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ANALOGUE COMPUTER SOLUTION OF THE
NONLINEAR REACTOR KINETICS EQUATION

by

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DEFINITIONS OF SYMBOLS

n	Prompt-neutron flux density
C_i	Flux density of i th delayed-neutron group
K_{ex}	Excess reactivity
β_i	The fraction of prompt neutrons entering the i th delayed-neutron group
$\beta = \sum \beta_i$	The fraction of prompt neutrons entering all delayed groups
ℓ	Prompt-neutron lifetime
λ_i	Decay constant of neutrons in the i th delayed group
t	Time
$n_1 = n(t)/n(0)$	The ratio of the prompt-neutron density at time t to the equilibrium value, $n(0)$
$C_{1i} = C_i(t)/C_i(0)$	The ratio of the density of neutrons in the i th delayed group to the equilibrium value, $C_i(0)$
Δn_1	The increment of n_1 from its equilibrium value
ΔC_{1i}	The increment of C_{1i} from its equilibrium value
$K_{lex} = K_{ex}/\beta$	Excess reactivity (in dollars)
$\left. \begin{array}{l} n_1' \\ C_{1i}' \\ t' \\ K_{lex}' \\ \Delta n_1' \\ \Delta C_{1i}' \end{array} \right\}$	Primes denote the computer voltages which represent the unprimed variables
$a = t'/t$	The ratio of machine time to actual time
$b = n_1'/n_1$	The ratio of the machine voltage representing n_1' to n_1
$d_i = C_{1i}'/C_{1i}$	The ratio of the machine voltage representing C_{1i}' to C_{1i}

DEFINITIONS OF SYMBOLS

$e = K'_{lex}/K_{lex}$ The ratio of the machine voltage representing K'_{lex} to K_{lex}

$\eta = \ln n_1$

$\epsilon = e^{-\eta} C_{1i}$

η' Equals machine value of η (in volts)

ϵ'_i Equals machine value of ϵ_i (in volts)

$\psi_i = \epsilon_i - 1$

ψ_i Equals machine value of ψ_i

ANALOGUE COMPUTER SOLUTION OF THE NONLINEAR REACTOR KINETICS EQUATION

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The Kinetics Equations

The reactor kinetics equations are:

$$\frac{dn}{dt} = \frac{1 - \beta}{\ell} K_{ex} n - \frac{\beta}{\ell} n + \sum \lambda_i C_i \quad ; \quad (1)$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\ell} K_{ex} n + \frac{\beta_i}{\ell} n - \lambda_i C_i \quad . \quad (2)$$

When the system is in equilibrium $\frac{dn}{dt} = 0$; $\frac{dC_i}{dt} = 0$; $K_{ex} = 0$. One then has, from (2),

$$\frac{\beta_i}{\ell} n(0) = \lambda_i C_i(0) \quad (3)$$

and since $\sum \beta_i = \beta$, Eq. (1) is also satisfied. In practice, a convenient arbitrary value, usually 1.000, is assumed for $n(0)$; the $C_i(0)$'s are then calculated from Eq. (3). Since the terms in Eqs. (1) and (2) differ considerably in magnitude, it is desirable to introduce new variables, defined as follows:

$$\left. \begin{aligned} n_1 &= n(t)/n(0) \\ C_{1i} &= C_i(t)/C_i(0) \\ K_{1ex} &= K_{ex}/\beta \end{aligned} \right\} \quad . \quad (4)$$

In many reactor systems ℓ/β is so small that $(\ell/\beta)(dn_1/dt)$ can be neglected. This is always true where, for this term to be significant, the capacity of the machine would have to be exceeded.

Substitution of Eq. (4) into Eqs. (1) and (2) leads to

$$\frac{\ell}{\beta} \frac{dn_1}{dt} = (1 - \beta) K_{1ex} n_1 - n_1 + \sum \frac{\beta_i}{\beta} C_{1i} \quad (5)$$

and

$$\frac{dC_{1i}}{dt} = \lambda_i \beta K_{1ex} n_1 + \lambda_i n_1 - \lambda_i C_{1i} \quad . \quad (6)$$

If $(\ell/\beta)(dn_1/dt)$ is neglected, Eqs. (5) and (6) become

$$0 = (1 - \beta)K_{lex}n_1 - n_1 + \sum(\beta_i/\beta) C_{1i} \quad (7)$$

and

$$\frac{dC_{1i}}{dt} = \lambda_i \beta K_{lex} n_1 + \lambda_i n_1 - \lambda_i C_{1i} \quad (8)$$

Other useful forms of the equations are found by introducing the deviations from equilibrium as new variables, as follows:

$$\left. \begin{aligned} \Delta n_1 &= \frac{n(t) - n(0)}{n(0)} = n_1(t) - 1 \\ \Delta C_{1i} &= \frac{C_i(t) - C_i(0)}{C_i(0)} = C_{1i}(t) - 1 \end{aligned} \right\} \quad (9)$$

Substituting Eq. (9) in Eqs. (5) and (6), there is obtained

$$\frac{\ell}{\beta} \frac{d\Delta n_1}{dt} = (1 - \beta)K_{lex}\Delta n_1 - \Delta n_1 + \sum \frac{\beta_i}{\beta} \Delta C_{1i} + (1 - \beta)K_{lex} \quad (10)$$

and

$$\frac{d\Delta C_{1i}}{dt} = \lambda_i \beta K_{lex} \Delta n_1 + \lambda_i \Delta n_1 - \lambda_i \Delta C_{1i} + \lambda_i \beta K_{lex} \quad (11)$$

These equations differ from Eqs. (5) and (6) only by the forcing function terms, $(1 - \beta)K_{lex}$ and $\lambda_i \beta K_{lex}$, which appear on the right-hand side.

Equations (10) and (11) with $(\ell/\beta)(dn_1/dt)$ neglected become

$$0 = (1 - \beta)K_{lex}\Delta n_1 - \Delta n_1 + \sum(\beta_i/\beta) \Delta C_{1i} + (1 - \beta)K_{lex} \quad (12)$$

and

$$\frac{d\Delta C_{1i}}{dt} = \lambda_i \beta K_{lex} \Delta n_1 + \lambda_i \Delta n_1 - \lambda_i \Delta C_{1i} + \lambda_i \beta K_{lex} \quad (13)$$

The Machine Equations

Even though the transformations (4) reduce the problem of scaling considerably, it is still necessary to set up an equivalence between machine voltages and the problem variables as follows:

$$\left. \begin{aligned} at &= t' \\ bn_1 &= n_1' \\ d_i C_{1i} &= C_{1i}' \\ eK_{1ex} &= K_{1ex}' \end{aligned} \right\} , \quad (9A)$$

where primes denote the machine voltages corresponding to the unprimed variable.

The kinetics equations then become

$$\frac{a\ell}{\beta} \frac{dn_1'}{dt'} = \frac{(1-\beta)}{e} K_{1ex}' n_1' - n_1' + \Sigma \frac{\beta_i b}{\beta d_i} C_{1i}' \quad (5')$$

$$\frac{dC_{1i}'}{dt'} = \frac{\beta \lambda_i d_i}{eab} K_{1ex}' n_1' + \frac{\lambda_i d_i}{ab} n_1' - \frac{\lambda_i}{a} C_{1i}' \quad (6')$$

$$0 = \frac{1-\beta}{e} K_{1ex}' n_1' - n_1' + \Sigma \frac{\beta_i b}{\beta d_i} C_{1i}' \quad (7')$$

$$\frac{dC_{1i}'}{dt'} = \frac{\lambda_i \beta d_i}{abe} K_{1ex}' n_1' + \frac{\lambda_i d_i}{ab} n_1' - \frac{\lambda_i}{a} C_{1i}' \quad (8')$$

$$\frac{a\ell}{\beta} \frac{d\Delta n_1'}{dt'} = \frac{(1-\beta)}{e} K_{1ex}' \Delta n_1' - \Delta n_1' + \Sigma \frac{b\beta_i}{d_i \beta} \Delta C_{1i}' + \frac{1-\beta}{e} b K_{1ex}' \quad (10')$$

$$\frac{d\Delta C_{1i}'}{dt'} = \frac{\lambda_i \beta d_i}{abe} K_{1ex}' \Delta n_1' + \frac{\lambda_i d_i}{ab} \Delta n_1' - \frac{\lambda_i}{a} \Delta C_{1i}' + \frac{\lambda_i \beta d_i}{ae} K_{1ex}' \quad (11')$$

$$0 = \frac{1-\beta}{e} K_{1ex}' \Delta n_1' - \Delta n_1' + \Sigma \frac{b\beta_i}{d_i \beta} \Delta C_{1i}' + \frac{(1-\beta)}{e} b K_{1ex}' \quad (12')$$

$$\frac{d\Delta C_{1i}'}{dt'} = \frac{\lambda_i \beta d_i}{abe} K_{1ex}' \Delta n_1' + \frac{\lambda_i d_i}{ab} \Delta n_1' - \frac{\lambda_i}{a} \Delta C_{1i}' + \frac{\lambda_i \beta d_i}{ae} K_{1ex}' \quad (13')$$

Initial Conditions

Initially when the reactor is in a steady state,

$$\left. \begin{aligned} \frac{dn_1}{dt} &= \frac{dC_{1i}}{dt} = K_{1ex} = 0 \\ n_1(t) &= 1 \\ C_{1i}(t) &= 1 \end{aligned} \right\} . \quad (14)$$

Substituting Eq. (14) in Eqs. (5) and (6), there is obtained

$$\sum (\beta_i/\beta) = 1 \quad ,$$

since

$$0 = -1 + \sum (\beta_i/\beta) \quad (5'')$$

$$0 = \lambda_i - \lambda_i \quad (6'')$$

The corresponding machine equilibrium conditions are

$$\begin{aligned} n_1'(0) &= b \\ C_{1i}'(0) &= d_i \end{aligned} \quad (15)$$

When the incremental values Δn_1 and ΔC_{1i} are used, the equilibrium conditions are

$$\left. \begin{aligned} \frac{d\Delta n_1}{dt} &= \frac{dC_{1i}}{dt} = K_{1ex} = 0 \\ \Delta n_1 &= 0 \\ \Delta C_{1i} &= 0 \end{aligned} \right\} \quad (16)$$

Substituting Eq. (16) in Eqs. (10) or (11), there is obtained

$$0 = 0 \quad (10'')$$

and

$$0 = 0 \quad (11'')$$

A further condition imposed on the incremental equations (10') and (11') is that when Δn_1 and ΔC_{1i} are equal to -1, then

$$\frac{d\Delta n_1}{dt} = \frac{d\Delta C_{1i}}{dt} = 0 \text{ for all } k_{ex}. \quad (17)$$

Substituting Eq. (17) in Eqs. (10') and (11'), then

$$0 = -(1 - \beta)K_{ex} + 1 - \sum (\beta_i/\beta) + (1 - \beta)K_{ex} = 0 \quad (10A)$$

and

$$0 = -\lambda_i \beta K_{1ex} - \lambda_i + \lambda_i + \lambda_i \beta K_{ex} \quad (11A)$$

Let

$$\left. \begin{aligned} \eta &= \ln n_1 \\ \epsilon_i &= e^{-\eta} C_{1i} \end{aligned} \right\} \quad (12A)$$

Substituting Eq. (12A) in Eqs. (5) and (6) ,

$$\frac{\ell}{\beta} e^{\eta} \frac{d\eta}{dt} = (1 - \beta) K_{1ex} e^{\eta} - e^{\eta} + e^{\eta} \sum \frac{\beta_i}{\beta} \epsilon_i \quad (18)$$

and

$$e^{\eta} \frac{d\epsilon_i}{dt} + \epsilon_i e^{\eta} \frac{d\eta}{dt} = \lambda_i \beta K_{1ex} e^{\eta} + \lambda_i e^{\eta} - \lambda_i e^{\eta} \epsilon_i \quad (19)$$

On collecting terms, Eqs. (18) and (19) become

$$\frac{d\eta}{dt} = \frac{\beta}{\ell} \left[(1 - \beta) K_{1ex} - 1 + \sum \frac{\beta_i}{\beta} \epsilon_i \right] \quad (20)$$

and

$$\frac{d\epsilon_i}{dt} = \lambda_i \beta K_{1ex} + \lambda_i - \epsilon_i \left(\lambda_i + \frac{d\eta}{dt} \right) \quad (21)$$

or, in terms of machine voltages,

$$\begin{aligned} \frac{a}{b} \frac{d\eta'}{dt} &= \frac{\beta}{\ell} \left[\frac{(1 - \beta)}{e} K_{1ex}' - 1 + \sum \frac{\beta_i}{\beta} \frac{\epsilon_i'}{d_i} \right] \\ \frac{a}{d_i} \frac{d\epsilon_i'}{dt} &= \frac{\lambda_i \beta}{e} K_{1ex}' + \lambda_i - \epsilon_i' \left(\frac{\lambda_i}{d_i} + \frac{a}{b d_i} \frac{d\eta}{dt} \right) \\ \frac{d\eta'}{dt} &= \frac{b}{a} \frac{\beta}{\ell} \left[\frac{1 - \beta}{e} K_{1ex}' - 1 + \sum \frac{\beta_i}{\beta d_i} \epsilon_i' \right] \end{aligned} \quad (22)$$

$$\frac{d\epsilon_i'}{dt} = \frac{\lambda_i \beta d_i}{a e} K_{1ex}' + \frac{d_i \lambda_i}{a} - \epsilon_i' \left(\frac{\lambda_i}{a} + \frac{1}{b} \frac{d\eta}{dt} \right) \quad (23)$$

when

$$\left. \begin{aligned} t = 0; \quad \eta = 0; \quad \epsilon_i = 1 \\ \text{or} \\ t = 0; \quad \eta' = 0; \quad \epsilon_i' = d_i \end{aligned} \right\} \quad (24)$$

Let

$$\epsilon_i - 1 = \psi \quad . \quad (25)$$

Substituting Eq. (25) in Eqs. (20) and (21),

$$\frac{d\eta}{dt} = \frac{\beta}{\ell} \left[(1 - \beta) K_{1ex} + \sum \frac{\beta_i}{\beta} \psi_i \right] \quad (26)$$

and

$$\frac{d\psi_i}{dt} = \lambda_i \beta K_{1ex} - \frac{d\eta}{dt} - \psi_i \left(\lambda_i + \frac{d\eta}{dt} \right) \quad . \quad (27)$$

In terms of machine voltages

$$\frac{a}{b} \frac{d\eta'}{dt'} = \frac{\beta}{\ell} \left[\frac{(1 - \beta)}{e} K_{1ex}' + \sum \frac{\beta_i}{d_i \beta} \psi_i' \right]$$

$$\frac{a}{d_i} \frac{d\psi_i'}{dt'} = \lambda_i \beta \frac{K_{1ex}'}{e} - \frac{a}{b} \frac{d\eta'}{dt'} - \frac{\psi_i'}{d_i} \left(\lambda_i + \frac{a}{b} \frac{d\eta'}{dt'} \right)$$

or

$$\frac{d\eta'}{dt'} = \frac{b}{a} \frac{\beta}{\ell} \left[\frac{1 - \beta}{e} K_{1ex}' + \sum \frac{\beta_i}{d_i \beta} \psi_i' \right] \quad (28)$$

$$\frac{d\psi_i'}{dt'} = \frac{d_i \lambda_i \beta}{ae} K_{1ex}' - \frac{d_i}{b} \frac{d\eta'}{dt'} - \psi_i' \left(\frac{\lambda_i}{a} + \frac{1}{b} \frac{d\eta'}{dt'} \right) \quad . \quad (29)$$

Let $a = \alpha(\beta/\ell)$, then

$$\frac{d\eta'}{dt'} = \frac{b}{\alpha} \left[\frac{1 - \beta}{e} K_{1ex}' + \sum \frac{\beta_i}{d_i \beta} \psi_i' \right] \quad (30)$$

and

$$\frac{d\psi_i'}{dt'} = \frac{d_i \lambda_i \ell}{\alpha e} K_{1ex}' - \frac{d_i}{b} \frac{d\eta'}{dt'} - \psi_i' \left(\frac{\ell \lambda_i}{\alpha \beta} + \frac{1}{b} \frac{d\eta'}{dt'} \right) \quad . \quad (31)$$

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POTENTIOMETER SETTINGS

PROBLEM NO. INS. - 2
 DRAWING NO. 1
 DATE 6/19/59

REACTOR KINETICS (n_1)

POTENTIOMETER NO.		MATHEMATICAL VALUE	VALUE	CORREC- TION	SETTING	SET	PARAMETERS
DRAWING	MACHINE						
	21	λ_1/a	.0125		.0125		$l = 2 \times 10^{-4}$
	22	λ_2/a	.0315		.0315		$\beta = .00755$
	23	λ_3/a	.1535		.1535		$\lambda_1 = .01246$
	24	λ_4/a	.4560		.4560		$\lambda_2 = .0315$
	25	λ_5/a	1.612		.1612 (10)		$\lambda_3 = .1535$
	26	λ_6/a	14.30		.9533 (15)		$\lambda_4 = .4560$
	27	$\beta_1 b/\beta d_1$	.0331		.0331		$\lambda_5 = 1.612$
	28	$\beta_2 b/\beta d_2$	.2199		.2199		$\lambda_6 = 14.30$
	29	$\beta_3 b/\beta d_3$	.2821		.2821		$\beta_1 = .00025$
	30	$\beta_4 b/\beta d_4$	.3192		.3192		$\beta_2 = .00166$
	31	$\beta_5 b/\beta d_5$	.1126		.1126		$\beta_3 = .00213$
	32	$\beta_6 b/\beta d_6$	.0331		.0331		$\beta_4 = .00241$
	33	$\lambda_1 d_1/a b$	.0125		.0125		$\beta_5 = .00085$
	34	$\lambda_2 d_2/a b$	.0315		.0315		$\beta_6 = .00025$
	35	$\lambda_3 d_3/a b$	.1535		.1535		$a = 1$
	36	$\lambda_4 d_4/a b$	.4560		.4560		$b = d_1 = 10$
	37	$\lambda_5 d_5/a b$	1.612		.1612 (10)		$e = 20$
	38	$\lambda_6 d_6/a b \times 10^{-1}$	1.430		.1430 (10)		
	39	.1 $\lambda_1 d_1/a b$	.0013		.0013		
	40	.1 $\lambda_2 d_2/a b$	.0032		.0032		
	41	.1 $\lambda_3 d_3/a b$	.0154		.0154		
	42	.1 $\lambda_4 d_4/a b$	.0456		.0456		
	43	.1 $\lambda_5 d_5/a b$	.1612		.1612		
	44	.1 $\lambda_6 d_6/a b \times 10^{-1}$	.1430		.1430		

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PROBLEM NO. INS. - 2
 DRAWING NO. 3
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REACTOR KINETICS (Δn)

POTENTIOMETER NO.		MATHEMATICAL VALUE	VALUE	CORREC- TION	SETTING	SET	PARAMETERS
DRAWING	MACHINE						
	21	λ_1/a	.0125		.0125		$t = 2 \times 10^{-4}$
	22	λ_2/a	.0315		.0315		$\beta = .00755$
	23	λ_3/a	.1535		.1535		$\lambda_1 = .01246$
	24	λ_4/a	.4560		.4560		$\lambda_2 = .0315$
	25	λ_5/a	1.612		.1612 (10)		$\lambda_3 = .1535$
	26	λ_6/a	14.30		.9533 (15)		$\lambda_4 = .4560$
	27	$b\beta_1/d_1\beta$	.0331		.0331		$\lambda_5 = 1.612$
	28	$b\beta_2/d_2\beta$	.2199		.2199		$\lambda_6 = 14.30$
	29	$b\beta_3/d_3\beta$	.2821		.2821		$\beta_1 = .00025$
	30	$b\beta_4/d_4\beta$	.3192		.3192		$\beta_2 = .00166$
	31	$b\beta_5/d_5\beta$	.1126		.1126		$\beta_3 = .00213$
	32	$b\beta_6/d_6\beta$	.0331		.0331		$\beta_4 = .00241$
	33	$\lambda_1 d_1/a b$	.0125		.0025 (5)		$\beta_5 = .00085$
	34	$\lambda_2 d_2/a b$	.0315		.0063 (5)		$\beta_6 = .00025$
	35	$\lambda_3 d_3/a b$	.1535		.0307 (5)		$a = 1$
	36	$\lambda_4 d_4/a b$	.4560		.0912 (5)		$b = d_i = e = 200$
	37	$\lambda_5 d_5/a b$	1.612		.1612 (10)		
	38	$\lambda_6 d_6/a b$	14.30		.9533 (15)		
	39	$\lambda_1 \beta d_1/a e$	.0000		.0000		
	40	$\lambda_2 \beta d_2/a e$	.0001		.0001		
	41	$\lambda_3 \beta d_3/a e$	.0003		.0003		
	42	$\lambda_4 \beta d_4/a e$	.0011		.0011		
	43	$\lambda_5 \beta d_5/a e$	.0014		.0014		
	44	$\lambda_6 \beta d_6/a e$	.0036		.0036		

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PROBLEM NO. INS. - 2
DRAWING NO. 4
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REACTOR KINETICS (Δn)[illegible]

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POTENTIOMETER SETTINGS

PROBLEM NO. INS. - 2
 DRAWING NO. 5
 DATE 6/19/59

REACTOR KINETICS ($\ln \eta$)

POTENTIOMETER NO.		MATHEMATICAL VALUE	VALUE	CORREC- TION	SETTING	SET	PARAMETERS
DRAWING	MACHINE						
	22	λ_1/a	.0125		.0125		$\ell = 2 \times 10^{-4}$
	23	λ_2/a	.0315		.0315		$\beta = .00755$
	24	λ_3/a	.1535		.1535		$\lambda_1 = .01246$
	25	λ_4/a	.4560		.4560		$\lambda_2 = .0315$
	26	λ_5/a	1.612		.3224 (5)		$\lambda_3 = .1535$
	27	λ_6/a	14.30		.9533 (15)		$\lambda_4 = .4560$
	28	$b \beta_1/d_1 \beta$	.0331		.0331		$\lambda_5 = 1.612$
	29	$b \beta_2/d_2 \beta$	.2199		.0220 (10)		$\lambda_6 = 14.30$
	30	$b \beta_3/d_3 \beta$	.2821		.0564 (5)		$\beta_1 = .00025$
	31	$b \beta_4/d_4 \beta$	.3192		.0638 (5)		$\beta_2 = .00166$
	32	$b \beta_5/d_5 \beta$	.1126		.1126		$\beta_3 = .00213$
	33	$b \beta_6/d_6 \beta$	.0331		.0331		$\beta_4 = .00241$
	34	$\ell a/\beta$	.0265		.0265		$\beta_5 = .00085$
	39	$d_1 \lambda_1 \beta/a e$	.0000		.0000		$\beta_6 = .00025$
	40	$d_2 \lambda_2 \beta/a e$	.0000		.0000		$a = 1$
	41	$d_3 \lambda_3 \beta/a e$	.0001		.0001		$b = d_1 = 10$
	42	$d_4 \lambda_4 \beta/a e$	.0003		.0003		$e = 100$
	43	$d_5 \lambda_5 \beta/a e$	.0012		.0012		
	44	$d_6 \lambda_6 \beta/a e$	.0108		.0108		
	45	$b(1-\beta)/e$	.0992		.0992		
	46	$1/b$	.1000		.1000		
	50	d_1/b	1.000		1.000		
	51	d_2/b	1.000		1.000		
	52	d_3/b	1.000		1.000		

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PROBLEM NO. INS. - 2
 DRAWING NO. 7
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REACTOR KINETICS (Δn)

POTENTIOMETER NO.		MATHEMATICAL VALUE	VALUE	CORREC- TION	SETTING	SET	PARAMETERS
DRAWING	MACHINE						
	21	λ_1/a	.0127		.0127		$l = 8 \times 10^{-8}$
	22	λ_2/a	.0318		.0318		$\beta = .00683$
	23	λ_3/a	.1153		.1153		$\lambda_1 = .0127$
	24	λ_4/a	.3110		.3110		$\lambda_2 = .0318$
	25	λ_5/a	1.400		.1400 (10)		$\lambda_3 = .1153$
	26	λ_6/a	3.870		.3870 (10)		$\lambda_4 = .3110$
	27	$b \beta_1/d_1 \beta$	.0343		.0343		$\lambda_5 = 1.400$
	28	$b \beta_2/d_2 \beta$	.2016		.2016		$\lambda_6 = 3.870$
	29	$b \beta_3/d_3 \beta$	.1840		.1840		$\beta_1 = .000234$
	30	$b \beta_4/d_4 \beta$	.4041		.4041		$\beta_2 = .001377$
	31	$b \beta_5/d_5 \beta$	.1426		.1426		$\beta_3 = .001257$
	32	$b \beta_6/d_6 \beta$	.0334		.0334		$\beta_4 = .00276$
	33	$\lambda_1 d_1/a b$	.0127		.0025 (5)		$\beta_5 = .000974$
	34	$\lambda_2 d_2/a b$	.0318		.0064 (5)		$\beta_6 = .000228$
	35	$\lambda_3 d_3/a b$	.1153		.0231 (5)		$a = 1$
	36	$\lambda_4 d_4/a b$	.3110		.0622 (5)		$b = d_1 = 200$
	37	$\lambda_5 d_5/a b$	1.400		.1400 (10)		$e = 200$
	38	$\lambda_6 d_6/a b$	3.870		.3870 (10)		$(1-\beta) = .99317$
	39	$\lambda_1 \beta_1 d_1/a e$	.0001		.0001		
	40	$\lambda_2 \beta_2 d_2/a e$	.0002		.0002		
	41	$\lambda_3 \beta_3 d_3/a e$	.0008		.0008		
	42	$\lambda_4 \beta_4 d_4/a e$	.0021		.0021		
	43	$\lambda_5 \beta_5 d_5/a e$	.0096		.0096		
	44	$\lambda_6 \beta_6 d_6/a e$	.0264		.0264

NON-LINEAR REACTOR KINETICS
SCHEMATIC FOR η'
FIGURE 1

FIGURE 2
 $\Delta \eta'$ REACTOR KINETICS SCHEMATIC

FIGURE 3

NON-LINEAR REACTOR KINETICS EQUATIONS

($\eta = \ln h_i$)

NON-LINEAR REACTOR KINETICS EQUATION

 n_1 vs. t

NON-LINEAR REACTOR KINETICS EQUATION NEGATIVE REACTIVITY, n_1 vs. t

NON-LINEAR REACTOR KINETICS EQUATION NEGATIVE REACTIVITY, n_1 vs. t

Δn vs. t

Δn vs. t

$\ln n$ vs. t

Δn vs. t

Δn vs. t