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THERMAL STRESSES IN RECTANGULAR STRIPS II

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ABSTRACT

Stresses and deformations in rectangular strips due to various longitudinal temperature distributions are presented in formulas, tables and graphs. The results are important for slabs, plate assemblies, rectangular ducts, tube sheet ligaments and various other structures.

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1. Discussion

Purpose of the present paper is to extend the theory of Reference 1, developed for the (vertical) step function temperature distribution illustrated in Figure 1a, to the practically more often recurring cases of a break in temperature as in Figure 1b, an inclined step of arbitrary length L as in Figure 1c (idealized by the dashed line), behavior near the end of a strip as in Figures 1d, 1e. Other cases may be handled by superpositions of the basic cases illustrated in Figures 1a, b, c, d, e.

In Figures 2a, b, c, d, e are shown, pictorially, the stresses σ_x, σ_y, τ and displacements u, v produced in a thin rectangular strip of width $2b$ when a temperature break T' degrees/inch is imposed at an internal cross-section ^{far from the ends}. The principal results, derived in Sections 2, 3, are*

$$\bar{\sigma}_x(0) = .098 S \quad S = E \alpha T' b$$

$$\bar{\sigma}_x(b) = \sigma_{x \max} = -.328 S$$

$$\bar{\sigma}_y(0) = \sigma_{y \max} = .141 S$$

$$\tau(.45b, .67b) = \tau_{\max} = .041 S$$

$$\bar{u}(0) = -.083 b^2 \alpha T'$$

$$\bar{u}(b) = u_{\max} = .167 b^2 \alpha T'$$

$$\bar{v}(b) = v_{\max} = .096 b^2 \alpha T'$$

[1]

* As in Reference 1, we use bars to indicate values at $x=0$, and the circumflex to indicate values for the extreme fiber, $y=b$. Thus: $\bar{\sigma}_x(y) = \sigma_x(0, y)$, $\hat{\sigma}_x(x) = \sigma_x(x, b)$.

The calculations were performed by 3 methods: (A) the Fourier integral method, (B) the biharmonic eigenfunction expansion method, and (C) the self-equilibrating function expansion method; the results are recorded in Tables I a, b, c, d, e. The self-equilibrating function method uses as starting point the precise $\bar{\sigma}_x(y)$ distribution, ~~as in 2.50~~, as obtained from a numerically evaluated Fourier integral, Eq. [9a], and then solves the problem as an end problem.

In Section 4 we superimpose the foregoing results (where available, the precise values were used), as illustrated schematically in Figure 3 for σ_x to treat the problem of the inclined-step, Figure 1c, for various lengths, L, of the step. Figure 4a plots, vs L/b, the curves of $\bar{\sigma}_x(0)$, $\sigma_{x \max} = \bar{\sigma}_x(b)$, $\sigma_{y \max} = \bar{\sigma}_y(0)$. The maximum shear stress, $\tau_{\max}$, as well as its location x_r, y_r are plotted in Figure 4b.

In Section 5 we treat the cases of Figures 1d, e where the temperature break occurs L = 0.4b distance away from the end. The stress results, except for sign, are identical for cases d, e; using subscripts d, e, we may write

$$\sigma_{x,e}(x,y) = -\sigma_{x,d}(x,y)$$

$$\sigma_{y,e}(x,y) = -\sigma_{y,d}(x,y)$$

$$\tau_e(x,y) = -\tau_d(x,y)$$

[2a]

The displacements for Cases d, e, differ only by very simple expressions,

$$u_e(x,y) = \frac{3y^2 - b^2}{6b} [1 - (x-L)\alpha T']^2 - u_d(x,y)$$

$$v_e(x,y) = -y(x-L)\alpha T' - v_d(x,y)$$

[2b]

The problem of Case d was solved by taking the stress values $\sigma(.4b, y)$, $\tau(.4b, y)$ for the Case of Figure 1b from Table I (in the self-equilibrating approximation), and fitting to these values, by least squares, the polynomials listed in Eqs. [18].

The Airy functions ϕ_σ, ϕ_τ corresponding to the values $-\sigma_x(.4b, y), -\tau(.4b, y)$ are next expanded into self-equilibrating functions, in Equations [20]. Superposition of the effect of the compensating edge stresses $\hat{\phi}_{\sigma,yy}, \hat{\phi}_{\tau,xy}$ on the Figure 1b stress-condition eliminates the unwanted stresses along edge $x = L$, and leads to Figures 5. (The arc above ϕ refers to the right edge.)

It is shown that the maximum stresses developed for cases d and e, when $L/b = 0.4$, are

$$\begin{aligned} \sigma_{x \max} &= \sigma_x(-.03b, b) = -.234 S, & \sigma_{y \max} &= \sigma_y(-.03b, 0) = .165 S \\ \tau_{\max} &= \tau(-.35b, .65b) = -.059 S & S &= E \alpha T' b \end{aligned} \quad [3]$$

It is somewhat surprising to see that the nearness of the temperature break to the free edge does not effect a greater than 30% reduction in the maximum stress. When $L/b \rightarrow 0$ all stresses become zero.

The foregoing results apply to the conditions of plane stress (the strip is very thin, its thickness $H \ll b$). For the case of plain strain ($H \gg b$, subscript II) the stress values are obtained in terms of the former ones (subscript I) by the formulas

$$\sigma_{II} = \frac{\sigma_I}{1-\mu}, \quad \tau_{II} = \frac{\tau_I}{1-\mu} \quad [4a]$$

i.e. simply by multiplying the results of Tables, I, II by

$$k = 1/(1-\mu) = 1.4 \quad (\text{for } \mu = 0.3)$$

as shown in Eqs. [30], [31] of Reference 3. The conversion of the displacements is somewhat more complicated. As shown in Eq. [33] of Reference 3, the

* Note the misprint in Equation [39] of Reference 3. Instead of $(k + 33)$ write $(k + 3)$.
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displacements u_I, v_I should be first recalculated for an effective Poisson ratio of

$$\nu = \frac{\mu}{1-\mu} = 0.43$$

and then should be multiplied by $1+\mu$, to give u_{II}, v_{II} . If we ignore the slight change that arises in the displacements u_I, v_I through a change of the Poisson ratio from .3 to .43, then we may use the simpler, though not quite accurate formulas

$$u_{II} = (1+\mu)u_I, \quad v_{II} = (1+\mu)v_I \quad [46]$$

The paper is concluded by an Appendix which provides convenient formulas for least square approximation of even and odd functions.

2. The Fourier Integral Expressions of the Stresses and Deformations Corresponding to a Temperature Break

The stresses and deformations for a step function temperature distribution were given, for $x \geq 0$, in [I.5].* We now apply the superposition principle: If the effect of the temperature step $T = 1$, applied to a 2-inch wide strip at $x = 0$, is $F(x)$ at location x , then the effect of the break T' degrees per inch is

$$G(x) = T' \int_x^\infty F(x) dx \quad [5]$$

* We use I to refer to equations in Reference 1. In the present and the next Section we shall assume, for simplicity of writing, that the semi-width b of the strip is unity.

By integrating the expressions [I.5] in the above fashion we arrive at the results for the temperature break T'

$$\sigma_x(x, y)/E\alpha T' = \frac{2}{\pi} \int_0^{\infty} [(A+2B) \cosh \lambda y + B\lambda y \sinh \lambda y] \frac{\cos \lambda x}{\lambda^2} d\lambda$$

$$\sigma_y(x, y)/E\alpha T' = \frac{2}{\pi} \int_0^{\infty} \left[\frac{1}{2} - A \cosh \lambda y - B\lambda y \sinh \lambda y \right] \frac{\cos \lambda x}{\lambda^2} d\lambda$$

$$\tau(x, y)/E\alpha T' = \frac{2}{\pi} \int_0^{\infty} [(A+B) \sinh \lambda y + B\lambda y \cosh \lambda y] \frac{\sin \lambda x}{\lambda^2} d\lambda$$

$$u(x, y)/\alpha T' = \frac{3y^2-1}{12} + \frac{2}{\pi} \int_0^{\infty} \left[-\frac{\mu}{2} + \{ (1+\mu)A + 2B \} \cosh \lambda y + (1+\mu)B\lambda y \sinh \lambda y \right] \frac{\sin \lambda x}{\lambda^3} d\lambda$$

$$v(x, y)/\alpha T' = \frac{2}{\pi} \int_0^{\infty} \left[\frac{\lambda y}{2} - \{ (1+\mu)A - (1-\mu)B \} \sinh \lambda y - (1+\mu)B\lambda y \cosh \lambda y \right] \frac{\cos \lambda x}{\lambda^3} d\lambda$$

where

$$A = \frac{\lambda \cosh \lambda + \sinh \lambda}{2\lambda + \sinh 2\lambda}, \quad B = \frac{-\sinh \lambda}{2\lambda + \sinh 2\lambda}$$

In the displacement expressions we have chosen the point $(\infty, 0)$ as fixed point and the x axis as fixed axis. The stresses and displacements for negative x are obtained from [6] in the form:

For $x = -X < 0$:

$$\sigma_x(x, y) = \sigma_x(X, y)$$

$$\sigma_y(x, y) = \sigma_y(X, y)$$

$$\tau(x, y) = -\tau(X, y)$$

[8]

$$u(x, y) = \frac{3y^2-1}{6} (1-x\alpha T')^2 \alpha T' - u(X, y)$$

$$v(x, y) = -y\alpha T' + v(X, y)$$

One verifies, as in Reference 1, that Equations [6] and [8] satisfy all boundary conditions of the problem.

For the $x = 0$ cross-section, Equations [6] lead to the particularly important results

$$\bar{\sigma}_x(y)/E\alpha T' = .09665 - .12722y^2 - .64847y^4 + .113632y^6 - .78332y^8 \quad [9a]$$

$$\bar{\tau}(y)/E\alpha T' = 0 \quad [9b]$$

$$\bar{u}(y)/\alpha T' = \frac{3y^2-1}{12} \quad [9c]$$

$$\bar{v}(y)/\alpha T' = .10844y + .00241y^3 - .05258y^5 + .03859y^7 - .00083y^9 \quad [9d]$$

The expressions [9a] , [9d] of $\bar{\sigma}_x$, $\bar{v}$ were obtained by fitting, by least squares, 8th and 9th degree polynomials, respectively, to the values derived from Formulas [6] for $x = 0$, by numerical integration at the ordinates $y = 0, .2, .4, .6, .8, .9, .95, 1.0$. (The pertinent formulas for least square fitting are listed in the Appendix.) As expected, the temperature break of Figure 1b gives rise at $x = 0$ to a precisely parabolic displacement $\bar{u}(y)$ which is independent of μ , and which has a curvature one half of that developed at $x = -\infty$.

When the strip width is $2b$ rather than 2 then all the foregoing results apply provided one makes the replacements

$$x \rightarrow x/b, y \rightarrow y/b, T' \rightarrow bT', u \rightarrow u/b, v \rightarrow v/b \quad [10]$$

3. Evaluation of Stresses and Displacements.

A. Numerical Integration

This is performed in the same manner as in Reference 1. The values obtained in this fashion are indicated in Table 1 by a star.

B. Biharmonic eigenfunction expansion

A few values of $\sigma_x(x, y)$, noted by # in Table 1, were obtained by expanding the Airy function of the problem into biharmonic eigenfunctions

$$\mathcal{G}(x, y) = E\alpha T' R \sum \frac{\sin^2 z_n}{\cos^3 z_n} \frac{\Phi_n(x, y)}{z_n} \quad (x \geq 0) \quad [11]$$

$$\Phi_n = \frac{e^{-z_n x}}{z_n^2} (\cos z_n y - y \cot z_n \sin z_n y)$$

and taking

$$\sigma_x = \mathcal{G}_{,yy} \quad [12]$$

In the above formula [11] the summation is carried out over the roots

$$z_n = \alpha_n + i\beta_n \quad [13]$$

of

$$\sin 2z + 2z = 0$$

[14]

which have positive real part. ($\Re$ = real part.)

C. Self-equilibrating function expansion

By double integration of the $x = 0$ boundary values [9a] of σ_x we find that the boundary value of the Airy function is

$$\begin{aligned} \bar{D}(y)/E\alpha T' &= \int_{-1}^y d\eta \int_0^{\eta} \bar{\sigma}_x(\eta) d\eta / E\alpha T' \\ &= -.027695 + .048324y^2 - .010601y^4 - .021616y^6 + .020291y^8 - .008704y^{10} \end{aligned} \quad [15]$$

and therefore the expansion of $D(x, y)$ into self-equilibrating functions is

$$\begin{aligned} D(x, y)/E\alpha T' &= \sum \langle \bar{D}/E\alpha T', f_k \rangle f_k g_k \\ &= -.025544 f_2 g_2 - .000751 f_4 g_4 - .000068 f_6 g_6 - .000030 f_8 g_8 - \dots \\ &= \sum c_k f_k g_k \end{aligned} \quad [16]$$

Stresses and deformations have now the expressions

$$\begin{aligned} \sigma_x &= D_{,yy}, \quad \sigma_y = D_{,xx}, \quad \tau = -D_{,xy} \\ u(x, y)/\alpha T' &= \sum c_k (f_k'' \tilde{g}_k - \mu f_k g_k') \\ v(x, y)/\alpha T' &= \sum c_k (\tilde{f}_k g_k'' - \mu f_k' g_k) \end{aligned} \quad [17]$$

and their numerical values are listed in Table 1 without markings. (The values of the functions $f_k''(y), \dots, g_k(x)$ are tabulated in Reference 4.)

4. The Inclined step temperature distribution

This case is solved by superposing two breaks, as illustrated schematically in Figure 3 for the case of σ_x . The developed stress maxima and the location of the maximum shear stress are shown in Figures 4a, b.

5. Temperature break near the end of a strip

Let the break occur a distance $L = 0.4b$ from the end. If there were no free then and the stresses σ_x and τ developed at $x = 0.4b$ would have the values ($\eta = y/b$)

$$\begin{aligned}\sigma_x(.4b, y)/E\alpha T'b &= .07869 - .19715\eta^2 - .06684\eta^4 + .01646\eta^6 - .01759\eta^8 \\ \tau(.4b, y)/E\alpha T'b &= .08589\eta - .04614\eta^3 - .01283\eta^5 - .02701\eta^7 + .00010\eta^9\end{aligned}\quad [18]$$

(8th and 9th degree)

These are least square fits to the "self-equilibrating function" values listed in Table 1.

We eliminate the stresses [18] by applying compensating tractions

$$\begin{aligned}\phi_{\sigma, yy} &= -\sigma_x(.4b, y) \\ -\phi_{\tau, xy} &= -\tau(.4b, y)\end{aligned}\quad [19]$$

at the edge of the strip. The corresponding Airy functions are seen to be $\phi_{\sigma} = \phi_{\tau}$

$$\begin{aligned}\phi_{\sigma}/E\alpha T'b &= -.01892 f_2 g_2 - .00023 f_4 g_4 - .00000 f_6 g_6 \\ \phi_{\tau}/E\alpha T'b &= .02414 f_2 h_2 + .00104 f_4 h_4 + .00006 f_6 h_6\end{aligned}\quad [20]$$

By superposing the stresses derived from [20] on those listed in Table 1, we obtain the stress distribution in the strip, as shown in Figures 5a, b, c.

Appendix. Least square polynomial approximation to a function.

Because of the frequent need in problems of the present type for least square approximation of functions $f(y)$, whose values are given at $y_i \geq 0$ ($i=1, 2, \dots, l$), (a) by an even polynomial

$$F(y) = a_0 + a_2 y^2 + a_4 y^4 + \dots + a_n y^n \quad [21a]$$

or (b), by an odd polynomial

$$F(y) = a_1 y + a_3 y^3 + a_5 y^5 + \dots + a_n y^n \quad [21b]$$

we list the results when $y_i = 0, .2, .4, .6, .8, .9, .95, 1.0$. One can show that

$$\{a_0, a_2, a_4, a_6, a_8\} = A_8 \{w_0, w_2, w_4, w_6, w_8\} \quad [22]$$

$$\{a_1, a_3, a_5, a_7, a_9\} = A_9 \{w_1, w_3, w_5, w_7, w_9\}$$

where $\{ \}$ stands for a column matrix,

$$w_k = \sum_{i=1}^l y_i^k f(y_i) \quad [23]$$

and

$$A_8 = \begin{vmatrix} .7341 & -9.8981 & 35.5790 & -46.9549 & 20.5868 \\ -9.8990 & 302.5151 & -1344.9705 & 1958.0309 & -908.4412 \\ 35.5858 & -1345.0515 & 6522.5257 & -9958.0536 & 4763.0808 \\ -46.9673 & 1958.2291 & -9958.4139 & 15639.774 & -7627.8555 \\ 20.5934 & -908.5608 & 4763.3729 & -7628.0381 & 3773.4283 \end{vmatrix} \quad [24a]$$

$$A_9 = \begin{vmatrix} 22.5565 & -135.1427 & 228.3153 & -118.3869 & 1.9681 \\ -135.1423 & 957.3055 & -1754.2097 & 965.7771 & -26.5619 \\ 228.3143 & -1754.2073 & 3387.6793 & -1974.9466 & 95.5344 \\ -118.3861 & 965.7742 & -1974.9437 & 1265.5895 & -126.1295 \\ 1.9680 & -26.5613 & 95.5333 & -126.1289 & 55.3152 \end{vmatrix} \quad [24b]$$

References

1. Thermal Stresses in Rectangular Strips I, by G. Horvay. Proc. 2nd Nat'l. Congress of Applied Mech., ASME 1954.
2. The End Problem of Rectangular Strips, by G. Horvay, J.A.M. 1953, (a) p. 88, (b) p. 576.
3. Transient Thermal Stresses in Disks and Cylinders, by G. Horvay, Trans. ASME, 1954, p. 127.
4. Tables of Self-Equilibrating Functions, by G. Horvay and J. S. Born, to be published.

Table 1
Stresses and deformations due to a temperature break

$$(a) \quad \sigma_x / E \alpha T' b$$

x/y	0	.2	.4	.6	.8	.9	.95	1.0
0	.0980*	.0895 *	.0621*	.0096*	-.0860*	-.1670*	-.2265 *	-.3277*
0 [#]	.1029#	.0841#	.0686#	.0015#	-.0766#	-.1637#	-.2343#	-.3208#
0	.0967	.0906	.0638	.0068	-.0843	-.1649	-.2291	-.3213
.1								-.2802*
.1	.0961	.0888	.0617	.0065	-.0858	-.1632	-.2197	-.2958
.2								-.2375*
.2	.0927	.0845	.0572	.0044	-.0850	-.1543	-.1999	-.2561
.4								-.1661*
.4	.0805#	.0720#	.0451#	-.0036#	-.0776#	-.1222#	-.1444#	-.1660#
.4	.0788	.0707	.0454	-.0004	-.0734	-.1237	-.1533	-.1864
.6	.0608	.0540	.0334	-.0026	-.0565	-.0912	-.1108	-.1320
.8	.0474#	.0408#	.0217#	-.0083#	-.0435 #	-.0597#	-.0665 #	-.0723#
.8	.0435	.0385	.0232	-.0028	-.0402	-.0636	-.0765	-.0903
1.0	.0292	.0258	.0154	-.0022	-.0270	-.0422	-.0505	-.0593
1.2	.0186	.0163	.0097	-.0015	-.0171	-.0266	-.0318	-.0372
1.4	.0111	.0098	.0058	-.0009	-.0102	-.0158	-.0189	-.0221
1.6	.0080#	.0065 #	.0025 #	-.0027#	-.0066#	-.0074#	-.0073	-.0069#
1.6	.0062	.0054	.0032	-.0005	-.0057	-.0088	-.0105	-.0123
1.8	.0031	.0027	.0016	-.0002	-.0028	-.0044	-.0052	-.0061
2.0	.0013	.0011	.0007	-.0001	-.0012	-.0018	-.0022	-.0026

Star* denotes value obtained by numerical evaluation of Fourier integral, sign# denotes value obtained by 4-term biharmonic eigenfunction expansion, unmarked values refer to results obtained by 4-term self-equilibrating function expansion.

Table 1

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(b) $\sigma_y / E \alpha T' b$

x/y	0	.2	.4	.6	.8	.9	.95	1.0
0	.1405*	.1355*	.1203*	.0939*	.0547*	.0295 *	.0153*	0*
0	.1379	.1376	.1217	.0859	.0494	.0166	.0079	0
.1	.0965	.0905	.0743	.0492	.0175	.0048	.0012	0
.2	.0597	.0546	.0416	.0239	.0054	.0004	-.0001	0
.4	.0095	.0077	.0033	-.0010	-.0022	-.0011	-.0004	0
.6	-.0154	-.0149	-.0131	-.0094	-.0038	-.0012	-.0003	0
.8	-.0242	-.0226	-.0181	-.0113	-.0039	-.0011	-.0003	0
1.0	-.0244	-.0226	-.0176	-.0105	-.0034	-.0010	-.0003	0
1.2	-.0208	-.0192	-.0148	-.0087	-.0028	-.0008	-.0002	0
1.4	-.0160	-.0148	-.0113	-.0066	-.0021	-.0006	-.0002	0
1.6	-.0116	-.0106	-.0082	-.0047	-.0015	-.0004	-.0001	0
1.8	-.0079	-.0073	-.0056	-.0032	-.0010	-.0003	-.0001	0
2.0	-.0051	-.0047	-.0036	-.0021	-.0007	-.0002	-.0000	0

(c) $\gamma / E \alpha T' b$

x/y	0	.2	.4	.6	.8	.9	.95	1.0
0	0	0	0	0	0	0	0	0
.1	0	.0047	.0120	.0169	.0190	.0181	.0131	0
.2	0	.0101	.0211	.0292	.0300	.0234	.0150	0
.4	0	.0167	.0312	.0399	.0352	.0232	.0133	0
.6	0	.0177	.0319	.0385	.0312	.0194	.0108	0
.8	0	.0154	.0274	.0320	.0249	.0151	.0082	0
1.0	0	.0120	.0211	.0244	.0185	.0111	.0060	0
1.2	0	.0086	.0151	.0174	.0131	.0078	.0042	0
1.4	0	.0058	.0102	.0117	.0088	.0052	.0028	0
1.6	0	.0037	.0065	.0075	.0056	.0033	.0018	0
1.8	0	.0023	.0039	.0045	.0034	.0020	.0011	0
2.0	0	.0013	.0022	.0025	.0019	.0011	.0006	0

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Table 1

(d) $u/b^2 \propto T'$

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x/y	0	.2	.4	.6	.8	.9	.95	1.0
0	-.0833*	-.0733*	-.0433*	.0067*	.0767*	.1192*	.1129*	.1667*
0	-.0784	-.0705	-.0453	.0006	.0720	.1225	.1542	.1920
.1	-.0723	-.0649	-.0419	-.0007	.0626	.1057	.1315	.1609
.2	-.0651	-.0584	-.0376	-.0012	.0537	.0897	.1105	.1333
.4	-.0498	-.0445	-.0285	-.0014	.0376	.0618	.0752	.0893
.6	-.0355	-.0317	-.0203	-.0014	.0248	.0405	.0489	.0577
.8	-.0239	-.0213	-.0137	-.0013	.0154	.0251	.0303	.0357
1.0	-.0152	-.0135	-.0088	-.0012	.0089	.0147	.0178	.0209
1.2	-.0091	-.0081	-.0053	-.0009	.0048	.0080	.0097	.0114
1.4	-.0051	-.0045	-.0030	-.0007	.0022	.0039	.0047	.0055
1.6	-.0025	-.0023	-.0016	-.0005	.0008	.0015	.0018	.0022
1.8	-.0011	-.0010	-.0007	-.0003	.0001	.0002	.0003	.0004
2.0	-.0003	-.0003	-.0002	-.0002	-.0003	-.0004	-.0004	-.0005

(e) $v/b^2 \propto T'$

x/y	0	.2	.4	.6	.8	.9	.95	1.0
0	0*	.0220*	.0431*	.0622*	.0789*	.0866*	.0908*	.0960*
0	0	.0219	.0435	.0621	.0778	.0852	.0888	.0931
.1	0	.0133	.0253	.0356	.0444	.0491	.0521	.0559
.2	0	.0062	.0115	.0162	.0212	.0250	.0276	.0310
.4	0	-.0028	-.0052	-.0065	-.0048	-.0021	-.0000	.0025
.6	0	-.0066	-.0121	-.0154	-.0150	-.0131	-.0116	-.0098
.8	0	-.0072	-.0133	-.0169	-.0172	-.0159	-.0149	-.0136
1.0	0	-.0064	-.0118	-.0150	-.0156	-.0147	-.0141	-.0133
1.2	0	-.0051	-.0093	-.0120	-.0126	-.0121	-.0117	-.0111
1.4	0	-.0038	-.0069	-.0088	-.0094	-.0091	-.0089	-.0086
1.6	0	-.0026	-.0048	-.0062	-.0066	-.0065	-.0063	-.0062
1.8	0	-.0017	-.0031	-.0041	-.0044	-.0043	-.0043	-.0042
2.0	0	-.0011	-.0020	-.0025	-.0028	-.0028	-.0027	-.0027

Fig. 1.

Fig. 2c.

119

Fig. 3.

122

Fig. 4a

8094
22
23

23

8094 234

24

Fig. 5a.

8094 24-25

25

Fig. 5 b

8034 25

26

Fig. 5c.

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