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EXPONENTIAL EXPERIMENTS ON
LIGHT WATER MODERATED 1 PER
CENT U-235 LATTICES

By
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November 28, 1952

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Exponential Experiments on Light Water Moderated, 1 Per Cent U-235 Lattices

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Contents

I. Introduction.	2
II. Experimental Facilities.	2
III. Buckling Measurements.	6
A. Theory	6
B. Experimental Techniques	8
C. Analysis of Measurements and Results	11
D. Cadmium Ratios	13
IV. Anisotropy Measurement.	13
A. Introduction	13
B. Theory	14
C. Experimental Methods	16
D. Results	17
V. Migration Area Measurements.	27
A. Introduction	27
B. Theory	27
C. Experimental Methods	28
D. Analysis of Data and Results	29
VI. Intracell Flux Traverses.	38
A. Introduction	38
B. Experimental Procedure	38
C. Analysis of Data and Results	40
VII. The Effect of Absorber Rods.	43
VIII. The Effect of a "Gap" on the Axial Relaxation Length.	45

Exponential Experiments on Light Water Moderated, 1 Per Cent U-235 Lattices

I. Introduction.

At the request of the Division of Reactor Development of the U.S.A.E.C., the Physics Division of the Nuclear Engineering Department at Brookhaven undertook a program of exponential experiments on ordinary water moderated, slightly enriched uranium lattices. The purpose of the measurements was to provide nuclear design information for the core of a reactor for the production of plutonium and power. In particular, nuclear data were to be provided on the basis of which the Atomic Energy Division of the H. K. Ferguson Co. - now the Walter Kidde Nuclear Laboratory - could prepare a feasibility report on such a reactor. Consequently, the program of the experiments was planned in close cooperation with members of the WKNL and cooperation was maintained during the course of the experiments.

The present report is a summary of the exponential experiment measurements which were made during the period November 15, 1951 to June 30, 1952, as part of a broader Reactor Components Testing Project. The experiments to be described include measurements of the critical buckling, the migration area, intra-cell flux traverses, effectiveness of control rod materials, the effect of removing uranium rods from the assembly, etc.

II. Experimental Facilities.

The facility for the exponential assembly was planned with great care in view of the difficulties met in earlier experiments on ordinary water moderated-natural uranium lattices at Oak Ridge¹.

1. CP-2842.

The measurements were carried out in a water tank placed on a thermal column which occupies a portion of the top of the Brookhaven pile shield. The thermal column consists of five one-foot thick layers of graphite, stepped from a 4' x 4' size at the bottom to 5' x 5' at the top. This stepped construction prevents fast neutron leakage up the sides of the thermal column, and the well-machined surfaces of the graphite blocks which make up the structure provide tight packing which ensures a well-moderated source of neutrons at the top surface. The cadmium ratio of this source as measured with five mil thick indium foils was better than 10^5 .

The water tank is cylindrical, six feet in diameter and six feet high. Surrounding it is a makeshift shield of three-inch steel armor plate and assorted blocks of Brookhaven concrete.

Inside the large water tank is a perforated aluminum cylinder, four feet in diameter and four feet high, which serves as a vertical support for the top tube plate system, from which the uranium rods are suspended. This top is a ring of three inch thick armor plate which holds the circular aluminum top tube plates. The inner tank with a tube plate is shown in Fig. 1.

The tube plates are made of 2S aluminum, the upper plate being 1.5" thick, and the lower plate .75" thick. In each are drilled 275 holes for the uranium rods, and 20 holes for the insertion of foils. Care was taken to locate these holes accurately, the tolerance being 0, -.5 mils for each hole diameter, and -.5 mils overall for the distance between holes. These low tolerances were maintained largely because we did not know the effect of variations of individual rod

positions on reactivity and the positioning of foils, and we preferred working with conditions which could be trusted².

The uranium was prepared at K-25, and was rolled into rods of .750" dia. at the St. Louis plant of the Malinckrodt Chemical Corporation. The Brookhaven metallurgy group then straightened the rods to within 15 mils lateral deviation in four feet of length, and clad them with 30 mil thick aluminum. Most rods are single four-foot lengths of uranium. A few are slightly under four feet in length, and a few are made of shorter sections matched to provide approximately four foot lengths.

Analyses of the uranium composition were made at Oak Ridge. The average composition is 1.027 per cent $\pm$.001 per cent by weight.

Distilled water purchased commercially was used throughout the experiment. Occasional spectroscopic tests of purity were supplemented by tests of reproducibility of the data. In fact, the criterion of reproducibility was used constantly as a check on the possibility that other factors might be influencing the measurements. For a large part of the time the water contained boric acid in solution. At all times the tank contained cadmium from the shutter (see below). There was always the possibility that boron or cadmium coatings might form on the aluminum cladding of the rods, thus changing the reactivity. For this reason, particularly, reproducibility of the data was held to be important. This reproducibility was maintained throughout the experiments.

Best results from an exponential experiment are obtained if the source of

-
2. Lack of accurate positioning of rods and lack of straightness of rods have been suggested as the source of errors in the Oak Ridge measurements (see reference 1).

Best results from an exponential experiment are obtained if the source of thermal neutrons feeding the reproducing lattice is made about the same size as the lattice itself. In this event the decay vertically of the flux harmonics is most rapid, and the region of flux harmonics corrections is smallest. For this reason the bottom of the large water tank was covered with a cadmium sheet having a central circular orifice. The hole in the cadmium had a radius about 7 cm larger than that of the maximum uranium loading. This defines the source size most suitable for rapid decay of harmonics, because the radial extrapolation distance of the thermal neutron flux in the reproducing lattice was found to be close to seven centimeters for those loadings investigated.

A shutter system of cadmium covered aluminum plates was designed to fit under the lattice assembly, to make possible turning off the source flux when irradiations were not being made. Because of corrosion and working under the weight of the uranium and support assembly, this shutter system never worked properly. In practice, the source flux was always turned off by the dropping of a cadmium sheet into the water tank, over the disc source, after the inner tank with its uranium loading was removed by crane.

The water tank contains a system of steam pipes which allow heating of the water. Thus the equipment has provision for finding temperature coefficients of the various quantities measured. No temperature coefficients have yet been measured, but it is planned to determine some in the near future.

III. Buckling Measurements.

A. Theory.

We have used throughout the course of the measurements a one-group view of the diffusion problem in the lattice. Accordingly, we suppose the neutron flux satisfies the relation

$$(1) \quad \nabla^2 \phi + B^2 \phi = 0,$$

and the critical equation is simply

$$(2) \quad k_{\infty} = 1 + M^2 B^2$$

The standard exponential experiment involves obtaining the Laplacian by differentiation of the measured thermal flux distribution. For a cylindrical array such as we used, the space dependence of the flux can be written as a series of Bessel functions:

$$(3) \quad \phi = \sum_{n=1}^{\infty} J_0(a_n \frac{r}{R+\lambda}) f_n(z)$$

where r and z are cylindrical coordinates in the lattice.

λ is the reflector savings, and a_n are the successive roots of

$$(4) \quad J_0(r) = 0$$

The coefficients f_n have the form

$$(5) \quad f_n = C_n \exp \left[\left\{ \left(\frac{a_n}{R+\lambda} \right)^2 - B^2 \right\}^{1/2} z \right] + D_n \exp \left[- \left\{ \left(\frac{a_n}{R+\lambda} \right)^2 - B^2 \right\}^{1/2} z \right]$$

the exponents being real for a subcritical assembly. C_n is small compared with D_n , since the first term is only important near the upper boundary of the lattice. Furthermore, since the rate of decay of the higher harmonics is greater than that

of the fundamental, above some value of z' , the flux can be represented closely by the fundamental alone:

$$(6) \quad \phi \sim J_1(a_1 \frac{r}{R+\lambda}) f_1(z), \quad z > z'.$$

Ordinarily, one would measure the radial and axial flux distributions in the region of validity of (6). The radial distribution would be fitted to $J_0(a_1 r/r_0)$, the axial distribution fitted to $C_1 \exp[z/L] + D_1 \exp[-z/L]$, and the buckling would be

$$(7) \quad B^2 = (a_1/r_0)^2 - 1/L^2.$$

Unfortunately, the small loadings we were working with did not permit accurate measurements of the radial flux distribution. Symmetry conditions made only five positions available for flux measurements on any one radius, and the outermost of these was so close to the essentially infinite water reflector³ as to make it useless. The effect of the reflector can be seen from a typical radial traverse, shown in Fig. 2. Thus only four points were available along any one radius, and a curve-fitting to these points was not accurate enough for our purpose⁴.

On the other hand, the axial flux distributions showed in every case the behavior characteristic of the presence of the fundamental only, from about 10 cm

-
- 3. The decision to use reflected lattices was made mainly because of the small amount of uranium available for the experiment. The reflector savings was equivalent to about 0.7 tons of extra uranium.
 - 4. The lack of accuracy does not arise simply from the fewness of points. Rather, it is caused by the fact that these points are on the flat part of the J_0 curve, and small experimental errors in the values of the foil activities cause large variations in the extrapolated end-point.

upward from the bottom of the lattice. Moreover, the end corrections were small below about 70 cm from the bottom of the lattice. Thus there existed about 60 cm of variation of z over which the flux decay was simply exponential, and so quite good measurements of L could be made. We therefore decided to try basing measurements of B^2 on measurements of L alone.

For such a determination of B^2 , one would measure L for a wide range of rod loadings. Each such loading would be idealized to a cylinder with the same loaded radius the case would have if the uranium were uniformly distributed at the same water-to-metal ratio. Least squares fits would then be made to λ and B^2 in

$$(8) \quad (a_1/R+\lambda)^2 - 1/L^2 = B^2 .$$

Such an analysis contains of course an assumption that λ and B^2 have the same value for all loadings used. This is perhaps no worse than the assumption ordinarily made in an exponential experiment on an inhomogeneous lattice - that the results can be interpreted in terms of fudged calculations modeled after homogeneous reactor theory, with a value of the buckling which does not depend on the size of loading. Nevertheless we have kept in mind the fact that this assumption underlies the measurement, and later in this report present experimental evidence that the assumption is probably not far wrong. This problem is also being investigated from a theoretical viewpoint.

B. Experimental Techniques.

The measurements of neutron flux distributions were made with indium foils, 0.220" in diameter and five mils thick. Each foil was counted in from three to six

counters, to a total of more than ten thousand counts, (1 per cent statistical accuracy) per foil. Counter resolving times were found by the two-source method, and in all counts resolving-time corrections were kept below 3 per cent. We believe that in this way we have kept to about .1 per cent of the total counts per foil the error introduced by having to make resolving time corrections. Count times were controlled by a preset timer which was accurate within .3 seconds (at most .5 per cent per count), about half systematic and half random. The estimated error due to timing is thus about .25 per cent. The overall estimated error in the activity of an individual foil is then about 1.3 per cent about the mean, when the conditions for accuracy are worst.

The foils were positioned in foil rods inserted through the top tube plate. At first thin-walled aluminum tubes were used, with slots to hold the foils punched at ten centimeter intervals. The foils were placed in thin aluminum foil covers, each being held together with Duco cement. The cement was removed by two washings in acetone before foil counting was begun. The maximum error in vertical positioning of the individual foils was about 1 mm. For the shortest relaxation length we measured, the maximum error in foil activity measured was thus about 0.7 per cent from this source. Each foil was used only once, so there were no errors from the presence of long-lived activities.

The aluminum foil positioning tubes were later replaced by machined lucite rods. Positioning error was thus reduced (to about 0.2 per cent at most). The use of Duco cement was avoided, and the replacement rate of foil rods, which had been large because of bending of the aluminum and rapid enlargement of the holes

for foils, was materially reduced.

Because of large local variation of flux, it was necessary that all foils be centered accurately in a lattice cell. This was accomplished through the use of lucite spacers located at one foot intervals in height throughout the lattice.

The foils were weighed to within 0.1 mg each. Their weights averaged 0.027 gms, with not much variation about the mean. Measurements of activity as a function of foil weight showed that corrections to the activity due to weight variation are about 20 per cent of the weight correction. Matched sets of foils, with weight variations limited to $\pm .1$ mg, were used at every stage. The estimated error caused by individual variations in foil weight was at most about .05 per cent. This contribution to the total error is negligible.⁵ The value of the indium half life was taken as 54.05 minutes for foil activity calculations. At the end of the buckling experiments a careful determination of the half-life was made, and gave a result $T_{1/2} = 54.14 \pm .07$ minutes. The error in all the bucklings introduced by this discrepancy would be about 0.3 per cent. A measurement of the indium half-life was suggested to us by A. Wattenberg, who pointed out the lack of agreement in previously published values.

The compounding of all these errors indicates inaccuracies of individual foil activities of at most 1.5 per cent about the mean. Statistical variations in

-
5. The variation with weight of the weight correction to the activity was measured. It was found that a 1 per cent weight variation was accompanied by only a 0.2 per cent change in foil sensitivity. This is much less than the direct proportionality usually assumed.

the measured quantities were consistent with this estimated error.

C. Analysis of Measurements and Results.

The relaxation lengths listed in Table 1 are obtained from least squares fits of exponentials to the axial flux traverses. Water-to-metal ratios of 1.5:1, 1.75:1, 2:1, 2.5:1, 3:1, and 4:1 are reported. These ratios are really those of water plus aluminum to uranium. Also given in Table 1 are the residuals of the least squares fits of B^2 and λ .

Table 2 gives the values of B^2 and λ which result from the least squares fits. Fig. 3 is a plot of B^2 vs water-to-metal ratio from Table 2. Fig. 4 is a plot of λ vs water-to-metal ratio from Table 2.

The estimated errors in every case refer to reproducibility of data. Thus they do not include systematic errors or limitations to the validity of the method.

The effect of having ignored end corrections was considered. To determine the magnitude of this effect the following procedure was used. Values of L were calculated from flux measurements over a range of 60 cm, beginning 10 cm from the bottom of the lattice. Least squares fits of B^2 and λ were made for this set of data. The calculation was then redone leaving out the points at 70 cm from the bottom of the lattice. Since end corrections are greatest at this point, the changes in computed values of B^2 and λ may be attributed to them. Small changes in B^2 were observed; they averaged about 0.5 per cent, in the direction consistent with the presence of end corrections. Since these changes are of an order of magnitude smaller than the accuracy of the measurements, it is felt that end corrections are not of sufficient importance to warrant further investigation. The

values given in Tables 2 and 3 refer to the six point axial attenuation measurements, which have the smaller end effect corrections.

At no stage were any effects observed which might be attributed to the presence of the harmonics in (3).

Since the method used to find the buckling is not standard, it was considered advisable to make some radial traverses in order to be able to compare results with those which would normally have been obtained. The best radial traverses were made with the 3:1 water-to-metal ratio, at a loading of 265 rods. At this loading the lattice was approximately an elliptic cylinder, the distance to the edge of the metal loading being different for the two directions along which radial traverses were made. The effective radii were found by fitting the measured foil activities to $J_0(a_1 \frac{r}{R_{\text{eff}}})$. The values of the reflector savings for the two radii were then found as the difference between values of R_{eff} and the actual distance from the lattice center to the edge of the metal loading. The average of six radial traverses gave in this way a value of $6.65 \pm .50$ cm for the reflector savings. This is to be compared with the value of $6.94 \pm .11$ cm determined by the method of axial bucklings only. The agreement of these values to within three millimeters is strikingly good.

A single measurement of the reflector savings was carried out in the 1.5:1 lattice, in connection with the anisotropy measurement discussed later in this report. The measured value of λ in this case was 7.67 cm, with somewhat uncertain accuracy. The value given in Table 2 is obtained by the method of axial bucklings is $7.71 \pm .14$ cm. Again the agreement is exceptionally good.

As a further check on the validity of the assumption of constant B^2 and λ , the buckling for the 3:1 lattice was recomputed, using only loadings between 181 and 263 rods. B^2 and λ were unchanged within the experimental error reported in Table 3.

Thus the experimental data available tends only to bear out the accuracy of the assumption underlying the method we have used. It would still be interesting to see the results of critical experiments made on these lattices.

If the slowing-down and diffusion of neutrons does not take place isotropically in the reactor core, the exponential experiment must be modified considerably. The effect of such anisotropy has been discussed by Young and Wheeler⁶; they showed that if the anisotropy is large it is possible to draw grossly inaccurate conclusions from a simple exponential experiment.

D. Cadmium Ratios.

To a certain extent, information on the energy distribution of low energy neutrons may be determined from the cadmium ratios. Therefore accurate measurements of the cadmium ratio were made at the center of a lattice cell, using indium and gold foils. The indium foils were five mils thick; the gold foils were 1.2 mils thick. Measured values of the cadmium ratio are listed in Table 3 and plotted in Fig. 5.

IV. Anisotropy Measurement.

A. Introduction.

As will be seen in Part V of this report, the migration area which we have measured is essentially a constant over the range of water-to-metal volume ratios investigated, and is in this region quite close to the migration area of fission

6. C-90.

neutrons in water alone. Thus it appears that because of high inelastic scattering, uranium in water may be considered quite closely equivalent volumewise (on the average) to the water it displaces, for the purposes of neutron slowing-down. It might be suspected that this condition makes the existence of anisotropic effects unlikely. Nevertheless, the situation seemed to warrant a search for such an anisotropic character of the migration area.

Young and Wheeler also reported and analysed the suggestion by Wigner that a "double exponential experiment" would yield the degree of anisotropy. It is such a double exponential experiment which we have carried out for the 1.5:1 water-to-metal volume ratio lattice, which should have the greatest anisotropy of the lattices we have used.

B. Theory.

For the general theory of the double exponential experiment, one should see the paper by Young and Wheeler. We give here a discussion suitable for our purposes.

We consider our subcritical lattice in the shape of a rectangular parallelepiped, with edges $l_x = l_y$; l_z . The fuel rods are supposed to lie in the z direction. If the thermal neutron source activating this array is placed at one end of the lattice (the side with edges l_x and l_y) the one-group critical equation will be

$$(9) \quad k_{\infty} - 1 = M_{\perp}^2 (B_x^2 - B_y^2) - M_{\parallel}^2 B_z^2 .$$

k_{∞} is of course the infinite pile reproduction factor.

$$(10) \quad B_x^2 = B_y^2 = \frac{\pi^2}{(\ell_x/2 + \lambda)^2}$$

$$(11) \quad B_z^2 = 1/L^2.$$

L is the relaxation length for decay of the fundamental in the z direction, λ is the reflector savings, and $M_{\parallel}^2$ and $M_{\perp}^2$ have physical meaning associated with the mean square distance a fission neutron travels in directions parallel to the rods and normal to the rods, respectively.

If, on the other hand, the source is placed on one of the sides of the lattice (that defined by y and z), we shall have in place of (9)

$$(12) \quad k_{\infty} - 1 = M_{\parallel}^2 b_z^2 - M_{\perp}^2 (b_y^2 - b_x^2)$$

where

$$(13) \quad b_z^2 = \frac{\pi^2}{(\ell_z/2 + \lambda)^2}$$

$$(14) \quad b_y^2 = B_y^2$$

$$(15) \quad b_z^2 = 1/(L^1)^2.$$

L^1 is now the relaxation length of the fundamental in the x direction, measured in this geometry. From (9) and (12) we obtain

$$M_{\perp}^2 (B_x^2 + B_y^2) - M_{\parallel}^2 B_z^2 = M_{\parallel}^2 b_z^2 + M_{\perp}^2 (b_y^2 - b_x^2)$$

or

$$(16) \quad M_{\perp}^2 / M_{\parallel}^2 = (B_z^2 + b_z^2) / (B_x^2 + b_x^2).$$

In practise, we took the first geometry to be cylindrical, so that instead of (9) one has

$$(17) \quad k_{\infty} - 1 = M_I^2 B_I^2 - M_{II}^2 B_Z^2$$

with

$$(18) \quad B_I^2 = (a_1/R + \lambda)^2,$$

a_1 being the first root of the Bessel function of order zero, and R being the radius of the loading cylinder. The relation which corresponds to (16) is derived from (12) and (17):

$$(19) \quad M_I^2 / M_{II}^2 = (B_Z^2 + b_Z^2) / (B^2 + b_X^2 - b_Y^2).$$

C. Experimental Methods.

B_I^2 and B_Z^2 can be found from the buckling measurements in cylindrical geometry described earlier. Thus to find the anisotropy it was only necessary to carry out the second measurement above - the determination of b_X^2 , b_Y^2 and b_Z^2 .

For this purpose a set of lucite support pieces was constructed; these permitted a horizontal layered construction of the lattice in a 16 x 16 rod array - thus a loading of 256 rods. Support was provided by the lucite at four points along the length of the rods, to prevent sagging, and the foils which were used to measure the neutron distribution were placed in machined holes in the surface of the lucite support structure. The cadmium which defined the shape of the thermal neutron source had cut in it a rectangular hole about 14 cm larger on each side than that of the array. This source shape was most suitable for the appearance only of the fundamental.

The gold foils which were used throughout were .220 inches in diameter, and were about one mil thick. They were chosen as a set matched in weight to about .5 per cent. The foil counting techniques are the same as those described earlier.

D. Results.

Equation (17) does not refer to any particular radius of the lattice, and as a result B_z^2 and B_l^2 may be taken in (19) for any loading. We have chosen rather arbitrarily to base the analysis on the 235 rod loadings, the results of which are given in Table 2. Here, L is 17.72 cm by measurement. Analysis of the least squares methods used to obtain L show an expected accuracy of about $\pm .10$. We allow for each of the two measurements double this error, so δL (the error in L) is assumed about $\pm .14$. The loaded radius at 265 rods is 23.087 cm. The reflector savings given by buckling measurements is 7.71 cm. Although this number is here being questioned (its derivation is based on use of the isotropic critical equation rather than on (17)), it is doubtful that it is wrong by more than $\pm .5$ cm. Thus we may take $R+\lambda = 30.80 \pm .50$ cm with some confidence. So far we have

$$(20) \quad \begin{aligned} B_z^2 &= 1/L^2 = 3.18 \times 10^{-3} \pm .052 \times 10^{-3} \text{ cm}^{-2} \\ B^2 &= (a_l/R+\lambda)^2 = 6.09 \times 10^{-3} \pm .055 \times 10^{-3} \text{ cm}^{-2}. \end{aligned}$$

With the lattice placed horizontally, a set of seven foils was placed in the y direction, to provide a determination of the flux distribution in that direction. A least squares fit to $A \cos bx$ gave $b = .0513 \pm .0046$, or $\ell_y/2 + \lambda = 30.62 \text{ cm} \pm 2.75 \text{ cm}$. Since $\ell_y = 32.95 \text{ cm}$, we have $\lambda = 7.67 \text{ cm} \pm 2.75 \text{ cm}$. Inspection of the least squares fit to the plot of the flux distribution given in Fig. 6 shows that the cosine curve was actually shifted somewhat from the assumed center of the lattice. If all foils are translated slightly in the same direction, the error is considerably reduced. During the measurement such a displacement seems to have occurred. We prefer therefore to accept the above value of λ , but

with an error closer to about .5 cm. Thus

$$(21) \quad b_y^2 = 2.63 \times 10^{-3} \pm .043 \times 10^{-3} \text{ cm}^{-2}.$$

The flux values obtained from this traverse are listed in Table 4.

Also, b_z^2 can be obtained accurately enough from just the rod length (four feet) and this reflector savings; we have then

$$(22) \quad b_z^2 = .523 \times 10^{-3} \pm .004 \times 10^{-3} \text{ cm}^{-2}.$$

The thirteen foils which were used to measure b_x^2 seemed to separate into two groups, the symmetry of the triangular lattice cell in which a given foil was placed determining its group. Those in lattice cells with a vertex upward lay on a different curve from those with a vertex downward. These two curves seemed to have quite different end corrections, but apparently could be fitted by the same value of L' . Least squares fits to each of $A \exp[x/L'] - B \exp[-x/L']$ agreed fairly well with $L' = 100 \pm 30$ cm. The large error is caused by the presence of sizeable end corrections, the thickness of the lattice in the x direction being only 45.90 cm. The results of the x traverse are given in Table 5 and shown in Fig. 7. From the value of L' we have

$$(23) \quad b_x^2 = 10^{-4} \pm 10^{-5} \text{ cm}^{-2}.$$

Substituting the values from (20), (21), (22), and (23) in (19), we obtain

$$(24) \quad M_{\perp}^2 / M_{\parallel}^2 = 1.039.$$

An estimate of accuracy can be obtained by differentiating (19). Putting for the moment

$$\xi = M_{\perp}^2 / M_{\parallel}^2$$

we get

$$d\xi = \frac{dB_z^2 + db_z^2 - \xi(dB_x^2 + db_x^2 - db_y^2)}{B_1^2 + b_x^2 - b_y^2}$$

The uncertainties in the various values can be considered as random; the uncertainty in ξ is then given by

$$(25) \quad \Delta\xi = \frac{\left\{ (dB_z^2)^2 + (db_z^2)^2 - \xi^2 [(dB_1^2)^2 + (db_x^2)^2 + (db_y^2)^2] \right\}^{1/2}}{B_1^2 + b_x^2 - b_y^2}$$

Insertion of the uncertainties given in (20), (21), (22), and (23) gives

$$(26) \quad \Delta\xi \sim .028$$

The anisotropy indicated by (24) is just barely outside the error limits set by (26); we conclude that within experimental error it is negligible. Thus there appears to be too little anisotropy to influence the validity of the exponential experiment too greatly.

Another search for anisotropy could be made, based on the recasting of (19) in the form

$$(27) \quad \frac{k-1}{M_{||}^2} = \xi B^2 - B_z^2$$

with a three constant fit of ξ , λ , and $k-1/M_{||}^2$ to measurements of B_z^2 as a function of loading. The agreement of values of λ obtained from the two constant fit with the values obtained from radial traverses indicate though that such a procedure would also show negligible anisotropy.

Table 1.

Relaxation Lengths: L (cm)
In Water-Slightly-Enriched Uranium Lattices

<u>No. of Rods</u>	<u>L (1st run)</u>	<u>L (2nd run)</u>	<u>Residuals of L (cm)</u>	
			<u>1st run</u>	<u>2nd run</u>
<u>1.5:1 Lattice</u>				
271	19.559	19.793	-.156	.126
253	18.733	18.492	.042	-.132
235	17.722	17.723	.009	.093
217	16.892	16.564	.121	-.111
199	15.865	15.741	.006	-.012
181	15.154	14.896	.183	.040
163	14.143	13.997	.043	.020
145	13.200	13.129	-.037	.020
127	12.271	12.259	-.104	.014
109		11.282		-.094
91	10.435		-.191	
73		9.665		.095
55	8.801		.071	
<u>1.75:1 Lattice</u>				
253	21.935	22.168	.097	.161
235	20.287	20.391	-.026	-.057
217	18.811	18.757	-.105	-.268
199	17.666	17.744	.044	.034
181	16.285	16.623	-.125	.143
163	15.215	15.237	-.045	-.080
145	14.257	14.203	.097	-.002
127	13.118	13.062	.024	-.068
109	12.071	12.134	.023	.060
91	11.060	11.045	-.037	.019
73	10.012	10.030	.065	.068

Table 1. (Continued)

<u>No. of Rods</u>	<u>L (1st run)</u>	<u>L (2nd run)</u>	<u>Residuals of L (cm)</u>	
			<u>1st run</u>	<u>2nd run</u>
<u>2:1 Lattice</u>				
271		29.087		-.188
265	27.297		-.122	
253	25.955	26.539	.262	.282
235	23.424	23.947	-.010	.180
217	21.551	21.510	.080	-.135
199	19.631	19.612	-.100	-.180
181		18.252		.113
169	16.815		-.372	
163		16.781		.145
145	15.109	14.905	-.266	-.341
127	14.557	14.102	.453	.156
109	13.028		.145	
<u>2.5:1 Lattice</u>				
245	30.472	30.021	.352	-.011
235	27.800	28.091	-.312	-.097
217	24.737	25.062	-.332	.041
199	22.886	22.763	.333	-.032
181	20.330	20.688	-.073	-.037
163	18.761	18.444	.249	-.152
145	16.950	16.867	.137	.214
127	15.147	15.317	-.108	.154
109	13.793	13.932	.002	-.037
91	12.341	12.400	-.060	-.003
73	11.058	11.119	.022	-.045

Table 1. (Continued)

<u>No. of Rods</u>	<u>L (1st run)</u>	<u>L (2nd run)</u>	<u>Residuals of L (cm)</u>	
			<u>1st run</u>	<u>2nd run</u>
<u>3:1 Lattice</u>				
265	36.647	36.774	-.153	.300
253	33.592	33.437	.145	-.180
235	29.448	29.460	.016	-.199
217	26.152	26.296	-.104	-.280
199	23.754	23.669	.134	.145
181	21.577	21.419	.211	.139
163	19.547	19.438	.164	-.100
145	17.311	17.653	-.287	-.022
127	15.821	16.016	-.139	-.174
91	13.006		.051	
55	9.933		-.127	
<u>4:1 Lattice</u>				
267	24.845	25.521	-.347	-.021
253	24.396	24.333	.350	.017
		22.808		-.019
235	22.546	22.888	-.105	.062
217	21.315	21.390	-.001	-.027
199	20.342	20.150	.307	.076
181	19.019	18.580	.224	-.203
163	17.525	17.690	-.060	.160
		16.589		.284
		15.999		-.307
145	16.016	16.237	-.379	-.069
109		13.936		.049

Table 2.

<u>Results of Least Squares Fits to B^2 and λ</u>						
	<u>B^2 (cm⁻²)($\times 10^3$)</u>			<u>λ (cm)</u>		
<u>Lattice</u>	<u>1st Run</u>	<u>2nd Run</u>	<u>Average</u>	<u>1st Run</u>	<u>2nd Run</u>	<u>Average</u>
1.5:1	2.85 $\pm$.06	2.94 $\pm$.05	2.89 $\pm$.05	7.86 $\pm$.14	7.56 $\pm$.11	7.71 $\pm$.14
1.75:1	3.453 $\pm$.034	3.486 $\pm$.053	3.470 $\pm$.033	7.16 $\pm$.08	7.15 $\pm$.12	7.16 $\pm$.10
2:1	3.65 $\pm$.09	3.86 $\pm$.07	3.75 $\pm$.08	7.23 $\pm$.26	6.75 $\pm$.20	6.94 $\pm$.23
2.5:1	3.700 $\pm$.055	3.647 $\pm$.025	3.673 $\pm$.048	6.81 $\pm$.17	6.99 $\pm$.09	6.90 $\pm$.16
3:1	3.304 $\pm$.028	3.271 $\pm$.006	3.288 $\pm$.018	6.82 $\pm$.11	7.05 $\pm$.09	6.94 $\pm$.11
4:1	1.76 $\pm$.10	1.88 $\pm$.04	1.86 $\pm$.06 *	6.83 $\pm$.52	6.37 $\pm$.19	6.42 $\pm$.22

*Averages for this lattice are weighted because one measurement is much poorer than the other.

Table 3.

<u>Lattice</u>	<u>Cadmium Ratios</u>	
	<u>Gold Cadmium Ratios</u>	<u>Indium Cadmium Ratios</u>
1.5:1	1.918 $\pm$.052	2.576 $\pm$.014
2:1	2.292 $\pm$.038	3.044 $\pm$.16
3:1	2.884 $\pm$.025	4.30 $\pm$.033
4:1	3.307 $\pm$.045	5.742 $\pm$.092

Table 4.

Neutron Flux Measurements from y-Traverse

<u>Distance From Assumed Center (cm)</u>	<u>Measured Foil Activity (units arbitrary)</u>	<u>Least Squares * Fits Residuals (ΔActivity)</u>
-15.776	583.3	17.9
-10.039	718.1	4.6
- 4.303	798.3	- 1.8
1.434	821.2	3.5
7.171	760.2	- 4.7
12.908	634.9	-11.6
18.645	464.7	- 8.0

* The residuals are those resulting from fitting column 2 to $A \cos Bx$. The best values, on which the residuals are based, are

$$A = 819.9 \pm 28.0$$

$$B = .0513 \pm .0046 \text{ cm}^{-1}$$

Table 5.

Neutron Flux Measurements from x-Traversal

	Distance From First Foil <u>(cm)</u>	Measured Foil Activity <u>(arbitrary units)</u>	Least Squares Fit Residuals <u>(Δ Activity)</u>
GROUP 1	0.0	1160	- 8*
	4.970	1028	5
	9.940	882.5	2.7
	14.910	758.3	21.6
	19.880	570.5	23.5
	24.850	446.9	- 4.5
	29.820	316.6	6.9
GROUP 2	0.0	1107	-15
	4.970	962.9	20.4
	9.940	Lost	
	14.910	652.1	.7
	19.880	490.2	- 9.2
	24.850	350.7	2.9

* Column 2 was fitted to $A e^{-ax} + B e^{+ax}$. Although the scattering of the few points made best values somewhat uncertain, these are about

	A	B	$a(\text{cm}^{-1})$
GROUP 1	2083	-911.5	.010
GROUP 2	2162	-1041	.010

IV. Migration Area Measurements.

A. Introduction.

The values of the migration area are based on measurements of the buckling in lattices with boron poisoned water, as a function of the boron concentration. The theoretical basis for the measurement is discussed in the next section.

B. Theory.

We may express k_{∞} in two well-known ways, either by the critical equation:

$$(1) \quad k_{\infty} = 1 + B^2(\tau + L^2)$$

or by the four-factor formula

$$(2) \quad k_{\infty} = f \eta \epsilon p$$

B^2 is the buckling, τ the age to thermal of fission neutrons, L^2 the thermal diffusion area, so $\tau + L^2 = M^2$ is the migration area; f is the thermal utilization, η the number of neutrons captured in uranium which cause fission (per neutron cycle), ϵ is the fast fission factor, and p is the resonance escape probability.

If the moderator is made more neutron absorbing by means of the addition of a poison, those quantities in (1) and (2) which will be changed are k_{∞} , B^2 , f , and L^2 , the others remaining constant; the main effect of the poison is to decrease f . Hence, if the measured values of B^2 are plotted against f , a straight line should be obtained, the slope of which gives M^2 . For f , theoretical values calculated by the Kidde group, were used to get preliminary values of M^2 . These will eventually be corrected by the use of "experimental" values of f obtained from measurements of intracell flux distributions.

C. Experimental Methods.

A typical measurement of the migration area involved dissolving successively increasing amounts of B_2O_3 in the water, and measuring the values of the buckling of a lattice in these poisoned moderators. At each stage the boron concentration was determined by the analytical group in the Brookhaven chemistry department. The bucklings were measured at three different boron concentrations for each lattice. When the measurements with unpoisoned water are included, the dependence of B^2 on boron concentration is thus found at four values of the boron concentration for each water-to-metal volume ratio.

The buckling was found by measuring the axial relaxation length as a function of the number of rods loaded, a method described earlier. For each poisoned lattice, measurements were made at only five loadings, because of the large number of bucklings which had to be measured in a short time.

Thermal neutron fluxes were measured with foils of five mil thick indium, .220 inches in diameter. The techniques used to position foils in the lattice and to measure their activities are described in Part III of this report.

Because of the possibility that boron might plate out of solution and onto the aluminum cladding of the rods, tests of reproducibility were made when possible. These consisted of measurements of the relaxation length in an unpoisoned lattice, and comparison of the buckling it indicated with previous measurements. The results of these test axials are given in Appendix 1. They lead to the conclusion that no important change in reactivity of the lattice occurred during the buckling and migration area measurements.

An earlier effort to measure the migration area by another means was unsuccessful. In this attempt, the uranium rods were covered with cadmium sleeves, to suppress neutron multiplication. A measurement of the thermal neutron flux distribution due to a known fission source was to have yielded the migration area. Low values of the flux from the fission source kept this measurement from being successful. This method of determining the migration area was used at Oak Ridge in an earlier investigation of this reactor type. The Oak Ridge results differ significantly from ours, particularly at the lower water-to-metal ratios. These discrepancies will be discussed in a later section.

D. Analysis of Data and Results.

The values of the relaxation lengths, as found from least squares fits to the axial flux measurements, are given in Table 6. Also given are the residuals resulting from least squares fitting to the buckling and reflector savings. The best values of B^2 and λ are listed in Table 7. The boron concentrations, calculated values of f , measured values of B^2 , and deduced values of M^2 are listed in Table 8, for water-to-metal volume ratios of 1.5:1, 2:1, 3:1, and 4:1. The values of B^2 when plotted against f for a given lattice yielded good straight lines; the value of M^2 was obtained by a least squares fit. A typical plot of B^2 against f is shown in Fig. 8. Also shown in Table 9 are the values of k indicated by the values of M^2 and B^2 for the four lattices.

The values of migration area and k_{∞} given by this report must be considered as tentative, because they are based on calculated values of f . There is some uncertainty about the results of the calculations because of the simplifying

assumptions about the geometry, the angular distribution of the flux, and the cross-sections. The value of the migration area as obtained from a poisoned lattice experiment is quite sensitive to the values of f used and it appears that accurate measured values of f must be obtained before the migration areas reported here can be made trustworthy. A preliminary estimate of the "experimental" values of f for the unpoisoned lattices is given in section VI of this report. These values differ significantly from the values provided by Kidde, and theoretical work on this problem is in progress at BNL. For these reasons, the errors cited with the values of the migration area must be considered to represent only measures of the internal consistency of the experiments and the possibility of a systematic error must be borne in mind.

The most striking result of these experiments is therefore the apparent constancy of M^2 over the range of values of the water-to-metal ratio studied. This result is quite different from that predicted by present theory, as can be seen from Fig. 9. When calculated values of L^2 are subtracted from M^2 (and L^2 is small in these lattices), the age is obtained. The resulting ages are plotted against water-to-metal volume ratios in Fig. 9, and compared with the theory of Soodak and Forman.

Table 6.

Measured Values of Axial Relaxation Length for Various Boron Concentrations.

<u>Number of Rods</u>	<u>Relaxation Length L(cm)</u>	<u>L Residual (cm)</u>
<u>1.5:1 Lattice, .216 Boron atoms/10³ water molecules</u>		
262	16.898	-.023
229	15.686	.063
193	14.213	-.011
157	12.735	-.085
121	11.430	.056
<u>1.5:1 Lattice, .563 Boron atoms/10³ water molecules</u>		
263	14.960	.101
229	13.879	-.054
193	12.823	-.090
157	11.710	-.124
121	10.828	.160
<u>1.5:1 Lattice, .860 Boron atoms/10³ water molecules</u>		
263	13.677	-.002
229	12.976	.020
211	12.538	-.015
193	12.121	-.013
175	11.710	.013
<u>2:1 Lattice, .359 Boron atoms/10³ water molecules</u>		
265	18.744	.073
229	16.750	-.254
193	15.288	.438
157	13.597	-.181
121	12.085	-.059
<u>2:1 Lattice, .590 Boron atoms/10³ water molecules</u>		
265	16.367	.061
229	14.867	-.261
193	13.899	.392
157	12.568	-.090
121	11.205	-.108

Table 6. (Continued)

<u>Number of Rods</u>	<u>Relaxation Length L(cm)</u>	<u>L Residual (cm)</u>
<u>2:1 Lattice, .824 Boron atoms/10³ water molecules</u>		
265	14.700	-.003
229	13.902	.064
193	12.862	-.048
157	11.848	-.057
121	10.850	.061
<u>3:1 Lattice, .174 Boron atoms/10³ water molecules</u>		
263	22.519	.084
229	19.978	-.131
193	17.794	-.041
157	15.767	.086
121	13.579	.005
<u>3:1 Lattice, .345 Boron atoms/10³ water molecules</u>		
263	18.128	-.043
229	16.825	-.001
193	15.467	.089
157	13.910	.028
121	12.225	-.074
<u>3:1 Lattice, .512 Boron atoms/10³ water molecules</u>		
263	15.659	.036
229	14.856	-.063
193	13.874	.013
157	12.794	.024
121	11.582	-.008
<u>4:1 Lattice, .074 Boron atoms/10³ water molecules</u>		
265	21.287	-.005
193	17.790	.069
157	16.010	.101
121	13.939	-.034

Table 6. (Continued)

<u>Number of Rods</u>	<u>Relaxation Length L(cm)</u>	<u>L Residual (cm)</u>
-----------------------	--------------------------------	------------------------

4:1 Lattice, .146 Boron atoms/10³ water molecules

265	18.895	-.008
229	17.624	.053
193	16.203	.022
157	14.571	-.136
121	13.202	.091

4:1 Lattice, .218 Boron atoms/10³ water molecules

265	17.097	-.020
229	16.187	.064
193	15.045	-.001
157	13.745	-.114
121	12.595	.073

Table 7.Buckling and Reflector Savings as a Function of Boron Concentration

<u>Lattice</u>	<u>Boron Concentration</u> <u>(B atoms/10³ H₂O molecules)</u>	<u>B²(cm⁻²) x 10³</u>	<u>λ(cm)</u>
1.5:1	.216	2.164 ± .082	7.61 ± .18
1.5:1	.563	1.107 ± .217	7.61 ± .46
1.5:1	.860	.243 ± .05	7.75 ± .12
2:1	.359	2.045 ± .255	7.45 ± .67
2:1	.590	1.272 ± .314	7.11 ± .78
2:1	.824	.326 ± .094	7.32 ± .22
3:1	.174	2.086 ± .052	6.79 ± .20
3:1	.345	1.137 ± .057	6.37 ± .19
3:1	.512	.019 ± .049	6.59 ± .17
4:1	.074	1.103 ± .050	7.16 ± .24
4:1	.146	.596 ± .067	6.60 ± .30
4:1	.218	-.054 ± .070	6.81 ± .32

Table 8.

Values of the Migration Area Determined by the Poisoned-Lattice Method

<u>Lattice</u>	<u>Boron Concentration (atoms per 1000 molecules of water)</u>	<u>Thermal Utilization (Theoretical)</u>	<u>Buckling ($\text{cm}^{-2} \times 10^3$: experimental)</u>	<u>Migration Area (cm^2)</u>
1.5:1	0	0.917	2.86	28.99 $\pm$ 0.37
	0.216	.900	2.20	
	.563	.874	1.19	
	.860	.852	0.33	
2:1	0	0.889	3.75	30.06 $\pm$ 1.21
	0.359	.851	2.05	
	.590	.828	1.27	
	.824	.808	0.33	
3:1	0	0.828	3.29	28.69 $\pm$ 1.23
	0.174	.803	2.09	
	.345	.779	1.14	
	.512	.758	0.02	
4:1	0	0.774	1.86	28.47 $\pm$ 1.43
	0.074	.761	1.10	
	.146	.748	0.60	
	.218	.735	-0.05	

Table 9.

Preliminary Values of k_{∞}

<u>Lattice</u>	<u>Buckling</u> <u>($\text{cm}^{-2} \times 10^3$)</u>	<u>Migration Area</u> <u>(cm^2)</u>	<u>k_{∞}</u>
1.5:1	2.86	29.0	1.083
2:1	3.75	30.1	1.113
3:1	3.29	28.7	1.094
4:1	1.86	28.5	1.053

APPENDIX 1

Results of Test of Axial Measurements of Reproducibility

The poisoned lattice measurements were performed on the 2:1, 4:1, 1.5:1, and 3:1 lattices, in that order.

Immediately after the 2:1 poisoned lattice measurements, a single axial measurement was carried out with the 4:1 lattice and pure water. Use of the measured relaxation length and the reflector savings from reference 1 gave a value of $B^2 = 1.816 \times 10^{-3} \text{ cm}^{-2}$, compared with the previously measured best value of 1.79×10^{-3} .

Immediately after the 1.5:1 poisoned lattice measurement, a single clean axial was measured with the 3:1 lattice. The value of B^2 thus indicated was 3.23×10^{-3} , compared with the best value¹ of 3.287×10^{-3} .

Just after the 3:1 poisoned lattice runs, a clean axial flux measurement was made with the 2:1 lattice. This gave $B^2 = 3.62 \times 10^{-3}$, compared with the best value¹ of 3.75×10^{-3} .

Within the accuracy of the measurements, these results imply reproducibility of the data throughout the course of buckling and migration area measurements.

VI. Intracell Flux Traverses.

A. Introduction.

The values of the migration area we have reported earlier depend on calculated values of the thermal utilization f . Since there is some uncertainty about the theoretical methods used to find these values, an attempt at measuring them seems useful. We have used the direct approach of measuring the thermal neutron flux distribution in the water and in the fuel rods; thus the values of f we obtain are still uncertain by an amount depending on the cross sections used and hence on the assumed neutron temperature.

Evaluation of f of course also has motivation from the desire to check experimentally the methods used to calculate k through the four-factor formula.

B. Experimental Procedure.

Intracell flux distributions were measured for 1.5:1, 2:1, 3:1, and 4:1 water-to-metal volume ratios. These are actually ratios of water plus aluminum to uranium. Because these lattices are so tightly packed, it was necessary to use very small detector foils, not only in the rods but also in the water, if any detail in the flux distribution was to be obtained.

The foils used were 1.5 millimeter diameter discs of lucite containing dysprosium oxide, about .5 millimeters thick. Dysprosium was considered ideal as a detector, because its lack of low energy resonances makes possible the measurement of thermal fluxes without the need for making cadmium difference measurements. Foils of this type have been used by the reactor physics group at Argonne, with considerable success.

The foils were punched out of a small sheet made in a hot press from a powder mixture of lucite polymer and dysprosium oxide. Because the activity desired ($\tau^{1/2} \sim 140$ minutes) was not reported in the literature with an accuracy sufficient for our purposes, a measurement of the half-life was carried out. The procedure and result, $\tau^{1/2} = 139.17 \pm .14$ minutes, are reported as a letter to the editor in the Physical Review. During the course of this measurement, it was found that no other activity which would interfere with the one desired was present in detectable amounts. Mass spectrographic analysis of the dysprosium showed only trace impurities, due mostly to other rare earths.

Flux traverses in the uranium were made using a split fuel rod, with nine holes of about 1.6 millimeter milled on each of two diameters. A drawing of a cross section of this rod is shown in Fig. 10. From the figure it is seen that the foils are quite close together compared to their diameters. They are replacing a medium (uranium) with nearly the same absorption, however, and so the influence of neighboring foils on each other is small.

The use of finite-sized foils of course introduces an error arising from the fact that the flux is not constant over their area. For the foil sizes used and the fluxes observed, it can be shown that the observed activity can be attributed to the flux at the foil center, with an error of at most .2 per cent. Since corrections for this effect would be small compared to errors in measurement, they have not been made.

The foils used in the water were positioned in a piece of lucite which ran through the lattice. The horizontal positions of these foils relative to those in the fuel rod were known to be within about five mils (.013 cm).

Foils in the uranium were not placed at the same vertical level as those in the water. The difference in elevation was measured, however, and the known axial relaxation length made it possible to correct the observed activities to those they would have had if they had been at the same height. The error introduced by this procedure is considered negligible.

Each foil used was counted to about seven thousand counts in each of six end window counters. Thus there was no need to make counter efficiency corrections, the activity of a given foil being determined from the total counts observed in all counters.

In all, six complete intracell flux traverses were measured, one in the 1.5:1, one in the 2:1, two in the 3:1, and two in the 4:1 lattices. An additional partial traverse was also obtained (in the metal) in the 1.5:1 lattice.

C. Analysis of Data and Results.

The experimental data obtained from the intracell flux traverses are shown in Figs. 11 - 16. In view of the triangular symmetry of the lattice the neutron flux in the moderator was measured along two lines, the first joining rod centers and the second along the median of the triangle. The most complete moderator data was obtained at the higher moderator to fuel volume ratios. Only a few points could be obtained at the 1.5:1 volume ratio because of the tightness of the lattice.

The flux data in the fuel rod were first fitted to the Bessel function $I_0(K_0 r)$ where r is the radial position of the foil from the rod center and $K_0 = 1/L_0$ is the reciprocal diffusion length. The function $I_0(K_0 r)$ represents the asymptotic form of the flux distribution in the fuel, but in order to obtain a

good fit of the flux data, a value of K_0 considerably higher than the theoretical value must be assumed. The results of the least square fit of the data are given in Table 10 below.

Table 10.

Bessel Function Fit of Flux Traverse in the Uranium Rod

<u>Vol. Ratio</u>	<u>K_0 (in⁻¹)</u>	<u>L_0 (cm)</u>	<u>F</u>
1.5:1	2.970	0.855 $\pm$ 4.7 per cent	1.147
1.5:1	2.818	0.901 $\pm$ 6.0 per cent	1.133
2:1	2.585	0.982 $\pm$ 8.0 per cent	1.113
3:1	2.659	0.955 $\pm$ 5.4 per cent	1.119
3:1	2.644	0.961 $\pm$ 7.0 per cent	1.118
4:1	2.988	0.850 $\pm$ 5.4 per cent	1.149
4:1	3.152	0.806 $\pm$ 2.9 per cent	1.165

The ratio F of the flux at the surface of the fuel rod to the average flux in the rod is given in the final column of Table 10. The values do not vary significantly with moderator to fuel volume ratio. The average value of $F = 1.135$ which corresponds to $K_0 = 1.116 \text{ cm}^{-1}$ was used in plotting the flux curve $I_0(K_0 r)$ for the fuel rod in Figs. 11 - 16.

The values of F obtained from the BNL experiments are much higher than would be expected from elementary diffusion theory. These results are in conformity with those obtained with uranium rods at other laboratories, notably at North American Aviation. Kidde has estimated the thermal utilization of our water lattices (HKF-1492D-151, Mar. 4, 1952) by the use of an elementary diffusion theory formula with the value of F corrected by means of a spherical harmonics calculation. Their estimates are given in Table 11.

Table 11.

<u>Vol. Ratio</u>	<u>f</u>
1.5:1	0.917
2:1	0.889
3:1	0.828
4:1	0.775

These values of f were used in the preliminary estimates of the migration area (Section V of this report).

The theoretical flux curves in the water obtained by the use of elementary diffusion theory are shown in Figs. 11 - 16, corresponding to a value of $F = 1.135$. The lower curve in each figure corresponds to the currently accepted value of $L_1 = 2.85$ cm for the diffusion length in water. On the basis of these flux distributions the relative absorption of thermal neutrons in fuel, cladding and moderator is shown in Table 12.

Table 12.

Thermal Neutron Absorption in a Unit Cell

<u>Vol.</u>			
<u>Ratio</u>	<u>f</u>	<u>f_{al}</u>	<u>f_{mod}</u>
1.5:1	.9169	.0050	.0781
2:1	.8873	.0048	.1079
3:1	.8298	.0045	.1657
4:1	.7759	.0042	.2199

Although the values of f agree with the Kidde estimates it may be seen that the theory greatly underestimates the neutron flux in the moderator. The reason for this discrepancy is clear from the experimental flux data. Extrapolation of these data to the moderator fuel interface would indicate a discontinuity in flux in this region. The apparent discontinuity arises from the neglect of the non-asymptotic solutions of the transport equation which are important near such an interface.

The required refinements of reactor lattice theory are not within the scope of this report. However, to obtain a better fit of the experimental neutron flux curves in the moderator and hence more realistic estimates of the thermal utilization, we need to modify the value of the diffusion length L_1 in the moderator. The modified diffusion theory curves are also shown in Figs. 11 - 16. The corresponding values of the thermal utilization are given in Table 13.

Table 13.

Modified Values of the Thermal Utilization

<u>Vol. Ratio</u>	<u>f</u>
1.5:1	.910
2:1	.871
3:1	.819
4:1	.755

These values of f are preliminary and are being refined by a more exact analysis.

VII. The Effect of Absorber Rods.

The effectiveness of a central absorber rod in lattices of the type under investigation was studied in another set of experiments. Cadmium tubes of different diameters were tested; the change in the axial relaxation length caused by replacing the central uranium rod by an absorber rod was measured. Hollow cadmium rods of different diameter were used; the wall thickness was 0.056 inch in each case. In most of the experiments, the rod was filled with water; in one case the rod contained a steel cylinder. The fractional change of the axial buckling was also calculated from the relaxation lengths.

To test the effectiveness of an absorber rod as compared to that of the removal of a fuel rod, the central fuel rod in the 3:1 lattice was removed, and the change in relaxation length measured. The value of the relaxation length was 32.32 cm with 254 rods in the lattice; when the central rod was removed, the relaxation length decreased to 32.25 cm, or a decrease of 0.07 cm. This decrease corresponded to a fractional decrease in the axial buckling of 0.0022 ± 0.0017 , or about 0.2 per cent. In the other experiments, the change in relaxation length corresponds to the replacement of the central fuel rod by the absorber rod.

The results of the experiments are given in Table 14. Estimates of the precision has been shown in two cases, which seemed to be typical. No attempt has been made at BNL to treat the data from a theoretical standpoint.

Table 14.

The Effect of Absorber Rods on the Axial Relaxation Length

<u>Lattice</u>	<u>Rod Loading</u>	<u>Outer Diameter of Cadmium Tube (in)</u>	<u>Type of Filling</u>	<u>Relaxation Length: L(cm)</u>	<u>Fractional Change in the Axial Buckling</u>
1.5:1	255	No tube	---	18.96	0.253
		0.81	H ₂ O	16.93	
2:1	263	No tube	---	26.89	0.335 ± 0.010
		0.95	H ₂ O	23.27	
3:1	254	No tube	---	32.32	0.461
		0.81	H ₂ O	26.77	
	253	No tube	---	32.25	0.439
		0.65	H ₂ O	26.88	
4:1	253	No tube	---	23.15	0.203
		0.95	H ₂ O	21.11	
		0.81	H ₂ O	21.71	
	263	No tube	---	25.18	0.2522 ± 0.0042
		0.75	Steel	22.40	

VIII. The Effect on the Axial Relaxation Length of a "Gap" in the Lattice.

In the design considered by the Walter Kidde Co. for a power-plutonium reactor, the lattice is to be divided into several groups of rods. When the fuel is ready for processing, the sub assemblies are to be separated before being removed from the reactor. In this case, it is desirable to know what change in reactivity may be expected when two sub-assemblies are separated, and a series of experiments was made in order to obtain some information of this kind. In these experiments, the relaxation lengths were measured for two different arrangements of 160 rods; this was done for each of the four usual lattices. In the first arrangement, the 160 rods were in a rectangle with 16 rows, each containing 10 rods. To form the second arrangement, the rods in one of the two central rows were removed and placed at the end of the rectangle. This resulted in 2 rectangles each of 8 rows with 10 rods per row, the two rectangles being separated by one row without rods. The second lattice may be regarded as a split lattice separated by a water gap. The axial relaxation length was measured for each arrangement; the results are listed in Table 15. The fractional change in the axial buckling is plotted against water-to-metal ratio in Fig. 17. The fractional change in the axial buckling is defined as

$$\left(\frac{1}{L_2^2} - \frac{1}{L_1^2} \right) \div \frac{1}{L_1^2} ,$$

where the subscripts refer to the two arrangements.

Table 15Effect of a Central Gap on the Relaxation Length

<u>Lattice</u>	<u>Arrangement</u>	<u>Relaxation Length (cm)</u>	<u>Fractional Change in in Axial Buckling</u>
1.5:1	No Gap	13.26	0.0367 (Gain in Reactivity)
	Gap	13.51	
2:1	No Gap	15.70	0.124 (Loss)
	Gap	14.80	
3:1	No Gap	17.73	0.331 (Loss)
	Gap	15.37	
4:1	No Gap	16.48	0.354 (Loss)
	Gap	14.16	

In three of the four lattices, the effect of the separation was to decrease the relaxation length (decrease the reactivity). In the tightest lattice, 1.5:1, there was a slight, about 2 per cent, increase in relaxation length, indicating that the separation caused a slight increase in reactivity.

Figure 1 - Picture of Tank. Negative No. 8-215-2

The inner tank with a tube plate and fuel rods.

-64-

FIG. 3

FIG. 4

FIG. 5

FIG. 6

FIG. 7

FIG. 8

FIG. 9

FOIL POSITIONS
FOR INTRA CELL FLUX TRAVERSE

FIG. 10

INTRACELL FLUX DISTRIBUTIONS OF WATER-URANIUM LATTICES

FIG. II

INTRACELL FLUX DISTRIBUTIONS OF WATER-URANIUM LATTICES

FIG. 12

INTRACELL FLUX DISTRIBUTIONS OF WATER-URANIUM LATTICES
RUN NO.1

FIG. 13

INTRACELL FLUX DISTRIBUTIONS OF WATER-URANIUM LATTICES
RUN NO. 2

FIG. 14

INTRACELL FLUX DISTRIBUTIONS OF WATER-URANIUM LATTICES
RUN NO.1

FIG. 15

INTRACELL FLUX DISTRIBUTIONS OF WATER-URANIUM LATTICES
RUN NO. 2

FIG. 16

FIG. 17