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QUARTERLY REPORT
APRIL, MAY AND JUNE, 1950

PHYSICS DIVISION

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REPORT FOR APRIL, MAY AND JUNE 1950

Physics Division

L. A. Turner, Division Director

Edited by: A. Wattenberg
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Louis A. Turner

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Group #1 - C. O. Muehlhause

1. The Neutron Cross Sections of Mn Below 5000 ev, (S. P. Harris, C. T. Hibdon, C. O. Muehlhause)

A. Introduction

At neutron energies below 5000 ev two principal resonances in manganese are known. These are principally scattering resonances¹, and are located² at 345 ev and ~ 2400 ev, respectively.³ However, a significant amount of resonance absorption ($\sim 3\%$) is observed to be associated with the 345 ev level. Other available data on manganese consist of measurements of the thermal scattering, the coherent thermal scattering, and the thermal absorption cross sections. Also, the thermal scattering phase of manganese is known to be negative^{3,4}. In the present work³ measurements of the resonance scattering integrals of each resonance as well as the absorption integral of the 345-ev resonance are presented. In addition self-indication measurements at 345 ev are given which determine the total angular momentum of the 345-ev level. This is also independently obtained from the resonance scattering data. Finally, measurements of the scattering cross section of manganese at specific energies below 150 ev are presented which are in complete agreement with the predicted values of the Breit-Wigner scattering formula.⁵

Verification of the behavior of a neutron cross section in the wings of a resonance is one of the principal purposes of this paper. Manganese offers an admirable opportunity for such a study since the low energy wing of the 345 ev level extends to zero energy. In this region resonance scattering detectors with high energy resolution¹ are available. These

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1. S. P. Harris, C. T. Hibdon, C. O. Muehlhause, Phys. Rev. 79, July 1, 1950.
2. C. T. Hibdon, C. O. Muehlhause, W. Selove, W. Woolf, Phys. Rev. 77, 730 (1950).
3. C. G. Shull, S. Siegel, Phys. Rev. 75, 1008 (1949).
4. E. Fermi, L. Marshall, Phys. Rev. 71, 666 (1947).
5. H. A. Bethe, Rev. Mod. Phys. 9, 152 (1937).

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enable one to make accurate manganese transmission measurements at definite energies.

B. Theory

The Breit-Wigner expression for a many-levels scattering cross section⁵ is given by:

$$\frac{\sigma_s}{4\pi} = \sum_j g_j \sum_r \left/ R + \frac{1}{2} \cdot \frac{\lambda_r \Gamma_{nr}}{E - E_r + i \Gamma_n / 2} \right/ ^2 \quad (1)$$

where:

J = total angular momentum of the compound nucleus ($J = I \pm \frac{1}{2}$ for S neutrons where I is the total angular momentum of the initial nucleus)

$g_j = \frac{1}{2} (1 \pm \frac{1}{(2I+1)})$ (statistical weight)

R = nuclear radius

λ_r = neutron wave length (divided by 2) at exact resonance

E_r = resonance energy

Γ_{nr} = neutron resonance width at exact resonance

Γ_r = total resonance width at exact resonance ($\Gamma_r = \Gamma_{nr} + \Gamma_{\gamma r}$)

where $\Gamma_{\gamma r}$ = radiation resonance width)

The numerator of the resonance terms is actually $\lambda \Gamma_n$ but can be set equal to the constant $\lambda_r \Gamma_{nr}$ if Γ_n varies directly as the neutron velocity, v . The correctness of this important feature is proven experimentally by the present work. That "R" should be unique and independent of J would offhand seem surprising, except perhaps in very heavy nuclei where the size of the nucleus is "fixed" by the many particles, and where the off-resonance range of the forces is only slightly sensitive to the incoming neutron spin orientation. If R does depend on J , a more general formula would be given by:

$$\frac{\sigma_s}{4\pi} = \sum_j g_j \sum_r \left/ R_j + \frac{1}{2} \cdot \frac{\lambda_r \Gamma_{nr}}{E - E_r + i \Gamma_r / 2} \right/ ^2$$

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In the present work it is shown that only a single radius need be assumed. However, in other cases such as sodium² and cobalt it appears necessary to allow for two radii, one for each possible neutron spin orientation.

Coherent scattering is given by the square of the sum of the real parts of all amplitudes (when $E_r \gg \Gamma_r$):

$$\frac{\sigma_{coh}}{4\pi} = \left/ \sum_j \sum_r g_j R + \frac{g_j}{2} R_e \frac{\lambda_r \Gamma_{nr}}{E - E_r + i \Gamma_r/2} \right/ ^2$$

$$\frac{\sigma_{coh}}{4\pi} = \left/ R + \frac{1}{2} \sum_r g_r R_e \frac{\lambda_r \Gamma_{nr}}{E - E_r + i \Gamma_r/2} \right/ ^2 \quad (2)$$

For the case in hand, manganese, only two resonances are known. In order to apply formula (1) it is important to know whether $J_1 = J_2$ or $J_1 \neq J_2$, since only states of the same J mix. In the experimental sections of this paper, it is shown that $J_1 \neq J_2$, so that (1) may be written as the sum of two Briet-Wigner one-level terms:

$$\frac{\sigma_s}{4\pi} = g_1 \left/ R + \frac{\lambda_1 \Gamma_{n1/2}}{E - E_1 + i \Gamma_1/2} \right/ ^2 + g_2 \left/ R + \frac{\lambda_2 \Gamma_{n2/2}}{E - E_2 + i \Gamma_2/2} \right/ ^2 \quad (3)$$

Formulae (2) and (3) reduce to the following expressions at thermal energies:

$$\frac{\sigma_{ths}}{4\pi} = g_1 \left/ R - a_1 \right/ ^2 + g_2 \left/ R - a_2 \right/ ^2 \quad (4)$$

$$\frac{\sigma_{th coh}}{4\pi} = \left/ R - g_1 a_1 - g_2 a_2 \right/ ^2 \quad (5)$$

where $a_r = \frac{\lambda_r \Gamma_{nr}}{2E_r}$ valid for $E_r \gg \Gamma_r$

$$\frac{\sigma_{th-inc}}{4\pi} = \frac{\sigma_{ths} - \sigma_{th coh}}{4\pi} = g(1-g)(a_1 - a_2)^2 \quad (6)$$

(since $g_1 + g_2 = 1$)

Here a_r is the resonance scattering amplitude at $E = 0$ when $E_r \gg \Gamma_r$. It

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should be observed that $g(1-g)$ is uniquely given by I , the initial angular momentum. If both resonances have the same J value, (6) becomes:

$$\frac{\sigma_{th inc}}{4\pi} = g(1-g)(a_1 + a_2)^2 \quad (7)$$

If two radii were assumed, formulae (6) and (7) would contain their difference.

These expressions provide valuable checks of the Breit-Wigner relations. Conversely, they may be taken to evaluate the effective nuclear radius.

It is also shown in the experimental sections of this paper that the thermal capture cross section of manganese is approximately given by the resonance parameters of the 345 ev level. This is reasonable since the contributing resonance terms in the expansion of the thermal capture cross section decrease more rapidly than in the case of scattering. The Breit-Wigner one-level capture cross section is given by:

$$\frac{\sigma_a}{\pi} = g_1 \lambda_1^2 \Gamma_{n1} \sqrt{\gamma_1} \frac{\sqrt{E_1/E}}{(E-E_1)^2 + \Gamma_1^2/4} \quad (8)$$

which for thermal energies (.025 ev) reduces to:

$$\begin{aligned} \frac{\sigma_{tha}}{\pi} &= g_1 \lambda_1^2 \Gamma_{n1} \sqrt{\gamma_1} \frac{\sqrt{4OE_1}}{E_1^2} \quad \text{or} \\ \frac{\sigma_{tha}}{4\pi} &= g_1 \frac{\Gamma_{\gamma 1}}{\Gamma_{n1}} \cdot \sqrt{4OE_1} a_1^2 \end{aligned} \quad (9)$$

It should be observed that successive terms in (9) decrease more rapidly than a_r^2 (which holds for scattering) if $\Gamma_{\gamma r} \approx \text{constant}$ and Γ_{nr} increases more rapidly with E_r than E_r . This is the situation for manganese and sodium².

Resonance scattering, Σ_s , and absorption, Σ_a , integral expressions are also obtained from the Breit-Wigner formulation.¹

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$$\sum_{s \text{ or } a} = \int \sigma_{\text{res}} \frac{dE}{E} \text{ or } \int \sigma_a \frac{dE}{E}$$

These can be measured by the method given in reference 1. The Breit-Wigner evaluations are:

$$\sum_{sr} = \frac{\pi}{2} (4\pi \lambda_r^2 g_r \frac{\Gamma_{nr}^2}{\Gamma_r^2}) \frac{\Gamma_r}{E_r} = 4\pi^2 \lambda_r^2 \frac{\Gamma_{nr}}{\Gamma_r} g_r a_r \quad (10)$$

$$\sum_{ar} = \frac{\pi}{2} (4\pi \lambda_r^2 g_r \frac{\Gamma_{nr} \Gamma_r}{\Gamma^2}) \frac{\Gamma}{E_r} = 4\pi^2 \lambda_r^2 \frac{\Gamma_r}{\Gamma} g_r a_r \quad (11)$$

$$\sum_r = \sum_{sr} + \sum_{ar} = 4\pi^2 \lambda_r^2 g_r a_r \quad (12)$$

It can be seen from (12) that the total resonance integral (for level r) yields the coherent thermal amplitude, $g_r a_r$, due to resonance "r".

One more relation of importance is employed in the experimental sections. This is the cross section for self-indication, σ_{self} . The transmission cross section measured by placing a thin resonance filter in a neutron beam wherein neutrons are detected by a thin foil of the same resonance material is given by⁵:

$$\sigma_{\text{self}} = \frac{1}{2} \sigma_{\text{max}} = 2\pi \lambda_r^2 g_r \frac{\Gamma_{nr}}{\Gamma_r} \quad (13)$$

Γ_{nr}/Γ_r (~ 1 for Mn) is obtained¹ from the ratio of (10) to (12) and λ_r is known from the resonance energy, E_r . Therefore (13) determines g_r and consequently J_r of the compound state*.

*That $\sigma_{\text{self}} = \sigma_{\text{max}}/2$ experimentally (section D) may be considered as partial proof of the Breit-Wigner formulation. For example, if the resonance shape were gaussian, $\sigma_{\text{self}} = \sigma_{\text{max}}/\sqrt{2}$.

C. Experimental Measurement of $\sum_{s1}$ and $\sum_{s2}$

The apparatus and method of measuring resonance scattering integrals is fully described in reference 1 and will not be given again here.

With the use of a very thick sodium filter ($E_F \approx 3000$ ev) and a resonance neutron beam from the Argonne heavy water reactor, a $1/E$ neutron flux can be obtained in which neutrons near 2400 ev are removed.* The resonance scattering cross section of manganese can, therefore, be measured with and without the presence of scattering at 2400 ev. After corrections are made for foil thickness and counter sensitivity, it is found that:

$$\sum_{s1} = 419 \text{ b} \qquad \sum_{s2} = 93.6 \text{ b}$$

where res. 1 is at 345 ev and res. 2 is at 2400 ev.

In section D it is shown that:

$$\sum_{a1} \approx 14 \text{ b} \qquad \sum_{a2} \approx 0$$

Therefore:

$$\sum_1 = 433 \text{ b} \qquad \sum_2 = 93.6 \text{ b}$$

$$\text{and: } g_1 a_1 = .439 \text{ b}^{\frac{1}{2}} \qquad g_2 a_2 = .278 \text{ b}^{\frac{1}{2}}$$

In the case of manganese $I = 5/2$ and

$$g = \frac{1}{2} \left(1 \pm \frac{1}{2I+1} \right) = \frac{7}{12} \text{ or } \frac{5}{12}$$

$$\therefore a_1 = \begin{cases} .753 \text{ b}^{\frac{1}{2}} & (J = 3) \\ 1.0535 \text{ b}^{\frac{1}{2}} & (J = 2) \end{cases} \qquad a_2 = \begin{cases} .476 \text{ b}^{\frac{1}{2}} & (J = 3) \\ .667 \text{ b}^{\frac{1}{2}} & (J = 2) \end{cases}$$

From the experiments of Fermi⁴, Shull⁵, and Bendt⁶, it is evident that

*The Na-Mn resonance overlap is $\sim 15\%$. The resonances are probably closer than 600 ev to cause this. $E_2(\text{Mn}) = 2400 \pm 500$ and $E(\text{Na}) = 3000 \pm 600$ ev.
6. P. J. Bendt, I. W. Ruderman, Phys. Rev. 77, 575 (1950).

$\sigma_{\text{ths}} \sim \sigma_{\text{th coh}}$. In an actual neutron crystal diffraction experiment, temperature effects and crystal imperfections increase $\sigma_{\text{th inc}}$. There are two sets of a_r values above that satisfy the condition $\sigma_{\text{ths}} \sim \sigma_{\text{th coh}}$. This may be expressed by stating $J_1 \neq J_2$. Application of formulae (6) and (7) demonstrates the above. In section E of this paper it is shown that any reasonable fit to the total cross section data yields $\sigma_{\text{ths}} = 2.13 \pm .1$ b.

The two possibilities ($J_1 = 3, J_2 = 2$) and ($J_1 = 2, J_2 = 3$) are now to be considered.

Case (3,2)

$$\frac{\sigma_{\text{ths}} - \sigma_{\text{th coh}}}{4\pi} = \frac{7}{12} \times \frac{5}{12} (.753 - .667)^2 \approx 0$$

$$\text{i.e. } \sigma_{\text{th coh}} \approx 2.13 \text{ b and}$$

$$-\sqrt{\frac{2.13}{4\pi}} = (R - .439 - .278) - \text{phase}$$

$$\text{i.e. } R = .717 - .412 = .305 \text{ b}^{\frac{1}{2}}$$

$$\text{and } 4\pi R^2 = 1.17 \text{ b}$$

Case (2,3)

$$\frac{\sigma_{\text{ths}} - \sigma_{\text{th coh}}}{4\pi} = \frac{7}{12} \times \frac{5}{12} (1.054 - .476)^2$$

$$\text{i.e. } \sigma_{\text{th coh}} = 2.13 - 1.02 = 1.11 \text{ b}$$

$$-\sqrt{\frac{1.11}{4\pi}} = (R - .439 - .278) - \text{phase}$$

$$\text{i.e. } R = .717 - .297 = .420 \text{ b}^{\frac{1}{2}}$$

$$\text{and } 4\pi R^2 = 2.22 \text{ b}$$

Both cases agree sufficiently well with $\sigma_{\text{th coh}}$ given in the literature^{3,4,6}. However, the next two experimental sections demonstrate that the case (3,2) is correct.

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D. Experimental Measurement of $\sigma_1 \text{ max}$

The peak cross section of the 345 ev resonance is determined by the method of self-indication as outlined in section B. Two thin manganese foils of mean thickness 7.097×10^{19} atoms/cm² serve as activation detectors in a cadmium filtered 1/E distributed¹ pile neutron beam. Two aluminum cells, one filled with D₂O and the other with D₂O-MnSO₄ (3.286×10^{19} atoms of Mn/cm²) serve as filters for the detectors. The cells and detector foils are mounted on a 4 ft long shaft the axis of which is parallel with the neutron beam. The shaft slowly rotates the cells and foils in and out of the neutron beam thereby insuring a self-monitoring system. That is, the neutron transmission of the MnSO₄ is given by I_1/I_2 , where foil #1 is behind the cell containing the MnSO₄ and foil #2 is behind the blank cell. Figure 1 schematically illustrates the experimental arrangement.

The foils are bombarded to saturation and are counted simultaneously for three half lives ($T_{1/2} \approx 2.59$ hr) on two well matched Geiger counters. To avoid asymmetry in the counting apparatus the experiment is rerun with the roles of the foils exchanged as to cell and counter. A geometric mean transmission is then calculated, the result being:

$$\sigma_{\text{self}} = 1490 \pm 250 \text{ b}$$

In the above, it is assumed that negligible absorption occurs at 2400 ev. This is justified further on in this section, as well as:

$$\Gamma_{\text{nl}}/\Gamma_1 = .968.$$

Three corrections to the expected value of σ_{self} have to be made all of which lower it from $\sigma_{\text{max}}/2$. These are: a) 4.43% Cu-Ni impurity, b) foil thickness, c) absorption resolution. The second of these depends on which J value is assumed for the 345 ev level, and the third of these is given by the fraction of resonance absorption to total epicadmium absorption (includes 1/v absorption). A cadmium ratio measurement of the thin manganese foils standardized by a cadmium ratio measurement of gold and boron (1/v absorber)

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FIG. 1

ROTATING CELL & FOIL HOLDERS

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enables one to evaluate the $1/v$ and resonance absorption integrals. This method is described in reference 1. The results are:

$$\begin{aligned}\sum 1/v &= 5.60 \text{ b} && 1/v \text{ abs. integral} \\ \sum a &= 14.7 \text{ b} && \text{res. abs. integral}\end{aligned}$$

and the resonance absorption fraction, r , is given by:

$$r = \frac{14.7}{5.6 + 14.7} = .724$$

Making the various corrections indicated permits two possible observable self-indication cross sections:

$$\sigma_{\text{self}} = \begin{cases} 1410 \text{ b} & J_1 = 3 \\ 1030 \text{ b} & J_1 = 2 \end{cases}$$

The self-monitoring transmission measurements yield 1490 ± 250 b indicating strong preference for $J_1 = 3$.

That almost all of the resonance absorption is associated with the 345 ev level is evident from application of formulae (9) and (11). In this calculation the thermal absorption cross section for manganese (12.05 b) is taken from pile oscillation values.⁸

$$\begin{aligned}\Gamma_{\gamma 1} &= .525 \text{ ev} & \Gamma_{n1} &= 21.2 \text{ ev} & g_1 \Gamma_{n1} &= 12.4 \text{ ev} \\ \sum a_1 &= \frac{\pi}{2} \cdot \frac{2.6 \times 10^6}{345} \cdot \frac{12.4 \times .525}{21.7 \times 345} = 10.3 \text{ b}\end{aligned}$$

In other words, the expected resonance absorption integral of the 345 ev level ($J_1 = 3$) accounts for most of the observed resonance absorption integral (14.7 b). Since the thermal absorption cross section is well determined and the contribution to it from the 2400 ev resonance is

$\sim .1$ b, it is reasonable to expect that the proper variation of σ_a at energies below 200 ev is given by:

$$\begin{aligned}\sigma_a(E) &= \frac{\sigma_{\text{tha}}}{\sqrt{40E}} \left(\frac{E_1}{E_1 - E} \right)^2 \\ &= \frac{12.05}{\sqrt{40E}} \left(\frac{345}{345 - E} \right)^2\end{aligned} \quad (14)$$

7. F. G. P. Seid., S. P. Harris, A. S. Langsdorf, Jr., Phys. Rev., 72, 168 A (1947)
8. Argonne Pile Oscillator - to be published.

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E. Experimental Measurement of $\sigma_T(E)$

The apparatus employed here is the same as that described in section C for measuring resonance scattering integrals. The method consists in using various resonance scattering detectors such as Co⁵⁹ (120 ev), W¹⁸⁶ (19.5)⁹, and Sm¹⁵² (10.0 ev) in order to select neutron energies at which manganese transmission cross sections can be measured. This method has been described by the authors in connection with similar transmission measurements made on chlorine.¹⁰ The total manganese cross section at the cobalt point is corrected for the fraction of cobalt resonance scattering of the total epicadmium cobalt scattering.¹¹ Formula (14) is used to obtain the net or scattering cross section at the various energies selected. Table I lists the data obtained:

Table I			
E ev	$\sigma_T(E)$ barns	$\sigma_a(E)$ barns	$\sigma_s(E)$ barns
10.0	2.90	.64	2.26
19.5	2.88	.51	2.37
120	6.36	.38	5.98

Preliminary fitting of the data in Table I for either case (3,2) or case (2,3) indicates $\sigma_{ths} = 2.13 \pm .1$ b. Now the resonance parameters obtained in sections C and D are used to evaluate $\sigma_s(E)$. To do this the two-level Breit-Wigner formula is considered:

$$\frac{\sigma_s(E)}{4\pi} = R^2 + \frac{2E_1g_1a_1}{4(E_1-E)^2} \left\{ 2E_1a_1 - 4R(E_1-E) \right\} + \frac{2E_2g_2a_2}{4(E_2-E)^2} \left\{ 2E_2a_2 - 4R(E_2-E) \right\} \quad (15)$$

(for $\Gamma_r \ll |E-E_r|$). Two sets of parameters must be tried in (15)

9. Argonne Resonance Time-of-Flight Velocity Selector - W. Selove

10. C. T. Hibdon, C. O. Muehlhause, Phys. Rev. 79, July 1, 1950

11. C. T. Hibdon, C. O. Muehlhause, Phys. Rev. 76, 100 (1949)

corresponding to the two cases: (3,2) and (2,3). Table II lists the observed and calculated scattering cross sections. It is evident by inspection that the case (3,2) fits better than case (2,3). This is in agreement with section D in which $J_1 = 3$ is also preferred.

Table II

E - ev	$\sigma_s(E)$ - barns		
	Measured	Calculated (3,2)	Calculated (2,3)
10.0	2.26	2.26	2.34
19.5	2.37	2.47	2.57
120	5.98	6.10	7.50

F. Discussion of Results

Independent measurements of the resonance scattering properties of manganese can all be fitted to a two-level Breit-Wigner formula. Only one parameter, the nuclear radius, need be adjusted to do this. When this is done it is possible to obtain:

- The thermal scattering cross section.
- The coherent thermal scattering cross section.
- The negative thermal scattering phase.
- The thermal capture cross section.
- The specific behavior of the cross section in the wings of the resonance.
- The angular momenta of the two levels.
- Justification of $\Gamma_n \propto \sqrt{E}$.

The last conclusion is a necessary consequence of the good fit. One important difficulty remains; Γ_r appears to vary as E_r . This is also

indicated by the case of sodium.² If this is true, the a_r series might not decrease rapidly with increasing r about zero neutron energy thereby making it doubtful whether enough resonance terms have been included in the thermal cross section expressions. The effect of higher or negative energy levels is to alter the effective value of the nuclear radius. Since bound states should have very little scattering, the most important a_r terms are for r positive, i.e. $E_r > 0$. Such resonance levels have the effect of decreasing the apparent nuclear radius, and also of perturbing the calculated thermal coherent scattering cross section in a non-linear way. In the case of manganese, the fitted radius is smaller than that obtained by the $A^{1/3}$ law.* This may or may not be taken seriously. At any rate, if one assumes a third higher energy resonance having thermal amplitude a_3 , an unknown but too small amount is contained in the resonance scattering integrals given in the text. In general, one can say that the effect of an $a_3 > 0$ would be to increase R and to lower $\sigma_{th\ coh.}$

* $R = .521$ b from $R = .137 A^{1/3}$

2. On Spin Dependent Nuclear "Radii". (C. O. Muehlhaue).

In applying the Breit-Wigner scattering formula to low energy neutron scattering ($J = I \pm \frac{1}{2}$) the nuclear radius, R , enters as a potential scattering amplitude. At thermal energies the total scattering cross section, σ_s , is given by:

$$\frac{\sigma_s}{4\pi} = g_+ \left| R + a_+ \right|^2 + g_- \left| R + a_- \right|^2$$

where the $+$ and $-$ refer to states having total angular momenta J_+ and J_- respectively. The a_+ and a_- are sums of all resonance amplitudes of states having J_+ and J_- respectively, and the g_+ and g_- are statistical weight factors corresponding to the J .

It is possible that the apparent size of the nucleus (i.e. R , the range of the infinitely repulsive potential) may depend upon the incoming neutron spin orientation. This would yield an R_+ and an R_- and require σ_s to be given by:

$$\frac{\sigma_s}{4\pi} = g_+ \left| R_+ + a_+ \right|^2 + g_- \left| R_- + a_- \right|^2$$

Similarly, the incoherent (spin effect) thermal scattering cross section, σ_{inc} , would be given by:

$$\frac{\sigma_{inc}}{4\pi} = g_+ g_- \left| R_+ + a_+ - R_- - a_- \right|^2$$

For isotopes in which there are no significant resonance contributions to the thermal scattering cross section, this reduces to:

$$\frac{\sigma_{inc}}{4\pi} = g_+ g_- (R_+ - R_-)^2$$

It is an observed fact that materials having no significant low energy neutron resonance scattering also have no incoherent scattering¹.

From this it immediately follows that the off-resonance radii, R_+ and

1. Compilation by E. O. Wollan, unpublished.

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R_- , are equal. However, such may not always be the case for materials having low energy neutron resonances. In the cases of vanadium² and manganese³ it is possible to fit observed cross section data using only a single nuclear radius. In the cases of sodium⁴ and cobalt⁵ a single radius fit does not seem possible. This implies that the radius appearing in the resonance formula may not, in general, be given by the $A^{1/3}$ rule or even be given uniquely; rather, it should be treated as a double parameter.

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3. Use of Scintillation Counters to Discriminate Between U^{235} and U^{238} .
(Carl Hibdon)

An examination of the radiation from U^{235} and U^{238} and their daughter products indicates a predominance of low energy gamma rays from U^{235} (0.160 Mev) as compared with the radiation from U^{238} (0.7 to 0.9 Mev). The radiation from the equilibrium products of these two isotopes does not alter the picture appreciably. To discriminate between samples of these two isotopes an anthracene crystal scintillation counter has been used. In this case a discrimination between the two isotopes can be made on the basis of the high sensitivity of the scintillation counter for low energy gamma rays, the higher disintegration rate ($\sim 5X$) of U^{235} , and the predominance of γ -rays in U^{235} as compared with U^{238} .

Samples of $U_3^{235}O_8$ and $U_3^{238}O_8$ contained in glass tubes 0.50 cm (I.D.) and 0.73 (O.D.) were used. These tubes were encased by a second glass tube 0.90 cm (I.D.) and 1.10 cm (O.D.) for precautionary measures against breakage. The net uranium content averaged about 0.60 gm per centimeter of length. The experimental arrangement of samples and counter used is indicated in the accompanying sketch. In a counting time of 5 minutes

502 counts (scale of 512) were observed for the U^{235} sample and 47 counts (scale of 512) for U^{238} yielding a ratio of 10.7 counts for U^{235} as compared with U^{238} .

An aluminum absorber of 0.085 in. thickness reduced these counts to 480 and 45 respectively but the ratio of 10.7 was maintained.

As a further indication cold rolled steel absorbers of 0.125 in. thickness were used. The reduction in counts for U^{238} was negligible but the counts for U^{235} were reduced by a factor of 0.8. The ratio of counts for U^{235} to U^{238} was reduced to 8.5.

A two minute activation of the U^{235} by a 5.3 mg Ra-Be source in H_2O did not change the observed counts. In this part of the experiment the accumulated counts at the end of each succeeding minute were recorded for ten minutes before and immediately after activation.

The results obtained by this method are in agreement with the decay rates and decay schemes. U^{238} has a half-life period approximately five times that of U^{235} . In the decay of U^{235} to Th^{231} ($U\gamma$) the 0.160 Mev gamma ray occurs 80% of the time, which contributes to the high counting rate observed for U^{235} . Th^{231} decays to Pa^{231} , which has a long half-life and effectively stops the chain for these considerations. The gamma rays in the early part of the U^{238} chain are principally of the order of 0.7 to 0.9 Mev.

4. Electron Spectrum of Pa²³³. (H. B. Keller* and J. M. Cork**)

The protoactinium used in this investigation was derived from the 23 minute thorium 233 which was produced in the Oak Ridge pile by neutron capture in stable thorium. It was kindly made available to us by Dr. M. H. Studier.

The existence of a 27.4 day activity in protoactinium was first proposed by Meitner, Strassman and Hahn¹; the assignment was later questioned by Hahn and Strassman² and re-assigned to an isotope of zirconium. The final assignment to an isotope of protoactinium of mass number 233 was made and verified by other investigators³. Haggstrom⁴ and Levy⁵ have both previously investigated the conversion spectra and the beta continuum using a 180° constant radius type beta spectrometer. Both found the end point of the beta spectrum masked by conversion electrons but estimated it to be about 200 kev. Fulbright⁶ proposed a beta component with end point about 700 kev.

In the present investigation, a permanent magnet type photographic beta spectrometer was employed. Field strengths of approximately 200 and 600 gauss were used which made it possible to cover continuously the energy range from 19 kev to 1.5 Mev. The photographic plates used were Eastman Kodak, Type NTB with emulsions 25 microns thick. They were processed in Eastman Kodak Type D-8 developer for maximum contrast. In all,

*Phoenix Project Fellow, University of Michigan.

**Department of Physics, University of Michigan.

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46 conversion and 3 weak Auger lines were observed; all of which are confined to energies less than 411 kev. Absorption of the gamma rays in lead revealed no gamma radiation harder than 400 kev. Absorption in beryllium and aluminum gives a beta end point agreeing with that found by Haggstrom⁴ and Levy⁵; no trace of a harder beta component such as that reported by Fulbright⁶ was found.

Since Pa²³³ decays by beta emission, the K-L-M X-ray differences employed in determining the energy of the gamma rays will be characteristic of uranium. In the low energy portion of the spectrum, where the resolution of the instrument used is greatest, the L shell fine structure can be resolved into two components whose separation is 0.8 kev. It is important to mention, however, that no conversion could be detected in the L_{III} sub-shell. A photographic reproduction of the low energy spectrum appears in Fig. 2; the observed electron lines along with their interpretation is summarized in Table I. No satisfactory interpretation could be found for the line at 139.5 kev. It was consequently deleted from the table. All gamma rays listed fit well into the proposed level scheme of Fig. 3. While no claim to uniqueness for such a scheme is possible at this time, it is, however, to be noted that all possible transitions were observed with two exceptions. The least energetic of these is dotted in on the level scheme, since it is necessary to the scheme if it is to be self-consistent. Unfortunately, this gamma ray lies below the lower detecting limits of the apparatus.

This investigation was made possible by the joint support of the Office of Naval Research and the Atomic Energy Commission. The facilities of the Argonne Laboratory were kindly put at our disposal for this work.

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PREDOMINANT LINES IN THE LOW ENERGY CONVERSION SPECTRUM
OF Pa²³³

FIG.2

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PROPOSED LEVEL SCHEME (kev) FOR ${}_{92}\text{U}^{233}$ FOLLOWING
BETA EMISSION FROM ${}_{91}\text{Pa}^{233}$

FIG. 3

TABLE I

Electron Energy	Proposed Interpretation	Energy Sum	Gamma Energy
23.5	$M_{I,II,III}^1$	28.9	
24.6	M_{IV}^1	28.9	
27.5	N^1	28.9	
28.5	O^1	28.9	28.9
19.6	L_{II}^2	40.6	
35.2	$M_{I,II,III}^2$	40.6	
39.1	N^2	40.5	40.6
36.3	L_I^3	58.1	
37.1	L_{II}^3	58.1	58.1*
53.8	L_I^4	75.6	
54.7	L_{II}^4	75.7	
70.2	$M_{I,II,III}^4$	75.6	
74.3	N^4	75.7	
75.3	O^4	75.7	75.7
65.2	L_I^5	87.0	
66.1	L_{II}^5	87.1	
81.7	$M_{I,II,III}^5$	87.1	
85.7	N^5	87.1	
86.8	O^5	87.2	87.1
82.6	L_I^6	104.4	
83.5	L_{II}^6	104.5	
99.0	$M_{I,II,III}^6$	104.4	
103.2	N^6	104.6	
104.1	O^6	104.5	104.5

*M or N lines could not be seen because of interference of the very strong lines at 53.8 and 54.7 kev.

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(Table I cont.)

Electron Energy	Proposed Interpretation	Energy Sum	Gamma Energy
156.6	K ⁷	272.6	
250.9	L ⁷	272.7	272.6
185.6	K ⁸	301.6	
279.6	L ⁸	301.4	
296.1	M ⁸	301.5	301.5
197.5	K ⁹	313.5	
291.3	L ⁹	313.1	
307.5	M ⁹	312.9	
311.5	N ⁹	312.9	313.1
226.6	K ¹⁰	342.6	
320.2	L ¹⁰	342.0	
336.0	M ¹⁰	341.4	
340.1	N ¹⁰	341.5	342.0
260.6	K ¹¹	376.6	
354.5	L ¹¹	376.4	376.5
283.8	K ¹²	399.8	
378.1	L ¹²	399.9	
394.6	M ¹²	400.0	399.9
300.4	K ¹³	416.4	
394.6	L ¹³	416.4	
411.0	M ¹³	416.4	416.4
88.8	Auger L _I	110.6	K-M
93.4	Auger L _I	115.2	K-N
108.8	Auger M	114.2	K-N

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Group #2 - A. Langsdorf, Jr.

1. Investigation of Fast Neutron Resonances in Beryllium by Means of the Scattered Neutrons (A. Langsdorf, Jr., and M. T. Burgy).

The study of the angular dependence of the differential cross sections for fast neutrons, begun during the preceding quarter (ANL-4437, sections II, 5c and XII) has been continued, using a different experimental arrangement. As shown in Fig. 4, a slab of scattering material is placed in front of the Van de Graaf target 5 cm away and the fast neutron collimator and counter assembly described previously (ANL-4397, II, 3, 4, 5 and ANL-4437, II, 5) is pointed at the scatterer to receive the scattered neutrons. The collimator shields the counters from the neutrons coming directly from the target and from neutrons scattered from material in the vicinity, aside from a small portion of the scattering slab. Thus, neutrons scattered through a small angular range about the chosen mean angle are selected by the collimator, making possible a measurement of the differential scattering cross section as a function of scattering angle. An example of the counting rate observed is 260 counts/min with a 7 kev lithium target and a 9 microampere beam giving 730 kev neutrons of which 21% of those incident on the slab were scattered. The corresponding background rate was 100 counts/min. Backgrounds were obtained by removing the scatterer. The angular range is $\pm 8^\circ$. In this use of the collimator and counter assembly, only the inner three banks of counters in the annular arrangement are used, and a paraffin cylinder 2" D x $5\frac{1}{2}$ " is used to scatter into the set of counters the neutrons that came down the collimator hole. A BF_3 counter, imbedded in paraffin, and set to one side of the target served as a monitor of the neutron flux.

In addition to these differential cross sections for scattered neutrons, the total cross sections of the scattering materials were measured. Although the total cross sections of these materials have been measured in other

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EXPERIMENTAL ARRANGEMENT
FOR SCATTERING NEUTRON MEASUREMENTS

FIG. 4

laboratories, their remeasurement here was desirable as a check and as a guide to the effect of our spread of neutron energy. The counting rate without sample, of the transmission measurements, was needed anyway as a measure of the incident intensity in order to calculate differential cross section values. The transmissions were measured with the collimator axis in line with the proton beam, the front of the collimator being about 25 cm from the target. This arrangement has several advantages over the usual method of measuring transmission values; the scattering-in correction becomes very small because of the narrowness of the collimator hole, scattering into the detector can occur only up to $\sim 1^\circ$ scattering angle; the efficiency of detection is high, $\sim 20\%$; and there is a possibility of making self detection measurements.

Beryllium and carbon were used as scatterers during this quarter's work. Beryllium is of interest because of its two known resonances, at 620 kev and 810 kev. These are considered to be p-resonances, from transmission measurements. Carbon was measured along with beryllium as a comparison. Earlier work in carbon (ANL-4437, II, 5) together with a subsequent check of the counter geometry gave evidence that carbon scatters isotropically in the center of mass reference system. The geometry check was made by determining the counting rate as a function of the position of a Ra-Be source along the axis of the annular counter. This gave a curve symmetric about the center of the counter, so that the slight asymmetry observed with carbon in ANL-4437, II, 5 can be accounted for as due to motion of the center of mass. Sufficient data have been taken to obtain values of the differential cross section for beryllium and carbon separately, but the analysis of the data has not been carried this far as yet. This report will be concerned with the relative scattering of the two, only. The scatterers were discs 2.54 cm in diameter, 0.32 cm thick for beryllium and

0.64 cm thick for carbon. The cylindrical axes of the discs were set so as to bisect the mean neutron scattering angle. In measuring the total cross sections of Be and C, the thicknesses were chosen to give approximately 37% transmission. These were about 4.5 cm for carbon and from 1.3 to 2.5 cm for beryllium. Several lithium targets were used during the course of the work, with thicknesses ranging from 7 to 13 kev. An increase in target thickness with age was found, presumably due to a carbon deposit. The greatest target thickness, including deposit, was 24 kev.

In Fig. 5 there are given the total cross section curves for beryllium and carbon as a function of neutron energy and also the ratio of counting rates with the beryllium scatterer to that with the carbon scatterer for mean scattering angles of both 27° and 84° . A comparison of our results for the total cross section of beryllium differ from those of the Wisconsin group in that our resonance peak values are smaller. This difference is to be expected from our use of a poorer energy resolution. A further difference is found in the values of the resonance energies, ours being about 5 kev higher. In the curves giving ratios of scattered neutron counts, both Be resonances appear. From the marked presence of the beryllium resonances in the curve for 84° scattering angle, an immediate conclusion can be drawn, namely, that the beryllium resonances are not simple p-resonances. An 84° scattering angle corresponds to 90° scattering by beryllium in the center of mass system of coordinates. The differential cross section of a simple p-resonance has the form $\sigma(\theta) = A + B \cos \theta + C \cos^2 \theta$ where A is the potential cross section and θ is the scattering angle in the C of M system. Thus the resonance part should be absent at an angle of 90° in the center of mass system. Moreover, this behavior at an angle of 90° in the center of mass system holds for d, f, or higher ℓ value simple resonances.

FIG. 5
TOTAL NEUTRON CROSS SECTION CURVES FOR Be & C
& RELATIVE INTENSITY WITH Be & C SCATTERERS
AT 27° & 84° SCATTERING ANGLE.

Two possibilities are suggested; one, that the resonances are s-resonances, and two, that the two resonances in beryllium are members of a multiplet. (The latter possibility was pointed out to us by M. Hamermesh.) The original evidence against these resonances being s-resonances was the absence of dips in the total cross section curve on the low energy sides of the resonances due to interference between the potential and resonance scattering amplitudes. The multiplet hypothesis is in qualitative accord with the presence of resonance peaks in the 84° curve and with p-neutrons being scattered at the resonances, since it introduces terms in $\sin^2 \theta$. The details of the multiplet hypothesis have not been sufficiently worked out at this time to permit a closer comparison with the data. The data are presented in another form in Fig. 6 by multiplying the Be to C counting rate ratios at the two scattering angles by the inverse ratio of the total cross sections. Denoting the incident neutron flux by f , the atomic density in atoms per cm^2 by N , the solid angle for scattering by $d\Omega$, one has

$$\frac{\text{Be rate}}{\text{C rate}} \frac{\sigma_{t,c}}{\sigma_{t,\text{Be}}} = \frac{fN_{\text{Be}}\sigma_{\text{Be}}(\theta)d\Omega}{fN_{\text{C}}\sigma_{\text{C}}(\theta)d\Omega} \times \frac{\sigma_{t,c}}{\sigma_{t,\text{Be}}} = \frac{N_{\text{Be}}}{N_{\text{C}}} \times \frac{\sigma_{\text{Be}}(\theta)/\sigma_{t,\text{Be}}}{\sigma_{\text{C}}(\theta)/\sigma_{t,c}}$$

i.e., the product of the ratio of the N values by the ratio of the two ratios of differential to total cross section. If the resonances in Be and the potential scattering in both Be and C involve only s-neutrons the ratios $\sigma(\theta)/\sigma_t = 1/4\pi$ times a factor taking center of mass motion into account, this factor being only 4% greater for Be than for C at 27° and 1.5% at 84° . The horizontal straight lines in Fig. 6 were calculated on this basis. In the vicinity of the 620 kev resonance, the points depart from the straight lines markedly. In the vicinity of the 810 kev resonance, the points deviate from the straight line for 27° scattering angle but not at 84° scattering angle. Moreover, at all energies used above 810 kev at 27°

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scattering angle the points lie above the straight line. The deviation of the points in the vicinity of the 620 kev resonance at 27° scattering angle is qualitatively in accord with expectation for a simple p-resonance, namely, negative on the low energy side of resonance and positive on the high energy side. It is clearly evident that further work, both theoretical and experimental is needed to establish the nature of the beryllium resonances.

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Group #3 - G. R. Ringo

1. Small Angle Scattering of Neutrons. (G. R. Ringo)

At the suggestion of Prof. George Vineyard of the University of Missouri, the work on small angle scattering of neutrons, done here some time ago (ANL-4277 and ANL-4323), was re-examined in the light of a theory for the scattering of electromagnetic waves by spheres developed by H. C. Van de Hulst¹ on the basis of Mie's well known formal solution of the problem. This theory, which treats the particle as a bounded continuous medium characterized by a certain index of refraction, avoids one difficulty of the Halpern and Gerjuoy theory² which assumed that the incident beam is not appreciably attenuated in one particle. This assumption is not very reasonable for some of the materials with larger particle sizes. In addition, the Van de Hulst treatment is more complete than the refraction theory of Von Nardroff's³, considered earlier, in that it includes diffraction effects.

The width of the multiply scattered beam was calculated for the three theories with the results shown in Table I. In the range of values of particle size and index of refraction used in our experiments, the three theories do not differ greatly and all are in reasonable agreement with the experiments considering the approximations used in all three theories. These approximations are involved in assuming simple shapes for the particles, independent scattering by each particle, etc.

In other experiments, however, such as measurements of singly scattered beams, it might be desirable to restrict the use of these theories to the limits of validity of their assumptions. Thus, the Halpern and

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Material	Sample Thickness	Particle Diameter	Experimental θ_c	Halpern & Gerjuoy θ_c	Van de Hulst θ_c	Von Nardroff θ_c
Aluminum	17.3 gm/cm ²	16 microns	0.014 radians	0.0123 radians	0.0114 radians	0.012 radians
CaCO ₃	4.19	5.6	0.035	0.0232	0.0208	0.024
Lampblack	.348	0.72	0.027	0.025	0.0245	0.036

Gerjuoy theory should not be used when the incident beam is significantly attenuated in one particle. The Von Nardroff theory should not be used where $2\pi r \delta / \lambda \leq 2$, (r is the particle radius, λ the neutron wave length and δ the neutron index of refraction minus one) and the Van de Hulst theory should probably not be used for particles containing less than 1000 atoms.

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2. Coherent Neutron-Proton Scattering by Liquid Mirror Reflection
(D. J. Hughes*, M. T. Burgy and G. R. Ringo)

By use of the method of analysis of the data of this experiment described in ANL-4437, the value of the coherent neutron scattering amplitude of the hydrogen atom was found to be $-3.79 (\pm .03) \times 10^{-13}$ cm. This value relies heavily on the very careful determination, by R. W. Bane of the Chemistry Division, of the hydrogen-carbon ratio of the liquids used as mirrors. It agrees reasonably well with the value $-3.75 (\pm .03) \times 10^{-13}$ obtained in the first experiment.¹

*Now at Brookhaven National Laboratory, Upton, Long Island, New York.

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IV. Bismuth Absorption Cross Section (C. Egger and D. J. Hughes)

In connection with some experiments which are being planned for reflection of neutrons from bismuth mirrors, it was important to know the absorption cross section for bismuth. Measurements of the bismuth absorption cross section in the past have sometimes given high results, probably caused by impurities. The most recent results obtained with supposedly pure bismuth were those of Langsdorf and Muehlhause. They obtained about 35 mb with the pile oscillator in CP-3. The activation cross section, on the other hand is about 15 mb, definitely less than the absorption cross section. Langsdorf felt that some of the difference could be ascribed to a slowing-down effect he had discovered with the pile oscillator. In the "Langsdorf effect" neutrons above the fission threshold for U^{238} , which would be able to produce neutrons by fast fission, are slowed down below the U^{238} threshold in the sample being measured, and hence, cannot produce neutrons by fast fission. The effect will appear as an absorption and will obviously depend on the geometry of the apparatus, that is, on the proximity of uranium lumps.

In order to check the various possibilities for the discrepancy between the absorption and the activation cross sections, (impurities or "Langsdorf effect"), a series of experiments were performed with CP-2. Four different bismuth samples were obtained, all supposedly of high purity, and the neutron absorption was measured by the standard danger coefficient method in CP-2. All the bismuth samples give an absorption cross section close to 33 mb. The activation cross section was then measured by irradiating some bismuth at a spot in CP-2 where the flux was simultaneously calibrated with standard gold foils. The result of the activation measurement was 16 ± 3 mb. The only activity obtained was the 5 day Bi^{210} period and the activation cross section for any other

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periods with half lives between 1 minute and 1 week, was less than 1 mb. The fact that no activation other than the 5 day was obtained and that four samples from this source gave the same absorption cross section, made it seem unlikely that impurities were present.

It was thought that the "Langsdorf effect" would be small in CP=2 because the uranium lumps were not close to the bismuth samples. However, the effect was investigated by measuring the absorption cross section in the graphite stringer 23 T, first with uranium lumps 1" from the bismuth samples, then with the uranium removed. The sensitivity of the pile was measured for each configuration with a lead and iron absorber (for which the "Langsdorf effect" would be quite small). The results of this test showed no change in the bismuth cross section depending on the presence of uranium lumps. It appears then, that the absorption cross section of the bismuth samples is certainly larger than the activation cross section and by about the same amount for the different samples. The samples used are now being analyzed spectrographically and additional samples, possibly of greater purity, are being obtained in order to get an accurate value for the absorption cross section. An isomeric state of Bi²¹⁰ has recently been discovered by Perlman¹ in pile irradiated material. This isomeric state could of course account for a difference in the activation and the absorption cross sections, but the isomeric cross section necessary to explain the activity obtained by Perlman, could be less than 1 mb (only a lower limit is known for the lifetime of the isomer).

1. H. M. Neumann, J. J. Howland, Jr. and I. Perlman, Phys. Rev. 77, 720 (1950).

V. The Neutron Spectrum of a Radium-Beryllium Photo Source (C. Eggler)

A photoneutron source has been proposed by the National Bureau of Standards as a possible primary and permanent standard for the measurement of neutron fluxes. A radium-beryllium photo source as described by Wattenberg¹ has tentatively been adopted as this primary standard. Figure 7 shows a diagram of the source.

In connection with the measurement of the total neutron emission of such a source, it seemed desirable and even necessary for source calibration corrections to measure the spectrum of the neutrons. The spectrum was measured with a cloud chamber using the recoil proton technique as described by Hughes and Eggler.²

Figure 8 gives the recoil proton spectrum as a histogram after being normalized for the various pressures employed in the determination and corrected for variation in the hydrogen scattering cross section at different energies and the geometrical probability of seeing a full length track within the chamber.

Although photo sources are usually considered as monoenergetic when a single gamma ray is involved, nuclear factors enter in which may cause considerable spread in the energies observed. This spread is further enhanced by the method of measurement, and in a cloud chamber the errors may be large, although explainable.

Strangely enough but little work has been done to measure the energies of the gamma rays of radium since the work of Ellis,³ other than the investigation of Latyshev.⁴ He found a more complex gamma ray series than did Ellis and also found significantly different results for the relative

1. A. Wattenberg, National Research Council Report Number 6 of Nuclear Science Series "Photo-Neutron Sources".

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3. C. D. Ellis, Proc. Roy. Soc. (London) 113A, 350 (1933).

4. G. D. Latyshev et al, Jour. Exp. Theor. Phys. USSR 10, 996 (1940).

PHOTO-NEUTRON STANDARD

SCALE 10cm = 1cm

C - RADIUM CAPSULE DENSITY 4.8

(1) - BERYLLIUM

(2) - "

FIG. 7

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FIG.8

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intensities and energies above 1.1 Mev.

The recoil proton spectrum as observed in the cloud chamber would indicate the presence of at least four neutron energy groups and possibly a fifth. If one assigns energy values to these proton groups, it is possible to calculate the energy of the neutrons and hence, the gamma rays. Table I gives a summary of the indicated neutron and gamma ray energies with the relative yields of the five neutrons groups. The two low energy groups are not readily resolved as to intensities.

Table I

E_n Kev	E_γ Mev	Relative Yield of Neutrons Percent
70	1.750	35
190	1.884	10
310	2.019	14
480	2.210	26
640	2.390	15

} Resolution
difficult

Table 2 gives a summary of the nuclear and experimental corrections to be expected.

Explanation of Table II.

Column 1 gives the most probable mean range of the recoil protons as observed in the cloud chamber which, from previous experiments and from an examination of the probable spreads of the neutron energies and allowable errors of measurement, is approximately at 40% of the group spread.

Column 2 converts the range of the track in a hydrogen-filled chamber to energy. Bethe's curve⁵ was used with a conversion factor of 5 for the

5. H. A. Bethe, Rev. of Mod. Phys. 22, 213 (1950).

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Table II

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Range Assumed mm	E_p or E_n Kev	E_γ Mev	$\pm \delta$ (max) Kev	I % Range	Source Geometrical Correction	Length Track Error	Angle of Recoil Error	Expansion Spread	RMS Error % (col 5-9)	RMS Spread mm
5.6	70	1.750	4.8	± 6.9	$\pm 6\%$	$\pm 20\%$	$\pm 6\%$	$\pm 12\%$	+ 25.8 22.8 18.5	7.1 4.3 16.5
13.9	190	1.884	6.8	3.6	6	10	6	12	13.9 15.4	11.0 27.5
23.8	310	2.019	11.7	3.8	6	3	6	12	9.6 15.6	21.4 44.0
38.0	480	2.21	15.58	3.3	6	4	6	12	10 15.3	34.2 66.1
57.2	640	2.390	19.7	3.1	6	3	6	12	9.5	51.7

-15-

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ratio of H_2 /air.

Columns 3, 4, and 5 give the calculated energy of the gamma ray from neutron energy and the spread of the neutron in kev and % of range of track.¹

$$E_n = \frac{A-1}{A} \left[E_\gamma - Q - \frac{E_\gamma^2}{1862(A-1)} \right] \pm 2E_\gamma \left[\frac{2(A-1)(E_\gamma - Q)}{931 \times A^3} \right]^{\frac{1}{2}}$$

where: E_n = neutron energy

Q - Be threshold

A - target mass

E_γ = gamma energy

Column 6 gives the geometrical error. The source has a finite size and in its location causes an error in the measurement of the angle of 5 degrees. This error is reflected as 6% when correcting by the square of the secant at a maximum of 20 degrees to the incident neutron beam; twenty degrees being the limit of recoil to neutron angle reported.

Column 7 gives the error of track measurement. It has been assumed that a track can be measured to 2 mm of its actual length in the cloud chamber by means of stereo-reprojection through the same optical system as was used to take the photograph. Percentage errors vary as different pressures were used to accommodate the various energy groups.

Column 8 gives the error in measurement of the angle between the recoil proton and the incident neutron. It is the error assigned to the actual measurement of the angle and is in addition to the geometrical error resulting from the finite size of the source.

Column 9 gives the error introduced by expansion within the cloud chamber and the effect on the range of the track. This is a positive error only as the stopping power of the chamber is computed on a compressed basis. The chamber was operated at the same expansion ratio and in a constant temperature room to keep other factors constant.

Columns 10 and 11 give the root-mean-square error in percent and in ranges of tracks. Perhaps several other errors such as straggling of protons and scattering within the beryllium source and lead shield surrounding

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the source should be included. These errors are, however, small and considered to be within the allowable error. In the latter two cases, the error is difficult to calculate.

Table III gives a comparison of the gamma-ray energies determined here and those reported by Ellis, Latyshev, Keller and Wolfson above the beryllium threshold of 1.67 Mev.⁶

Table III

This Report	Ellis	Latyshev	Keller ⁷	Wolfson ⁸
Nil		1.69	Nil	
1.75 Mev		1.75	1.761	
1.88	1.80	1.82		
2.02		2.09		
2.21	2.2	2.20		2.208
2.39		2.42		2.452

A very low pressure run was made to look for evidence of the 1.69 Mev line reported by Latyshev. This gamma ray should give rise to neutrons of ~ 17 kev. Figure 9 gives a histogram of the low energy protons observed. Although some uncertainty exists in the absolute value of the range energy curve for protons of this energy, it seems safe to conclude that protons corresponding to 17 kev neutrons were not observed.

6. R. C. Mobley and R. A. Laubenstein, ANL-4468.

7. H. B. Keller - Private Communication, Preliminary Results.

8. J. L. Wolfson, Phys. Rev. 78, 176 (1950).

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$\frac{N}{\Delta E}$

RECOIL PROTON SPECTRUM

LOW ENERGY REGION

Ra-Be γ_n SOURCE

FIG. 9

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VI Photographic Plate Techniques Applied to Gamma Ray Studies (Bernard Hamermesh)

The D₂O-soaked plate technique for studying neutron capture gamma rays has been applied to lanthanum and silver and the results are reported herewith.

The method of presenting the results has been changed somewhat. All of the background data obtained with the elements previously studied has been used to obtain an average background. This average background has been used to correct the data. Histograms of the spectra have then been redrawn using the average background. Finally, the data has been smoothed by taking the average of the number of tracks on two adjacent blocks of the above histograms. A point corresponding to this average number of tracks was then plotted versus the average of the energy represented by the two corresponding blocks of the histogram. The points were then joined by a straight line. The smoothed spectra for all of the previously reported elements have been published in ANL-4447.

a) The Neutron Capture Gamma Ray Spectrum of Lanthanum. Fig. 10

The spectrum has an intensity maximum at 4.6 Mev. The high energy cutoff is between 7.5 and 8.0 Mev. There is evidence of a possible second peak at about 6.0 Mev.

b) The Neutron Capture Gamma Ray Spectrum of Silver Fig. 11

The spectrum has a maximum of intensity at 5.4 Mev. There is evidence of another peak at about 6.2 Mev. The high energy cutoff is at approximately 8.0 Mev.

The results for silver should be compared with the cadmium spectrum (ANL-4277 and 4447). In cadmium nearly all of the capture gamma rays are emitted by Cd¹¹⁴ and in silver by Ag¹⁰⁸ and Ag¹¹⁰. The great difference in the types of spectra emitted by two elements that are adjacent in the periodic table but which are of even-even type and odd-odd type

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respectively is in line with the suggestion (ANL-4437) that the nuclear type plays an important role in the emission of the capture gamma radiation.

VII. Activation Cross Sections Measured with Sb-Be Photoneutrons
(Virginia Hummel and Bernard Hamermesh)

The method of measuring activation cross sections with Sb-Be neutrons, reported in ANL-4437, has been used to study various substances. The following table includes some results which were reported as preliminary ones in the previous report:

Table
Activation Cross Sections for Antimony-Beryllium
Neutrons

Isotope	Half Life of A+1 Isotope	Natural Atom Cross Section (Millibarns)	Percentage* Error
Al ²⁷	2.4 min.	1.6	24.
V ⁵¹	3.9 min.	59.	13.
Mn ⁵⁵	2.59 hrs.	78.	9.
Cu ⁶⁵	5. min.	17.	20.
Zn ⁶⁸	57. min.	6.1	17.
Rh ¹⁰³	4.2 min.	2.0×10^2	14.
Ag ¹⁰⁷	2.3 min.	1.1×10^3	17.
In ¹¹⁵	54. min.	1.0×10^3	8.
Pt ¹⁹⁸	31. min.	20.	8.
Au ¹⁹⁷	2.7 days	1.5×10^3	11.

*Assuming Seren's values of the thermal cross section are correct, Seren's values actually have an uncertainty of about 20%.

VIII. Progress Report on a High Yield Multiple Plate Fission Detector for the Crystal Spectrometer (J. D. Richards)

The fission cross section of any fissionable material can be measured with a fission chamber, provided the monoenergetic neutron flux falling on the chamber as well as the total amount of material present in the chamber is accurately known. It is important to count quantitatively the number of fissions occurring in the material. The number of fissions recorded per second is equal to the product $nv N \sigma \epsilon$ where nv is the number of neutrons/cm² sec falling on the chamber, N is the total number of fissionable atoms present in the chamber, σ is the fission cross section, and ϵ is the counter efficiency.

It has been suggested that the fission cross section of U²³³ could be measured on the Argonne crystal spectrometer if a detector of sufficient yield could be built to operate on a relatively low flux. The enriched CP-3' reactor should make available a monoenergetic flux of about 300 neutrons/cm² sec at thermal energy and 15 neutrons/cm² sec at 10 ev. Because the flux, as well as the fission cross section, is low at the higher neutron energies, it is desirable to have as much material as possible in the chamber in order to have a high counting yield. However, there is an upper limit on the allowable thickness of the material. Too thick a layer increases the energy loss that fragments undergo in passing through the fissionable material. The plateau on a bias curve would disappear because of the smooth distribution of fragment energies ranging from 0 to 110 Mev. Thus, to obtain a high counting yield without sacrificing the plateau, it is necessary to spread this large amount of material over a greater area. This, however, increases the capacity of the chamber which lowers the ratio of pulse height to noise. The problem is then to build a fission detector which is an optimum compromise of the above prerequisites.

The design finally chosen is similar in principal to the detector used by E. E. Anderson et al¹ who used a BF_3 counter sandwiched between two multiple plate fission chambers. The BF_3 section was used as a monitor to determine the average nv falling on the front and rear fission chambers.

A simplified cross section view of the proposed detector is shown in Fig. 12. This chamber will use electrodes made from aluminum foils .002" thick, 1 cm wide and $\sim 4\frac{1}{2}$ " long. These will be alternately connected by a set of solder lugs which will be in contact with the foils and held $1/8$ " apart in a clamping arrangement of polystyrene spacers and tie bolts which will run the height and length of each section, as shown in Fig. 12. The monitoring section will consist of alternately connected B^{10} coated aluminum foils which will be mounted between $1/4$ " polystyrene spacers and clamped the same as the fission sections. Both the front and rear fission sections and the central B^{10} monitor will be mounted inside a single gas tight chamber which will be filled with argon and CO_2 at atmospheric pressure. This will greatly simplify the construction of the counter and will make possible the counting of fissions and the monitoring of the beam in the same counting atmosphere.

A working model of a portion of one fission section has been built using 78.6% enriched U^{235} as the fissionable material. The uranium in the form of uranyl nitrate was dissolved in alcohol mixed with dilute zapon and painted on the center 10 cm of the foils in the manner described by Rossi and Staub.² After being painted with a single stroke of a soft brush, the foils were then baked at 550°C for three or four minutes to drive off the zapon and convert the nitrate to UO_3 . This process was repeated until .96 mg of UO_3/cm^2 or a total of 270 mg was deposited on both sides of 13 foils

1. E. E. Anderson et al, LA-158

2. B. B. Rossi, H. H. Staub, Ionization Chambers and Counters, Experimental Techniques, Nat'l. Nuclear Energy Series, 1st Edition, McGraw Hill Book Company, Inc. 1949.

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HIGH YIELD MULTIPLE PLATE FISSION DETECTOR

FIG. 12

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and one side only of the front and rear foils. The foils were mounted and alternately connected so that they formed two sets, one of which served as a collecting electrode, the other as the high voltage electrode, and were connected to the preamplifier as shown in Fig. 12.

The output of the preamplifier was fed to a B1 amplifier. This in turn triggered a scaling unit. Collecting voltage was delivered by an auxiliary HV supply. The chamber worked nearly equally well on either positive or negative high voltage on the central electrodes but seemed to give an $\sim 10\%$ higher yield with the polarity as shown. Fig. 13 is a plot of counting rate vs amplifier gain for fissions, α 's, and noise. Capacity of the test chamber is $300 \mu\text{mf}$. Fig. 13 describes the operating characteristics of one section of the fission chamber only. The proposed chamber, when completed, will consist of seven of these sections. To simulate the capacity of the completed chamber, a $.0016 \mu\text{f}$ mica condenser was tied across the chamber electrodes to see how much the additional capacity would lower the ratio of signal to noise. This effect is observed by comparing Fig. 13 and Fig. 14. In Fig. 14 there is a general shift to the right of both the fission and alpha curves while the noise level remains substantially unchanged. The increased capacity has also decreased the ratio of fission pulse height to alpha pulse height until their separation in the region of the fission plateau becomes a problem. This situation will become worse with U^{233} or with the addition of more α -emitting material because of alphas which would pile up to a size comparable to the size of a fission pulse within the resolving time of the amplifier. An obvious remedy is to shorten the resolving time of the amplifier. An alternative would be to find experimentally a more efficient electrode spacing for the greatest ratio of fission pulse height to alpha pulse height for thickness of material used.

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An investigation is now being made to determine what pressure of argon and CO₂ provides the greatest ratio of fission pulse height to alpha pulse height.

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IX. Low Voltage Accelerator Activities. (L. S. Goodman and T. R. Robillard)

The low voltage (100 kv) accelerator is in satisfactory operating condition. The $T(D,n)He^4$ reaction is producing a total source strength of about 10^7 14 Mev neutrons per second. Plans are being made to improve the Zr-T target to increase this flux.

A 5819 phototube with an anthracene crystal together with an amplifier and pulse height discriminator designed and built by Robert Swank is a relatively efficient detector. By discrimination against low amplitude pulses, the counting rate for both the direct neutrons and the room scattered neutrons is decreased. Since, however, the room scattered neutrons have lower energy, their counting rate is decreased more rapidly. In normal operation, room scattering contributes only 4% of the neutrons counted.

A similar scintillation counter, built under the supervision of Thomas Brill, is being installed to monitor the α particles from the reaction. The α particles are to be counted with good geometry by means of a natural $CaWO_4$ crystal which has been cut and polished by Les Trater of the optics shop. The crystal is inside of the vacuum system and is optically coupled to the 5819 photo tube through a thin pyrex window and mineral oil.

Preliminary measurements on the total cross section of Be for 14 Mev neutrons yield a value of $\sigma = 1.5$ b which is considerably higher than Amaldi's value of $\sigma = 0.65$ b which is reported in the compilation of neutron cross sections by Goldsmith et al¹.

Samples are in preparation for further total cross section studies for 14 Mev neutrons.

1. Goldsmith et al, Rev. Mod. Phys. Vol. 19, No. 4, p. 264, Oct. 1947.

Absolute activation cross section measurements will be made as soon as the α monitor has been checked and proven operable.

The relative activation cross section of $\text{Al}^{27}(\text{n,p})\text{Mg}^{27}$ and $\text{Al}^{27}(\text{n},\alpha)\text{Na}^{24}$ for 14 Mev neutrons have been measured by counting β particles with a Geiger counter and $\sigma_{\text{np}}/\sigma_{\text{n}\alpha}$ has been found to equal 0.85 ± 0.08 .

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SOME PARTIAL DIFFERENTIAL EQUATIONS SOLVABLE BY THE

REEVES ELECTRONIC ANALOGUE COMPUTOR

G. W. Evans II

INTRODUCTION: Essentially the Reac (Reeves Electronic Analogue Computor) is capable of performing the basic operations of addition, inversion, multiplication, and integration. The machine addition of two functions $\phi(t) + \psi(t)$ gives the negative sum, $-\left[\phi(t) + \psi(t)\right]$; by inversion the machine will change $+\phi(t)$ to $-\phi(t)$; machine integration of the function $\phi(t)$ produces the negative of the integral,

$$-\int_0^t \phi(\tau) d\tau;$$

and machine multiplication of the function $\phi(t)$ by $\psi(t)$ gives one-one hundredth of the product, $[\phi(t) \psi(t)] / 100$, where 1 volt is used to represent $t=1$. The negative of one-one hundredth of the product may also be produced by interchanging the "Hi" and "Lo" connections indicated in Figure 15. The integrators and summers are also capable of producing four and ten times the negative integral and negative sum respectively. By rather simple circuits the machine may be made to subtract, divide, and take a derivative.

The symbols used in drawing circuit diagrams for solving problems on the Reac are given in Figure 15. Figure 16 shows an example of such a circuit diagram. It is the circuit diagram for the solution of the ordinary differential equation

$$\frac{d^2 y}{dt^2} + a \frac{dy}{dt} + \frac{bt}{y^2} = 0$$

where a and b are constants.

This report serves as a preliminary investigation into the possibilities of solving partial differential equations with the Reac. It is also intended to familiarize people who either have access to or expect to have access to

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FIG. 15 REAC SYMBOLS

The diagram illustrates the analog computer simulation of a two-degree-of-freedom system. It features several key components and signal paths:

- Input and Initial Conditions:** The input is -1 VOLT with $I.C. = 0$. This signal is integrated to produce $\ddot{y}$, which is then integrated again to produce $-\dot{y}$ (I.C. = $-\dot{y}(0)$).
- Gain and Integration:** The $-\dot{y}$ signal is multiplied by 10 (X10) to produce $10\dot{y}$ (I.C. = $10\dot{y}(0)$).
- Servo Motor (C):** The $10\dot{y}$ signal is fed into a servo motor (C), which drives a load L_o and a spring H_i to produce the output y^2 .
- Recording:** The output y^2 is recorded on a recorder, labeled "TO RECORDER [RECORDING $10\dot{y}(t)$]".
- Feedback and Summing:** The output y^2 is fed back through a summing junction (a) to produce $+\dot{y}$ and $+\ddot{y}$.
- Internal Simulation Blocks:**
 - Servo Motor (A):** Drives a load L_o and a spring H_i to produce $100\dot{y}^2$.
 - Servo Motor (B):** Drives a load L_o and a spring H_i to produce $100\dot{y}^2$.
 - Gain and Integration:** The $100\dot{y}^2$ signal is multiplied by 10 (X10) and then by 4 (X4) to produce $400\dot{y}^2$.
 - Summing Junction:** The $400\dot{y}^2$ signal is summed with the $10\dot{y}$ signal to produce the final output y^2 .

- FIG.16 REAC CIRCUIT DIAGRAM FOR $y=-(ay+bt/y^2)$**

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or expect to have access to a Reac with the possibilities of such a machine.

SOLUTIONS OF FIRST ORDER PARTIAL DIFFERENTIAL EQUATIONS: The Reac is especially well suited to solving systems of ordinary differential equations. This immediately suggests the characteristic method⁽¹⁾ of solving partial differential equations of first order in one dependent variable, z , and n independent variables x_i . By setting

$$\frac{\partial z}{\partial x_1} = p_1, \frac{\partial z}{\partial x_2} = p_2, \dots, \frac{\partial z}{\partial x_n} = p_n$$

we may represent such a partial differential equation by

$$f(x_1, x_2, \dots, x_n, z, p_1, p_2, \dots, p_n) = 0. \quad (1)$$

The characteristic strips of equation (1) satisfy the $2n+1$ ordinary differential equations

$$\begin{aligned} \frac{dx_i}{dt} &= f_{p_i} \\ \frac{dz}{dt} &= \sum_{i=1}^n p_i f_{p_i} \end{aligned} \quad \frac{dp_i}{dt} = -f_{x_i} - p_i f_z. \quad (2)$$

where t is a parameter. The system of ordinary equations (2) is applicable to the Reac, and any solution of this system,

$$x_i = x_i(t), \quad z = z(t), \quad p_i = p_i(t), \quad (3)$$

based on the initial condition

$$x_{i0}, \quad z_0, \quad p_{i0}, \quad (4)$$

where (4) satisfies equation (1), is a characteristic strip.

If the desired solution is to pass through a given curve C ,

$$x_i = x_i(\gamma), \quad z = z(\gamma) \quad (5)$$

(not a characteristic curve) then one chooses values of x_{i0} , z_0 satisfying (5) for a given value of γ . The p_{i0} must then satisfy both (5) and (1). By varying γ one gets a series of characteristic strips which are tangent to a particular solution of equation (1). The limit of increasing the

(1) L.R. Ford, "Differential Equations", pp 205-231, McGraw-Hill Book Co., Inc., New York, 1933.

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number of strips ad infinitum while decreasing their widths to zero is, of course, that particular solution.

The two factors which determine the possibility of solving the system of differential equations (2) are:

- a) that the system (2) satisfies a proper Lipschitz condition, and
- b) that there are enough components of the Reac to set up the equations.

If we reduce the problem to two independent variables, $n=2$, then equation (1) becomes

$$f(x, y, z, p, q) = 0 \quad (1')$$

where $x=x_1$, $y=x_2$, $p=p_1$, and $q=p_2$. The system of ordinary equations for the characteristic strips becomes

$$\begin{aligned} \frac{dx}{dt} &= f_p \\ \frac{dy}{dt} &= f_q \\ \frac{dz}{dt} &= p f_p + q f_q \\ \frac{dp}{dt} &= -(f_x + p f_z) \\ \frac{dq}{dt} &= -(f_y + q f_z) \end{aligned} \quad (2')$$

The curve C would then be

$$x = x(\tau), \quad y = y(\tau), \quad z = z(\tau), \quad (5')$$

and p and q must satisfy along C

$$\frac{dz}{d\tau} = p(\tau) \frac{dx}{d\tau} + q(\tau) \frac{dy}{d\tau} \quad (6)$$

and

$$f[x(\tau), y(\tau), z(\tau), p(\tau), q(\tau)] = 0. \quad (7)$$

Example: Find the integral surface of

$$z = px + qy + pq$$

passing through the parabola

$$x = \tau, \quad y = \tau, \quad z = \tau^2$$

The equations of (2') become

$$\begin{aligned}\frac{dx}{dt} &= x + q, \quad \frac{dy}{dt} = y + p, \\ \frac{dz}{dt} &= px + qy + zpq, \\ \frac{dp}{dt} &= 0, \quad \text{and} \quad \frac{dq}{dt} = 0.\end{aligned}$$

Now, by choosing $T=1$, then $x_0=1$, $y_0=1$, $z_0=1$; and p_0 and q_0 must satisfy

$$z = p_0 + q_0$$

and

$$1 = p_0 + q_0 + p_0 q_0.$$

A possible choice is $p_0 = 1 + \sqrt{2} \approx 2.414$ and $q_0 = 1 - \sqrt{2} \approx -0.414$. The React circuit is shown in Figure 17, and the solution of the characteristic curve through the point $(1,1,1)$ is shown in Figure 18. The desired characteristic strip for the curve of Figure 18 is the one for which $p=2.414$ and $q=-0.414$ for every point of the curve.

The computing machine's solution may be checked in this case, in the following way. By inspection, we know a complete integral to be

$$z = ax + by + ab \quad (8)$$

(note that $dq/dt=0$ and $dp/dt=0$ give $p=a$ and $q=b$). Now, p and q must be chosen so that they satisfy equation (8) along the initial condition curve, i.e.,

$$T^2 = aT + bT + ab. \quad (9)$$

If we write equation (8) as

$$F(x, y, z, a, b) = ax + by + ab - z = 0,$$

then p and q must also satisfy

$$\begin{aligned}F_x[x(\tau), y(\tau), z(\tau), a, b] x'(\tau) + F_y[x(\tau), y(\tau), z(\tau), a, b] y'(\tau) \\ + F_z[x(\tau), y(\tau), z(\tau), a, b] z'(\tau) = 0\end{aligned}$$

or

$$a + b - z = 0. \quad (10)$$

By eliminating T from (9) and (10) (solutions good for any T), one obtains the following relation between a and b (or p and q)

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FIG.17 REAC CIRCUIT FOR THE SYSTEM OF EQUATIONS (2')

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FIG.18 A CHARACTERISTIC CURVE OF
 $z = px + gy + pg$

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$$b = \phi(a) = a(\pm 2\sqrt{z} - 3). \quad (11)$$

Inserting (11) into (8) gives a one-parameter family of surfaces

$$F[x, y, z, a, \phi(a)] = 0. \quad (12)$$

whose envelope is found by eliminating a from (12) and

$$F_a[x, y, z, a, \phi(a)] = 0.$$

This envelope is either of the following

$$z = \frac{[x - y(3 \pm 2\sqrt{z})]^2}{4(3 \pm 2\sqrt{z})}. \quad (13)$$

If the negative sign of equation (13) is used and equation (13) is solved for x , the result is

$$x = \sqrt{.688z} + .172y. \quad (14)$$

This form of the equation minimizes errors in reading the values of x , y , and z from the curves of Figure 18. If the values of y and z are taken from the Reac solution and substituted into equation (14), a value for x is obtained which may be compared with the value from the Reac. The following table shows such a comparison for the four values indicated in Figure 18. x_r is the value taken from the $x(t)$ curve of Figure 18 and x_s is that computed by equation (14).

	y	z	x_s	x_r
(1)	21.1	2.0	4.8	4.6
(2)	28.2	2.5	6.3	5.9
(3)	37.1	2.8	7.8	7.4
(4)	47.3	3.3	9.6	9.5

Finally, it should be pointed out that should equation (1) be linear, i.e.,

$$P_1 p_1 + P_2 p_2 + \dots + P_n p_n = R, \quad (15)$$

where $P_1, P_2, \dots, P_n, R$ are functions of $x_1, x_2, \dots, x_n, z$, one needs only the first $n+1$ equations of (2) to define the characteristic curves. This

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advantage is greatly appreciated in setting up the computing machine.

SOLUTION OF THE HEAT EQUATION $\alpha u_{xx} = u_t$ AND THE DIFFUSION EQUATION

$\alpha u_{xx} + \beta u = u_t$: The solutions of these two equations are carried out by the Schmidt⁽²⁾ numerical or graphical process using the Reac for a convenient graphing process. This requires two "Input-output" tables with a slight modification in one. These tables allow a solution of the Reac to be graphed or a function otherwise not expressible by the Reac to be inserted into the machine.

The numerical process involved in the heat equation where u represents the temperature, x the space coordinate, and t the time, is hueristically derived in the following manner. One writes the equation

$$\alpha u_{xx} = u_t, \quad (16)$$

where $u = u(x, t)$, $u_t = \partial u(x, t) / \partial t$, and $u_{xx} = \partial^2 u(x, t) / \partial x^2$,

as

$$\alpha \left\{ \lim_{\Delta x \rightarrow 0} \left[\frac{u(x + \Delta x, t) - u(x, t)}{\Delta x^2} - \frac{u(x, t) - u(x - \Delta x, t)}{\Delta x^2} \right] \right\} = \lim_{\Delta t \rightarrow 0} \left[\frac{u(x, t + \Delta t) - u(x, t)}{\Delta t} \right] \quad (17)$$

Now, if Δx and Δt are taken small enough so that equation (17) is considered to hold without the limits, one obtains

$$u(x + \Delta x, t) - 2u(x, t) + u(x - \Delta x, t) = \frac{(\Delta x)^2}{\alpha \Delta t} [u(x, t + \Delta t) - u(x, t)] \quad (18)$$

To simplify the numerical solution of equation (18), choose a Δt and determine the Δx so that

$$\Delta x = \sqrt{2\alpha \Delta t} \quad (19)$$

Equation (18) then reduces to

$$u(x, t + \Delta t) = \frac{u(x + \Delta x, t) + u(x - \Delta x, t)}{2} \quad (20)$$

Equation (20) states that if the temperature distribution is given at any specified time, t , it may be determined at a time Δt later. Continuing

(2) W.H. McAdams, "Heat Transmission", pp. 39-43, McGraw-Hill Book Co., Inc., New York, 1942.

this process, temperature distributions may be determined for times $n\Delta t$ later, where $n=1, 2, \dots$.

A similar treatment for the equation

$$\alpha u_{xx} + \beta u = u_t \quad (21)$$

may be performed giving

$$u(x+\Delta x, t) + u(x-\Delta x, t) + \left[\frac{\beta(\Delta x)^2}{\alpha} - 2 \right] u(x, t) = \frac{(\Delta x)^2}{\alpha \Delta t} [u(x, t+\Delta t) - u(x, t)]. \quad (22)$$

In making numerical calculations from equation (22), one chooses a Δt and determines Δx so that

$$\Delta x = \sqrt{\frac{2\alpha}{\frac{1}{\Delta t} + \beta}}, \quad (23)$$

and the resulting formula is

$$u(x, t+\Delta t) = \frac{u(x+\Delta x, t) + u(x-\Delta x, t)}{2 - \frac{\beta(\Delta x)^2}{\alpha}} \quad (24)$$

In equations (19) and (23) α is considered positive. In equation (23) if $\beta < 0$, then Δt must be chosen such that

$$\frac{1}{\Delta t} + \beta > 0. \quad (25)$$

It is easily seen that the solutions of equations (16) and (21) may be carried out in regions which contain subregions with different values of α , but in which α is a constant in each subregion. This is done by changing the value of Δx as one enters a new region. In equation (21) β may also change from subregion to subregion.

The numerical calculations may be carried out on the Reac with the use of two "Input-output" tables where one is used as an output table and the other as an input table. The input table must be modified by adding a resistor pickup arm designed by the Rand Corporation⁽³⁾ with a wire graph on the drum and by maintaining the manual graph-following device of the table. The position of the resistor pickup arm must be adjustable so that

(3) W. F. Gunning and A. S. Mengel, "Summary of Reac Experience, RM-236", pp. 19-24, Santa Monica, Calif. 8 Sept. 1949.

it may be displaced a distance $2 \Delta x$ from the manual following device. On the input table is placed the wire graph of $u(x,t)$ while the output table is used to record the curve $u(x,t + \Delta t)$.

Using the notation shown in Figure 15, the schematic diagram for equations (20) and (24) are shown in Figure 19. The wire curve $u(x,t)$ is placed on the input drum and the Rand resistor is placed a distance $2 \Delta x$ from the manual graph following device. The potentiometer A is regulated to drive the drums of the input and output tables at an angular speed convenient for the manual following device of the input table. The potentiometer B is set at

$$0.5 \quad \text{for equation (20)}$$

and

$$\frac{1}{\frac{\beta(\Delta x)^2}{\alpha} - 2} \quad \text{for equation (24).}$$

The y-scale of the "Input-output" tables must be chosen so that $u(x,t)$ will remain between ± 100 volts, whereas the x-scale should utilize the full rotation of the drums. The initial condition on the integrator is used to align the drums with the point $(0, t_0)$ of the graph.

The initial and boundary values best applicable to these problems when using the Reac are

$$u(x, t_0) = U(x), \quad U(0) = u(0, t), \quad u(L, t) = U(L),$$

where $t \geq t_0$ and $L > 0$.

It should be pointed out, if it is not already obvious, that the use of the Reac provides a superior method of solution over the numerical and graphical methods in that it scans continuously in the x-direction.

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FIG. 19 SCHEMATIC DIAGRAM FOR EQUATIONS (20) AND (24)

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ENERGY LEVELS IN A NUCLEUS OF RECTANGULAR WELL SHAPE WITH
FINITE DEPTH, 50 NEUTRONS IN NUCLEUS

A. Radkowsky

The problem discussed here is to find the unoccupied neutron energy levels in a nucleus containing 50 neutrons, assuming that the neutron potential is a rectangular well shape of finite depth.

In the case of a nuclear potential well of infinite depth, the nucleon energy levels¹ are given by the roots of

$$J_{l+\frac{1}{2}}(k r_0) = 0 \quad (1)$$

where r_0 is the radius of the hole⁴ $\cong 1.7 \times 10^{-13} A^{1/3} = 7.9 \times 10^{-13} \text{ cm}$ for the present case of $A \sim 100$.

l is the azimuthal quantum number, an integer
 and $k = \frac{2}{\hbar^2} ME$, M being the mass of the nucleon and E the energy of the nucleon in the state l , - energies being measured from the bottom of the well.

Where there are several solutions of E for a single l , the corresponding levels are designated, analogously to atomic electrons, both by a small letter denoting the angular momentum and a number prefix corresponding to the 'radial quantum number'.

Now consider a well having a finite depth B

Let
$$K^2 = \frac{2}{\hbar^2} M(B-E)$$

The energy levels corresponding to azimuthal quantum numbers, l , are now given by those values of E satisfying the equation:

$$\frac{J_{l+3/2}(k r_0)}{J_{l-1/2}(k r_0)} = \frac{H_{l+3/2}(i K r_0)}{H_{l-1/2}(i K r_0)} \quad (2)$$

1. H. Margenau, Phys. Rev. 46, 613 (1934)

4. Motz and Feenberg, Phys. Rev. 54, 1055 (1938)

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Writing the right hand side of (2) as $-R_l(x)$, one finds² that R can be expressed as a rational fraction in powers of x , which greatly simplifies the computations.

Put $k r_0 \equiv Z = \sqrt{\frac{2}{\hbar^2} M E} r_0$

and $l \equiv \sqrt{\frac{2}{\hbar^2} M B} r_0$

(2) becomes

$$\frac{J_{l+\frac{1}{2}}(Z)}{J_{l-\frac{1}{2}}(Z)} = R_l(\sqrt{l^2 - Z^2}) \quad (3)$$

It will be noted that the energy of a state, l , depends upon the depth of well B . A rough idea of the minimum well depth can be obtained from computing the minimum depth necessary to bind all the particles in the largest known nuclei. This yields a minimum depth of approximately 21 Mev.

A better procedure is based upon use of the neutron binding energy which is known to be approximately 8 Mev for nuclei containing 100 nucleons. The well depth can then be determined by requiring that the energy difference between the highest occupied state and the well depth be ~ 18 Mev.

In the case of the well of infinite depth, the order of levels (in ascending values of energy) and the number of particles, ν , contained in each level (according to the formula $2(l+1)$) is:

level	1s	2p	3d	2s	4f	3p	5g	4d	6h	3s
l	0	1	2	0	3	1	4	2	5	0
ν	2	6	10	2	14	6	18	10	22	2
ϵ (Mev)	3.3	6.7	11	13	16.1	19.7	22.2	27.3	29.0	29.6

ϵ is the energy of the level measured from the bottom of the well.

Thus, the highest occupied level in the case of 50 neutrons would be the 5 g .

In the case of a finite well depth, the maximum energy a bound level

2. Jahnke and Emde, Tables of Functions, 4th edition, ppl36 & 137

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can have is, of course, limited by the well depth. Thus, if in equation (3), $Z \approx l$ the right hand side of the equation becomes $= -\infty$ so the denominator of the left hand side must vanish:

$$J_{l-\frac{1}{2}}(Z) = 0, \quad Z \approx kr_0$$

Thus, by comparison with (1) it is seen that the maximum energy of the highest bound level corresponding to a particular l is the same as that for a state with $l' = l - 1$ in the case of the infinite well depth. The lower bound states for the finite well are somewhat raised in energy as compared with the corresponding state, $l - 1$, of the infinite case but the order of the energy levels remains the same up to at least the 5g level as may be readily seen by inspection of the curves of fractional Bessel functions².

In the problem at hand, then, the procedure is first to find the energy of the 5g level, assuming it were at the top of the well. It would then have the same energy as the 4f level for the infinite well or 16.1 Mev. Adding 8 Mev to this value will give us a first, but too low, approximation to the well depth since, as previously mentioned the 5g level will not be depressed as much (in comparison with the corresponding infinite well 5g level) when it is not the highest bound level. By numerical solution of equation (3) the well depth is adjusted until the 5g level is approximately 8 Mev below the top of the well. Having fixed the well depth, equation (3) can then be used to find the levels of the other bound states. The results are:

Well depth = 25.5 Mev

State	Energy in Mev	
6h	22.6	Unoccupied states
3s	22.0	
4d	21.0	
5g	17.4	Highest occupied state

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Two interesting points are brought out from this study:

- (1) The small number of empty levels in this model.
- (2) The shift in relative energy of the $6h$ and $3s$ level. In the infinite well the $3s$ level has the higher energy while here the $6h$ is higher. In the oscillator potential case³ the $6h$ level is the higher. This shows that, as would be expected, the finite well model is closer to the oscillator potential than to the infinite well potential.

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SOLUTION OF TWO-GROUP DIFFUSION EQUATIONS ON

I.B.M. MACHINES

H. H. Hummel

In the previous Quarterly Report, the solution of single and coupled diffusion equations by transforming them to difference equations was discussed¹. In the following the results obtained for several two-group pile problems on I.B.M. machines will be described.

The equations being solved are, for a multiplying region,

$$\nabla^2 \phi - \frac{\phi}{\tau} + \frac{\eta \sum_a^{25}}{D_f v_f} \psi = \frac{1}{D_f v_f} \frac{\partial \phi}{\partial t}$$

$$\nabla^2 \psi - \frac{\psi}{L^2} + \frac{D_f}{D_s} \frac{\phi}{\tau} = \frac{1}{D_s v_s} \frac{\partial \psi}{\partial t}$$

For a non-multiplying region the term in $\sum_a^{25}$ is, of course, absent.

ϕ = fast flux

ψ = slow flux

τ = Fermi age

L = thermal diffusion length

D_s = slow diffusion coefficient

D_f = fast diffusion coefficient

v_f = fast neutron velocity

v_s = slow neutron velocity

$\sum_a^{25}$ σ_a^{235} thermal absorption cross section, macroscopic

Since the interest here is only in the critical solution, where

$$\frac{\partial \phi}{\partial t} = \frac{\partial \psi}{\partial t} = 0, \text{ the equations are simplified by defining } v_f = \frac{1}{D_f}$$

and $v_s = \frac{1}{D_s}$. For a two-dimensional problem in the x, y plane, with a Laplacian ∇^2 for the z direction, the equations will be put into

1. Numerical Determination of Fundamental Modes-Flanders & Shortley in ANL-4437

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difference form using a square network of points with spacing h . Assuming time dependence e^{at} for ϕ and ψ , and defining $\lambda = 1 + \frac{a h^2}{4}$, the resulting equations are

$$\frac{\sum \phi_{x,y}}{4} - \frac{h^2}{4} \left(\frac{1}{\tau} + \kappa_z^2 \right) \phi_{x,y} + \frac{\eta \sum_{a=25} h^2}{4 D_f} \psi_{x,y} = \lambda \phi_{x,y}$$

$$\frac{\sum \psi_{x,y}}{4} - \frac{h^2}{4} \left(\frac{1}{\tau^2} + \kappa_z^2 \right) \psi_{x,y} + \frac{D_f h^2}{4 D_s \tau} \phi_{x,y} = \lambda \psi_{x,y}$$

$\phi_{x,y}, \psi_{x,y}$ are flux values at point (x, y)
 $\sum \phi_{x,y}, \sum \psi_{x,y}$ are the sum of flux values for points $(x, y-1), (x, y+1), (x-1, y), (x+1, y)$.

The solution of these equations is carried out as described in the previous report. One starts with a set of flux values $\phi_{x,y}$ and $\psi_{x,y}$, applies to them the operations indicated on the left-hand side of the above equations simultaneously for all points of the network, and obtains a new set of flux values which would be for all points the same constant λ times the old values if the starting function were a solution.

For a three dimensional problem with spacing l in the z direction, the κ_z^2 term is replaced by one involving l and the z neighbors.

At an interface between different regions of the pile, the regular diffusion boundary conditions are applied. In difference form, if the boundary runs in the x direction, y is the boundary point, $y-1$ the neighboring point in region 1, and $y+1$ the neighboring point in region 2,

$$D_1(\psi_{y-1} - \psi_y) = D_2(\psi_y - \psi_{y+1})$$

where ψ is either fast or slow flux. In the machine calculation the new value of ψ_y is obtained as

$$\frac{\frac{D_1}{D_2} \psi_{y-1} + \psi_{y+1}}{1 + \frac{D_1}{D_2}}$$

where ψ_{y-1} and ψ_{y+1} are the old values. This procedure is acceptable when $\lambda \approx 1$.

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This process of obtaining a new set of flux values from an old set at all points of the network constitutes an iteration. By performing a series of such iterations and, during the process, taking proper linear combinations of flux functions thus obtained as a starting point for further iterations, the desired solution can be reached.¹

The method of finding the critical solution, for which $\lambda = 1$, is to assume a U^{235} concentration and carry the solution through until a satisfactory result is obtained, corresponding to some value of λ . If $\lambda > 1$ the concentration is lowered and the process repeated, until $\lambda = 1$ is bracketed and the critical concentration obtained by interpolation.

In the solution on I.B.M. machines, decks of cards are made up with one card assigned to each point of the network. The calculations required in performing an iteration are carried out by a sequence of reproducing, sorting, and calculating operations involving these decks, with two decks required per iteration.

The time required per iteration varies greatly depending on the number of points in the network. For a two dimensional problem of 200 or 300 points the actual machine and card handling time would only be about ten or fifteen minutes, while for a three dimensional problem of several thousand points the better part of a day would be required.

Unfortunately, the time required to work a problem cannot be calculated by multiplying the number of iterations by the theoretical time per iteration, because delays because of machine errors, human errors, and non-availability of machines will lengthen the average time required by a factor of at least two or three.

The usual procedure in solution of a problem is to start with an estimated flux distribution and first obtain an approximate solution in a coarse network, for which the iterations are faster and fewer are required. The

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flux and critical mass thus obtained are then used as starting values for a more accurate solution in a finer network.

Early in the year a three-dimensional calculation was started involving the one-gang Naval Reactor. A coarse network was used consisting of 540 points, the intention being to take the results of this calculation as the starting point for a fine mesh calculation involving about eight times as many points. The work was stopped after the coarse mesh calculation was completed because more important problems arose.

The next problem undertaken concerned the two-gang Naval Reactor. A two-dimensional geometry was used (Figure 20), representing a horizontal section of the pile with all rods withdrawn, leaving water holes, corresponding to maximum xenon. A leakage term was put in for the axial direction equivalent to a bare height of 122.5 cm.

The purpose of the calculation was to find the effect of the water holes on the slow flux and on reactivity of the pile.

Two mesh sizes were employed, as indicated in Figure 20, with 3.30 cm. and 316 points, and with $h = 1.65$ cm. and 1264 points. Other constants were

Core

$$D_f = 1.178 \text{ cm.}$$

$$\tau = 85.05 \text{ cm}^2$$

$$D_s = 0.3948 \text{ cm.}$$

$$\sum_a \text{ for moderator and poison } 0.02085 \text{ cm}^{-1}$$

$$\sum_a^{25} 0.03635 \text{ cm}^{-1}$$

$$\text{and } 0.02908 \text{ cm}^{-1}$$

Reflector

$$D_f = 1.391 \text{ cm.}$$

$$\tau = 48.22 \text{ cm}^2$$

$$D_s = 0.2329 \text{ cm.}$$

$$L^2 = 18.96 \text{ cm}^2$$

The value $\sum_a^{25} 0.03635 \text{ cm}^{-1}$ corresponded to the critical mass obtained by the Naval Reactor Division for a homogeneous core using the same constants

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FIG. 20
TWO GANG NAVAL REACTOR
HORIZONTAL SECTION WITH RODS WITHDRAWN
(ONE FOURTH OF PILE)

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as those above except for $\sum_a^{25}$, which was assumed spread over the whole core. On the basis of a cell calculation involving a water hole they estimated that the critical mass with water holes would be about 5% higher than in the homogeneous case.

The machine calculations in the coarse mesh indicated a critical mass about 20 to 25% below that for the homogeneous case. In the fine mesh, however, the indication seems to be that the critical mass will be about the same as for the homogeneous case. Work on determining the critical concentration has not been quite completed. Contour lines for the slow flux are plotted in Figure 20.

About 30 iterations were required in the coarse mesh and 24 in the fine mesh. About six weeks were required to do this.

Another problem solved for the Naval Reactor Division has been a cell calculation for a cross shaped control rod (Figure 21). This was to be compared by the Naval Reactor Division with an analytical cell calculation for a round rod, in order to see how big a round rod would be equivalent to the cross rod.

The constants used were as follows:

$$\sum_a \text{ for moderator + poison} = 0.0181$$

$$\tau = 85.05$$

$$D_s = 0.3957$$

Equivalent bare height 122.5 cm.

$$D_f = 1.178$$

$$h = 1 \text{ cm. Rod 8 cm. across}$$

$\sum_a^{25}$ at critical was found to be 0.0276 cm^{-1} . The round rod calculation has not been completed.

Contour lines for fast and slow flux are shown in Figure 21. These are based on an early iteration, which explains why the flux is not completely symmetrical about the diagonal symmetry line.

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CELL CALCULATION FOR CROSS SHAPED CONTROL ROD
WITH FLUX CONTOURS (ONE FOURTH OF CELL)

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Experience with this method of solving pile problems suggests that it is most practicable where the number of points in the network, and, therefore, the number of cards in a deck, is not larger than of the order of a thousand. A three-dimensional calculation with a mesh spacing comparable to that in the fine mesh water holes calculation would require ten to twenty thousand cards per deck, depending on the detail required in the axial direction. Such a calculation would take many months, and the card handling problems would be formidable. The conclusion is that the I.B.M. machines can be used most effectively for two-dimensional problems or for calculations on small portions of three-dimensional piles.

REF ID: A55710

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TEMPERATURE DEPENDENCE OF THERMAL TRANSPORT MEAN FREE PATH

A. Radkowsky

A. INTRODUCTION: In view of recent discussions concerning the necessity of measuring the transport mean free path, λ_T , of neutrons in water at high temperatures and pressures, such as those to be encountered in the Navy pile, the writer has examined the questions of how accurately λ_T must be known in pile calculations and whether λ_T cannot be calculated sufficiently accurate from existing data, thus avoiding the necessity for difficult and costly experiments.

B. CONCLUSIONS:

- (1) In order to obtain a 5% accuracy in calculations of $(k-1)$ and of the flux it is necessary to know the transport m.f.p. and thermal diffusion length within about the same accuracy. The latter quantities can be calculated from existing data to well within this accuracy provided that the neutron energy distribution (e.g. Maxwellian) is known. Calculations are included herein of the diffusion length of light water as a function of temperature and the diffusion constant of a homogeneous mixture of zirconium and water in equal volumes at various temperatures.
- (2) The phenomenon of neutron "hardening", i.e. preferential absorption of slower neutrons, leads to deviations from Maxwellian distributions which may cause serious errors in calculations of diffusion constants, not taking this effect into account. An example is given herein. The calculation of the true energy distribution of thermal neutrons in the presence of an absorbing medium is now under study.

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C. ACCURACY WHICH IS NEEDED IN KNOWLEDGE OF THERMAL TRANSPORT M.F.P. AND DIFFUSION LENGTHS

The two most important quantities calculated in pile physics are usually considered to be the reproduction factor, k , and the thermal flux distribution. According to members of the Naval Reactor Group it would be highly desirable to keep to within 5% the errors in $(k-1)$ and in the value of the average thermal flux and in the location of flux maxima. The accuracy of these quantities as a function of the thermal diffusion length and transport m.f.p. will now be examined.

(1) Accuracy of $(k-1)$

In a pile with reflector the critical equation for determination of k is obtained by satisfying continuity conditions at the boundary of the core and reflector. For one-group theory from equation 13.9 in AECD-2201 (Elementary Pile Theory-Soodak and Campbell) with a slight change of notation,

$$\lambda_{\tau_0} K_0 \tanh K_0 a = \lambda_{\tau_1} K_1 \coth K_1 \tau \quad (1)$$

Here the subscript, 0, refers to the core and the subscript, 1, to the reflector.

$$K_0^2 \approx \frac{k-1}{M^2} \quad (2)$$

where M^2 is the migration area, a is the radius of the core; τ , the thickness of the reflector

$$K_1^2 = \frac{\Sigma_{a_1}}{\lambda_{\tau_1}} = \frac{1}{L_1^2} \quad (3)$$

where Σ_{a_1} is the macroscopic thermal absorption of the reflector; λ_{τ_1} is the thermal transport m.f.p. of the reflector; and L_1 is the thermal diffusion length of the reflector. λ_{τ_0} is the transport m.f.p. in the core.

In the Naval reactor the reflector is of water and is so thick that

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$\coth \kappa_1 \tau \sim 1$, since τ is 20 cm. (8 inches) and even at the highest temperatures $L_1 = \frac{1}{\kappa_1}$ does not exceed 4.

From (1), $\kappa_0 a \sim \tan^{-1} \left(\frac{\lambda_{\tau_1}}{\lambda_{\tau_0}} \frac{\kappa_1}{\kappa_0} \right)$ and from (2) one obtains:

$$(\kappa - 1)^{\frac{1}{2}} \sim \frac{M}{a} \tan^{-1} \left(\frac{\lambda_{\tau_1}}{\lambda_{\tau_0}} \frac{\kappa_1}{\kappa_0} \right) = \frac{M}{a} \tan^{-1} \left(\frac{\lambda_{\tau_1}}{\lambda_{\tau_0}} \frac{M}{L_1 (\kappa - 1)^{\frac{1}{2}}} \right)^{(4)}$$

Since the core of the Naval reactor is composed approximately half of water and half of zirconium and since the scattering of zirconium is small compared with that of water, the ratio of $\frac{\lambda_{\tau_1}}{\lambda_{\tau_0}}$ will be approximately independent of the errors in computation of each quantity.

M^2 , the migration area will be little affected by uncertainties in L_0^2 , for $M^2 = \tau$, the age, $+ L_0^2$, the square of the thermal diffusion length in the core. Typical values at operating temperatures are:

$$\begin{aligned} \tau &\approx 80 \\ L_0^2 &\approx 5 \end{aligned}$$

Since L_0^2 is so small compared with τ , it would require an error of 80% in L_0^2 to change $(\kappa - 1)$ by 5%.

From (3) and (4) one sees that a variation in κ_1 , due to an error in λ_{τ_1} , the transport m.f.p. in the reflector will require a corresponding change in $(\kappa - 1)$ since $(\kappa - 1)$ appears in κ_0 . However, a 10% change in λ_{τ_1} will only cause a 5% change in κ_1 . The change in $(\kappa - 1)$ will be much less since $\frac{\lambda_{\tau_1}}{\lambda_{\tau_0}} \frac{\kappa_1}{\kappa_0} \sim 3$, and $\tan^{-1}(x)$ has a very small derivative when $x \approx 3$. Thus, one concludes that for 5% accuracy of $(\kappa - 1)$ it would be sufficient to know λ_{τ} and L within about 10%.

(2) Accuracy of Average Thermal Flux

The average thermal flux may also be estimated from one-group theory.

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For a spherical pile ϕ_0 (in core) = $\frac{\sin \kappa_0 r}{\kappa_0 r}$, taking the value of the flux as unity at $r=0$.

$$\text{average } \phi \equiv \bar{\phi}_0 = \frac{\int_0^a 4\pi \frac{\sin \kappa_0 r}{\kappa_0 r} r^2 dr}{\int_0^a 4\pi r^2 dr} = \frac{3}{(\kappa_0 a)^2} \left[\frac{\sin \kappa_0 a}{\kappa_0 a} - \cos \kappa_0 a \right] \approx -3 \cos \kappa_0 a$$

since in most cases $\kappa_0 a$ will be quite close to π .

$$\frac{d\bar{\phi}_0}{d\kappa_0} = a \left(\frac{3}{B^2} \left[\frac{\cos \beta}{\beta} - \frac{\sin \beta}{\beta^2} + \sin \beta \right] - \frac{6}{B^3} \left(\frac{\sin \beta}{\beta} - \cos \beta \right) \right),$$

$$\beta \equiv \kappa_0 a$$

$$\approx \frac{a 9 \cos \kappa_0 a}{(\kappa_0 a)^3}$$

thus $\left| \frac{d\bar{\phi}_0}{\bar{\phi}_0} \right| \approx \frac{3 d\kappa_0}{\kappa_0}$, or the fractional error in the average flux is three times that of κ_0 . Since κ_0 in this case is very little affected by errors in L_0 , $\bar{\phi}_0$, according to one-group theory, is also little affected by errors in L_0 . This conclusion is changed by the more accurate two-group theory as indicated below.

(3) Two-group Discussion

In the case of two-group theory one finds little change in the situation as far as $(k-1)$ is concerned.

In the case of the thermal flux one finds the typical second rise in the flux near the edge of the pile with a peak in the reflector.

This second rise aids in improving the flux distribution. It is necessary to know the magnitude of the second peak and its location.

The situation is somewhat difficult to discuss quantitatively because of the complexity of the two-group equations. Qualitatively, the second peak in the thermal flux is due to thermalization of

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fast neutrons entering the reflector. Thus the peak would be determined by the distance the fast neutrons must travel in slowing down plus a thermal diffusion length. The distance the fast neutrons must travel is much less than would be given by the square root of the age in the core since the neutrons reaching the core boundary would usually be already considerably moderated. Also, the age in the reflector is much less than in the core and the thermal diffusion length much greater. This all leads to the conclusion that the location and magnitude of the flux peak in the reflector will be strongly influenced by the thermal diffusion length which should therefore be known to approximately the same degree of accuracy (5%) as it is desired to calculate the former quantities.

D. METHOD OF CALCULATION OF DIFFUSION LENGTH AND TRANSPORT MEAN FREE PATH FROM EXPERIMENTAL DATA ON SCATTERING CROSS SECTION OF WATER

Since the experimental data on the scattering cross section of water for various neutron energies was obtained at room temperature, one first inquires as to whether there would be any reason to expect that the intrinsic scattering properties of water would change with temperature. The answer is negative for the following reasons:

- (1) Only a small part of the scattering cross section of water is coherent, approximately 2 barns whereas the cross section at room temperature for .025 ev neutrons is about 50 barns. Thus, diffraction effects from neighboring atoms and molecules will be inappreciable.
- (2) The rotational energy levels of the water molecule are spaced so closely that, even at room temperature, the molecule is practically rotating freely. Thus, (H. Sponer, Molekulspektren, p. 73) gives the lowest moment of inertia of water as $\sim 1 \times 10^{-40}$. The first

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rotational energy level is only $\frac{\hbar^2}{I} \sim .006$ ev. On the other hand, the lowest vibrational level is 1600 cm^{-1} or about .2 ev. Not even at the highest pile temperature contemplated will any of the vibrational levels be excited.

One concludes that the scattering cross section is a function only of the neutron energy. Let it first be assumed that neutrons in equilibrium with a moderator have the same temperature and temperature distribution (Maxwellian) as the moderator. Since,

$$L^2 = \frac{\lambda_t}{3 \Sigma_a} = \frac{1}{\Sigma_s (1 - \overline{\cos \theta}) 3 \Sigma_a}, \text{ where } (5)$$

L = diffusion length

λ_t = transport mean free path

Σ_a = macroscopic absorption cross section of water.

Σ_s = macroscopic scattering cross section of water.

$\overline{\cos \theta}$ = average cosine of angle through which neutron is scattered in the laboratory system.

Σ_s is well known as a function of neutron energy (e.g. see E. Melkonian, AECD-2675).

Σ_a is also known since σ_a equals about .325 b. at .025 ev and is usually assumed to have a $\frac{1}{v}$ variation.

The difficulty in calculating L or λ_t as a function of neutron energy is then caused by the lack of knowledge of $\overline{\cos \theta}$ as a function of neutron temperature.

L has been measured in water at various temperatures up to 100°C by Wilson, Bragdon, and Kanner (CP-2306) and thus, from equation (5), a check is available on the correctness of a calculation of $\overline{\cos \theta}$.

One may attribute the asymmetry of scattering to the zero point vibrations of the water molecules which effectively increase the molecular dimensions. From equation (454), H. A. Bethe, R. Mod. Phys. Vol. 9-2, April, 1937, one obtains for a particular model of molecular binding

$$\overline{\cos \theta} = \frac{\int_0^\pi \sin \theta \cos \theta e^{-(2-2\cos \theta) E/\hbar \omega} d\theta}{\int_0^\pi \sin \theta e^{-(2-2\cos \theta) E/\hbar \omega} d\theta}$$

where E is the neutron energy and ω the average circular frequency of the vibrational modes of the molecule.

$$\overline{\cos \theta} = \frac{\cosh \frac{2E}{\hbar \omega} - \frac{\hbar \omega}{2E} \sinh \frac{2E}{\hbar \omega}}{\sinh \frac{2E}{\hbar \omega}}$$

$$\approx \frac{1}{8} \left(\frac{2E}{\hbar \omega} \right)^2 \text{ when } \frac{E}{\hbar \omega} \text{ is very small as in this case. One obtains } \overline{\cos \theta}$$

only .0005, for neutron energies already twice that of room temperature.

This value of $\overline{\cos \theta}$ is far too small to check the experimental data.

From the fact that the scattering from hydrogen in water is mostly incoherent which should be isotropic in the center of mass system one would conclude that the value of $\overline{\cos \theta}$ in the laboratory could be determined if the effective mass of the scattering atom were known.

Thus for isotropic elastic scattering in the C. M. system, $\overline{\cos \theta} = \frac{2}{3A}$, where A is the number of mass units of the scattering atom.

The effective A can be determined in this case from the scattering cross section data since, as pointed out by Bethe (p. 122 l.c.), the Born approximation is applicable here.

$$\text{Thus } \sigma_s = M \mu^2,$$

where M is proportional to the transition probability and μ is the reduced mass of neutron and scattering atom.

$$\mu = \frac{A m}{A+1}$$

where m is the proton mass.

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The value of σ_s is known to be 20 barns when the neutron energy is so high that the hydrogen atom is effectively free, $A = 1$.

So at any lower energy
$$\frac{\left(\frac{A}{A+1}\right)^2}{\frac{1}{4}} = \frac{\sigma_{s \text{ observed}}}{20} \quad (6)$$

In using this formula to obtain A and thus $\cos \theta$ and obtain a check with the observed diffusion length from equation (5) one must take into account that the measured diffusion length is an average over thermal energies and is obtained from the equation.

$$\frac{\lambda_t}{3} v \Delta n - n v \Sigma_a = 0 \quad \text{solved in a diffusing medium.}$$

Here λ_t must be such that the average neutron current density leaving a volume is given by $\frac{\lambda_t \bar{v} \Delta n}{3}$ where $\bar{v}$ is the average neutron velocity. Similarly Σ_a must be such that the average number of neutrons absorbed per second is $\Sigma_a \bar{v} n$.

Thus, the correct equation for L^2 is:

$$L^2 = \frac{\lambda_t}{3 \Sigma_a} \quad (5)'$$

where
$$\bar{\lambda}_t = \frac{\int_0^\infty \lambda_t v M(v) dv}{\int_0^\infty v M(v) dv} \quad (5)''$$

and
$$\bar{\Sigma}_a = \frac{\int_0^\infty \Sigma_a v M(v) dv}{\int_0^\infty v M(v) dv}$$

$M(v)$ is the Maxwell distribution, the quantities λ_t and Σ_a appearing in the integrand are those applying to neutrons of a particular velocity.

For H_2O λ_t in (5)'' must be calculated at each velocity from the relation:

$$\lambda_t = \frac{1}{N_{H_2O} [2\sigma_H + \sigma_\theta] \left[1 - \frac{2}{3A} \left(\frac{2\sigma_H}{2\sigma_H + \sigma_\theta} \right) \right]} \quad (7)$$

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where N_{H_2O} number of water molecules per cc for water of unit density
 $= .0334$

σ_H is the microscopic cross section of hydrogen obtained from Melkonian
 AECD-2675.

σ_O is the microscopic cross section of oxygen assumed constant at 3.73
 barns.

For oxygen it may be assumed that the scattering is isotropic for prac-
 tical purposes.

It is possible to show in an elementary way that in an isotropic medium
 having two or more kinds of scattering atoms the proper $\overline{\cos \theta}$ appearing in
 the denominator of the expression for λ_t is the average cosine of scatter-
 ing from each type of atom weighted by the associated scattering probability.

If the neutron makes its first collision at point 1, traveling a distance
 l , and its second collision at point 2, after which it leaves at an angle
 θ with the vector $\vec{l}_{12}$, the neutron will, after an infinite number of
 collisions, have traveled an average distance equal to λ_t , by definition,
 in the direction $\vec{l}_{23}$.

The average distance traveled in the original direction $\vec{l}_{12}$ must also
 equal λ_t . Also the value of $\cos \theta$ depends only upon the type of
 nucleus which happens to be located at point 2 and is independent of the
 distance l_{12} . Averaging over all quantities, one obtains

$$\lambda_t = \lambda + \lambda_t \overline{\cos \theta}, \quad \lambda = \text{mean free path}$$

where
$$\overline{\cos \theta} = \frac{\sum_{s_1} \cos \theta_1 + \sum_{s_2} \cos \theta_2 \dots + \sum_{s_n} \cos \theta_n}{\sum_{s_1} + \sum_{s_2} \dots + \sum_{s_n}}$$

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where, assuming η different types of atoms,

$\sum S_1$ = macroscopic scattering cross section of atom type 1

$\sum S_2$ = macroscopic scattering cross section of atom type 2

etc. and $\theta_1, \theta_2, \dots, \theta_n$ are the associated scattering angles.

The results of the calculation for water are shown in the accompanying graph. The value of .325 barns was used for the microscopic absorption cross section of hydrogen. The agreement with experiment is well within the experimental error of 3.5% given in CP-2306. Somewhat better agreement could be obtained by using a 1% smaller value of the hydrogen absorption cross section which, in fact, is only known to within about 3%. It should be further remarked that the calculations assume that the hydrogen absorption cross section obeys the $\frac{1}{v}$ law.

All points on the graph refer to water of unit density, since the actual density of water is a function of pressure as well as temperature, both of which vary in various reactors.

At the request of the Naval Reactor Division some calculations also have been made for homogeneous mixtures of equal volumes of water and zirconium. The diffusion coefficient, $\frac{\lambda_t}{3}$, rather than the diffusion length, has been calculated since the latter involves the macroscopic absorption cross section of the pile which is a function of the concentration of 25.

The formula for λ_t at any velocity which was used in equation (5) in this case is

$$\frac{1}{\lambda_t} = \left(\frac{\rho(.0334)}{2} (2\sigma_H + \sigma_\theta) + \frac{N_{Zr}}{2} \sigma_{Zr} \right) \left(1 - \frac{2}{3A} \frac{[2\sigma_H \rho]}{[2\sigma_H + \sigma_\theta]\rho(.0334) + N_{Zr}\sigma_{Zr}} \right) \quad (8)$$

where ρ is the density of water for various specified conditions of pressure and temperature,

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VARIATION OF THERMAL DIFFUSION LENGTH OF
NEUTRONS IN WATER VS TEMPERATURE

FIG. 22

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CONTRIBUTION OF FAST FISSIONS TO k

ADDITIONAL EFFECT DUE TO FAST FISSIONS IN NEIGHBORING RODS

A. Radkowsky

1) The contribution of fast neutron fission (i.e., fission caused by neutrons before slowing down) to the multiplication factor, k , of a lattice has been previously discussed in CP-644 and MT-199. The procedure in these papers is to start with a fast neutron, born in a particular rod as a result of a thermal fission, and to compute the average total number of neutrons per each such thermal fission fast neutron, which escape from or are slowed inelastically in the same rod.

³⁵ In lattices in which the rods are close together there is an additional effect due to the fact that a fast neutron born in one rod may cause a fission in some neighboring rod before being slowed down. The purpose of the present paper is to outline methods of computing this additional effect. The paper consists of three parts; first (see section 2) below) the quantity Σ , which is the contribution of the fast fission to the multiplication factor, is expressed in terms of the total probability that a fast neutron will make its first collision in metal (i.e. either in the rod in which it originates or in some other rod of the lattice); second, (see section 3) below) methods are given for calculating the above collision probability which is a purely geometric factor; third (see section 4) below) a discussion is given as to available data for the average values of fast neutron cross sections in natural uranium and water.

The writer prepared this study while on loan from the Argonne National Laboratory to H. K. Ferguson Co. and is much indebted to Dr. Karl Cohen of this company for his encouragement and helpful suggestions.

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$N_{Zr} =$ number of zirconium atoms per cc = .0425

$\sigma_{Zr} =$ microscopic scattering cross section of $Zr = 6.4$

The density of zirconium was assumed constant at 6.44.

The results of the calculation are as follows:

Temp. °F	Density of water	Calculated Diffusion Constant = λ_{τ}
59	1	.255
250	.94	.305
475	.827	.378
600	.550	.527

E. EFFECT OF NEUTRON SPECTRUM DIFFERENT FROM MAXWELLIAN

As previously mentioned, the presence of an absorber causes the neutron energy spectrum to differ from a Maxwellian. The effect is discussed in a paper by Wigner and Wilkins (AECD-2275) for the case of a hydrogen gas moderator wherein it appears that the principal change in the neutron energy distribution is for velocities exceeding v_{RT} . Such a change in energy distribution will have an important effect on λ_{τ} , which is greater at the greater velocities.

AECD-2275 presents a series of curves of energy distributions, corresponding to various values of $\beta\gamma = \frac{\sigma_a v}{\sigma_s \sqrt{2} RT}$ (mass of neutron taken as unity). Taking this constant equal to about .1, which is of the order of magnitude encountered in reactors, the diffusion constant at 475°F for a zirconium water mixture was found to be 9.7% greater than that calculated for the Maxwell distribution.

The above, of course, is merely an indication of the possible magnitude of the effect and the need for a more accurate calculation which it is hoped to do in the future.

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2) The formula for the Fast Effect, ϵ , in the Handbook CL697-IV E takes into account only the effect of fissions in the rod in which the neutron originates. A similar formula will now be developed taking into account the probability of fission in neighboring rods. In this calculation the distribution of thermal fission fast neutrons will be assumed to be flat since: (1) as pointed out in MT-199, the values of the experimental constants concerned are not known with sufficient precision to justify the refinement of distinguishing between thermal and flat distributions; (2) it appears from the results of CP-644 that for small rod diameters of the order of magnitude usually used in a closely packed lattice, it makes very little difference in the results for a single rod whether one considers a thermal or a flat distribution of the source neutrons; hence the effect of the difference in distributions will be certainly negligible as far as neighboring rods are concerned.

Let ϵ , be the total fast effect; P , the total probability of a fission neutron colliding on first flight in the rod in which it is born; and P_n the total probability of a fission neutron colliding on first flight with metal elsewhere in the lattice (i.e. in any other rod of the lattice).

Start with 1 fast neutron produced by thermal fission. Let σ_t , σ_i ,

σ_e , σ_f , and σ_c be the average fast fission microscopic cross sections of the rods, where the subscripts t, i, e, f, and c denote total, inelastic, elastic, fission and capture respectively. ν is the average number of neutrons produced per fission.

After 1 collision there will be

$$((1 - (P + P_n)) + \frac{(\sigma_i)(P + P_n)}{\sigma_t})$$

neutrons which have entered the slow- (1)
ing down region
(escaped into mod-
erator or slowed
inelastically)

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$$\left(\frac{\nu \sigma_f + \sigma_e}{\sigma_t} \right) (P + P_N)$$

fast neutrons capable of producing further fission

After 2 collisions there are an additional

$$\left(\frac{\nu \sigma_f + \sigma_e}{\sigma_t} \right) (P + P_N) \left[(1 - (P + P_N)) + \left(\frac{\sigma_i}{\sigma_t} \right) (P + P_N) \right] \quad (2)$$

neutrons entered the slowing down region

and

$$\left(\frac{\nu \sigma_f + \sigma_e}{\sigma_t} \right)^2 (P + P_N)^2$$

neutrons capable of producing further fission;

After r collisions there are an additional

$$\left(\frac{\nu \sigma_f + \sigma_e}{\sigma_t} \right)^{r-1} (P + P_N)^{r-1} \left[(1 - (P + P_N)) + \left(\frac{\sigma_i}{\sigma_t} \right) (P + P_N) \right]$$

neutrons entered the slowing down region;

and

$$\left(\frac{\nu \sigma_f + \sigma_e}{\sigma_t} \right)^r (P + P_N)^r$$

neutrons capable of producing further fission.

The fast effect, ϵ , is the total number of neutrons which enter the slowing down region per neutron produced by thermal fission.

$$\epsilon = \left[(1 - (P + P_N)) + \left(\frac{\sigma_i}{\sigma_t} \right) (P + P_N) \right] \left[1 + \left(\frac{\nu \sigma_f + \sigma_e}{\sigma_t} \right) (P + P_N) \right] \quad (3)$$

$$\begin{aligned} & + \left(\frac{\nu \sigma_f + \sigma_e}{\sigma_t} \right)^2 (P + P_N)^2 \\ & + \left(\frac{\nu \sigma_f + \sigma_e}{\sigma_t} \right)^3 (P + P_N)^3 \dots] \\ & = \frac{[1 - (P + P_N)] + \left(\frac{\sigma_i}{\sigma_t} \right) (P + P_N)}{1 - \left(\frac{\nu \sigma_f + \sigma_e}{\sigma_t} \right) (P + P_N)} \end{aligned}$$

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or, neglecting powers and products of P and P_N , and using

$$\sigma_t = \sigma_i + \sigma_e + \sigma_f + \sigma_c,$$

$$\epsilon \approx 1 + \frac{(P+P_N) \left[(\nu-1) \sigma_f - \sigma_c \right]}{1 - \left(\frac{\nu \sigma_f + \sigma_e}{\sigma_t} \right) (P+P_N)} \quad (3)'$$

where P and P_N are sufficiently small.

Attention is invited to the assumption that a fast neutron which makes its first collision with a moderator atom has entered the slowing down region and need no longer be considered as capable of producing fast fissions. It is conceivable that the moderator might contain an element with relatively poor slowing down properties for which the above assumption would not be true. While this situation is not considered in detail herein, the following points may be mentioned:

- (a) In most cases, if the moderator has relatively poor slowing down properties, the ratio of moderator to metal is relatively large so that the entire effect discussed in this paper is negligible.
- (b) The collision probability with the rods for those fast neutrons remaining fast after the first collision with the moderator might be approximated by assuming a uniform equivalent 'production' of such neutrons in the moderator and using a method similar to that of Dancoff in CP-2157.

This matter is discussed further in section 4) herein. In section 3) any fast neutron which makes its first collision in the moderator is considered to have entered the slowing down region.

3) The essentially geometrical problem of computing P_N will now be considered. If P_{AB} is the average probability that a neutron born in rod A will make a collision on its first flight in rod B, then

$$P_N = \sum_B P_{AB}$$

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To compute P_{AB}

Assume very long rods of length L and of radius R . Let d be the distance between the axes of rods A and B .

Let Σ_1 and Σ_2 be the total macroscopic fast cross sections for metal and moderator, respectively.

The first case to be discussed will be that in which there are no rods intervening between A and B and in which the distance between rods is so large compared with their radii that it is sufficiently accurate to assume each rod concentrated on its axis.

Take the Z direction parallel to the rod axes. Take points 1 in rod A and point 2 in rod B with corresponding volume elements d_{1A} and d_{2B} , respectively.

Let ρ be the distance from point 1A to point 2B and θ the angle between ρ and the plane perpendicular to the rod axes.

The probability that a neutron originating in d_{1A} will make its first collision in d_{2B} is

$$\frac{\Sigma_2 e^{-\Sigma_2 \rho} d_{2B}}{4\pi \rho^2}$$

Taking the density of neutrons created in rod A as unity,

$$P_{AB} = \frac{\Sigma_1}{4\pi} \frac{\int_{\text{cylinder B}} \int_{\text{cylinder A}} \frac{e^{-\Sigma_2 d \cdot \sec \theta}}{(d \cdot \sec \theta)^2} d_{1A} d_{2B}}{\text{volume of cylinder A}}$$

now the volume element $d_{1A} = d(\tan \theta) \cdot \pi R^2 \cdot d$;

the volume element $d_{2B} = \pi R^2 dz$

so

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$$P_{AB} = \frac{\sum_i}{4} \int_{-\pi/2}^{\pi/2} \frac{(R^2)e^{-\sum_i d \sec \theta}}{d^2} d\theta$$

$$= \frac{\sum_i R^2}{2d} K_{i_1}(\sum_i d)$$

(4)

where $K_{i_1}(x)$ is defined as $\int_x^\infty K_0(y) dy = \int_0^\infty \frac{e^{-x \cosh \mu}}{\cosh \mu} d\mu$

By substituting $\cosh \mu = \sec \theta$ the integral on the right becomes

$$\int_0^{\pi/2} e^{-x \sec \theta} d\theta$$

$K_0(x)$ is the Bessel function of zero order of the second kind with "imaginary" argument. $K_{i_1}(x)$ is tabulated in LAMS-728.

As a second case, assume that the rods are sufficiently close so that it is no longer accurate to consider the rods as line sources. Assume also that both the metal and moderator have the same total macroscopic cross section denoted by Σ . As explained in the next section, this assumption seems to be nearly true for natural uranium and water at room temperature. In this case, it makes no difference if one or more rods intervene between A and B.

Let the vector from point 1 in rod A to point 2 in rod B be denoted by $\vec{r}_{1A,2B}$.

The rod axes will be assumed vertical. Take a horizontal plane through the rods. (See sketch). The angle between $\vec{r}_{1A,2B}$ and this plane will be called θ . The projection of $\vec{r}_{1A,2B}$ on this plane will be called ρ .

Let a be the intersection of the axis of rod A with the plane. Draw the line $\vec{r}_{a,2B}$ joining a to point 2 of the rod B. Call ϕ the angle between ρ and $\vec{r}_{a,2B}$.

$$\text{Let } r \equiv |\vec{r}_{a,2B}|$$

The probability that a neutron originating at point 1A will make its

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PLAN

AXIS ROD A

AXIS ROD B

ELEVATION

FIG. 23

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first collision in a volume element dr_{2B} at point 2B is:

$$\frac{\Sigma_e |\vec{r}_{1A,2B}| d\vec{r}_{2B}}{4\pi |\vec{r}_{1A,2B}|^2}$$

The density of neutrons created in rod A will be assumed to be unity.

Then

$$P_{AB} = \frac{\Sigma_e \int_{\text{cylinder B}} \int_{\text{cylinder A}} \frac{e^{-\Sigma |\vec{r}_{1A,2B}|}}{|\vec{r}_{1A,2B}|^2} d\vec{r}_{1A} d\vec{r}_{2B}}{\text{volume of cylinder A}} \quad (5)$$

The volume element, $d\vec{r}_{1A}$, at point 1A, is

$$P d\phi \cdot d\rho \cdot \rho d(\tan \theta)$$

Substituting $|\vec{r}_{1A,2B}| = \rho \sec \theta$

$$P_{AB} = \frac{1}{4\pi(\pi R^2 L)} \int_{\text{cyl B}} \int_{\Sigma(r-R)}^{\Sigma(r+R)} \int_{-\phi_{\max}}^{\phi_{\max}} \int_{-\pi/2}^{\pi/2} e^{-\Sigma \rho \sec \theta} d(\Sigma \rho) d\phi d\theta d\vec{r}_{2B} \quad (6)$$

For fixed ρ , $\phi_{\max}$ is the angle opposite R in a triangle with sides r, R, and ρ , - and is independent of θ . Hence, integrating with respect to ϕ , and taking account of symmetry with respect to ϕ and θ ,

$$P_{AB} = \frac{1}{\pi^2 R^2 L} \int_{\text{cyl B}} \int_{\Sigma(r-R)}^{\Sigma(r+R)} \int_0^{\pi/2} \phi_{\max} e^{-\Sigma \rho \sec \theta} d(\Sigma \rho) d\theta d\vec{r}_{2B} \quad (7)$$

again substitute

$$\int_0^{\pi/2} e^{-\Sigma \rho \sec \theta} d\theta = K_i(\Sigma \rho)$$

Now

$$\int_{\Sigma(r-R)}^{\Sigma(r+R)} K_i(\Sigma \rho) \phi_{\max} d(\Sigma \rho) \equiv f(\Sigma r) \quad (8)$$

must be evaluated numerically.

In the integration over cylinder B,

$$P_{AB} = \frac{1}{\pi^2 R^2 L} \int_{\text{cyl B}} f(\Sigma r) d\vec{r}_{2B} \quad (9)$$

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it is convenient to take the volume element, dr_{2B} , as equal to

$$r \alpha_{\max} d \geq d r$$

where α is the angle between r and a line of length d joining the points of intersection of the cylinder axes with the horizontal plane.

$\alpha_{\max}$ is the angle opposite R in a triangle with sides r , R , and d .

Performing the integration over Z , and taking account of the symmetry with respect to α

$$P_{AB} = \frac{2}{\pi^2 (\sum R)^2} \int_{\sum(d-R)}^{\sum(d+R)} (\sum r) \alpha_{\max} f(\sum r) d(\sum r) \quad (10)$$

This integration must also be performed numerically.

To find P_n , the total collision probability in metal other than in own rod (rod A), one must then take the summation of the various P_{AB} over all other rods (rods B).

At sufficiently great separations (10) may be replaced by (4) by setting $\sum_1 = \sum_2 = \sum$ and in (4) $\kappa_{i_1}(x)$ may be replaced by its asymptotic expansion

$$\kappa_{i_1}(x) \sim e^{-x \left(\frac{\pi}{2x} \right)^{\frac{1}{2}}} \quad (11)$$

Then since the number of rods in a layer equidistant from rod A is $k(\sum d)$, where k is a constant depending upon the geometry, and d is the distance between the axis of rod A and the axes of the rods considered, the residual portion of P_n is

$$\frac{k}{2} \left(\frac{\pi}{2} \right)^{\frac{1}{2}} (\sum R)^2 \sum_{a=a_0}^{a=\infty} e^{-\sum a p} (\sum a p)^{-\frac{1}{2}} \quad (12)$$

where d has been replaced by (ap) , a being an integer, and p the distance between axes of immediately neighboring rods. $(a_0 p)$ is the rod separation at which the use of (11) is permissible.

The summation in (12) will be approximated by the integral

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$$\int_{a_0 - \frac{1}{2}}^{\infty} e^{-\sum a p} (\sum a p)^{-\frac{1}{2}} da = \frac{1}{\sum p} \int_{\sum (a_0 - \frac{1}{2}) p}^{\infty} e^{-t} t^{-\frac{1}{2}} dt \quad (13)$$

Substituting $t = y^2$, (13) becomes

$$\frac{2}{\sum p \sqrt{\sum (a_0 - \frac{1}{2}) p}} \int_0^{\infty} e^{-y^2} dy$$

The last integral¹ = Erfc $(\sqrt{\sum (a_0 - \frac{1}{2}) p}) = \sqrt{\frac{\pi}{2}} - \text{Erf}(\sqrt{\sum (a_0 - \frac{1}{2}) p})$

(12 becomes

$$\frac{k}{\sum p} \left(\frac{\pi}{2}\right)^{\frac{1}{2}} (\sum R)^2 \left[\sqrt{\frac{\pi}{2}} - \text{Erf}(\sqrt{\sum (a_0 - \frac{1}{2}) p}) \right] \quad (14)$$

where

$$\text{Erf}(X) = \int_0^X e^{-t^2} dt \quad \text{and has been tabulated.}$$

Thus the procedure in finding P_n is to evaluate (10) numerically and sum over rods at various values of d until a rod separation is reached at which the error in using (4) and (11) is acceptable. The remaining portion of P_n can be approximated by (14).

It is also possible to solve for P_{AB} by a surface integration, instead of a volume integration, using a method similar to that in CP-644. Differentiate (5) with respect to $\sum$ under the integral sign. The integral

$$\left(\frac{1}{4\pi} \int \left[e^{-\sum |\vec{r}_{1A,2B}|} / |\vec{r}_{1A,2B}| \right] d\vec{r}_{1A} \right)$$

over cylinder A will be denoted by $w(r_{2B})$ and will be a solution of

$$\Delta w(\vec{r}_{2B}) - \sum^2 w = 0 \quad \text{outside of rod A, and of} \quad (15)$$

$\Delta w(\vec{r}_{2B}) - \sum^2 w = -1$, inside of rod A, w and its first derivative to be continuous at the surface of cylinder A.

$$w \text{ is found to be equal to } \frac{R}{\sum} I_1(\sum R) K_0(\sum r) \quad (16)$$

1. Whittaker and Watson, 4th Ed. p. 341

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Using (15) it is found that

$$P_{AB} = \frac{\Sigma}{\pi R^2} \int_{-\infty}^{\infty} \frac{1}{\Sigma^2} \int_{\text{surface of rod B}} \text{grad } w \cdot d\vec{\sigma} \text{ rod B } d\Sigma \quad (16)$$

where $d\vec{\sigma}$ is the surface element of rod B.

$$P_{AB} = -\frac{\Sigma}{\pi R} \left[\int_{-\cos^{-1} \frac{R}{d}}^{\cos^{-1} \frac{R}{d}} \int_{\Sigma} \frac{1}{\Sigma^2} I_1(\Sigma R) K_1(\Sigma r_2) r_2 d\Sigma d\alpha \right. \\ \left. - \int_{-\cos^{-1} \frac{R}{d}}^{\cos^{-1} \frac{R}{d}} \int_{\Sigma} \frac{1}{\Sigma^2} I_1(\Sigma R) K_1(\Sigma r_1) r_1 d\Sigma d\alpha \right] \quad (17)$$

where r_2 and r_1 are respectively the larger and smaller of the two possible sides of a triangle containing the angle α with opposite side R and one adjacent side d . $I_1(x)$ and $K_1(x)$ are Bessel functions of first order of the first and second kinds respectively with imaginary argument. Unfortunately, the integral with respect to Σ has not been tabulated, although a rather complicated series expression has been given in equations (34) to (36) of CP-644. It would be necessary to know the value of this integral very accurately to determine P_{AB} since P_{AE} is composed of the very small differences between two expressions containing these integrals. Therefore it is considered that the use of equation (10) is more suitable for computation.

When the macroscopic cross sections of the metal and moderator differ, consider first the case where the difference is small and suppose, for definiteness that the cross section of the moderator, Σ_2 , is smaller than Σ_1 , the cross section of the metal.

The solution for P_{AB} is then the same as it would be were $\Sigma_2 = \Sigma_1$ and the rods brought closer together. The distance of 'closest approach' of the two rods is $(d-2R)$ (See sketch). Take the distance between the rod axes as d' where

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$$\sum_1 (d' - zR) = \sum_2 (d - zR) \quad (18)$$

and use (10) to obtain P_{AB} taking $\sum_1$ as macroscopic cross section of both metal and moderator. This will give too low a result since a neutron leaving the rod A outside of the plane containing the rod axes will not have its path in the moderator reduced sufficiently to justify use of $\sum_1$ instead of $\sum_2$ as the moderator cross section.

On the other hand, if d' is chosen on the basis of

$$\sum_1 d' = \sum_2 d \quad (19)$$

and the procedure repeated, the value of P_{AB} will come out too large since the average neutron path in the moderator will be reduced too greatly.

Thus, upper and lower limits may be fixed for P_{AB} and the correct value may be approximated by interpolating within these limits.

If greater accuracy is required, the best procedure is to assume each rod composed of a finite number, N , of smaller rods (subrods), all having parallel axes. The probability that a neutron of subrod n of rod A will make its first collision in subrod j of rod B will be denoted by

$$P_{i,j}$$

Then

$$P_{AB} = \sum_{i=1}^N \sum_{j=1}^N P_{i,j}$$

From (4)

$$P_{i,j} = \frac{\sum_1 \cdot (R')^2}{2 P_{i,j}} K_{i,j} (\sum_1 \cdot (x_i + x_j) + \sum_2 \cdot (P_{i,j} - x_i - y_j))$$

where R' is the radius of the subrod,

$P_{i,j}$ is the shortest line between the two subrods

x_i is the shortest distance a neutron going from subrod i of

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rod A to subrod j of rod B travels in rod A.

y_i : corresponding shortest distance traveled in rod B.

This procedure can also be adjusted to any degree of accuracy desired to the case where another rod intervenes between A and B.

4) The writer has computed from the best available experimental data the effective fast cross sections of metal (28) and of the moderator (H_2O). By effective cross section is meant here the neutron atomic cross section averaged over the neutron energy spectrum resulting from 25 fission, i.e.

$$\sigma_{\text{effective}} = \int \sigma(E) N(E) dE / \int N(E) dE$$

where $\sigma(E)$ represents the measured values of the particular cross section at various neutron energies, and $N(E)$ the number of neutrons, resulting from 25 fission, per unit energy interval.

The above integral was evaluated numerically.

For hydrogen and oxygen total cross sections, only, were computed, $\sigma(E)$ being taken from the curves given in Nuclear Science and Engineering, edited by Clark Goodman. For 28 the values of σ_t , σ_f , σ_c , and σ_i given in ORNL-86 were used. The values of σ_e were computed by subtracting ($\sigma_f + \sigma_c + \sigma_i$) from σ_t . The subscripts represent total, fission, capture, inelastic, and elastic, respectively. The values of $N(E)$ were obtained from LA-715, LA-718, and CP-3800.

The values of effective cross section thus computed are listed in the accompanying table, together with corresponding values given in the Project Handbook (CL697-IV E) and those given in CP-644.

It will be noted that the values of σ_f and σ_c found here are very close to those given in the Handbook whereas the values of σ_t and σ_e differ considerably.

Examination of the formula for ϵ , the fast multiplication factor a

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single rod: (in (3) set $P_n = 0$) indicates that as long as $(\epsilon - 1)$ and P are small, ϵ is to a good approximation, a function of $\sqrt{\sigma_f}$ only. Thus, σ_c is very small compared to σ_f and P is an average of such factors as $(1 - e^{-\sum d}) \approx \sum d$ when P is small. Here d represents various distances from the periphery of the rod to points inside, and $\sum$, the macroscopic cross section of the rod, equals σ_t multiplied by N_{28} , the number of 28 atoms per cc. The new values of effective cross sections computed here do not, then, imply any substantial change in ϵ for a single rod from the values computed on the basis of the data given in the Handbook.

An interesting point brought out from the study of fast cross sections herein is that the total macroscopic cross section, $\sum$, for 28 and for H_2O (of specific gravity 1) are closely equal, being approximately $.34 \text{ cm}^{-1}$ each. This, as has been seen, greatly simplifies the calculation of the effect of the neighboring rods on the fast multiplication factor since it is unnecessary to take specific account of which parts of the neutron paths lie in metal and which in water.

It should be noted that the resonances at high energies introduce some uncertainty in the cross section values for oxygen.

With further reference to the justification for the assumption that a neutron striking an oxygen atom can be considered to have lost sufficient energy to no longer play a part in fast fission, it will be noted from ORNL-86 that the fission threshold for 28 sets in very abruptly about 1.4 Mev and from CP-3800, the number of fission neutrons is decreasing quite sharply with energy at this point, thus even a small loss of energy will reduce the majority of those fission neutrons which originally are above 1.4 Mev to below the fission threshold.

It also appears from the work of L. E. Beghian et al (Phys. Rev. 77,

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2, 286 of January 15, 1950) that in a large number of light elements the cross section for inelastic scattering in the neighborhood of 2.5 Mev is almost half the total cross section and the energy loss is sufficient to put the neutron well below the 28 fission threshold.

Average Values of Fast Constants

<u>Material</u>	<u>Computed Here</u>	<u>Handbook</u>	<u>CP-644</u>
Natural U.	$\sigma_t = 7.11$	4.3	4.39
	$\sigma_e = 5.21$	1.5	1.5
	$\sigma_c = 0.08$	0.06	
	$\sigma_i = 1.64$		2.4
	$\sigma_f = 0.284$	0.29	0.49 or 0.4
	$\Sigma t = 0.336$, based on N = 0.0473, density = 18.7		
H	$\sigma_t = 3.78$		
O	$\sigma_t = 2.44$		
H ₂ O	$\Sigma t = \text{for density 1} =$ $(0.0334)(2 \times 3.78 + 2.44) =$ 0.334		

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