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A STUDY OF PLANE STRESS FRACTURE
IN THE LARGE-SCALE PLASTIC YIELDING REGIME

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A STUDY OF PLANE STRESS FRACTURE IN THE
LARGE-SCALE PLASTIC YIELDING REGIME*

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ABSTRACT: Three independent methods for measuring plane stress fracture resistance in the large-scale plastic yielding regime were tested and were found to give equivalent results. The three methods were the J integral technique, a method based on the increase in compliance that accompanies the development of a plastic zone, and a method based on the direct measurement of the plastic work that must be done to cause fracture.

KEY WORDS: plane stress fracture, fracture testing, nonlinear fracture analysis

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LIST OF SYMBOLS

J	J integral value or pseudo strain energy release rate, $J \cdot m^{-2}$.
J_{crit}	Value of J at fracture.
$\mathcal{G}$	Strain energy release rate, $J \cdot m^{-2}$.
$\mathcal{G}_{crit}$	Value of $\mathcal{G}$ at fracture.
σ	Remotely applied gross area stress, MPa.
F	Applied force, N.
Δ	Displacement measured remote from crack plane along the line of loading, mm.
v	Displacement measured at crack mouth, mm.
U	Pseudo potential energy or work done in loading a specimen to a given displacement, J.
a	Crack length, mm.
h	Specimen thickness, mm.
W	Specimen width, mm.
E	Modulus of elasticity, MPa.
σ_{ys}	Uniaxial tensile yield strength, MPa.
δ	Displacement of crack surfaces, mm.
δ_t	Displacement of crack surfaces at crack tip.
δ_t^*	Value of δ_t at fracture.
$\sigma(\delta)$	Flow strength of the metal within the plastic zone as a function of the separation of imaginary crack surfaces extending through the zone, MPa.
Y	Same as $\sigma(\delta)$, but assumed independent of δ .
r_y	Irwin estimate of plastic zone size, mm.
ρ	Plastic zone length from Dugdale model, mm.

INTRODUCTION

In their original development of the J integral approach to fracture testing, Begley and Landes [1] experimentally verified the significance of critical J by showing that it was equal, within limits imposed by variation in materials, to a critical σ found from large linear elastic specimens of the same material. Such comparisons are not always possible, either because of the absence of linear-elastic data or because extremely large specimens would be required to develop the linear-elastic data. In the case of very tough materials that fail in plane stress, for example, the specimen size required for macroscopic linear-elastic behavior can be immense, and such specimens are all but impossible to acquire or test. It was considered desirable, therefore, to examine other independent approaches to nonlinear analysis of plane stress fracture so that the veracity of critical J results might be tested experimentally in the absence of linear-elastic data. The two new approaches to nonlinear fracture analysis reported in the present paper are a compliance technique and a plastic work method. The results from these are described, and they are compared to the results from critical J tests.

EXPERIMENTAL DETAILS

Material

Thin, cross-rolled-ingot sheet beryllium was chosen for the present study because this material can exhibit large

nonlinear deflections in the absence of stable crack growth. Thus, the complexities that attend the presence of stable crack growth were avoided. A rather thin sheet (0.80 mm thick) was chosen for the study because it was believed that fracture would occur in plane stress in this thin section. The beryllium sheet was annealed at 780°C for 1 h after warm cross-rolling. This resulted in a fully recrystallized microstructure and an intercept grain size of 49 μm . The material was found to have a high degree of in-plane isotropy as indicated by the basal plane pole figure given in the Appendix (see Fig. A-1). The chemical composition of the sheet is also recorded in the Appendix.

The uniaxial, engineering tensile properties were determined; the averaged results from two specimens were 151 MPa yield strength, 302 MPa ultimate stress, and 7.8% elongation.

Fracture Resistance Test Procedures

Specimen Types — Single-edge notched (SEN) specimens of two different sizes were used in the present study. The SEN specimens were 31.75 mm and 95.25 mm wide. Other dimensions were in accord with suggested geometry [2]. Machined edge slots in the specimens were extended, using electrical discharge machining (EDM), by a minimum of 1.3 mm to produce a relatively small slot root diameter (~ 0.12 mm). Fatigue cracks were extended from the EDM slots using procedures described below. Four small SEN specimens were prepared with edge slots of different lengths (11.43, 12.70, 13.97, and

15.87 mm — measured to the end of the EDM slot) to facilitate J integral testing. The edge notch length to the end of the EDM slot in the large SEN specimen was 34.29 mm.

Two double-edge notched (DEN) specimens were also tested. These were prepared by using EDM to introduce a slot opposite the edge notch in the small SEN-type specimen described above. The lengths of the two edge slots were adjusted so that a net ligment, 4.57 mm wide, remained centered on the specimen axis. The reasons for this particular specimen design are discussed later in the section on "Results and Discussion."

Fatigue Precracking — All fracture test specimens were fatigue precracked using a 200-kN capacity electro-hydraulic machine in tension-tension cycling to develop a sharp crack front. In the case of the SEN specimens cycling was carried out at a calculated stress intensity range of 2 to 15 MPa·m^{1/2}. The relation between stress intensity and specimen geometry given by the calibration equation on page 12 in Ref. 3 was used for the calculation. The fatigue cycling caused between 0.13 and 0.25 mm of fatigue crack growth in fewer than 10⁵ cycles. The unusual design of the DEN specimens, however, made it necessary to fatigue precrack above the general yield stress. Reasons for this are discussed further in the "Results and Discussion" section.

Crack Mouth Displacement — In all cases except the J integral tests, the displacement in the fracture test was measured at the crack mouth. The crack mouth displacement

gage that was used is described in Ref. 3 on pages 35-37. Briefly, the displacement is measured by following the output of a foil resistance strain gage bridge mounted on the arms of the displacement gage. The output of the gage is calibrated in a fixture incorporating a barrel micrometer. The sensitivity of the method is somewhat better than 0.0025 mm.

Load Point Displacement — In the case of the J integral tests, it was necessary to measure the displacement along the line of loading in order to obtain a well-defined energy input to the specimen. To accomplish this the beam gage method illustrated in Fig. 1 was used. Figure 1 shows the gage sprung between two stops cemented onto the specimen. The gage parameters were designed to maximize linearity and sensitivity over the range of 0.25 mm displacement. The displacement is measured by following the output of foil resistance strain gages bonded to the mid-section of the beam gage. The length over which the displacements were measured was chosen equal to twice the specimen width (63.5 mm) because the work by Srawley, Jones, and Gross [4] indicated that the rate of change of compliance with crack length is independent of gage length for gage lengths of $2 \times$ width and greater (for fully elastic SEN specimens). Two foil resistance gages were mounted on each side of the gage to form a full bridge. The outputs of two such gages on each side of the specimen were averaged to obtain the load point displacement. The sensitivity of the method is better than 0.002 mm.

Fracture Test Procedures - All of the fracture tests were carried out at a constant cross head velocity of $2 \times 10^{-6} \text{ m} \cdot \text{s}^{-1}$. The J integral tests were carried out on a screw driven, 600-kN-capacity machine, and the other tests were carried out on a 200-kN electro-hydraulic machine. Force and displacement were recorded over the course of the test.

In preliminary studies it was established that no detectable stable growth occurred prior to fast fracture in the present ingot sheet in the 780°C condition of heat treatment. This was concluded following microscopic examination of the crack tip during the course of the test. The absence of stable growth was confirmed by a later test in which the output of a strain gage mounted at the tip of the precrack indicated a gradual increase in strain as force increased with no indication of cracking prior to unstable fracture. It was not necessary, therefore, to develop a sophisticated technique to monitor stable crack extension. Visual observation was used during each test, however, to verify that no observable stable growth had occurred.

RESULTS AND DISCUSSION

Nonlinear Methods of Fracture Analysis

J Integral Method - The approach to the J integral tests taken in the present study followed that first

described by Begley and Landes [1]. The force-displacement behavior for four small SEN specimens, each containing a different crack length, is given in Fig. 2. The J integral values corresponding to the three shortest crack lengths were determined for a number of different displacements. To do this, the area between adjacent curves was measured with a planimeter to obtain a value for the pseudo energy release, dU , for various displacements. This energy value was then divided by the difference in crack length, da , and thickness to obtain a value for J (a pseudo energy release rate) at each displacement. The resulting J-vs-displacement relationships for three different crack lengths are given in Fig. 3. The critical J value is obtained at the point of fracture, and the values corresponding to three crack lengths are indicated in Fig. 3 (6160, 5510, and $5420 \text{ J}\cdot\text{m}^{-2}$). These values are considered to be within normally expected material variation.

Begley and Landes found that the value of J_{crit} for the steel they investigated was the same within material variation as a linear elastic value of σ_{crit} determined for the same steel by a different investigator using much larger specimens. Begley and Landes thus concluded that the J approach to nonlinear analysis yielded a result equivalent to a linear-elastic test. The mathematical basis for the J test was fully discussed by Begley and Landes and is not repeated here. Suffice it to say that

J describes the elastic-plastic stress field in the same way that σ describes the elastic field in a linear-elastic situation. The values of J_{crit} and σ_{crit} are measures of the plastic work done at the crack tip up to the point of fracture.

Regley and Landes pointed out that minimum specimen dimensions (including remaining ligament and crack length) required for a valid J result are determined by the dimensions over which the J parameter describes the crack tip stress field. It is necessary, therefore, to verify the validity of the J result by independent methods until sufficient experience and/or theoretical understanding is acquired to allow minimum specimen size requirements to be defined. In the present instance, two other independent, nonlinear methods were investigated for comparison with the J_{crit} results.

Compliance Method - Using the expression [5] that gives an approximate dimension of the plane-stress plastic zone,

$$r_y \approx \frac{1}{2\pi} \left(\frac{E\sigma}{\sigma_{ys}^2} \right), \quad (1)$$

and assuming that $\sigma_{crit} = J_{crit}$, a zone size of 11 mm is obtained at fracture for the present material. Since this is a very large multiple of the thickness of the present sheet (0.80 mm thick), it is reasonable to conclude that fracture occurs in plane stress. This makes it possible to test a second method of nonlinear analysis based in part on the Dugdale plane-stress model.

Records of force vs crack mouth displacement for both the large and small SEN specimens are given in Fig. 4. A method of analyzing these records is obtained if it is assumed that all of the deviation from linearity is due to the development of a plastic zone ahead of the crack tip. This is reasonable, since no stable crack growth was observed. If it is assumed that the development of a plastic zone ahead of the crack tip has the same effect on compliance as an amount of crack extension of equal length, then the plastic zone length can be deduced from the apparent compliance at the point of fracture and a prior correlation between compliance and a/W . As noted in Fig. 4, the apparent compliance is deduced from the reciprocal slope of the line drawn through the fracture point and the origin. A value of compliance compensated for modulus (taken as $290 \times 10^9 \text{ Nm}^{-2}$ for beryllium) and thickness (0.80 mm) is then computed to give a value for E_{vh}/F . A value for $(a + \rho_{app})/W$ (where ρ_{app} is the apparent length of the plastic zone) is found from the correlation between E_{vh}/F and a/W given in Ref. 3 on page 37. The apparent plastic zone lengths at fracture found using this method are 10.7 mm for the large SEN and 6.08 mm for the small SEN. It is possible to compute the remote applied stress required to develop these same plastic zone lengths in infinite specimens if it is assumed that the Dugdale [6] elastic-plastic model for plane stress is applicable. This model, originally developed by Dugdale,

subsequently extended by Goodier and Field [7], and used by Hahn and Rosenfield [8] to account for the magnitude of plane stress fracture toughness, has proven very useful in the interpretation of the origin of fracture resistance. The basis for and the principle results of this model are thoroughly described in the previous reference and are not repeated here. The appropriate equation for calculation of the remote stress is, from the Dugdale model,

$$\sec \frac{\pi \sigma}{2Y} = \frac{\rho}{a} + 1. \quad (2)$$

It is necessary to choose a value for the cohesive stress or the strength of the plastic metal (Y) within the zone for this calculation. A value for Y was taken as an average of the yield and ultimate strengths in uniaxial tension ($Y = 230$ MPa). The values of the remote stress so computed are 162 MPa ($\rho = 10.07$ mm, $a = 34.7$ mm) and 127 MPa ($\rho = 6.08$ mm, $a = 11.7$ mm). Values for the fracture resistance can be found from these stresses using the relationship between σ , τ , and a that is appropriate for the infinite SEN specimen in plane stress loading [9]:

$$\sigma = \frac{\sigma^2 \pi a}{E}. \quad (3)$$

Since ρ is a large fraction of a in both cases, a plastically corrected value of σ is calculated using (3), but with a

replaced by $a + \rho$, following a suggestion by McClintock and Irwin [5]. This results in critical values of $5050 \text{ J}\cdot\text{m}^{-2}$ for the large SEN and $3100 \text{ J}\cdot\text{m}^{-2}$ for the small SEN.

The plastic zone length and critical fracture resistance found from the test on the small SEN specimen are significantly lower than the results for the larger SEN specimen. This difference probably results from the assumption that the plastic zone has the same effect on compliance as an equivalent amount of crack extension. Actually, because a cohesive force acts on the elastic-plastic boundary, the specimen will be less compliant than one with the plastic zone replaced by an equivalent amount of crack extension. Thus, the actual plastic zone length will be greater than that found using the method described above. This effect is expected to be greater for smaller specimens in which the zone will occupy a greater fraction of the net ligament width. The fracture resistance result obtained using the compliance result from the large SEN is similar to the magnitude of the J_{crit} results given in the previous section.

PLASTIC WORK METHOD

The third method of fracture analysis is based on Rice's [10] proof that

$$G_{\text{crit}} = \int_0^{\delta_t^*} \sigma(\delta) d\delta. \quad (4)$$

This proof applies strictly only to the Dugdale model, in which the plastic zone is a constant height strip that extends directly ahead of the crack tip [10]. Assuming the fracture of the present sheet can be reasonably approximated by such a strip model, it is apparent that the value of σ_{crit} can be obtained by measuring the area under the cohesive stress-crack opening displacement curve. It was considered possible to measure the crack opening displacement directly at the crack mouth if the specimen could be made small enough so that the elastic contribution to crack mouth displacement was small compared to the contribution from crack tip plasticity. To accomplish this it was appropriate to choose a specimen with a net ligament smaller than the plastic zone size at fracture so that the zone would span the entire ligament width. On this basis the net ligament width chosen was 4.57 mm, considerably smaller than the plastic zone length estimated previously (11 mm). A double-edge notched specimen was chosen so that $\sigma(\delta)$ could be simply obtained from the applied force and net section area. The principle of the technique is illustrated in Fig. 5. It is evident that the crack tip displacement, δ_t , indicated in Fig. 5 will be the same as the displacement of the upper and lower boundaries of the plastic zone and that this displacement, except for a small elastic contribution will be equal to the displacement measured at the crack mouth. This is most easily visualized if the plastic zone is imagined to be a hole in the specimen.

It was necessary to fatigue the two DEN specimens above the general yield stress in order to develop a

reasonable amount of fatigue crack growth in a reasonable time. This results because very low stress intensities are developed with this specimen design, even at net section stresses approaching the ultimate tensile stress. The areas under the curves in Fig. 6 were measured with a planimeter to obtain values of G_{crit} for the two specimens. As indicated in Fig. 6, the values obtained were 5256 and $4660 \text{ J}\cdot\text{m}^{-2}$. Of these, the first is considered to be the more correct value since it is evident that the extreme fatigue history is probably responsible for the lower second result.

CONCLUSIONS

The most correct results from each of the methods of analysis are summarized in Table 1. It is evident that all three methods of nonlinear analysis resulted in values that can, in view of normally expected material variation, be considered the same. This correspondence is taken as evidence that the methods used resulted in correct values for the fracture resistance. It might be expected that the same value would result from a linear-elastic test of a very large specimen of the same thickness. The demonstration of this was, however, beyond the scope of the present program.

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APPENDIX: THE BASAL PLANE POLE AND THE CHEMICAL
COMPOSITION OF THE BERYLLIUM SHEET

The extent of in-plane isotropy of the annealed
beryllium sheet is shown by the basal plane pole in Fig. A-1.

The chemical content is given below:

C	0.069	Co	<0.001
Fe	0.138	Cu	0.007
Al	0.072	Pb	<0.001
Mg	<0.001	Mn	0.013
Si	0.061	Mo	<0.003
Ca	<0.003	Ni	0.015
Cr	0.012		

Table 1. Summary of fracture resistance values found using three independent methods of nonlinear fracture analysis.

	J_{crit} or G_{crit} Jm^{-2}
J integral method	6160, 5510, 5420
Compliance method, large SEN	5050
Plastic work method	5260

FIGURE CAPTIONS

- Fig. 1. Photographs of the load point displacement gages in place on the small SEN specimen.
- Fig. 2. Force-vs-load point displacement results for the J integral specimens.
- Fig. 3. J vs load point displacement for the small SEN specimens.
- Fig. 4. Force-vs-crack mouth displacement behavior for SEN specimens of two sizes.
- Fig. 5. Principle of plastic work method for measuring fracture resistance.
- Fig. 6. Net section stress vs displacement for double-edge notched specimens used to determine fracture resistance by plastic work method.
- Fig. A-1. Basal plane pole and chemical composition of beryllium.

Tardiff -
Fig. 1

Tardiff -
Fig. 2

Tardiff -

Fig. 3

Tardiff -
Fig. 4

Tardiff —
Fig. 5

Fatigue history:

36 to 134 MPa — 70,000~

36 to 170 MPa — 25,000~

Growth: 0.08 and 0.10 mm

Fatigue history:

36 to 170 MPa — 30,000~

Growth: 0.18 and 0.20 mm

CRACK MOUTH DISPLACEMENT $\approx$ CRACK OPENING DISPLACEMENT, mm

Tardiff —

Fig. 6

Tardiff -
Fig. A-1