3} \right] \quad (2)$$

$$\text{for } 560^\circ\text{F} \leq T_B \leq 635.8^\circ\text{F}$$

For local boiling beginning at bulk temperature above 560°F.

$$f = f_{iso} \text{ for } T_B \leq T_{lb}$$

$$f = f_{iso} \left[1 + \frac{T_B - 560}{126.3} \right] \quad (3)$$

$$\text{for } T_{lb} \leq T_B \leq 635.8^\circ\text{F}$$

Observe that the above equations express the local boiling friction factor as a function of only the bulk water temperature, or indirectly the degree of sub-cooling. The equations were empirically obtained. From the equations it is seen that λ/λ_{iso} during local boiling is unity until the bulk water temperature reaches 560°F. From 560°F to 635.8°F the factor increases linearly from 1 to 1.6. It should be mentioned that the above equations create a discontinuity in λ/λ_{iso} which probably does not exist since for any given test λ/λ_{iso} increases in a continuous fashion.

More recently the Bettis Plant has reviewed available void data and incorporated the data in an overall pressure drop analysis.¹⁵ The expected preliminary results indicate that local boiling pressure drop can be treated on exactly the same basis as bulk boiling pressure drop if the necessary void information is available.

In addition an interim procedure was developed for estimating pressure drop in rectangular channels at 2000 psia independently of the experimental void fraction. The relations are presented in equation form which compensate for uncertainties due to a lack of knowledge of the exact effect of the several variables included in these correlations. This correlation was presented at the Nuclear Conference held in March, 1955 in Chicago.¹⁵

$$H_B \leq H_{sat}$$

$$f/f_{iso} = \left[C_1 \right] \left[1 + 0.912 \left(\frac{10^6}{G} \right)^{2/3} \psi \right] \quad (4)$$

where $C_1 = 1.05 - 0.00262 \Delta T^*$

when $\left[1.05 - 0.00262 \Delta T^* \right] \leq 1.0$

and $C_1 = 1.0, \psi = f(\Delta T^*)$,

when $\left[1.05 - 0.00262 \Delta T^* \right] > 1.0$

Conditions were:

Geometry:	Rectangular Channels
Static Pressure:	2000 psia
Mass Velocity:	0.7×10^6 to 5.0×10^6 lbs/hr-ft ²
Heat Flux:	
(axially uniform)	0 to 1×10^6 Btu/hr-ft ²
Inlet Temperature:	400° to 625°F

The University of California carried out extensive boiling studies¹⁶ recently and reported an increase in pressure drop of the order of 10% over that for single phase flow for cases where the difference between the saturated water temperature and the bulk water temperature (i.e. sub-cooling) where local boiling begins is less than 36°F.

Rohsenow and Clark in their experiments¹⁷ observed that when surface boiling begins the pressure drop began to rise slightly as the heat flux is increased. A velocity, a liquid sub-cooling and an absolute pressure effect were discerned. It appeared that the effect of liquid sub-cooling was more pronounced than the effect of absolute pressure. (Figure 7 is a graph of the local boiling to single phase friction factor ratio versus enthalpy of the coolant used in the analysis for this report).

Experimental evidence indicates that local boiling pressure drop is an inverse function of the total mass flow.¹⁵ However, in the present state of our knowledge the cause of this dependancy has not been satisfactorily explained.

Other local boiling pressure drop correlations exist.¹⁸ However, most of the experimental work was conducted at atmospheric pressure and extrapolation to higher pressures is not advisable.

X. Two-Phase Pressure Drop

A. Introduction

Pressure drop in two-phase flow is greater than that for the flow of either phase alone for several reasons. The increase is due to the irreversible work done on the gas by the liquid. In addition the presence of the second fluid reduces the flow cross section for the first fluid. Thus, during two phase flow the individual hydraulic diameters based on the liquid and gas flow area are always less than the pipe diameter. This reduction in equivalent diameter increases the pressure drop. For both two-phase flow and local boiling there is still the increase due to additional shear in the liquid near the wall caused by the presence of bubbles. However, very little information exists on the magnitude of the bubble shear.

B. Martinelli Correlation

Of the many proposed methods for calculating two phase pressure drop that appear in the literature, the Martinelli method has been used most successfully to cover a wide range of steady flow. In spite of its limitations such as the inadequate correlation of the system pressure and total flow rate, the Martinelli method has received impressive support through accumulation of flow studies and partial theoretical studies. (Figure 6 gives the two phase pressure drop for the 0-5% quality region.)

The postulates upon which the Martinelli correlation is based are:

1. Static pressure drop for the liquid phase must equal the static pressure drop for the gaseous phase regardless of the flow pattern, as long as an appreciable radial static pressure difference does not exist:
2. The volume occupied by the liquid plus the volume occupied by the gas at any instant must equal the total volume of the pipe. Thus, it is assumed that the flow pattern does not change along the tube length and slug flow is eliminated from consideration.

Martinelli has plotted the two-phase friction factor ratio versus steam quality using absolute pressure as the parameter. The curve as plotted is for the two-phase friction factor and not for total

Figure 6

Martinelli Two-Phase Pressure Drop Correlation for the Zero to 0.05 Quality Region

pressure drop. The momentum term is calculated from a separate expression. The curve predicts a value of f_{TP}/f of unity at zero quality. Hence the curve cannot be used to calculate the local boiling friction factor since the curve precludes a value greater than unity of the local boiling friction factor. The curve was extrapolated from 4% to zero per cent quality because the original data were for 4 to 100 per cent exit qualities.

XI. Graphical Method for Determining the Extent of Local Boiling in a Reactor Core

A. Heat Transfer and Surface Temperature Correlations

The Dittus-Boelter heat transfer correlation is used in conjunction with the Jens and Lottes local boiling correlation¹⁹ to determine the point where local boiling can begin. Surface temperatures have been correlated by means of the Jens and Lottes correlation to within $\pm 15^\circ\text{F}$ over a wide range of conditions in a constant-heat flux system where nucleate boiling is present. Other equations can be found in the literature giving the value of the surface temperature during local boiling and any one of these could have been used. However, the Jens and Lottes correlation is simple and expresses the surface temperature at a given pressure as a function of the heat flux. (The McAdams correlation is of the same form but the experimental pressure range is 30 to 90 psia).

The Dittus-Boelter Equation

$$h = 0.023 \frac{k}{D_e} (N_{Re})_B^{0.8} (Pr)_B^{0.4}$$

is combined with the Jens and Lottes correlation

$$\Delta T_{\text{sat}} = \frac{60 \left[\frac{\phi}{(10^6)} \right]^{1/4}}{e^{P/900}} = 1.9 (\phi)^{1/4} e^{-P/900} \quad (5)$$

$$\text{to give } T_B + \frac{\phi}{h} = T_{\text{sat}} + 1.9 (\phi)^{1/4} e^{-P/900}$$

$$\text{or } T_B + \Delta T_0 = T_s = T_{\text{sat}} + \Delta T_{\text{sat}}$$

The two terms on the left hand side of the equation are the bulk water temperature and the drop across the film. Furthermore, the surface temperature which is the sum of the bulk water temperature and film drop temperatures can be expressed by the following equation:

$$T_s = \left\{ T_{Bo} + \frac{\Delta T_{max}}{2} \left[1 + \frac{\sin\left(\frac{\pi Z}{L}\right)}{\sin\left(\frac{\pi}{2} \frac{L}{L'}\right)} \right] + \theta_{max} \cos\left(\frac{\pi Z}{L}\right) \right\} J_0\left(2.405 \frac{r}{R'}\right) \quad (6)$$

where Z is measured vertically from the center of the reactor.

B. Flux Distribution and Local Boiling Geometry

As a first approximation the flux pattern in a cylindrical reactor can be assumed to follow a zero order Bessel function radially, and a cosine function longitudinally. Because of this non-uniform flux pattern the flux, the fuel rod surface temperature and the bulk water temperature are functions of r, the radius measured horizontally from the center of the core, and Z, the vertical distance along the fuel rods measured from center of the core.

Because of the symmetry imposed by the assumed flux distribution, the possible extent of the local boiling region is symmetrical about the longitudinal axis. Moreover, since the theoretical hot channel is assumed to occur along the longitudinal axis, the length of the core in local boiling should be greatest in the hot channel and should taper off because of the nature of the Bessel function. Based on this reasoning the limits of the local boiling region should be the surface of an ellipsoid whose major axis is the hot channel. The minor axis of the ellipsoid should occur at a distance measured from the inlet where the fuel rod surface temperature is a maximum.

C. Engineering Hot Channel Factor Consideration

It should be clearly stated that the actual volume of the core in local boiling depends upon the local physics and engineering conditions. The present analysis gives the maximum extent of the local boiling region. Local boiling can occur anywhere within this region but nowhere outside of the region. Indeed, local boiling can occupy the entire region within the calculated ellipsoid boundary only if the engineering hot channel factors are superimposed on the physics factors. If the engineering hot channel factors are considered point functions, local boiling will be a local or point condition. Any application of the engineering hot channel factors between the two extremes will cause a different percentage of the calculated ellipsoid to be in local boiling.

LOCAL BOILING FRICTION FACTOR
SINGLE PHASE FRICTION FACTOR

Figure 7

Ratio of the Local Boiling Friction Factor to the Single Phase Friction Factor

Figure 8

Surface Temperature Along the Center Axis (Radius = Zero Feet)
(Second Parameter Study)

Figure 9
Surface Temperature at a Radius of 0.5 Feet.

Figure 10

Surface Temperature at a Radius of one Foot

- 32 -

Figure 11

Surface Temperature at a Radius of 1.2 feet

288 033

Figure 12

Surface Temperature at a Radius of 1.24 ft.

Figure 13

Surface Temperature at a Radius of Zero Feet
(First Parameter Study)

Figure 14

Surface Temperature at a Radius of
1.16 Feet
(First Parameter Study)

D. Graphical Procedure

To obtain the possible extent of the local boiling region:

1. Plot the fuel rod surface temperature at various radii by means of Eq. 6.
2. Plot the function ΔT_{sat} (Jens and Lottes correlation) at various radii.
3. The longitudinal distance between the intersection of the plotted curves at a given radius is the maximum possible length of the core in local boiling at that radius.
4. A smooth curve drawn through the points of intersection will give the limits of the local boiling region. See Figures 8 to 14.

E. Radial and Axial Limits of Local Boiling

If a surface temperature is assumed at the inception of local boiling, the equation for the surface temperature can be solved using values of the radial distance as the parameter. The value of the Jens and Lottes correlation does not change greatly at 2000 psi. In fact, for the two parameter studies analyzed the maximum value of the Jens and Lottes correlation (ΔT_{sat}) is approximately 5.3°F and the minimum value is 2.8°F .

The equation for the point in the reactor where the surface temperature is a maximum is

$$Z_{\text{s max}} = \frac{L'}{\pi} \arctan \frac{\Delta T_{\text{max}}/2}{\theta_{\text{max}} \sin \left(\frac{\pi}{2} \frac{L}{L'} \right)} \quad (7)$$

For the second parameter study

$$\begin{aligned} Z_{\text{s max}} &= \frac{9.00}{3.14} \arctan \frac{125/2}{99.2 \times 0.995} \\ Z_{\text{s max}} &= 2.87 \arctan 0.65 \\ &= 2.87 \times 0.576 = 1.65 \text{ ft.} \end{aligned}$$

Since Z is measured vertically from the center of the reactor, the maximum surface temperature occurs at a distance of 5.9 feet from the entrance to the core. Using $Z = 1.65$ feet and assuming that local boiling begins when the surface temperature reaches 641°F , it follows that:

$$641 = 495 + \left\{ 62.5 \left[1 + \frac{0.545}{0.995} \right] + 99.2 \times 0.838 \right\} J_0 \left(2.405 \frac{r}{R'} \right)$$

$$0.8152 = J_0 \left(2.405 \frac{r}{R'} \right)$$

$$0.719r = 0.881$$

$$r = 1.23 \text{ feet (radial limit)}$$

To obtain the axial limits proceed as follows: Use $J_0 \left(2.405 \frac{r}{R'} \right) = 1$, since the maximum length of the core in local boiling occurs in the central or hot channel. Assume as before that local boiling occurs at a surface temperature of 641°F . Substituting these values in the surface temperature equation reduces the equation to a trigonometric quadratic.

$$\cos \frac{\pi Z}{L'} = \frac{2.38 + \sqrt{5.68 - 3.45}}{3.97}$$

$$\cos \frac{\pi Z}{L'} = 0.97605 \text{ or } 0.22292$$

$$\frac{\pi Z}{L'} = 0.218 \text{ or } 1.345 \text{ radius}$$

$$Z = 0.637 \text{ or } 3.86 \text{ ft.}$$

Therefore the axial limits of the local boiling region are 3.61 feet and 8.11 feet. This agrees with the value of the axial extent of local boiling found from the graphical method.

XII. Analytical Method for Determining the Extent of a Core in Local Boiling

A. Introduction

In conjunction with calculating the local boiling pressure drop, the extent of local boiling in the core should also be determined. In contrast to the graphical method for determining the volume of a core in local boiling, an analytical method which is more succinct can be used. The assumptions are

1. The surface temperature at which local boiling takes place throughout the core can be judiciously estimated.
2. The radius of the volume of the core in local boiling is small, such that all terms after the second in the series for the J_0 Bessel function can be neglected.

3. The radial and axial flux distribution can be expressed by well defined functions. (i.e., "chopped" Bessel radially and "chopped" cosine axially).
4. Local boiling begins when $T_B + \Delta T_\theta = T_S + \Delta T_{J\&L}$, where θ , T_S and $\Delta T_{J\&L}$ are functions of Z and L .

B. Development of Expressions and Equations

Expressing the surface temperature by

$$T_S = T_{Bo} + \left\{ \frac{\Delta T_{max}}{2} \left[1 + \frac{\sin \left(\frac{\pi Z}{L'} \right)}{\sin \left(\frac{\pi}{2} \frac{L}{L'} \right)} \right] + \theta_{max} \cos \frac{\pi Z}{L} \right\} J_0 \left(2.405 \frac{r}{R'} \right) \quad (6)$$

$$T_{(surface)} = T_{(inlet\ water)} + \Delta T_{(water\ rise)} + T_{(film)} \quad (7)$$

Then expand the J_0 function to obtain

$$J_0(X) = 1 - \frac{X^2}{2^2} + \frac{X^4}{2^4(2!)^2} - \frac{X^6}{2^6(3!)^2} + \dots + (-1)^k \frac{X^{2k}}{2^{2k}(K!)^2} + \dots \quad (8)$$

If X is less than one, the terms in the series diminish rapidly. For the Yankee reactor $X = 0.719 r_{LB} < 1$.

By neglecting all terms after the second in Eq. 8, and substituting the desired temperature values in Eq. 6.

$$1 - Mr^2 = \frac{M}{K + S \sin \frac{\pi Z}{L'} + N \cos \frac{\pi Z}{L'}} \quad (9)$$

Using new constants for simplification

$$r^2 = \frac{1}{A} - \frac{B}{C + D \sin \frac{\pi Z}{L'} + E \cos \frac{\pi Z}{L'}} \quad (10)$$

Equation 10 is the desired relationship between r , the radius, and Z , the longitudinal distance measured from the center of the core, since r is expressed as an explicit function of Z . It will be recalled that the volume of a body of revolution of the type in question is

$$V = \int_{Z_1}^{Z_2} \pi r^2 dz$$

The longitudinal axis is the axis of symmetry and the volume of the core in local boiling is an ellipsoid rather than a spheroid.

On changing the variable from X to Z and inserting the longitudinal local boiling limits the following equation is obtained,

$$V = L' \int_{Z_2}^{Z_1} \left(\frac{1}{A} - \frac{B}{C + D \sin \frac{\pi Z}{L'} + E \cos \frac{\pi Z}{L'}} \right) dz \quad (11)$$

Now that the entire integral is in standard form, the solution to the second integral is

$$\frac{1}{\sqrt{D^2 + E^2 - C^2}} \ln \left| \frac{D - \sqrt{D^2 + E^2 - C^2} + (E-C) \tan \frac{\pi Z}{2L'}}{D + \sqrt{D^2 + E^2 - C^2} + (E-C) \tan \frac{\pi Z}{2L'}} \right| \Bigg|_{(Z)_2}^{(Z)_1} \quad (12)$$

The volume in boiling can then be readily calculated.

C. Analytical Step-by-Step Procedure

In summary, an analytical solution for determining the volume of the core in local boiling is as follows:

1. From the Jens and Lottes local boiling correlation estimate the average temperature at which local takes place throughout the core. (For the Yankee core a value of the surface temperature super heat equal to 90% of the maximum value of the Jens and Lottes gives good results.) At 2000 psia the maximum value of the Jens and Lottes local boiling correlation is approximately 5.3°F.

Figure 15

Maximum Volumetric Limits of Local Boiling
(First Parameter Study)

2. Determine the limits on Z in the hot channel by using Eq. 6 with the assumed Jens and Lottes temperature difference, a zero radius, and the values of the maximum temperature rise of the coolant and maximum film temperature drop as set by the latest Thermal Design Data.
3. Solve Eq. 11 using Expression 12.
4. To find the radial limits of local boiling determine the value of Z corresponding to the maximum surface temperature. With this value of Z and the local boiling surface temperature estimated in Step 1, calculate the maximum radius of local boiling. Check the calculated value of the radius with the assumption that $0.719 r_{LB} \leq 1$. See Figure 15.

XIII. Determination of the Local Boiling Friction Factor

A. By Means of the Two-Phase Pressure Drop Correlation

The first approach used to obtain a value for the local boiling friction factor was to investigate correlations for two-phase pressure drop. Since local boiling is defined as the region of nucleate boiling in which no net steam or very little net steam is generated it follows that the maximum local boiling friction factor should be always less than the two-phase friction factor under similar conditions.

The two phase friction factor was calculated using the Martinelli correlation for a four per cent quality mixture and the average of this value which is 1.3 was the original value used for the local boiling friction factor.

The final value used for the local boiling friction factor was obtained as follows. A graph was plotted using the enthalpy of the fluid and λ_{LB}/λ as coordinates. At the enthalpy which corresponds to the inception of local boiling the value of λ_{LB}/λ was taken as unity and a straight line was drawn to the value of λ_{TP}/λ at the enthalpy of 5% quality steam for a mass flow rate of 1.5×10^6 lbs/hr-ft².¹⁵ The advantage of using the enthalpy as one of the coordinates instead of the temperature of the coolant is that no discontinuity in the friction factor ratio occurs at the transition between single and two phase flow.

B. Calculation of the Local Boiling Friction Factor From the Heat Transfer Momentum Analogy²⁰

From the previous discussion it can be seen that values of λ_{LB}/λ reported in the literature show considerable scatter and range from 1.1 to 2 at 2000 psia. No data have been reported for flow parallel

to cylindrical rods at 2000 psia. For these reasons the friction factor ratio was calculated by means of a momentum-heat transfer analogy. Bubble dynamics was assumed to influence the shear mechanism in the boundary layer only by reducing the effective film thickness and increasing the turbulence. The film coefficient for local boiling is probably more complex than that for non-boiling heat transfer. A survey of current literature reveals that the local boiling heat transfer is proportional to the difference between the surface temperature and the saturation temperature or

$$h \approx \frac{q}{A} \approx c (\Delta t)^n \quad (5a)$$

From the Jens and Lottes correlation

$$T_{sat} = (T_s - T_{sat}) = 1.9 \frac{\phi_{max}(r, z)^{0.25}}{eP/900} \quad (5)$$

$$\text{or } \phi_{max}(r, z) = Ce^{P/225} [T_s - T_{sat}]^4$$

In the non-boiling region

$$h = \frac{\phi_{max}(r, z)}{(T_s - T_B)} \quad (13)$$

equating Eq. 5 and 13 gives

$$h_{LB} = Ce^{P/225} \frac{(T_s - T_{sat})^4}{T_s - T_B} \quad (14)$$

where

$$Ce^{P/225} = 570$$

Equation 14 is of the same general form as Eq. 5a. Values of h in Eq. 14 cannot be readily calculated from the physical properties or the geometry of the system as can the heat transfer coefficient in the non-boiling region. However, in the present case the surface temperature and the bulk water temperature can be calculated for any point in the core if the flux distribution is known.

Figure 16

Local Boiling Friction Factor Based on
Momentum-Heat Transfer Analogy
(Second Parameter Study)

Since the Prandtl number for water at the operating temperature in the reactor is approximately unity, the Reynolds analogy should predict the friction factor accurately, within the limitation of the assumption. The modified equation of Von Karman was used, however, since it includes the effect of the buffer layer on the heat transfer coefficient. Use of this equation also assumes that the bubbles cause no great change in the universal velocity - distribution curve.

$$Nu = \frac{(f/2)(Re)(Pr)}{1 + 5 \sqrt{f/2} \left[Pr - 1 + \ln \left(1 + \frac{5}{6} (Pr - 1) \right) \right]} \quad (15)$$

Using values of the heat transfer coefficient calculated from Eq. 14 in Eq. 15 the friction factor is calculated by trial and error solution.

As yet it has not been determined whether bubble motion at a solid surface causes a great change in the velocity distribution curve, especially in the region near the solid wall. Equation 15 is based on a velocity distribution curve which is equally valid for rough or smooth surfaces (i.e., "a universal distribution curve")

The second parameter study shows higher pressure drops and in general is more critical from an overall hydraulic standpoint. The weighted average local boiling to isothermal friction factor ratio is 1.25 for the second parameter study.

XIV. Flow Redistribution Caused by Local Boiling

A. Geometrical Considerations²¹

As discussed in the previous section, local boiling can exist in the steady operation state of both core parameter studies. Sub-cooled, or local boiling, is associated with an increase in pressure drop which influences the flow distribution across the core.

The Yankee core consists of a matrix of cylindrical fuel elements in which the only resistance to transverse flow between straps is the spacing between the fuel rods and the transverse resistance caused by the control rods. The transverse resistance offered by the control rods is discontinuous whereas the spacing between the rods can be considered uniform across the core. In any event an additional degree of freedom for flow re-distribution is introduced.

The heat generation in the core has been assumed to be a "chopped" cosine distribution axially and a "chopped" Bessel function radially. Therefore, the generation varies on both the longitudinal and radial

axes and the sub-cooled boiling as previously shown is non-uniform and a transverse shift takes place because of the change in the friction coefficient and the overall pressure drop balance. An open parallel lattice arrangement implies that the pressure drop between two elevations is constant for the corresponding lateral positions. For such an arrangement the pressure gradient as determined by the pressure drop equation for an average channel would apply laterally across the reactor. Using this pressure gradient the pressure drop equation can be applied to the hot channel to obtain the corresponding mass flow rates. The water mass velocity through the region of more intense local boiling is decreased, resulting in a greater rate of decrease in sub-cooling which increases the local boiling intensity further up the channel. In general, flow distribution at any axial position depends on the previous history of the flow from the channel entrance to the point in question. Flow redistribution and sub-cooling are important in determining whether burnout conditions exist.

B. Flow Analysis Assumptions

A complete solution to the flow redistribution problem is of an iterative nature and would involve the use of a computer. For a more direct solution, the problem of flow redistribution due to local boiling and fluid property variations is approached on the basis of a two channel study; a hot channel and an average channel. It is assumed that the hot channel occurs in the longitudinal channel at the center of the reactor. The remainder of the reactor core is considered to consist of average channels. Furthermore, it is assumed that the physical properties of the water and the mass flow rate in the average channels are not affected by the flow from the hot channel. Finally, it is assumed that no change in flux pattern occurs due to the flow redistribution. The restriction between two adjacent fuel rods is not considered in the analysis.

C. Pressure Drop Analysis

The general momentum equation for steady incompressible turbulent flow in a channel is

$$\frac{\partial}{\partial X} \left(\frac{G^2}{\rho} \right) = - \frac{\partial P}{\partial X} - g \rho - \frac{f}{XDe} \frac{G^2}{\rho} \quad (16)$$

The pressure drop along a channel is obtained by integrating $\partial P / \partial X$ along the length X to give

Figure 17

Pressure Gradient in a Average Channel
(First Parameter Study)

Figure 18
Pressure Gradient in an Average Channel

$$P_2 - P_1 = - \left[\frac{G^2}{\rho_g} \right]_1^2 - \int_1^2 \rho \, dX - \int_1^2 \frac{f G^2 \, dZ}{2 \rho_g \, De} \quad (17)$$

The total fuel rod length is divided into seven increments in the first parameter study and eight in the second study. Average values of the fluid properties for a given increment are used in the associated analogue for that length. The four components of pressure drop are:

$$\text{Density-variation} \quad - \left[\frac{G^2}{\rho_g} \right]_1^2$$

Observe that this term cannot be accurately calculated in the local boiling region because of the difficulty encountered in determining how the bulk density is affected by the bubbles. Therefore, in the local boiling region the term is not truly the total momentum change of the fluid. The problem of accurately determining the acceleration term is closely related to the problem of measuring the void fraction in local boiling which was previously discussed.

$$\text{Elevation head} \quad \int_1^2 \rho \, dX = \rho_{av} (X_2 - X_1)$$

$$\text{Friction head} \quad \int_1^2 \frac{f G^2 \, dX}{2 \rho_g \, De} \approx \sum \frac{f G (\Delta X)}{2 \rho_g \, De}$$

$$\text{Form head} \quad - \sum K_{form} \left(\frac{G^2}{2g\rho} \right)$$

Although the density variation in the absence of void fraction consideration is small, it is appreciable enough to be included in the analysis.

Friction is the major contribution to pressure drop and is given detailed investigation. For a typical increment of length where local boiling is possible, four different sections are considered. The first is the friction drop along the tube without local boiling. Where local boiling is possible the total friction factor is determined, as previously discussed, from the plot of the enthalpy of the water versus the local boiling to single phase friction factor ratio.

Calculation of pressure drop along the section containing straps is complicated by the geometry at these sections. The sections could be analyzed by considering the section as a combination cross-rod and strap matrix and using porosity as one of the correlating parameters. Figures 17 and 18 are graphs of the value of the pressure drop versus distance from the core inlet.

Figure 19

Mass Flow Rate in the Hot Channel
(First Parameter Study)

Figure 20
Mass Flow Rate in the Hot Channel
(Second Parameter Study)

Figure 21

Water Temperature vs Distance from Core Inlet
(First Parameter Study)

However, the approach taken is direct and considers the strap region as two characteristic cross sections with corresponding equivalent diameter, flow areas, and weighted relative surface roughness.

For a typical incremental length dx , the friction drop across the straps is larger than an equal length without straps because of the smaller equivalent diameter. There is also a form loss at the straps because of the change in flow area. As in the usual case, the mass flow rate in the strap region can be related to the non-strap region by the use of an area ratio. Although the form loss is approximately one-fourth of the total pressure drop, its relative independence of local boiling helps reduce the mass flow rate from the hot channel.

The elevation head is about one fifth of the total head loss. Since the elevation head loss is less in the hot channel than in an average channel the elevation factor acts to maintain stability and tends to prevent the flow redistribution from diverging.

D. The Effect of Fuel Rod Supports (i.e. Ferrules and Straps) on Flow Redistribution

The occurrence of local deviations from a normally open lattice caused by obstructions in the flow path similar to the straps which may be used in the Yankee reactor demands additional consideration. The strap configuration adds form losses which are virtually independent of local boiling effects. If the mass flow rates in the hot channel are determined by the pressure gradient of the average channel, the mass flow through the straps in the local boiling region of the hot channel are higher than the mass flows before and after the straps. The method used to determine the mass flows in the hot channel was to assume that the pressure was constant laterally across the reactor at a level midway between two adjacent strap sections. The elevations midway between two strap sections comply closest to the open lattice type of arrangement. It should be noted that the existence of form losses in the local boiling region reduce the mass efflux from the hot channel; a most desirable condition. This is an effect similar to orificing the coolant flow and helps to insure adequate flow through the hot channel.

E. Magnitude of Flow Redistribution

The mass flow rates through the hot channel were investigated for the first and second parameter studies of the Yankee core. The pressure drop in the average channel was used as the reference pressure drop across the fuel rod assembly. Mass flow rate in the hot channels are calculated for each of the two parameter studies.

If the discontinuity of the mass flow at the straps is neglected, the maximum loss of flow through the hot channel is calculated to be 17.6%. When the second approach of taking the pressure drop across the most lattice positions between straps is used, the calculated loss of flow in the hot channel is 12.0%. A similar analysis for the second parameter study gives a loss of 23.8% of the flow through the hot channel if the straps are neglected, and 16.2% if they are included.

Figures 19 and 20 are graphs of the mass flow rates in the hot channel for the first and second parameter studies. The broken line in Figure 19 which is shown as a discontinuity of mass flow rate through the straps can be represented by peaks in the flow curves.

Figure 21 shows that the outlet water temperature in the hot channel increases from 603°F to 610°F if the flow redistribution caused by local boiling is considered. For the second parameter study the outlet temperature is 628°F.

This analysis is conservative because of the following reasons; (a) the control rods help to reduce flow redistribution along the fuel rods, (b) the space between two adjacent fuel rods acts as a restriction thereby reducing flow redistribution, (3) the indicated analysis is fundamentally a two channel study.

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