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FLOW REGIMES IN A SIMULATED PWR DOWNCOMER ANNULUS

by

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January 1974

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ABSTRACT

This paper presents the results of an experimental and theoretical investigation of the possible flow regimes which could be present in the downcomer region of a pressurized water nuclear reactor during the emergency core coolant injection period of a postulated loss of coolant accident. Because a simplified flow geometry was studied, the results presented here have a general usefulness apart from the nuclear reactor modeling intended. The possible application of the results to flow behavior in a nuclear reactor is discussed.

Water was injected transversely from a horizontal tube into the region formed by two parallel, transparent vertical plates. The flow patterns produced as the water impacted on these plates and fell under the influence of gravity were studied experimentally and theoretically. Several distinct flow patterns were observed; their size and shape are predictable by several analytical models.

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NOMENCLATURE

a	typical length scaling the water profile
A	dimensionless vertical momentum flux, parameter in potential flow model
B	" horizontal " "
C	source strength
d	tube diameter
f	friction factor
f'	" " on the liquid band
F_o	Froude Number, F_o' modified Froude Number, $F_o (s/d)^{-0.35}$
g	acceleration due to gravity
$\bar{g}$	vector " " " "
h	height of top of liquid above tube axis
K	a constant
dL	element of length
p	pressure
Q	overall volumetric flow rate
Q_b	volumetric flow rate in the liquid band
r	radius
s	spacing between plates
t	$\tan \psi$
U	scale of velocity in potential flow analysis
V_o	average velocity in pipe
$\bar{V}$	vector velocity
w	width of an element, velocity potential
w_o	" " " " at the beginning of the film
x, y	coordinates
x', y'	dimensionless coordinates
z	complex number (x+iy)
z'	dimensionless form of z

δ	film thickness
δ_0	starting film thickness
θ	angle from vertical
μ	viscosity
ρ	density
σ	surface tension
τ_w	wall shear stress
$\bar{\tau}_w$	vector " "
ψ	angle between tangent to liquid band and horizontal

Dimensions are in inches

INTRODUCTION

During the emergency core cooling system (ECCS) injection period of a postulated loss of coolant accident (LOCA) in a pressurized water reactor (PWR), cooling water flows through a cold leg pipe into a narrow annular region between the reactor core and the reactor vessel. The purpose of these ECCS systems is to deliver the cooling water to the lower plenum of the reactor vessel and hence to the reactor core. Because there is counter-current upward steam flow tending to oppose the downward motion of this water during a portion of the water-injection period, the flow patterns in the downcomer annulus which are produced by the injected water have an important effect on the speed of the water delivery to the lower plenum and hence the rapidity of the core cooling processes.

The flow regimes possible with only water flow in the downcomer annulus were studied in a simplified geometry. The extension of this work to the cases with upward gas flow and heated walls remains to be done. However, knowing the possible water flow regimes in this simple, single phase experiment, provides a starting point for developing better understanding of the effect of upward steam flow on the water delivery rates under the more complicated conditions.

The reader who is not interested in the details of the theoretical developments may wish to skip pages 10-16 and 18-28 on the first reading. Pages 17 and 29 provide approximations to the important results which may be useful for design purposes.

EXPERIMENTAL APPARATUS

A scale model of an "unwrapped" downcomer annulus was constructed using transparent materials (plexiglas). Since this initial effort was aimed at investigating the water flow patterns alone, a single cold leg was used to inject the water into the "annulus". The experimental geometry is shown schematically in Figure 1. It consisted of two flat transparent plates with a single inlet cold leg. The principal experimental variables were:

1. Cold leg diameter ($1/2$, 1, 2, and 4 inch),
2. Gap spacing ($1/16$, $1/8$, $3/16$, $1/4$, $3/8$, $1/2$, $1\ 1/2$, 2, $3\ 1/2$ inches), and
3. Injected water flow rate.

The spacing between the plates was readily varied using different thickness shims which also served to close the side and top edges of the plates. The bottom edge of the plates was open to allow the water to flow into a catch basin. Several other geometric modifications were also explored as will be discussed later in this paper.

DIMENSIONAL ANALYSIS

The variables which might be expected to influence this physical situation include the geometrical parameters, d and s , the average velocity V_o in the inlet pipe, the fluid parameters of density ρ , surface tension σ , and viscosity μ and the acceleration due to gravity g .

An obvious requirement for similarity as the scale is changed is that the ratio s/d be kept constant.

Gravity can never be completely ignored - it is the force which causes the liquid to flow out of the bottom of the apparatus rather than the top or sides, in all cases.

The fluid properties appear in the inertia, viscous and surface "forces". The ratio between the first pair results in the Reynolds Number which in these experiments was well over 1000 in almost every case, by any means of definition. We therefore expect the influence of viscosity to be small except perhaps when d or s are at their smallest and V_o is also small.

Surface tension effects are confined to curved boundaries. These occur in the gap and have a maximum curvature scaled by $(1/s)$. An equivalent liquid head introduced by these forces is of the order of $(\frac{\sigma}{\rho g s})$ which is equal to less than 0.2 inches for the minimum value of $s = 1/16$ inches. In a profile measuring several inches in all directions this effect should be minimal. The effect on stability may be more marked, however, with the smaller gap sizes.

The balance between the major forces of gravity and fluid inertia (which also scales friction to a large extent) can be represented by a Froude Number:

$$F_o = \frac{V_o}{\sqrt{g d}} \quad (1)$$

This should be the major dimensionless group characterising the force balance.

If we are interested in the profile size or shape we may define a characteristic dimension, such as the height h above the tube axis, and the coordinates x and y of other points on the profile. These can be rendered dimensionless as (h/d) , (x/d) and (y/d) .

We should therefore expect that a purely empirical correlation of data could be sought by plotting (h/d) etc. against F_0 for various values of s/d . Other distinctive features, such as flow regime should also be functions of s/d and F_0 .

In plotting data in this way we should be on the lookout for systematic deviations which might be explained in terms of the forces which have been ignored.

We shall see that theoretical considerations later allow us to combine these dimensionless groups into composites which characterise individual flow regimes more economically than could be done using two independent groups.

FLOW REGIMES

As the independent variables, flow rate, tube diameter and gap spacing were varied, numerous flow regimes, each with a distinct pattern, were observed. Although more detailed subdivisions are possible, we have chosen to define six regimes, numbered from 0 to 5.

Regime 0 covers several flow patterns characteristic of low liquid flow rate. The tube does not "run full" and water either dribbles out of the end or forms a narrow jet which jumps across the gap and flows as a rivulet down the far wall.

Regime 1, which we also call the "river" regime, consists of a solid water stream which fills the cross-section of the gap. The stream is broadest a little below the injection point and then contracts as it accelerates under the influence of gravity. Eventually, when friction balances gravity, the river reaches a constant asymptotic thickness. This regime is sketched in Figure 2a. Actual experimental profiles are shown in Figure 3.

Several variations of Regime 1 were also observed. Sometimes a bubble of arbitrary size could attach itself to the lower rim of the injection point, causing the water to flow continuously around it (Figure 3d). The bubble could develop either from the evolution of dissolved air, a slight air leak where the pipe joined the flat plate, or from the starting transient when the flow was first turned on.

It was also observed, usually at the higher flowrates, that the edges of the stream could go unstable some distance below the injection point. This instability started close to the inflection in the surface profile.

Regime 2, illustrated in Figure 4, is a flow pattern which might not be predicted apriori. The upper part of the water jet fills the gap, as in Regime 1. The lower part separates from the near wall, jumps

the gap and forms a spreading film on the far wall. This film falls under gravity directly below the injection point. Elsewhere it spreads outward until it meets the liquid streams flowing down from the top part of the tube. Most of the liquid streamlines which start in the film are eventually captured by the outer streams. Sometimes one or more small rivulets may form below the tube inlet (Figure 4).

Regime 3 is an outgrowth from Region 2 as the flow rate is increased. The jet separates completely from the near wall, impinges on the far wall and spreads out in all directions (Figure 5). Most of the liquid streamlines in this film eventually terminate in a "hydraulic jump" where they expand to fill the gap. The film region is surrounded by a broad band of liquid filling the gap to form a continuous arch.

Regime 4 is the limit of Regime 3 in which the arch or band of liquid is thin enough to be considered as a single streamline, rather than a two-dimensional continuum flow (Figure 6).

Regime 5 occurs when the gap size is large, compared with the pipe diameter, and the flow rate is low. The jet impinges on the far wall and spreads to form an outer band which does not fill the gap but falls partially away from the wall, as shown in Figure 2b.

The regimes which are most clearly defined are regimes 1 and 4. We refer to them as the "river" and "film" regimes. Regimes 2 and 3 are transition regimes containing some of the features of both extremes. Regime 5 is a degenerate case of Regime 4.

FLOW REGIME MAP

A flow regime map was developed from observations taken with the four pipe sizes, a range of s/d from $1/32$ to 1 and Froude Numbers F_o from 0.1 to 10 . The result is shown in Figure 7. The boundaries are approximate since some are qualitatively defined. However, the map should be useful as a basis for deciding under which circumstances the various theoretical models, to be developed later, should be valid.

The limit of Regime 0 at high s/d is a Froude Number of 0.65 . At flow rates greater than this the pipe "runs full". At Froude Numbers below 0.54 a long bubble is able to propagate up the pipe from the exit and a stratified flow develops (it is not always stable, waves and surges tend to occur). This is consistent with Benjamin's analysis¹ of the motion of large bubbles in horizontal tubes. For the range $0.54 < F_o < 0.65$ the pipe runs full up to a point close to the exit; there is then a short section of stratified flow as the issuing jet develops.

The line through the middle of the figure has the equation

$$F_o = 3 \left(\frac{s}{d} \right)^{0.65} \quad (2)$$

The distinction between Regimes 3 and 4 is qualitative, and one might expect that a theory valid for Regime 4 might give a reasonable description of Regime 3 in the neighborhood of the indicated boundary.

Over part of the range of s/d two regimes (at least) are stable. It is possible to flip from one regime to the other by suitable manipulation of the flow rate or by temporarily inserting an object into the flow. These overlapping regimes are again somewhat indistinct. For example, it seemed to be impossible to maintain a stable Regime 1 at an s/d of $1/8$ with the 4-inch pipe, but it could be easily obtained at this s/d with smaller pipes. This could be due to surface tension effects.

We concentrated our analytical effort on obtaining theoretical predictions for the well-defined Regimes 1 and 4. These theories will be presented in the following sections, compared with data and used as a guide for the development of simple correlations for use in engineering design.

REGIME 1Theory

Apart from a region near the pipe outlet, Regime 1 consists of a steady plane flow, under gravity, between flat plates. The equation of motion of a fluid element is:

$$\rho \bar{V} \cdot \nabla \bar{V} = - \bar{\nabla} p + \rho \bar{g} - \frac{2 \bar{\tau}_w}{s} \quad (3)$$

where $\bar{\tau}_w$ is the shear stress on either wall and $\bar{V}$ is the vector average velocity across the gap. Equation (3) should then be solved simultaneously with the continuity equation, the boundary conditions at the pipe and the condition of constant pressure at the surface of the stream. For the range of parameters with which we are concerned, the flow will be largely turbulent and $\bar{\tau}_w$ will be given by

$$\bar{\tau}_w = \frac{1}{2} f \rho \bar{V} |\bar{V}| \quad (4)$$

Rather than solving (3) in its entirety, we look at two limiting cases:

- 1) Frictionless flow.
- 2) Flow in which friction is large.

It is not immediately obvious what conditions are necessary for either limit to be approached. However, all else being equal, we should expect friction to become more and more significant as s is decreased.

Case 1 Frictionless flow

We neglect the final term in equation (3) and are left with a conventional potential flow. The boundary condition at the surface can be converted to the condition $V^2 = 2g(h-y)$, where h is the maximum height of the surface above the origin (Figure 8).

An approximate analysis can be performed by assuming that the top of the "river" has a semi-circular shape centered at the origin. The velocity at the outer edge of the equator is then $\sqrt{2gh}$. If the downwards velocity across the equator varies linearly from zero at the center to $\sqrt{2gh}$ at the outside the total flow across the equator from above is

$$Q_e = \frac{2sh\sqrt{2gh}}{2} \quad (5)$$

This is one half of the total flow, if the streamlines spread uniformly from the hole, therefore

$$Q = 2sh\sqrt{2gh} \quad (6)$$

For the inlet pipe,

$$Q = (\pi d^2/4) V_o \quad (7)$$

Therefore, from (6) and (7)

$$\begin{aligned} \frac{h}{d} &= \left(\frac{V_o}{\sqrt{gd}} \right)^{2/3} \cdot \left(\frac{d}{s} \right)^{2/3} \cdot \left(\frac{\pi^2}{128} \right)^{1/3} \\ &= 0.426 \left(F_o \frac{d}{s} \right)^{2/3} \end{aligned} \quad (8)$$

The form of this equation is correct, only the coefficient and the profile shape are approximate. (For example, if we had chosen to assume a uniform velocity across $y = 0$ the coefficient in (8) would have been 0.27).

To get a more realistic model we construct a potential flow which has the correct overall features and contains parameters which can be chosen to fit the boundary conditions approximately.

The complex velocity potential

$$w = \frac{U}{a} (z - ai)^2 - C \ln \frac{z}{a} \quad (9)$$

represents the superposition of a source at the origin and a corner flow centered at $z = ai$. The streamlines from the source are enclosed by a boundary ($\psi = C\pi/2$) with the general features desired. The total flow from the source is

$$Q = 2\pi Cs \quad (10)$$

Equation (9) can be put into dimensionless form as

$$w' = (z' - i)^2 - A \ln z' \quad (11)$$

with $w' = \frac{w}{Ua}$, $z' = \frac{z}{a}$, $A = \frac{C}{Ua}$

The complex velocity everywhere is $\frac{dw}{dz} = U \frac{dw'}{dz'}$. (12)

The boundary condition to be satisfied at the surface is

$$v^2 = 2g(h - y) \quad (13)$$

or $\left(\frac{dw'}{dz'}\right)^2 = \frac{2ga}{U^2} (y_0' - y')$ (14)

where y_0' is the value of y' at the highest point of the river ($y_0' = h/a$, $y' = y/a$).

We now endeavor to find a value of A so that (14) can be approximately satisfied over a wide range of y' . By choosing a series of values

for A and plotting $(\frac{dw'}{dz'})^2$ on the surface versus $(y_0' - y')$ it is found that a nearly linear relationship is obtained if $A = 0.35$. The corresponding value of $(2ga/U^2)$ in (14) is 20. Combining these results with (10) we get

$$\begin{aligned} Q &= 2\pi C s = 2\pi (UaA) s = 2\pi a s (0.35) \sqrt{\frac{ga}{10}} \\ &= 0.695 s (ga^3)^{1/2} \end{aligned} \quad (15)$$

$$\text{or } a = 1.275 \left(\frac{Q^2}{gs} \right)^{1/3} \quad (16)$$

The surface shape computed from (9) with $A = 0.35$ is shown in Figure 9. To compare this with measured values we can use (16), remembering that $x = x'a$, $y = y'a$.

The asymptotic solution far below the source can be analysed one-dimensionally. From Bernoulli's equation we have

$$V = \sqrt{2g(y_0 - y)} \quad (17)$$

And from continuity

$$2Vxs = Q \quad (18)$$

The equation of the surface is then

$$Q^2 = 8x^2 gs^2 (y_0 - y) \quad (19)$$

Using (18) and the computed value $y_0 = 0.226$ this may be transformed to

$$x'^2 (0.226 - y') = \frac{(0.695)^2}{8} = 0.0605 \quad (20)$$

Equation (20) is compared with the profile, computed from (9), in Figure 9 and is seen not to be too far away from it.

If representation in terms of the Froude Number is preferred we can use

$$Q^2 = V_o^2 \frac{\pi^2 d^4}{16} = F_o^2 \frac{\pi^2}{16} g d^5 \quad (21)$$

in (16) to get

$$\frac{a}{d} = 1.085 (F_o \frac{d}{s})^{2/3} \quad (22)$$

In particular, for the top of the profile

$$\frac{h}{d} = y_o \cdot \frac{a}{d} = 0.246 (F_o \frac{d}{s})^{2/3} \quad (23)$$

which has the same form as (8).

Case 2 Friction large

When the plate spacing is small we should expect friction to assume significant importance. The solution then becomes more complex but should involve the same dimensionless groups, $F_o, \frac{s}{d}, (\frac{h}{d}, \frac{x}{d}, \frac{y}{d})$ as before together with the friction factor f . To get an idea of the scale of the flow we may consider the asymptotic parallel flow far below the pipe. Here friction balances gravity, there are no pressure gradients, and we have, simply,

$$\rho g = \frac{2 \tau_w}{s} = \frac{f \rho V^2}{s} \quad (24)$$

$$\text{Therefore } V = \sqrt{\frac{g s}{f}}$$

The volumetric flow rate is

$$Q = 2 V s x \quad (25)$$

$$\text{Therefore } Q = 2 s x \sqrt{\frac{g s}{f}} \quad (26)$$

Using (26) and (21) gives

$$\frac{x}{d} = \frac{\pi}{8} \sqrt{f} F_o \left(\frac{d}{s} \right)^{3/2} \quad (27)$$

Since the asymptotic thickness is given by (27) we may expect the whole profile to be scaled similarly. For example, with $f = .005$ we might

$$\text{expect } \frac{h}{d} = K F_o \left(\frac{d}{s} \right)^{3/2} \quad (28)$$

K being a constant with order of magnitude .03.

Case 3 The complete solution

The complete solution will be some compromise between the extremes. A reasonable way to represent data might then be to plot (h/d) versus $(F_o d/s)$ for various values of (d/s) .

On log paper, equation (23) is a straight line of slope $(2/3)$ while equation (28) represents a family of lines with intercepts on any vertical axis proportional to $\sqrt{d/s}$. The lines intersect within the range of experimental interest so we might expect much of the data to lie within a "transition" region.

Experimental Results

Experiments were performed with pipes of diameters $1/2$, 1, 2 and 4 inches (actual measured diameters were 0.49, 0.995, 1.97 and 3.96 inches) and various gap sizes. The flow rate was varied from the point where the pipe first ran full up to the condition where the size of the profile

approached the limits of the apparatus. The shape of the water surface was recorded, either photographically or by tracing directly onto a piece of transparent paper attached to the far wall. With the latter method care was taken to avoid optical distortion by suitable arrangement of the lighting system (e.g. placing a flood light a long way from the apparatus on the tube axis). From these profiles the dimension of the water surface at any chosen location could be determined. In particular we measured "h", the maximum height of the interface above the centerline of the tube. Data were then plotted as (h/d) versus $(F_o d/s)$ as shown in Figures 10, 11 and 12.

The most comprehensive and accurate series of data were obtained with $s/d \approx 1/8$. The results are shown in Figure 10 and can be correlated quite closely with the equation,

$$\frac{h}{d} = 0.27 (F_o \frac{d}{s})^{0.79} \quad (29)$$

The slope of this line is intermediate between the slopes predicted by (23) and (28). The figure also shows the predictions of (23) and (28), using $K = .03$.

Data for lower values of s/d are shown in Figures 11 and 12. There is more scatter than in Figure 10. Some of this is due to experimental errors which increase as the spacing between the plates is decreased. In particular, any distortion of the plates due in part to the forces imposed on them by the flowing water, becomes more significant when s is lower. This effect could not be entirely eliminated in spite of a series of modifications to the apparatus.

If "best lines" are drawn through the data in Figures 11 and 12 they are each found to have a slope of about 0.95. This is consistent with the idea that friction should become more important as s/d decreases. In fact, equation (28) with $K = 0.03$ is not unreasonably far from the data in either case.

Some of the scatter possibly results from variation in the turbulence level in the inlet pipe, due to different piping systems for the different diameters, some of which contained bends or changes of section close enough to the injection point to influence f , and hence K .

It seems reasonable to conclude that the data are in a transition region in which neither friction nor inertia dominate the motion but both are significant.

If an equation is desired for engineering design purposes, it is possible to use equation (29) over the range $5 < F_o d/s < 100$ for the whole set of Regime 1 data presented here, with extreme errors of $\pm 30\%$ being expected.

Though the potential flow theory does not predict the size of the profile well it does provide a good representation of the shape. If we change the scale of Figure 9 by plotting (y/h) versus (x/h) and plot sample data points on the same graph the agreement is pleasing (Figures 13 and 14). The largest deviations occur for the smallest value of s/d or at the lowest Froude Numbers for which the effects of finite tube size are significant.

Approximate Equations for Design Purposes

The height of the profile may be calculated from (29),

$$\frac{h}{d} = 0.27 (F_o \frac{d}{s})^{0.79}$$

The shape then follows from Figure (13) which shows (y/h) versus (x/h) . In particular, the maximum horizontal spread (which blocks the annulus and restricts the available area for steam flow) is given by

$$\frac{x_{\max}}{d} = 0.38 (F_o \frac{d}{s})^{0.79}$$

and occurs at

$$\frac{y}{d} = - 0.4 (F_o \frac{d}{s})^{0.79}, \text{ below the injection point.}$$

REGIME 4

Regime 4 is another "well-defined" flow pattern. The jet separates completely from the near wall and spreads as a thin film on the far wall, terminating in a hydraulic jump to form a thin band of liquid which fills the gap.

Again, we solve two ideal problems.

- 1) Frictionless film flow, followed by a jump.
- 2) Film flow dominated by friction, followed by a jump.

Before proceeding with these analyses we need to solve one of the boundary conditions, namely "what is the initial radial velocity with which the film leaves the region near the pipe?" This velocity depends both on V_o and on s/d . Moving the plates closer together causes the streamlines to be constricted near the pipe circumference and increases the initial film speed (Figure 15).

We solved this problem empirically rather than theoretically. It was noticed that the liquid band had the same shape at different values of s/d but that its size depended on both V_o and s/d . We studied which combination of V_o and s/d would give the same values of h/d and found that data for all spacings could be brought together by plotting in terms of the product $F_o (s/d)^{-0.35}$. It was therefore only necessary to solve the problem for one particular value of s/d which we chose to be $s/d = 1$. It was also assumed that at this large spacing the initial film velocity was equal to V_o .

Theory (for $s/d = 1$)

Case 1 Frictionless Film Flow

The film surface is all exposed to the air and is at a constant pressure. The fluid particles therefore follow trajectories characteristic of free flight under gravity. For profiles much bigger than the tube diameter we assume these trajectories all start at the pipe centerline,

which we shall use as the origin, with speed V_0 . The trajectories, for an initial angle θ to the vertical have the equation

$$y = x \cot \theta - \frac{1}{2} g \frac{x^2}{V_0^2 \sin^2 \theta} \quad (30)$$

If it were not for the intersection of trajectories and the formation of the liquid band, the envelope of the fluid paths would be the "bounding parabola" with equation

$$\frac{y}{y_0} = 1 - \left(\frac{x}{x_0} \right)^2 \quad (31)$$

where $y_0 = V_0^2 / 2g$, $x_0 = \sqrt{2} V_0^2 / g$. The maximum liquid height would be given by Bernoulli's equation;

$$\frac{h}{d} = \frac{V_0^2}{2gd} = \frac{F_0^2}{2} \quad (32)$$

In reality the trajectories are cut short before this envelope and share their momentum with the liquid band. Let there be a flow rate Q_b in the band with speed V_b at an angle ψ to the horizontal at the point where it intersects the trajectory which starts at θ to the vertical (Figure 16). From continuity we have

$$Q_b = \frac{\theta}{2\pi} Q \quad (33)$$

The momentum equations in the x- and y-directions for the short length of band (assumed to have zero thickness) shown in the figure are

$$d(Q_b V_b \cos \psi) = V_0 \sin \theta dQ_b \quad (34)$$

$$d(Q_b V_b \sin \psi) = \left(V_0^2 \cos^2 \theta - 2gy \right)^{1/2} dQ_b + gA_b dL \quad (35)$$

A_b is the cross-sectional area of the band and equals Q_b / V_b .

From geometry we have

$$\tan \psi = - dy/dx \quad (36)$$

$$dL = \frac{-dy}{\sin \psi} = \frac{dx}{\cos \psi} \quad (37)$$

The positive sign is used in equation (35) if the trajectory has passed its maximum value of y , i.e. if

$$x > \frac{V_o^2 \sin \theta \cos \theta}{g} \quad (38)$$

Combining equations (33) and (34) we get

$$V_o \sin \theta \frac{Q d\theta}{2\pi} = d(Q_b V_b \cos \psi) \quad (39)$$

which integrates to give

$$\frac{Q V_o (1 - \cos \theta)}{2\pi} - Q_b V_b \cos \psi = \frac{Q \theta}{2\pi} V_b \cos \psi \quad (40)$$

Therefore

$$V_b = V_o \frac{(1 - \cos \theta)}{\theta \cos \psi} \quad (41)$$

We now use equations (41) and (33) in (35) to get

$$d(V_o (1 - \cos \theta) \tan \psi) = \frac{1}{2} (V_o^2 \cos^2 \theta - 2gy) d\theta + \frac{g \theta^2 dx}{V_o (1 - \cos \theta)} \quad (42)$$

Equations (30), (36) and (42) now represent three equations linking changes in x , y , θ and ψ . ψ only appears as $\tan \psi$. We tidy up the mathematics by letting $t = \tan \psi = - dy/dx$, $y' = yg/V_o^2$, $x' = xg/V_o^2$. The resulting three equations are:

$$\text{from (36)} \quad dy' = -t dx' \quad (43)$$

$$\text{from (30)} \quad dx' (t + \cot \theta - \frac{x'}{\sin^2 \theta}) = d\theta \frac{x'}{\sin^2 \theta} (1 - \frac{x'}{\tan \theta}) \quad (44)$$

$$\text{from (42)} \quad dt(1 - \cos \theta) = d\theta [-t \sin \theta + (\cos^2 \theta - 2y')^{1/2}] \quad (45)$$

with the positive sign used if $x' > \sin \theta \cos \theta$.

These equations can now be solved numerically if a suitable starting point can be found. After some algebra and a thorough study of the apex of the liquid band at $x = 0$ we find that $y' = 5/18$ there and that near this point we must have $x' = \theta/3$ (the reader who wishes to check this by means of a momentum balance for the apex must remember that all the film streamlines except the stagnation streamline are curved). We then start the numerical solution a little bit away from the apex at $y' = 5/18$, $\theta = \delta \theta$, $x' = \delta \theta/3$, $t = 0$ and integrate the simultaneous linear differential equations, using θ as the independent variable, to get the result shown in Figure 17. The liquid band is much smaller than the bounding parabola because of hydraulic losses in the jump.

In terms of the variables which were used previously we have

$$\frac{h}{d} = \frac{5}{18} \frac{V_o^2}{g d} = \frac{5}{18} F_o^2 \quad (46)$$

for the particular case $s/d = 1$. For other values of s/d we have, empirically

$$\frac{h}{d} = \frac{5}{18} F_o^2 \left(\frac{s}{d}\right)^{-0.7} = \frac{5}{18} F_o'^2 \quad (47)$$

Case 2 Friction Dominates Gravity in the Film

At high film velocities the wall shear stress exerts a much greater force per unit mass on a fluid particle than does gravity. In this limiting case the liquid flows out radially in straight lines from the impinging jet but continually decelerates until it reaches the surrounding band.

We first analyse flow in the film (Figure 18) and then consider the dynamics of the band (Figure 21).

Assume that, for a value of $s/d = 1$, by the time the liquid gets out to a radius d from the tube axis it has a uniform radial velocity equal to V_0 . From continuity the film thickness there is

$$\delta_0 = \frac{d}{8} \quad (48)$$

Now consider a pie-shaped piece of film which has width w (much less than r) at a distance r from the axis. Let w_0 be the value of w at $r = d$. Then from geometry, since the streamlines are straight,

$$w = \frac{w_0 r}{d} \quad (49)$$

From continuity

$$\delta w V = \delta_0 w_0 V_0 = \frac{d w_0 V_0}{8} \quad (50)$$

Combining (49) and (50) we get

$$\delta = \frac{d^2 V_0}{8 V r} \quad (51)$$

The momentum equation for a particle moving in the film with friction the only significant force is

$$\rho V \frac{dV}{dr} = - \frac{\tau_w}{\delta} \quad (52)$$

Introducing the friction factor $f = \tau_w / \frac{1}{2} \rho V^2$ and using (51) we get

$$V \frac{dV}{dr} = - \frac{4fV^3 r}{d^2 V_0} \quad (53)$$

Integrating from $r = d$, $V = V_0$ to $r = r$, $V = V$ gives

$$\left(\frac{r}{d}\right)^2 = 1 + \frac{1}{2f} \left(\frac{V_o}{V} - 1\right) \quad (54)$$

$$\text{or } \frac{V}{V_o} = [2f \left\{ \left(\frac{r}{d}\right)^2 - 1 \right\} + 1]^{-1} \quad (55)$$

This equation gives the average film velocity as a function of distance from the tube axis.

Before analysing the entire band we can derive an expression for h , the value of y at the apex. Figure 19 shows the situation for a small piece of band subtending an angle $d\theta$ near $\theta = 0$. From continuity the volumetric flow rate leaving the band is

$$dQ_b = V_o \delta_o d \quad d\theta \quad (56)$$

The momentum balances for the volume enclosing the piece of band are:
Horizontally;

$$\rho dQ_b V_b = \rho dQ_b \frac{V d\theta}{2} \quad (57)$$

$$\text{or } V_b = \frac{V d\theta}{2} \quad (58)$$

Vertically;

$$V \rho dQ_b = \rho g A_b h d\theta \quad (59)$$

The band area A is given by

$$A_b = \frac{dQ_b}{V_b} \quad (60)$$

Combining (56), (58) and (60) we get

$$A_b = \frac{2 V_o \delta_o d}{V} \quad (61)$$

while using (58), (59) and (60) gives

$$\frac{V^2}{gh} = 2 \quad (62)$$

which can be expressed in the alternative form,

$$\left(\frac{V}{V_0}\right)^2 F_0^2 = 2 \frac{h}{d} \quad (63)$$

Equation (63) can now be solved simultaneously with (55), putting $r = h$, to give (h/d) as a function of F_0 and f . The result is shown in Figure 20 for values of f of 0.005 and 0.01. The transition from frictionless to friction-dominated flow appears almost continuous. Apparently losses in the "shock" dominate near the transition. When F_0 is large the solution approaches the value $(h/d) = (F_0^2 / (2\sqrt{2}f))^{2/5}$. When F_0 is small it approaches (32).

To get the shape of the band we proceed as before, writing momentum and continuity equations for the element shown in Figure 21.

The continuity equation may be derived directly in integral form;

$$Q_b = \frac{\theta}{2\pi} Q = \frac{\theta}{2\pi} V_0 \pi \frac{d^2}{4} = \frac{V_0 \theta d^2}{8} \quad (64)$$

The momentum equations, neglecting friction on the sides of the band, are:

$$\text{Horizontal: } d(Q_b V_b \cos \psi) = V \sin \theta dQ_b \quad (65)$$

$$\text{Vertical: } d(Q_b V_b \sin \psi) = -V \cos \theta dQ_b + g A_b r d\theta / \cos(\theta - \psi) \quad (66)$$

$$\text{with } A_b = \frac{Q_b}{V_b} \quad (67)$$

Using (64) and (67), (65) and (66) can be rearranged to give a set of dimensionless equations:

$$dB = \left(\frac{V}{V_0}\right) \sin \theta d\theta \quad (68)$$

$$dA = \left(\frac{r}{d} \frac{\theta}{F_0^2} \frac{V_0}{V_b \cos(\theta - \psi)} - \frac{V}{V_0} \cos \theta\right) d\theta \quad (69)$$

where $A = \theta \sin \psi V_b/V_o$, $B = \theta \cos \psi V_b/V_o$.

If A and B are known, ψ and V_b follow as

$$\psi = \tan^{-1} (A/B) \quad (70)$$

$$\frac{V_b}{V_o} = \frac{A^2 + B^2}{\theta} \quad (71)$$

From geometry, the change in the polar coordinate r is

$$d\left(\frac{r}{d}\right) = \frac{r}{d} \tan(\theta - \psi) d\theta \quad (72)$$

We can therefore solve (68), (69) and (72) simultaneously with θ as the independent variable, making use of (55) and (71) to get V and V_b as required.

This numerical integration was carried out, using a fourth order Runge-Kutta technique, starting from the apex, $x = 0$, $y = h$ (from (55) and (63)), and proceeding until the angle θ reached 3 radians.

The profile is a function of f and F_o . Since $f = 0.005$ appeared to correlate most of the data, profiles are shown for this value in Figure 22.

For values of (s/d) between $1/8$ and 1 we may anticipate that F_o should be replaced by the empirical function introduced earlier, i.e. by $F_o (s/d)^{-0.35}$, which we shall call F_o' , a "modified Froude Number".

If the shear stresses between the walls and the band are considered, the appropriate friction factor being f' , a term $-\frac{r}{s} f' V_b Q_b \frac{\cos \psi}{\cos(\theta - \psi)} d\theta$ is added to the right hand side of (65) and a term

$-\frac{r}{s} f' V_b Q_b \frac{\sin \psi}{\cos(\theta - \psi)} d\theta$ is added to the right hand side of (66). Equation (68) then gains a term $-\frac{r}{d} \cdot \frac{d}{s} \cdot \frac{f' B}{\cos(\theta - \psi)} d\theta$ and (69) gains a term

$-\frac{r}{d} \frac{d}{s} \frac{f' A}{\cos(\theta - \psi)} d\theta$ on their respective right hand sides. When the numeri-

cal integration was carried out for the values of d/s of interest and with $f' = f$, the results only differed very slightly (changes in x/d and y/d by less than 1%) from the previous calculations which neglected friction on

the sides of the band. However, this refinement of the model might perhaps be worthwhile if greater precision is required in extreme cases (e.g. very high values of x/d).

Experiments

Experiments were performed over the range of parameters mentioned previously. The resulting values of h and the detailed profiles were then compared with the theoretical predictions.

Figures 23, 24, 25, and 26 show h/d plotted versus $(F_o^2/2)(s/d)^{-0.7}$ for the four pipe sizes tested. The upper line on each graph represents equation (32) and the lower line equation (47). The factor $(s/d)^{-0.7}$ is added to bring the data together over the range of (s/d) tested. The curve is the result of solving (63) and (55) simultaneously for $f = 0.005$.

For $s/d \geq 0.15$ and h/d larger than about 2 most of the data are close to the curve. At lower values of s/d the prevailing flow pattern is Region 3, yet the data deviate from the theory only by about 20%.

At lower values of h/d the data show more scatter. None of the idealised theories would be expected to be too good in this range because of the effect of finite tube size. Except for extreme cases, however, the data lie between the predictions of equations (32) and (47) (Bernoulli's equation for frictionless flow and the theory which allows for losses in the hydraulic jump).

There is a general tendency for the data to drift lower as the pipe diameter decreases. This could well be due to an increased friction coefficient for the smaller pipes, particularly at the lowest velocities. At the same value of Froude Number, the Reynolds Number for the 4 inch pipe is almost 23 times as large as for the 1/2 inch pipe which should lead to a friction factor ratio of about 2.3, if Blasius' equation is valid.

For the large pipe diameters typical of nuclear reactors or civil engineering installations it would appear that the theory with $f = 0.005$, or perhaps a little lower, should be applicable.

Some liquid profiles at low Froude Number are shown in Figure 17 compared with various theoretical expressions.

The "bounding parabola", equation (31), overestimates the size of the liquid band, as might be expected since no account is taken of frictional losses or mixing losses.

The theory which ignores friction in the film is only a reasonable approximation over a limited range of F_o' .

Data for the 4 inch pipe are represented remarkably closely by the theory which ignores gravitational forces on the film if a value of $f = 0.005$ is assumed. This result is somewhat fortuitous, with the effects of the ignored features perhaps cancelling each other out. Data for the 1/2 inch pipe at low Froude Number are more satisfactorily correlated using a friction factor of 0.02 which is not unreasonable for a Reynolds Number of about 10,000 in the pipe and lower in the film (based on film thickness).

Figures 27, 28, and 29 show data corresponding to higher values of Froude Number for pipe diameters of 1/2, 1 and 2 inches at one particular spacing for each diameter. Data for the 1 and 2 inch diameters are not far from the theory using $f = 0.005$. In the case of the 1/2 inch diameter, deviations are greater but could be considerably reduced by choosing a larger value of friction factor at the lower Reynolds Numbers. The precision of the data is not adequate for a conclusive study of this, relatively small, Reynolds Number effect.

The shapes of the measured profiles are generally close to the predictions. There is a tendency for the experimental profiles to "droop" at the edges more than the analysis predicts. This is to be expected in view of the neglect of gravitational forces in the film and

the neglect of friction on the narrow band. At large values of θ (or $-y$) it would, in fact, be expected that all of the x -momentum would be destroyed, leaving the liquid band to fall vertically under gravity. Since the spread of the band in the x -direction proceeds slowly after $\theta \simeq 170^\circ$ this effect might reasonably be neglected unless flow over a long (relative to tube diameter) vertical wall is of interest.

Approximations for Engineering Design Purposes

It is awkward to have to solve a computer program or interpolate on a graph in order to make predictions. Accordingly we have devised approximate, simple correlations which give the shape of the liquid band within the accuracy of the original theory for $f = 0.005$.

For the height of the band we have:

$$F_o' < 15 \quad \frac{F_o'^2}{2} = \frac{h}{d} + 0.1 \left(\frac{h}{d}\right)^{3.5} \quad (73)$$

$$15 < F_o' < 80 \quad \frac{h}{d} = 4.2 \left(\frac{F_o'^2}{2}\right)^{0.25} \quad (74)$$

$$80 < F_o' \quad \frac{h}{d} = 6.3 \left(\frac{F_o'^2}{2}\right)^{0.2} \quad (75)$$

The shape of the band may then be approximated by a parabola with the equation

$$2 < F_o' < 20, \quad \frac{y}{h} = 1 - 0.25 F_o'^{0.157} \left(\frac{x}{h}\right)^2 \quad (76)$$

$$20 < F_o', \quad \frac{y}{h} = 1 - 0.4 \left(\frac{x}{h}\right)^2 \quad (77)$$

which gives x/h to within 10%, at a given y/h , out to $x/h = 3$. At greater distances from the origin both the experimental data and the theoretical values "droop" progressively further below the approximate solution. F_o' is a "modified Froude Number" equal to $F_o (s/d)^{-0.35}$.

IMPLICATIONS FOR REACTOR GEOMETRIES

One important measure of the ability of the water to penetrate to the lower plenum is the horizontal width or compactness of the flow regime pattern. The more compact (the narrower the horizontal width) the flow regime, the higher the flow rate to the lower plenum expected in the presence of upward steam flow. Examining Figures 30a and 30c (with identical cold leg diameters, identical gap spacings, and identical flow rates) reveals a large difference in compactness between the river and film flow patterns. (These figures also illustrate the double-valuedness of the flow regimes in the range $1/16 < s/d < \frac{1}{8}$. The parameter affecting this is not well understood at the present time; however, it appears related to the rate of change of water flow rates).

Figures 30a and 30d have identical cold leg diameters and identical flow rates, but Figure 30d has a gap twice as large as Figure 30a. However, since this larger gap results in a film flow regime, the width of the pattern in 30d is considerably wider than in 30a at the same flow rate. It is interesting that as one reduces the gap spacing and in so doing crosses the boundary into the river regime, the horizontal width of the flow regime decreases significantly. This effect can also be seen by comparing Figures 31a and 31b. Again (this time with a two inch cold leg), the smaller gap results in a more compact flow pattern*.

It is also interesting to compare Figures 30c and 31c. Both of these flow patterns have about the same horizontal width in the region of the cold leg (although 30c has a much greater width several diameters below the cold leg). However, even though the gap size is the same in Figures 30c and 31c, the flow rate in Figure 31c is $2\frac{1}{2}$ times greater than in Figure 30c. Again we see the possible "advantage" of a river flow regime - if one becomes established.

* (In Figure 31b, flow down the side walls can be seen. The tests run with the two inch cold leg had a slightly smaller than properly scaled horizontal width. However, this has essentially no effect on the results presented here.)

Figure 3ld shows the effect of a horseshoe-shaped baffle placed above the cold leg. This photo shows a flow rate of 42 gpm and should be compared with Figures 3la and 3lb which have a flow rate of only 24 gpm, but a much broader flow pattern. In addition to compactness, the river regime has a relatively small surface which would be exposed to upcoming steam; the film regime has a relatively large surface area exposed to the upcoming steam. Both the compactness and the low surface area of the river regime lead one to believe that, with countercurrent upward steam flow, the river regime would penetrate through to the lower plenum at a higher steam velocity than would the film regimes. Said another way, one is led to believe that the river regime would fill the lower plenum and core quicker than would the film regimes.

If one assumes that the proper scaling of dimensions is:

$$\frac{s}{d})_{\text{model}} = \frac{s}{d})_{\text{reactor}}, \quad \frac{x}{d}, \frac{y}{d})_{\text{model}} = \frac{x}{d}, \frac{y}{d})_{\text{reactor}}$$

$$\frac{L}{d})_{\text{model}} = \frac{L}{d})_{\text{reactor}}$$

and of flow rate is:

$$F_{o\text{ model}} = F_{o\text{ reactor}}$$

$$\text{that is } \frac{V_o}{\sqrt{gd}})_{\text{model}} = \frac{V_o}{\sqrt{gd}})_{\text{reactor}}$$

$$\text{or } \frac{Q_{\text{model}}}{d^{5/2}_{\text{model}}} = \frac{Q_{\text{reactor}}}{d^{5/2}_{\text{reactor}}}$$

then a 1" model cold leg implies a 1/30 linear scaling of a reactor with a 30" cold leg and a model flow rate of 3 gpm corresponds to a reactor flow on the order of 15,000 gpm and 6 gpm corresponds to about 30,000 gpm.

Figure 32 shows two scaled drawings each of a typical two-loop reactor, three-loop reactor, four-loop reactor, and the Semiscale vessel. Superimposed on these outlines are two different flow regimes observed with the 1" pipe at 3 gpm water flow each. Figure 33 shows similar patterns at 6 gpm. In these figures the "slice" made in order to unwrap the annulus was taken through the broken cold leg where the steam will vent. The water flow patterns are shown in the absence of upward steam flow. In Figures 32 and 33, the river patterns were obtained with a 1 in. diameter cold leg and a 1/8 in. gap; the film flow regimes were obtained with a 1 in. diameter cold leg and a 1/4 in. gap.

It should be pointed out that we know of no concrete evidence that rivers can occur in any full scale reactor. In fact Figure 7 shows that Regime 1 is unlikely at current values of s/d . Since the presence of a river flow pattern could have important implications with respect to reactor safety (making them safer), it may be important to conduct tests which will determine the conditions under which reactors might possess a river regime. Perhaps even more important is the redesign of reactor geometries to stimulate the river regime. However, it is clear that much additional work needs to be done before this recommendation can be firmly made. The effects of countercurrent upward steam or air flows and of hot walls on these flow regimes also require investigation.

Regarding the applicability of these results to a cylindrical geometry, only one observation is presently known to the authors. This was a brief experiment conducted by the authors on ANC's transparent vessel. With a 1.5 inch cold leg and a 0.353 inch gap, we were able to establish both flow regimes 1 and 2 in the ANC transparent vessel. Flow Regime 2 appeared to occur "naturally"; Flow Regime 1 had to be somewhat artificially established (at this "large" s/d of 0.235 - compare with Figure 7). However, once it was established it was quite stable over a wide range of countercurrent upward air flow rates.

SUMMARY

This report has shown that over a limited range of dimensions and flow rates the major flow regimes occurring in an "unwrapped" vertical annulus with water injection from a horizontal tube can be described in terms of three dimensionless variables, s/d , F_o and f .

Figure 7 presented an approximate flow regime map in which the major regimes of interest are Regimes 1 and 4. The other Regimes are transition regimes.

Both Regimes 1 and 4 were analysed by techniques which used basic fluid mechanical principles as much as possible rather than empirical correlations. Surface tension was neglected and f was taken as approximately equal to 0.005. A slight dependence of f on Reynolds Number (viscosity effect) was anticipated and observed in practice.

Two limiting theories were developed for Regime 1. One was based on frictionless two-dimensional potential flow while the other assumed that friction was dominant over inertia. The data were found to lie in a transition region between these theories. The height of the profile could be represented quite closely by the empirical equation (29) while the potential flow theory, if scaled to the measured height, gave a close representation of the shape.

The profile in Regime 4 was closely predicted by a theory which took account of frictional forces acting on the liquid film together with mass and momentum conservation in the surrounding liquid band. Approximate equations, (73) to (77), for engineering purposes give the profile in terms of a modified Froude Number, $F_o' = F_o (s/d)^{-0.35}$.

These results should be suitable for preliminary assessment of some of the phenomena which are likely to occur in a reactor annulus during ECC water injection. They provide a starting point for the an-

alysis of the effects of hot walls or countercurrent steam flow on ECC delivery to the lower plenum.

Since the results are based both on basic theoretical models, containing a minimum of empiricism, and on a wide range of data they should provide a close description of behavior in larger geometries. The neglected effects of surface tension and viscosity should be even less important as the scale is increased.

REFERENCE

1. Benjamin, T.B., "Gravity Currents and Related Phenomena", J. Fluid Mech., Vol. 31, part 2, pp. 209-248, 1968.

Figure 1. Schematic of the Experimental Apparatus

Figure 2a. Regimes 0, 1 and 2

Figure 2b Regimes 3, 4 and 5

a). $d=1$ in., $s=1/16$ in.,
 $Q=2$ gpm

b). $d=1$ in., $s=1/16$ in.,
 $Q=3$ gpm

c). $d=1$ in., $s=1/16$ in.,
 $Q=4$ gpm

d). $d=2$ in., $s=\frac{1}{8}$ in.,
 $Q=6$ gpm

Figure 3. Regime 1.

a). $d=1$ in., $s=3/16$ in.,
 $Q=4$ gpm

b). $d=1$ in., $s=3/16$ in.,
 $Q=4.3$ gpm

c). $d=2$ in., $s=\frac{1}{8}$ in.,
 $Q=4$ gpm

d). $d=2$ in., $s=\frac{1}{8}$ in.,
 $Q=6$ gpm

Figure 4.

Regime 2.

a). $d=1$ in., $s=1/18$ in.,
 $Q=4$ gpm

b). $d=1$ in., $s=1/18$ in.,
 $Q=4.6$ gpm

c). $d=1$ in., $s=1/18$ in.,
 $Q=6$ gpm

d). $d=1$ in., $s=1/18$ in.,
 $Q=7.9$ gpm

a). $d=1$ in., $s=\frac{1}{4}$ in.,
 $Q=5$ gpm

b). $d=1$ in., $s=\frac{1}{4}$ in.,
 $Q=6$ gpm

c). $d=1$ in., $s=\frac{1}{4}$ in.,
 $Q=8$ gpm

d). $d=1$ in., $s=\frac{1}{4}$ in.,
 $Q=10$ gpm

Figure 6. Regime 4.

Figure 7 Approximate Flow Regime Map

Figure 8 Analysis of Regime 1 as a Potential Flow

Figure 9 Interface Profile For Regime 1 (Frictionless)

— Computed
 --- Asumptotic (eq. 20)

The coordinates are $x' = x/a$, $y' = y/a$ where a is given by equation (16).

Figure 10 $\frac{h}{d}$ versus $F_0 \frac{d}{s}$ for Regime 1 with $\frac{s}{d} \approx \frac{1}{8}$

Figure 11 $\frac{h}{d}$ versus $F_o \frac{d}{s}$ with $\frac{s}{d} = \frac{1}{16}$

Figure 12 $\frac{h}{d}$ versus $F_o \frac{d}{s}$ for Regime 1 with $\frac{s}{d} = 1/32$

Figure 13 Regime 1 Profiles Normalised as y/h versus x/h

Figure 14 Regime 1 Profiles normalized as y/h versus x/h

Figure 15 Flow near the pipe outlet in Regime 4

Figure 16 Control Volume for Regime 4 Analysis with Frictionless Flow in the Liquid Film.

Figure 17 Profiles for Regime 4 at Low Froude numbers compared with various theories

Figure 18 Flow in a piece of the film for the Friction-Dominated Case.

Figure 19 A Small Piece of Liquid Band near the Apex.

Figure 20 Dimensionless band height in Regime 4 predicted from simultaneous solution of Equations (55) and (63) for $s/d=1$.

Figure 21 Control volume for Analysis of Regime 4 with Friction Dominant in the Liquid Film.

Figure 22 Dimensionless Profiles for Regime 4 (obtained by solving (68), (69) and (72) with $f=0.005$).

Figure 23 Profile height data in Regime 4 for the 1/2 inch pipe (compared with various theories).

Figure 24 Profile height data in Regime 4 for the 1 inch pipe.

Figure 25 Profile height data in Regime 4 for the 2 inch pipe.

Figure 26 Profile height data in Regime 4 for the 4 inch pipe.

Figure 27 Typical Regime 4 Profiles.

--- Data ——— Theory with $f = 0.005$
 1/2" diameter Pipe 3/8" gap $s/d = 3/4$

Figure 28 Typical Regime 4 Profiles.
 - - - - Data, ——— Theory, with $f = 0.005$
 1" diameter pipe 3/8" gap $s/d = 3/8$

Figure 29 Typical Regime 4 Profiles
 - - - - Data, ——— Theory with $f = 0.005$
 2" diameter Pipe 1/2" gap $s/d = 1/4$

a. Regime 1. ($d=1$ in.,
 $s=\frac{1}{8}$ in., $Q=6$ gpm)

b. Regime 2. ($d=1$ in.,
 $s=\frac{1}{8}$ in., $Q=3$ gpm)

c. Regime 3. ($d=1$ in.,
 $s=\frac{1}{8}$ in., $Q=6$ gpm)

d. Regime 4. ($d=1$ in.,
 $s=\frac{1}{4}$ in., $Q=6$ gpm)

Figure 30. Comparison between Several Flow Regimes at Similar Flow Rates.

a. $d=2$ in., $s=\frac{1}{8}$ in., $Q=24$ gpm

b. $d=2$ in., $s=\frac{3}{8}$ in., $Q=24$ gpm

c. $d=1$ in., $s=\frac{1}{8}$ in., $Q=15$ gpm

d. $d=2$ in., $s=\frac{3}{8}$ in., $Q=42$ gpm

Figure 31. The Effect of a Baffle on the Width of the Flow Pattern.

Figure 32 Comparison of River and Film Flow Patterns in Various Geometries. (Both Scaled From $d=1"$ and $Q=3\text{gpm}$ DATA).

Figure 33. Comparison of River and Film Flow Patterns in Various Geometries. (Both Scaled from $d=1"$ and $Q=6\text{gpm}$ DATA)

APPENDIX A

CONDITIONS FOR A COLD LEG TO "RUN FULL"
DURING ECC INJECTION

ABSTRACT

This Appendix discusses the condition necessary for a horizontal pipe to "run full" when liquid flows from it into a gap between vertical plates. These conditions could describe one of the important flow regime boundaries for emergency core coolant injection into the annulus region of a pressurized water reactor.

The result of this work is a more thorough definition of the boundary of Regime 0 than is given in the main report.

NOMENCLATURE FOR APPENDIX A

A	area of flow cross-section
A_o	area of pipe cross-section
d	pipe diameter
F_o	Froude Number, $V_o/\sqrt{gd}$
g	acceleration due to gravity
K	a coefficient in(2) representing friction
Q	volumetric flow rate
r	radius
s	gap or spacing between plates
V	average water velocity
V_o	water volumetric flux, Q/A_o
W	water line width of flow
x	horizontal coordinate
y	vertical coordinate
Z	conversion factor in equation (13)
Re	Reynolds Number $\rho Vd/\mu$
μ	viscosity
ρ	density
$\Delta\rho$	density difference
σ	surface tension
Σ	dimensionless surface tension, $\sigma/(\Delta\rho gr^2)$

INTRODUCTION

In the main report the regime in which the water did not fill the injection pipe (cold leg) was designated "Regime 0". The transition from Regime 0 to various other regimes was correlated in terms of a Froude Number, F_o , and the gap to diameter ratio, s/d . This transition was only approximately defined since the major emphasis of that report was on the other regimes.

The present work was conceived and carried out in order to better define the transition region between Regime 0 and the other regimes under consideration.

Flow of water was observed in tubes of the same diameters as those used in the previous experiments, the rate of flow beginning at zero and gradually increasing to the point where the fluid completely filled the pipe. This is not quite as trivial as one might think at first glance. Qualitatively, it ought to be observed that as the fluid flow in the tube is increased slowly, three things occur in succession:

- 1) The level of the water in the tube increases with increasing flow rate to a point where -
- 2) A stationary "bubble" is formed in the tube and eventually -
- 3) This "bubble" is washed out the end of the pipe with small further increases in the flow rate, and the pipe runs full.

In describing the boundary between the Regime 0 and the other regimes it is the transition point from flow in a partially filled tube to flow in a filled tube that is of interest.

Further, since the boundaries of the other regimes are correlated in terms of the Froude number, it is desirable to present this transition on the same basis.

The first step, after qualitatively examining this flow in a partially filled tube, was to generate a theory which would explain the transition from "open channel flow" to the type of flow described as incorporating a "bubble" - that is, the tube is filled behind the bubble, but only partially filled at the end of the tube.

THEORY

Critical Flow

To describe the flow prior to the formation of the bubble, the theory of critical flow in an open channel was adapted to the tubular geometry. Since the water does not hit the top of the tube anywhere prior to bubble formation, it seems reasonable to assume that the tube would act as an open channel and that critical flow will be achieved at the outlet.

We start from the one-dimensional equation of continuity:

$$Q = VA \tag{A-1}$$

The area of the tube (in cross-section) can be determined for any flow rate from geometrical considerations, as will be demonstrated in a moment. In order to determine Q however, an expression for the velocity, V , of the fluid is needed.

The following presentation is derived from notes by Professor Graham B. Wallis for a course on Fluid Mechanics at Dartmouth College, concerning specifically "Steady Flow with Friction and Changes in Width and Slope in a Channel of any Cross-Section".

The one-dimensional equation of motion in the presence of gravitational and frictional forces reduces to:

$$V \frac{dy}{dx} = -g \frac{dy}{dx} - KV^2 \tag{A-2}$$

under the conditions that flow is open to the atmosphere, the channel is horizontal, flow is steady, and friction can be represented by a turbulent friction factor. K is some function of y .

Equation (A-1) can be written in differential form, noting that A is a function of y , as

$$\frac{1}{V} \frac{dV}{dx} + \frac{1}{A} \frac{dA}{dx} = \frac{1}{V} \frac{dV}{dx} + \frac{1}{A} \frac{\partial A}{\partial y} \frac{dy}{dx} = 0 \quad (\text{A-3})$$

By eliminating dV/dx in (A-2) and (A-3), and rearranging, we get

$$\frac{dy}{dx} \left[1 - \frac{V^2}{Ag} \frac{\partial A}{\partial y} \right] = - \frac{KV^2}{g} \quad (\text{A-4})$$

Since the sign of dy/dx in (A-4) changes at the point where

$$V = \sqrt{\frac{Ag}{(\partial A / \partial y)}} = \sqrt{\frac{Ag}{W}} \quad (\text{A-5})$$

changing arbitrarily without reference to any other variables, it then becomes clear that (A-5) is a general expression for the velocity of propagation of small waves in channels of any shape.

As a further point, a little thought will reveal that the term $\partial A / \partial y$ as it appears above, amounts to the width of the fluid at the surface in the channel, and henceforth this term will be symbolized by the notation W .

Equation (A-5) is then a suitable expression for the critical velocity of the fluid in the tube under open channel flow conditions, and is substituted in (A-1) to give

$$Q = A^{3/2} g^{1/2} / W^{1/2} \quad (\text{A-6})$$

It will be seen in equation (A-6) that Q is now expressed, except for constants, in terms of A , the cross-sectional area of the filled portion, and y , the height of the fluid in the channel. A can further be related to y using a system of equations related by the angle θ (measured from the horizontal axis). Consider the geometry represented in the figure below:

From the above geometrical considerations

$$A = \theta r^2 + \pi r^2/2 + r^2 (\cos \theta) (\sin \theta) \quad (\text{A-7})$$

$$W = 2r (\cos \theta) \quad (\text{A-8})$$

and $y/d = [(\sin \theta) + 1] / 2 = y' \quad (\text{A-9})$

These expressions are then solved together with (A-6) to determine the flow rate, Q , as a function of fluid height.

Finally, to state the result in terms of the Froude Number, begin with the definition

$$F_o = V_o / (gd)^{1/2} \quad (\text{A-10})$$

where $V_o = Q/A_o$ and A_o is the cross-sectional area of the tube.

$$F_o = \frac{4Q}{g^{1/2} d^{5/2} \pi} \quad (\text{A-11})$$

when the substitution is made in (A-10).

And then by substituting (A-6) in (A-11), we get

$$F_o = \frac{4A^{3/2}}{d^{5/2} W^{1/2} \pi} \quad (A-12)$$

Thus, by simultaneously solving (A-12) along with (A-7), (A-8), and (A-9), F_o can be obtained as a function of dimensionless fluid height in the tube (y'). This is plotted dimensionlessly as F_o versus y/d in Figure (A-1), and designated as the Critical Flow Theory in that and subsequent Figures.

Qualitatively, one may observe that as W tends to zero (as the tube begins to fill to the top) F_o should tend to infinity. This should motivate the thinking that some transition must take place as the flow increases.

[To aid in any analysis, note that equation (A-11) can be written

$$F_o = Q/Z \quad (A-13)$$

where Z is a constant, which depends upon the tube diameter:

$$Z = \frac{g^{1/2} d^{5/2} \pi}{4} \quad (A-14)$$

which has a value for a 4, 2, 1, and 1/2-inch diameter tube of 128, 22.7, 4.0, and .71 gal/min respectively. This number is handy because it converts the Froude number into the corresponding flow rate, in the case of the diagrams, or vice versa in the construction of the diagrams.]

Bernoulli Flow

To describe the flow situation in which a bubble exists, a simple one-dimensional model, ignoring surface tension and friction and based upon Bernoulli's equation, is adopted.

The physical situation can be visualized as sketched below.

One-Dimensional Model for Analysis of Flow with a "Bubble"

In this case, the velocity of the liquid on the surface at the end of the tube is given by Bernoulli as

$$V^2 = 2 g (d-y) \quad (\text{A-15})$$

And then (assuming one-dimensional flow) by substituting this expression for V in the equation of continuity

$$Q = A \sqrt{2 g (d-y)} \quad (\text{A-16})$$

The flow rate can again be related to a Froude number, in the same manner as for the Critical Flow theory, such that (using (A-13) and (A-16)):

$$F_o = \frac{\sqrt{2 g d (1 - \frac{y}{d})}}{Z} \quad A = \frac{A}{Z} \sqrt{2 g d (1 - y')} \quad (\text{A-17})$$

and F_o is expressed in terms of y/d . This is plotted in Figure A-2, and designated as the Bernoulli theory in that and subsequent Figures. The Figure is generated using Equations (A-7), (A-9), (A-14) and (A-17).

It ought to be observed here that as y/d approaches unity, the predicted flow rate tends toward zero. This suggests, in the light of the trend in Critical Flow, that some type of "choking" may take place as the fluid height in the tube increases.

Note that in the subsequent Figures, the Bernoulli Theory has been drawn as a dashed line below y/d approximately .5 because the theory does not really apply if there is no bubble in the tube!

Experimental Procedure

The apparatus, as sketched in Figure A-3, consisted of large, thick plexiglas plates, supported in a vertical position, separated by appropriate spacers, and clamped in place. A hole was machined in the center of one sheet to allow a plexiglas tube to penetrate one sheet and sit flush with the inner surface. The water introduced could fall through the gap downward into a basin below. The set-up can be considered to simulate a cold leg injection of water into an annulus which has been "unwrapped".

The flow was regulated by a valve upstream of the inlet tube. Flow measurements were made with a manometer in the case of the 4-inch tube, rotameters in the cases of the 2, 1, and 1/2-inch diameter tubes, and by bulk volume measurement for very small flow rates in all the cases.

The procedure followed was to increase the flow of water into the inlet tube stepwise and measure the corresponding height of the water level (from the side of the tube) at the tube exit with a ruler appropriately placed and levelled. If a bubble existed, the distance the bubble penetrated down the length of the tube was also measured.

Thus, y was experimentally determined by direct measurement and divided by d to give $y' = y/d$ for plotting purposes. The corresponding flow rate was then used to get the value of the Froude Number by $F_o = Q/Z$ as given in equation (A-13).

Effect of Tube Size

The diameter of the tube used in the experiment has an influence upon the results obtained, especially when the tube size is small. Consider for the time being, only those experimental curves representative of large gap sizes in Figures A-4, A-5 and A-6. (In Figures A-5 and A-6 this refers to the data near the critical flow theory. In Figure A-4, both measurements were taken at large gap sizes.)

Figure A-4 presents the graphs of F_0 versus y/d at large gap sizes for the 1/2-inch and 1-inch diameter tubes. It appears as though the flow in the two tubes does not follow either the Critical Flow theory, or the Bernoulli theory. In both cases, a stationary bubble is present at all data points, a fact which is not allowed for in the simple theories at low values of y/d . We shall show that this is a result of the influence of surface tension.

Zukoski¹ has researched bubble propagation in tubes that are open at one end and closed at the other. He filled the tubes, and then allowed them to empty at various levels of inclination, measuring the velocity of the bubble propagated in the tube. One might speculate that viscosity and surface tension are playing some role in the results presented above (Figure A-4) which cause the smaller tubes not to behave like open channels. Zukoski's work can be helpful in interpreting those results, and although his situation involved bubble propagation in an emptying, closed tube, those results should be applicable to filling a tube, as far as conditions for the formation of a bubble are concerned.

In regard to the effects of viscosity, Zukoski indicates that above a Reynolds number of 200, viscosity has little effect on the resulting bubble propagation. One can estimate the Reynolds number for the flow in the 1/2-inch tube:

$$\text{Re} = \frac{\rho dV}{\mu} = \frac{\rho d^{3/2} \sqrt{g} F_o}{\mu}$$

$$= (3.5 \times 10^5) d^{3/2} F_o \quad (d \text{ in feet})$$

Using the data in Figure A-4, according to which the range of the Froude number for the 1/2-inch tube is $F_o = 0.1$ to 0.4 , the Reynolds number then varies from 300 to about 1200. This would indicate, so Zukoski shows, that friction effects are minimal in the 1/2-inch tube, and consequently, larger tubes.

Surface tension may then be the offending factor.

Zukoski presents his results pertaining to surface tension by letting the symbol Σ , a dimensionless parameter, be the variable representing those surface tension effects. Σ is defined as $\Sigma = \sigma / (\Delta \rho g r^2)$, where σ is the interfacial tension, $\Delta \rho$ is the density difference between the liquid (water) and bubble fluid (air), and r is the tube radius. This term relates the surface tension forces to the gravity forces for the fluid in the tube. Σ is also the reciprocal of the Bond Number, a number that is more commonly used.

In his experiments, Zukoski determined the Froude number for which a bubble propagates in horizontal tubes of different sizes. Since the Reynolds number above 200 represents the case where viscosity effects are unimportant, we can use Zukoski's results in which the surface tension parameter was varied to determine the Froude number at which a bubble should propagate in the horizontal tubes which we used.

Figure A-7 graphically displays the Froude numbers at which bubbles are propagated. The solid line for each of the pipe sizes is Zukoski's result. The x's are the experimentally observed results, i.e., the Froude Number at which a bubble was first observed. (Note that all of Zukoski's work was carried out with tubes open to the atmosphere, or in other words, with an infinitely large gap spacing. Therefore, there

are no results available from his work for small gap sizes so only the experimental data for large gap sizes are represented in Figure A-7.)

Zukoski's predictions lie within 10% of the experimental results for each of the tubes. The discrepancies may be a result of experimental error, upstream conditions in the tube, and two-dimensional effects in combination.

The values of the surface tension parameter for each tube size are listed in Figure A-7. In an ideal tube, Benjamin² postulated bubble propagation at a Froude Number of .54, and Zukoski's work indicates that a bubble is propagated at $F_o = .54$ in a 17.8 cm. tube. This size then approaches an ideal tube, and the Σ for a tube this size is .001. For any smaller tube, Σ will have a larger value, as shown in Figure A-7, and one would expect surface tension effects to have a bearing on the Froude Number at which a bubble is propagated, namely, a bubble should appear at a lower Froude Number in the 2-inch pipe than in the 4-inch pipe, and so on.

Further, the results indicate that the 4-inch pipe is close to having a minimum effect from surface tension, but it still would be expected to have some. Only at tube sizes over about 7 inches ($\Sigma < 0.001$) will the surface tension effects be negligible when considering bubble propagation.

Gap Size and Bubble Formation

The gap size in the annulus also affects the formation of a bubble and its subsequent life in the tube. If the gap is wide, the liquid falls free once it leaves the tube. On the other hand, if the gap is narrow, the fluid fills the gap and interferes with the flow in the tube.

This essentially means that the fluid height in the tube is greater, at lower values of F_0 , in the arrangement with the smaller gap.

An interesting observation arises in the data, however, in this respect. It turns out that the value of F_0 where the data intersects the curve of Bernoulli's theory also turns out to be the point (within experimental error) that the bubble is first observed in the tube - at all different gap sizes.

In some instances, discrepancies could be rationalized, apart from experimental error, by noting that upstream conditions and instabilities could influence the formation of the bubble.

Figure A-8 is a plot of the gap size in Figures A-5 and A-6 versus the corresponding Froude number at which a bubble is first observed in the tube. These points are the circled data points in Figures A-5 and A-6. The F_0 's at which bubbles first appear in the 1- and 1/2-inch tubes (at large gap sizes) are presented for comparison.

This dimensionless curve for the 2-inch and 4-inch tubes shows a fairly consistent result, on the whole, although the two "plateaus" are slightly different as a result of surface tension considerations.

The plot roughly forms a curve which indicates that above s/d about .375, the gap size no longer affects the flow in the tube significantly, but below that s/d , gap size is a factor. In any case, the point at which a bubble will form in a large tube can be approximately predicted for a variety of gap sizes.

Bubble Profile and Washout--End Effects

Figure A-9 is a plot of the profile of the bubble in a 4-inch pipe as it is in the process of being washed out.

A problem arises here: at what point do we say that the tube is flowing full?

A choice is necessary because of the end effects at the point of very nearly complete washout of the bubble. In taking experimental readings, as is possible to get data points that lie anywhere above the Bernoulli curve (speaking of large gap sizes) and in the range of $.63 < y/d < 1$.

Using Figure A-9 as a reference in explaining this result, it is seen that the flow proceeds with the characteristic "tongue" shape of the bubble from $F_o = .59$ up to around $F_o = .67$, but as the penetration of the bubble in the x-direction approaches 1 diameter (and this is also true for the 2-inch pipe) the bubble loses its shape and becomes a sloping plane. In addition to that, it develops that a relatively large increase in flow rate is then required to have the flow in the tube hit the top of the tube at the very exit.

Figure A-10 presents the same data as Figure A-9, and in addition, similar data for other gaps (still large, $s/d > .375$) in the 2- and 4-inch tubes. It appears as if the graph approaches some limiting value of y/d where the bubble exists infinitely far down the tube.

The theory of Benjamin² (large gap) for flow of fluid out of a hollow tube indicates that a bubble cannot exist in water flowing out of a tube below an angle $\theta = 7.3^\circ$. This translates into a y/d of .625. Furthermore, Benjamin's theory says that this is a unique solution to flow from a tube. Figures A-9 and A-10 support that finding, although the y/d at which bubbles existed in this experiment is $y/d = .64$, or close to that. Assuming that the fluid height at which a bubble exists in the tube remains essentially

constant as long as the bubble is able to exist, argues that the sloping profile of $F_o = .70$ is not a bubble, but merely the result of two dimensional effects at the pipe outlet. In actuality, for the above reasons, it has been assumed in this analysis that when the bubble profile is lost and the fluid height makes a transition from y/d approximately .64 to a value somewhat greater (like .70 in Figure A5 - that is quite a jump from the previous data point) - a transition has been made.

This then, probably gives a better approximation of where the transition to the tube being full actually occurs, and it helps to eliminate the scattering of data that could occur if no choice was reasoned and one attempted to take data until the tube was completely filled at the very exit.

Bubble Washout -- the Regime Boundary

Sufficient background has now been developed such that the primary point of this investigation can be examined, that is, the boundary between Regime 0 and the other Regimes, especially at large gap sizes.

The data in Figures A-5 and A-6 (for the large gap sizes) can be seen to follow the critical flow theory closely from low y/d to near the intersection of the two theories. Since the critical flow theory is based upon flow in open channels, this would seem eminently reasonable.

When the bubbles begin to appear, things become a little less nice.

The first appearance of the bubble correlates fairly well with Zukoski's results and with the Bernoulli theory at large gap sizes. At smaller gap sizes, the agreement with the Bernoulli theory is also good. The appearance of the bubble does not, however, mark the boundary between Regime 0 and the other regimes because the tube is not yet running full.

As the reader should be aware by now, there is a further transition region in which the bubble, formed at a certain value of F_O , is gradually washed out the end of the tube with increasing flow rate (hence increasing F_O). The point where it has been considered washed-out is discussed in the previous section and is taken to be when $x/d < 1$, in general, and at $F_O = .67$ in the example described.

To explain this situation in a little more detail, Figure A-11 shows data taken with the 4-inch pipe at Froude Numbers above the bubble formation point. Until the bubble forms, the pipe does not run full anywhere along its length. A small increase in flow rate once the bubble has formed leads to the pipe running full within less than one diameter of the exit plane, although at the exit plane y/d has not changed significantly. Further increase in flow rate eventually moves the bubble stagnation point up to the exit plane. There is some unsteadiness and oscillation of the interface in this "sloping plane" regime.

Figures A-12 and A-13 show the Froude number at which the bubble is washed-out of the tube (compare this to bubble formation - Figure A-8).

The main point of interest in each case is the F_O where the plateaus are reached in Figure A-12 -- these are the Froude Numbers for which the bubbles are washed-out of the various tubes at the large gap sizes ($s/d > 3/8$). As with bubble formation, the wash-out occurs at lower values of F_O with smaller tube sizes, and, similarly, at higher values of the parameter Σ with smaller tube sizes. The Σ for each tube, as estimated from Zukoski's graphs, is indicated on the Figure.

Note that approximately the same relative spacing of the plateaus occurs in the Figure for bubble formation (A-8) and the Figure for bubble wash-out (A-12).

It must also be mentioned that upstream conditions may have an influence on events in the tube because of the set-up of the apparatus -- the flow in the case of the 4-inch tube was forced around a bend at

fairly high speed, resulting in some sloshing on the wall of the bend that could influence results. The two-inch tube was about two feet long and connected to the inlet by a rubber hose -- there could be upstream effects there.

The data may also be slightly in error owing to 2-dimensional effects in the tube -- due to surface tension having a slightly more pronounced effect depending upon the tube size.

Figure A-13 compares the results of this work for the large pipes with the boundary of Regime 0 presented in the main report. The agreement is good.

UNCERTAINTY ANALYSIS

The magnitudes of the errors in the data are estimated below in the manner that they depend upon the uncertainties in the experimental measurements.

Fluid height readings: The uncertainty in measuring the fluid height in the 2- and 4-inch tube experiments is $\pm 1/16$ inch in the 4-inch and $\pm 1/32$ inch in the 2-inch tube. Thus, the y/d readings may be displaced $\pm .015$ in either case.

The comparable uncertainties for the 1/2- and 1-inch tubes are: $\pm .05$ inch in the height readings, and $y/d \pm .1$ and $\pm .05$ respectively.

Manometer readings: A manometer was used to get a measurement of the flow rate in the case of the 4-inch tube. The uncertainty there is $\pm .1$ inches of water, or $\pm .001$ ft³/sec, or .4 gal/min, which results in an uncertainty in F_o of $\pm 4\%$.

Rotameter readings: Two rotameters were used in the rest of the measurements. The 6.3 gpm manometer had an uncertainty of about $\pm .2\%$, and that for the 12.4 gpm was $\pm 1\%$. Both of these are then $\pm .1$ gpm, which gives $F_o \pm 1\%$. Therefore the accuracy in the 2-inch diameter tube may be slightly better than that in the 4-inch tube.

Actual Pipe Size: The diameter of the nominal 4-inch diameter tube was checked and found to be more like 3.96 inches. An uncertainty enters into calculating F_o due to this discrepancy. The value for the parameter Z , rather than being 128.46 for 4 inches diameter, could be 125.27 for 3.96 inches diameter. This results in an uncertainty in F_o of about $\pm .2$, and is the largest cause of uncertainty in that value.

REFERENCES

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2. Benjamin, T. Brooke, "Gravity Currents and Related Phenomena", J. of Fluid Mechanics, Vol. 31, part 2, pp. 209-248, 1968.

FIGURE A-1 CRITICAL OPEN CHANNEL FLOW IN A HORIZONTAL TUBE
(derived from equation A-12)

FIGURE A-2 FLOW PAST A STATIONARY "BUBBLE",
THE "BERNOULLI THEORY"

FIGURE A-3 APPARATUS

FIGURE A-4 REGIME 0 FLOW 1/2-inch PIPE
1-inch PIPE, $s = 1.5$ in.

FIGURE A-5 REGIME 0 FLOW 2-inch PIPE
(The circled points indicate the onset of bubble formation)

FIGURE A-6 REGIME 0 FLOW 4-inch PIPE
(The circled points indicate the onset of bubble formation)

FIGURE A-7 COMPARING THE OBSERVED FROUDE NUMBER FOR BUBBLE FORMATION AT LARGE s/d WITH ZUKOSKI'S RESULTS — Zukoski, $\diamond$ data

FIGURE A-8 EFFECT OF GAP SIZE ON BUBBLE FORMATION

FIGURE A-9 SCHEMATIC OF BUBBLE WASHOUT PROFILE AS FROUDE NUMBER IS INCREASED (4 inch pipe)

FIGURE A-10 DISCHARGE DEPTH AS A FUNCTION OF LOCATION OF BUBBLE "NOSE"

FIGURE A-11 AN EXAMPLE OF BEHAVIOR IN THE SLOPING PLANE REGIME FOLLOWING BUBBLE WASHOUT

TUBE DIAMETER = 4"
GAP = .75"

FIGURE A-12 FROUDE NUMBER FOR BUBBLE WASH-OUT vs. GAP SIZE

◇ - 2" PIPE
 △ - 4" PIPE

FIGURE A-13 APPROXIMATE FLOW REGIME MAP

◇ - 2" PIPE
 ○ - 4" PIPE

APPENDIX B

SIMULATION OF HOT LEGS AND THEIR EFFECT
ON FLOW PATTERNS

INTRODUCTION

The main report describes experiments conducted with only a single cold leg inlet represented by the apparatus. In the actual reactor, the many cold and hot legs may create a different flow pattern than has been found previously. Since the hot legs penetrate the core barrel and the pressure vessel, they occupy space in the annular gap -- possibly to the effect of interfering with the flow of incoming water. Many possible variations of the hot leg-cold leg-hot leg alternating arrangement may be used. Incoming water from cold legs could possibly interfere with incoming water from other cold legs -- as well as with hot legs.

The purpose of this additional set of trials was to explore the effect of two hot leg-cold leg geometries on the flow pattern of the water in the absence of hot wall or countercurrent flow effects (the latter to be considered shortly in an additional report). The two geometries looked at were hot leg-cold leg-hot leg in a symmetrical design, and a hot leg-cold leg asymmetrical arrangement. The experiments concentrated mainly upon water flows (represented by the Froude number) which are in the range of actual Emergency Core Cooling system values.

SCALING

The experiment was designed to be approximately geometrically similar to an actual reactor in order to make the results more applicable to a real situation. The dimensions of the Trojan reactor, scaled by $1/7$ provided some basis for scaling the apparatus.

The dimensions of the Trojan reactor pressure vessel, hot legs, and cold legs are sketched in Figure B-1. The cold legs used in the experiment were 4" in diameter; the hot legs were 6" diameter (slightly large scale for the Trojan, though about right for a typical Westinghouse reactor--OD 46").

Evenly spaced around the pressure vessel, the center-to-center and edge-to-edge dimensions for this reactor are 80" and 49" respectively.

Thus, the scaled edge-to-edge spacing representing Trojan reactor geometry would be 7". During the course of the experiment, spacings of 1, 3, 5, 7, and 9 inches were considered.

As for water flow rates, a Froude number on the order of .5 is typical of ECC flow, so several flow rates in the range $F_o = .55$ to .75 were used, along with a high value, $F_o = 1.09$, for purposes of comparison.

The gap in the annulus for the experiment remained at one inch throughout.

APPARATUS

The experimental set-up consisted of the same two plexiglas plates as before, supported vertically, with the four inch plexiglas inlet tube connected to the variable water supply. Appropriate one-inch spacers were placed, and in addition, two disks of wood 6" in diameter and 1" thick were placed in the gap. Their centers were at the same vertical level as the center of the inlet tube, and spaced horizontally according to the scheme mentioned before. The disks were clamped in place from above by specially made C-clamps with extra-long grips.

Of course, in the asymmetrical case, only one disk was placed in the gap.

PROCEDURE

In describing the results of these trials, we sought for some other measure, besides the flow rate of the water from the inlet, which would better describe the sometimes complicated flow patterns that evolved (see the photographs). Composite drawings of flow patterns for increasing flow rates did not seem to adequately convey the situation.

A way of introducing some quantification of the results was devised. Three catch basins were placed below the flow, each twelve inches wide. The flow was allowed to establish itself fully at the various rates, and then the catch basins were utilized to capture the water in the following ways:

--Flow was captured in the central 12" region of the pattern, below the pattern (Q_c) .

--also in a 12" region to the left of and immediately adjacent to the central portion (Q_1) .

--and in a twelve inch length to the right and immediately adjacent to the central region. (Q_r).

Thus, a 36" width (centralized) of the flow pattern was captured. Any flow outside the 18" to each side of the centerline which would not be caught in one of the basins is designated as the remainder ("rem"). (In the data this is listed as divided by a factor of two. For the symmetrical case this is very reasonable. It may be misleading in the asymmetrical case however.)

Because the width of the flow changed during the course of the experiments, the amount of water flowing in a given 12" section had to be expressed by a measure which took this variation into account. The amount of water captured in a basin was represented by the expression:

$$R = \frac{\frac{\text{captured flow}}{\text{total flow}}}{\frac{\text{basin width}}{\text{total width}}}$$

This ratio would be 1 in the case where the flow would be considered evenly divided--that is, it would be 1 in all regions. It would be less than one if a region receives less water than would be expected in even distribution; greater than one if more water is received in a given region.

The procedure consisted of measuring the flow pattern

width at catch depth, capturing water in the three basins representing the different regions (for 10 sec. intervals), measuring the amount of water contained in each basin for different flow rates and disk spacings, and calculating the above ratio.

RESULTS

The results -- particularly these calculated ratios for the various geometries -- are given in Tables B-1 to B-10. A few words about these must be said in advance. The signification "NC" alerts the reader to the fact that there was no contact between the incoming water and the simulated hot legs. The readings were taken for NC's at the various flow rates, but are not repeated in several cases to avoid redundancy.

A few photographs of the flow patterns in selected cases are also included here, and will be referred to later. (For flows with Froude numbers ranging from .55 to 1.1.)

Qualitatively, with the help of the pictures, some observations about the flow patterns may be made. First, if the disks are far apart, water may not contact the disks at all. In these instances, flow is in one of the Regimes as considered in the main portion of this report, however; if flow is increased, or if the disks are closely spaced, several different patterns are observed.

The pattern where no contact is achieved consists of a thin film of liquid in the middle, with a band of liquid at the edges. As this flow pattern begins to impinge on the simulated hot legs, water begins to be stored on the side of the disks towards the incoming flow. (Follow the four series of photographs 1-4, 5-7, 8-10, and 11-12 in Figure B-4 to observe the changes in the flow pattern as they occur.) Some water flows around and under the wooden disks, and the overall impression one

gets is that the shape of the pattern--except for the backed up water--is the same as if no disk were present. A further increase in flow results in a bigger backlog of water upwards from the centerline between the tube and the disks, and eventually some (at first very little) water begins to flow over the top of the disks. This flow generally falls straight downward after flowing around the contour of the hot legs; other water flows around the bottom of them as before. At this point, disturbances are introduced beneath the disks as flow continues to increase. These disturbances gradually widen, starting from the outer periphery of the flow, inward, until at still higher flow rates, the total shape of the sub-pattern now looks almost as if the hot leg is the source of an incoming flow, if the water that flows over the disk is also considered. The total pattern then appears almost as one would expect if three (or two) sources existed. It will be seen in the data that the widths of the patterns are somewhat reduced by the disks, but the same shape is maintained as if there were no disks. Photographs 13-15 show very well the inward growth of the disturbances and this apparent pattern that is generated. Further increases in the incoming water do not change the type of pattern--merely widen it.

As for the quantitative measure of the flow as expressed by the calculated ratios: it is helpful to have some standard upon which to compare the results. The base data upon which

to compare was selected as the 7-inch disk spacing and the $F_0=.55$ flow rate, since this is felt to correspond closely to the Trojan reactor ECC conditions.

The symmetrical case will be considered first. Observations may be made in two ways 1) with increasing disk spacing and constant flow rate and 2) increasing flow rates and constant disk spacing. The ratio R_c refers to water caught in the central region. The ratio R_{1r} signifies water caught on either side of the central region, since the amount is ostensibly the same on either side because of symmetry.

With the 7-inch disk spacing, the water first begins to impinge on the wooden disks at $F_0 = .94$. Thus, below $F_0 = .94$ the flow pattern is the same, under these given conditions, as reported in the body of this paper. When flow rates are reached in which water is reflected back to the center of the pattern (for example photographs 4, 7, 10, 12, and 15) more water is measured in the central region. Before contact with the disks (7-inch spacing) values of R_c were found to be .543, .656, .567, and .62 for the F_0 's .55, .625, .695, and .755. When F_0 is 1.09 however, $R_c = .88$ (Fig. B-2). Other disk spacings show increases, the greatest of which occurs in the 5-inch case.

The exception is the 9-inch spacing, which shows a decrease because no reflection occurs at the measured flow rates.

In most cases, the proportion of water to the adjacent regions also showed an increase. This may indicate that the

band of liquid from the "no contact" flow has been broken up and distributed--indeed, very little of the flow is in the remainder which is not captured.

The results just mentioned were fairly obvious and consistent, but a word of caution must be applied in attempting to draw any conclusions from the data. At times, the pattern of the flow, especially the location of bands of liquid on the periphery or the width of the central disturbances can cause a jump in the values measured at increasing flow rates--in the previous paragraph, such a jump in the amount of water measured was what was sought, but at other times, results may appear peculiar.

Considering varying disk spacing for a given flow rate, while the flow increases from $F_0 = .55$ to $.755$, the flow to the central region again decreases and that to adjacent regions generally increases or remains the same. This is because flow reverts to NC band type of flow if the disks are moved outward. This may not be true at close spacings and high flow rates--one of the factors which precludes wider generalization.

Turning to asymmetrical flow (Fig. B-3) three R's (R_c , R_l , and R_r) are calculated, because each of the three regions will have a different flow rate in this case. It ought to be noted first, though, that the right hand region of the flow, unaffected by disks, produces results very close to those for the symmetrical cases where no contact was made--in other words, the flow to the unobstructed side was the same as in the symmetric "no contact" patterns.

The flow to the adjacent regions to the left side in the asymmetrical case agreed closely with the results of the symmetrical case for different flow rates and disk spacings--an exception possibly being the $F_0 = 1.09$ data, although the results are in the ball park. It may be that the asymmetry does become more pronounced at the higher flow rates.

The flow to the central regions in the asymmetrical geometry is reduced in almost all instances in comparison to the symmetric results, probably as a result of the pattern of the flow being much increased towards the open side. Of the two exceptions where higher values are observed, the results may be due to experimental error--the values are within 10% of the symmetrical case data.

UNCERTAINTY

The error in the time during which flow is captured in the basins may be estimated at .5 sec (out of 10 sec.). This results in $\pm 5\%$ error in the amount of water measured. (The error in the actual measurement of captured water is negligible.)

The errors in the catch basin width and the flow pattern width are negligible.

The error in reading the manometer (for water flow rates) is .1 inches. The greatest possible error occurs at lower flow rates, such as 10" H_2O where an error ± 3 gpm is possible. This error is also $\pm 5\%$ (at the greatest). F_0 is then $\pm 5\%$ too.

The calculated value of the ratio R may be in error by $\pm 10\%$ at most.

CONCLUSIONS

Although some qualifications may be necessary because of a particular pattern of flow, it may be stated that flow to the central region directly below the inlet pipe is generally enhanced by the impinging of the water on simulated hot legs in the gap, when such contact occurs in both symmetrical and asymmetrical geometries.

In instances where no (or minimal) disk contact is made, water flow to the outer periphery of the pattern is enhanced. This is because the flow tends to form a band to the outside of the pattern which carries more of the flow than the central region (as reported in the main portion of this report).

The different patterns the flow may follow are best presented in the photographs, without attempting to repeat a lengthy discussion of them in words.

These results are important for indicating various causes of non-uniformity in the flow distribution in the annulus which may have to be taken into account when analysing steam-water interaction phenomena.

Table B-1
ANNULUS INJECTION EXPERIMENT
HOT LFG SIMULATION
Symmetrical

Total Flow = 70 gpm.

$F_o = 0.55$

Catch Basin Width = 12"

Tube to disk edge spacing	Pattern width at catch depth (inches)	Q_c gpm	Q_{lr} gpm	Total gpm	$\frac{\text{rem.}}{2}$	%width basin	% Q_c	% Q_{lr}	$\frac{\% \text{rem.}}{2}$	R_c	R_{lr}
1"	38	24	21	66	2	32	34	30	3	1.06	.938
3"	36	18	27	72	-	33	26	39	-	.788	1.18
5"	34 NC	13.6	30	73.6	-	35	19	43	-	.543	1.23
7"	" NC					"					"
9"	" NC					"					"

Table B-2
ANNULUS INJECTION EXPERIMENT
HOT LEG SIMULATION
Symmetrical

Total Flow = 80 gpm.

$F_o = 0.625$

Catch Basin Width = 12"

Tube to disk edge spacing	Pattern width at catch depth (inches)	Q_c gpm	Q_{lr} gpm	Total gpm	$\frac{\text{rem.}}{2}$	%width basin	% Q_c	% Q_{lr}	$\frac{\text{rem.}}{2}$	R_c	R_{lr}
1"	44	24.6	24.6	73.8	3.1	27	31	31	4	1.15	1.15
3"	38	18	31.2	80.4	-	32	23	39	-	.719	1.22
5"	38 NC	16.8	31.8	80.4	-	32	21	40	-	.656	1.25
7"	" NC					"					"
9"	" NC					"					"

Table B-3
ANNULUS INJECTION EXPERIMENT
HOT LFG SIMULATION
Symmetrical

Total Flow = 89 gpm.

$F_o = 0.695$

Catch Basin Width = 12"

Tube to disk edge spacing	Pattern width at catch depth (inches)	Q_c gpm	Q_{lr} gpm	Total gpm	$\frac{\text{rem.}}{2}$	%width basin	% Q_c	% Q_{lr}	$\frac{\% \text{rem.}}{2}$	R_c	R_{lr}
1"	46	27.6	24.6	76.8	6.1	26	31	28	7	1.19	1.07
3"	38	22.2	34.8	91.8	-	32	25	39	-	.781	1.22
5"	40 NC	15	28.8	72.6	8.2	30	17	32	9	.567	1.06
7"	" NC					"					"
9"	" NC					"					"

Table B-4
ANNULUS INJECTION EXPERIMENT
HOT LEG SIMULATION
Symmetrical

Total Flow = 96.6 gpm.

$F_o = 0.755$

Catch Basin Width = 12"

Tube to disk edge spacing	Pattern width at catch depth (inches)	Q_c gpm	Q_{lr} gpm	Total gpm	$\frac{\text{rem.}}{2}$	%width basin	% Q_c	% Q_{lr}	$\frac{\text{rem.}}{2}$	R_c	R_{lr}
1"	48	28.8	25.8	80.4	8.1	25	30	27	8	1.2	1.08
3"	38	30.6	27	84.6	6.0	32	32	28	6	1.0	.875
5"	42 NC	17.4	23.4	64.2	16.2	29	18	24	12	.62	.828
7"	" NC					"					"
9"	" NC					"					"

Table B-5

ANNULUS INJECTION EXPERIMENT
HOT LEG SIMULATION
Symmetrical

Total Flow = 139 gpm.

$F_o = 1.09$

Catch Basin Width = 12"

Tube to disk edge spacing	Pattern width at catch depth (inches)	Q_c gpm	Q_{lr} gpm	Total gpm	$\frac{\text{rem.}}{2}$	%width basin	% Q_c	% Q_{lr}	$\frac{\% \text{rem.}}{2}$	R_c	R_{lr}
1"	44	27	46.8	120.6	9.2	27	19	34	7	.704	1.25
3"	44	38	33.6	105	17	27	27	24	12	1.0	.889
5"	48	42	28.2	98.4	20.3	25	30	20	15	1.2	.80
7"	48	30	25.3	81.6	28.7	25	22	19	21	.88	.76
9"	54	18	24	66	36.5	22	13	17	26	.591	.773

Table B-6
ANNULUS INJECTION EXPERIMENT
HOT LEG SIMULATION
Asymmetrical

Total Flow = 70 gpm.

$F_o = 0.55$

Catch Basin Width = 12"

Tube to disk edge spacing	Pattern width at catch depth (inches)	Q_c gpm	Q_l gpm	Q_r gpm	Tot. gpm	$\frac{\text{rem.}}{2}$	% width basin	% Q_c	% Q_l	% Q_r	$\frac{\% \text{rem.}}{2}$	R_c	R_l	R_r
	(left--right)													
1"	18 + 17	21.6	22.8	26.4	70.8	-	34	31	33	38	-	.91	.97	1.1
3"	18 + 17	20.8	26.4	28.8	76	-	34	30	38	41	-	.88	1.1	1.2
5"	NC													NC
7"	NC													NC
9"	NC													NC

Table B-7

ANNULUS INJECTION EXPERIMENT
HOT LEG SIMULATION
Asymmetrical

Total Flow = 80 gpm.

$F_o = 0.625$

Catch Basin Width = 12"

Tube to disk edge spacing	Pattern width at catch depth (inches)	Q_c gpm	Q_l gpm	Q_r gpm	Tot. gpm	$\frac{\text{rem.}}{2}$	% width basin	% Q_c	% Q_l	% Q_r	$\frac{\% \text{rem.}}{2}$	R_c	R_l	R_r
	L. R.													
1"	19 + 18	18	30	28.8	76.8	1.6	32	23	38	36	2	.72	1.2	1.13
3"	19 + 18	21.6	27	24	72.6	3.7	32	27	34	30	5	.84	1.1	.94
5"	NC													NC
7"	NC													NC
9"	NC													NC

Table B-8

ANNULUS INJECTION EXPERIMENT
HOT LEG SIMULATION
Asymmetrical

Total Flow = 89 gpm.

$F_o = 0.695$

Catch Basin Width = 12"

Tube to disk edge spacing	Pattern width at catch depth (inches)	Q_c gpm	Q_l gpm	Q_r gpm	Tot. gpm	$\frac{\text{rem.}}{2}$	% width basin	% Q_c	% Q_l	% Q_r	$\frac{\% \text{rem.}}{2}$	R_c	R_l	R_r
	L. R.													
1"	20 + 19	21	25.8	25.8	72.6	8.2	31	24	29	29	9	.77	.94	.94
3"	20 + 19	19.8	29.4	24	73.2	7.9	31	22	33	27	9	.71	1.1	.87
5"	NC													NC
7"	NC													NC
9"	NC													NC

Table B-9

ANNULUS INJECTION EXPERIMENT
HOT LEG SIMULATION
Asymmetrical

Total Flow = 96.6 gpm.

$F_0 = 0.755$

Catch Basin Width = 12"

Tube to disk edge spacing	Pattern width at catch depth (inches)	Q_c gpm	Q_l gpm	Q_r gpm	Tot. gpm	$\frac{\text{rem.}}{2}$	% width basin	% Q_c	% Q_l	% Q_r	$\frac{\% \text{rem.}}{2}$	R_c	R_l	R_r
	L. R.													
1"	20 + 21	21.6	28.8	26.4	76.8	9.9	29	22	30	27	10	.76	1.0	.93
3"	20 + 20	22.2	25.8	25.2	73.2	12	30	23	27	26	12	.77	.90	.87
5"	20 + 20	16.8	28.8	25.8	71.4	13	30	17	30	27	13	.57	1.0	.90
7"	NC													NC
9"	NC													NC

Table B-10

ANNULUS INJECTION EXPERIMENT
HOT LEG SIMULATION
Asymmetrical

Total Flow = 139 gpm.

$F_o = 1.09$

Catch Basin Width = 12"

Tube to disk edge spacing	Pattern width at catch depth (inches)	Q_c gpm	Q_l gpm	Q_r gpm	Tot. gpm	$\frac{\text{rem.}}{2}$	% width basin	% Q_c	% Q_l	% Q_r	$\frac{\% \text{rem.}}{2}$	R_c	R_l	R_r
	L. R.													
1"	20 + 28	23.4	36	30	89.4	25	25	17	26	22	18	.68	1.0	.88
3"	20 + 28	28.2	37.2	24.6	90	25	25	20	27	18	18	.80	1.1	.72
5"	24 + 28	28.2	30	22.8	81	29	23	20	22	16	21	.87	.96	.70
7"	24 + 28	24	28.2	19.8	72	35	23	17	20	14	25	.74	.87	.61
9"	26 + 28	18	25.2	24	67.2	36	22	13	18	17	26	.59	.82	.77

FIGURE B-1 HOT AND COLD LEG PENETRATIONS IN THE TROJAN REACTOR

HOT LEG

ID = 29"

OD = 33.86"

COLD LEG

ID = 27.5"

OD = 29"

FIGURE B-2 MEASURED WATER FLOWING IN TWO REGIONS OF THE FLOW PATTERN FOR TWO SYMMETRICAL DISK SPACINGS

FIGURE B-3. MEASURED WATER FLOWING IN TWO REGIONS OF THE FLOW PATTERN FOR TWO ASYMMETRICAL DISK SPACINGS

FIGURE B-4

Photographs 1-15 on the following pages show the appearance of the flow for various Froude Numbers and arrangements of the discs.

Photograph 1
1" Disk Spacing -- $F_o = .55$

Photograph 2
1" Disk Spacing - $F_o = .77$

Photograph 3
1" Disk Spacing - $F_o = .95$ ASYMMETRICAL

Photograph 4
1" Disk Spacing - $F_o = 1.1$

Photograph 5
1' Disk Spacing - $F_O = .55$

Photograph 6
1" Disk Spacing - $F_O = .95$

Photograph 7
1" Disk Spacing - $F_O = 1.1$

SYMMETRICAL

Photograph 8
Asymmetrical . 7" Spacing - $F_o = .77$

Photograph 9
Asymmetrical . 7" Spacing - $F_o = .95$

Photograph 10
Symmetrical . 7" Spacing - $F_o = 1.1$

Photograph 11
Symmetrical . 7" Spacing - $F_o = .95$

Photograph 12
Symmetrical . 7" Spacing - $F_O = 1.1$

Photograph 13
Symmetrical . 5" Spacing - $F_O = .77$

Photograph 14
Symmetrical . 5" Spacing - $F_O = .95$

Photograph 15
Symmetrical . 5" Spacing - $F_O = 1.1$