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ABSTRACT

The design of internally cooled electrical coils for the production of high intensity magnetic fields presents many new aspects and combinations of the familiar modes of heat transfer. However, the customary methodology appears to be sufficient for preliminary analysis and understanding of those problems. This methodology comprises the derivation of a qualitative, approximate equation expressing the relative performance of the various parts of a system, followed by an examination of this equation in order to locate the limiting features of the system. These features are then investigated by more powerful methods, which in turn provide guidance for development research in the laboratory.

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In order to illustrate the problems, the two-layered, annular coil shown schematically in Fig. 1 was selected for analysis. The coils are helically close-wound with wire of rectangular cross section. Adjacent turns are separated by high temperature insulating material having low thermal conductivity. The inner and outer circumferences of the coils are in direct contact with the cooling fluid, which enters through numerous openings and flows across the four exposed faces of the coil.

The magnetic field strength may be estimated from the relation¹:

$$B = C \left[\frac{(1-f)P}{\bar{\rho}} \right]^{1/2}, \quad (1)$$

where B = magnetic field strength,
 P = electrical power expended as heat in the coil,
 $\bar{\rho}$ = average electrical resistivity of the metal in the coils,
 f = fraction of the volume of the coil occupied by coolant passages,
 C = "solenoid" constant, related to the "Fabry" factor and conversion factors.

This equation does not display the heat transfer characteristics of the system. In order to introduce these, we must first select the limiting variables. The choice is arbitrary; let us suppose the supply of electrical and pumping power is unlimited and take as the limiting conditions the inlet coolant temperature, the maximum permissible temperature in the metal, the physical properties of the fluid and metal, the flow conditions, and the shape of the coil.

We begin with the difference between maximum metal temperature, T_m , and inlet coolant temperature, T_1 . This difference may be resolved into three temperature differences:

$$T_m - T_1 = (T_m - T_s) + (T_s - T_o) + (T_o - T_1) , \quad (2)$$

where T_s denotes the temperature at the interface between metal and coolant at the outlet end, and T_o denotes the outlet coolant temperature.

The difference $(T_m - T_s)$ is easily computed from a consideration of the heat flow in the metal. Consider the heat flow Q_x at some section x in the metal. We have, from Fourier's law²

$$Q_x = -k_m \frac{dT}{dx} , \quad (3)$$

where k_m is the thermal conductivity of the metal. Now, Q_x is proportional to the power density and the volume of metal enclosed in the section x ; thus

$$Q_x = \frac{P}{(1-f)V} x , \quad (4)$$

where V is the total volume (including both metal and coolant) of the coil. Substituting Eq. (4) into (3) and integrating between the limits 0 and $W_m/2$ gives

$$T_m - T_s = \left[\frac{P}{(1-f)V} \right] \frac{W_m^2}{8k_m} , \quad (5)$$

where W_m is the width of the metal turns in the coil.

The second temperature difference in (2) may be evaluated from a consideration of the heat transfer at the boundary between metal and coolant. The heat flow here is governed by the expression

$$Q_s = h(T_s - T_o) \quad , \quad (6)$$

where h is the film coefficient of heat transfer, which is related to fluid properties, flow conditions, and geometry by the semi-empirical relation³

$$hW_c/k_c = 0.012 \left[\frac{C_p \mu}{k_c} \right]^{0.4} \left[\frac{2W_c u d}{\mu} \right]^{0.8} \quad , \quad (7)$$

where W_c is the width of the coolant passages, k_c is the thermal conductivity of the fluid, u is the velocity, d is the density, C_p is the specific heat, and μ is the viscosity. This expression is too awkward to be inserted into Eq. (5); instead the symbol h will be retained. From (4) we obtain Q_s by setting x equal to $W_m/2$. Substituting this into (6) and solving gives

$$T_s - T_o = \left[\frac{P}{(1-f)V} \right] \frac{W_m}{2h} \quad (8)$$

The last term in (2) is found by noting that the energy dissipated in the coil must be removed by the coolant:

$$P = fA u d C_p (T_o - T_1) \quad , \quad (9)$$

where A is the cross sectional area of the coil. By rearranging Eq. (9) to read

$$T_o - T_i = \frac{P}{fA\mu C_p} \quad (10)$$

substituting Eqs. (5), (8), and (10) into (2), solving for P; and using the result to eliminate P from (1), we have an expression for the magnetic field strength, B:

$$B = C' \left[\frac{(T_m - T_i)/\bar{\rho}}{\left[1 + \frac{W_c}{W_m}\right]^2 W_m \left[\frac{W_m}{8k_m} + \frac{1}{2h} + \frac{1}{W_c \mu C_p}\right]} \right]^{1/2} \quad (11)$$

In Eq. (11), A, f, and V were eliminated by using the relations:

$$V = AL \quad (12)$$

where L is the channel length, and

$$\bar{\rho} \approx \frac{W_c}{W_m + W_c} \quad (13)$$

For reference, we put (7) into the form:

$$h = 0.020 c_p^{0.4} k_c^{0.6} u^{0.8} d^{0.8} / W_c^{0.2} \mu^{0.4} \quad (14)$$

ANALYSIS

The relative performance of the multilayered coil under various combinations of conditions will now be examined by means of Eqs. (11) and (14). Consider first the term $(T_m - T_i / \bar{\rho})$. We see at once that a low electrical resistivity is desirable, and, hence, choose copper for the metal in the coils. Let us suppose that T_i is limited to about 100°F by direct discharge of heat in the coolant to the atmosphere. The approximate, relative performance of the system corresponding to various maximum metal temperatures is shown in Table I. (The resistivity of the copper at the maximum temperature was used instead of a mean resistivity.)

TABLE I
Relative Performance of Multilayered
Magnetic Coil at Various Maximum Metal Temperatures
Inlet Coolant Temperature = 100°F

T_m (°F)	$\bar{\rho}$	$\left[\frac{(T_m - T_i)}{\bar{\rho}} \right]_{rel}$	B_{rel}
200	2.17	1.0	1.0
300	2.55	1.7	1.3
400	2.91	2.2	1.5
600	3.67	2.9	1.7
1000	5.19	3.7	1.9

It is seen that increases in performance are dissappointingly small and that the return diminishes rapidly in the neighborhood of 1000°F.

This comes in part from the square root dependence of B and in part from the rapid increase in resistivity of copper with temperature. The latter disadvantage can be avoided and turned to an advantage by lowering T_1 . This will require refrigeration, and thus more equipment and more power.

Next, we consider the terms in the right-hand bracket in the denominator in Eq. (11). From left to right, these represent the relative resistance to heat transfer in the metal, at the interface, and the coolant thermal capacity, respectively. Design and development efforts should be directed toward reducing the largest of these. From the first term, we conclude that the thickness of the metal should be small and the thermal conductivity large. From the second term, we see that a large h is desirable. Referring to Eq. (14) we see that this implies a high velocity, u . Having narrow coolant channel widths, W_c , helps some, but the dependency is weak. In regard to the fluid properties C_p , d , k_c , and μ , it is found that water provides the best overall combination. From the last term, we see that the channels should be short and wide, and the velocity high. Again, of the available coolants, water provides the largest product dC_p .

Now consider the first bracket in the denominator of (11). For a given coolant channel thickness, W_c , it is seen that the product $(1 + \frac{W_c}{W_m}) W_m$ is a minimum when the metal thickness, W_m , is equal to the coolant channel thickness. As we have seen the coolant channel should have thickness such that the third term in the second bracket of the denominator is approximately equal to the other two terms. The designer must judiciously adjust the variables in the denominator to obtain the smallest value compatible with engineering feasibility.

EXAMPLE

Let us apply (11) to the analysis of a DCX-type of magnetic coil⁴. In a typical case, the copper thickness is about 1 in., and the coolant channels are 1/2 in. thick. The coolant (water) enters at about 80°F and leaves at about 90°F. The velocity is about 15 fps, the density is 62.4 lb/ft³, the specific heat is 1 Btu/lb.°F, the thermal conductivity is 0.34 Btu/hr.ft.°F, and the viscosity is 1 centipoise. The coolant channel length, L, is about 1 ft. The thermal conductivity of copper is approximately 210 Btu/hr.ft.°F at these temperatures. By substituting into Eq. (11), we find that the product $1 + \frac{W_c}{W_m} \frac{W_m}{W_c}$ has a value of 1.6 in. This could be reduced to 1 in. by making $W_m = W_c = 1/4$ in. The terms in the second bracket have the values, from left to right, 5×10^{-5} , 25×10^{-5} , and 1.4×10^{-5} hr.ft².°F/Btu. Reducing the thickness of the metal, W_m , to 1/4 in. would reduce the first term by a factor of 4 and make it comparable to the last term in magnitude. However, the thermal resistance at the solid-fluid interface (second term) is controlling, and greatest gains are to be obtained by increasing h. Turning to Eq. (14), we see that large increases in C_p , k_c , or d, or large decreases in μ , would increase h. In general, the product $C_p d$ for other fluids is less than for water. However, k_c for liquid metals is orders of magnitude greater. Unfortunately, liquid metals are electrically incompatible with the system. This leaves us the variables u (coolant velocity) and W_c (coolant channel thickness) to work with. The dependence of h on W_c is very weak. The most profitable course is to increase the coolant velocity. This can most conveniently be done by connecting the coolant passages in series instead of in parallel, and thus putting the entire flow through each channel. This increases the coolant channel length, L, in the third term, but it increases u in the same proportion, and therefore the last term is unaffected.

These changes would convert a two-layered coil having a coolant velocity of 15 fps and a channel length of 1 ft into a five-layered coil having a coolant velocity of 60 fps and a channel length of 5 ft, and would make possible an increase in B by a factor of 2 without increasing the maximum metal temperature.

CONCLUSIONS

The approximate, relative performance of the various parts of internally cooled magnetic coils can be studied by means of the familiar methodology, and the effectiveness of design changes can be evaluated. Once the system has been brought into approximate balance, it becomes appropriate to apply more rigorous methods and more realistic assumptions. For instance, in a second analysis, the temperature dependence of the thermal and electrical conductivity of copper would be taken into account in a detailed analysis of the temperature field. But these methods should not be applied until the range of the design variables has been narrowed by the approximate methods.

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SYMBOLS

A	Cross sectional area of magnetic coil
B	Magnetic field strength
C	Solenoid constant
C_p	Specific heat of coolant
d	Density of coolant
f	Fraction of volume of coil occupied by coolant passages
h	Film coefficient of heat transfer
k_c	Thermal conductivity of coolant
k_m	Thermal conductivity of metal in coils
L	Length of coolant passages
P	Total electrical power expended in coil as heat
Q_s	Heat flux at surface of coils, heat units per unit time per unit of surface
T_o	Temperature of coolant at outlet of coil
T_i	Temperature of coolant at inlet of coil
T_s	Temperature of metal surface near coolant outlet
T_m	Maximum (midpoint) metal temperature near coolant outlet
u	Velocity of coolant in channels
V	Total volume of magnetic coil, including metal and coolant
W_c	Width of coolant channels in multilayered coil
W_m	Width of metal turns in multilayered coil
x	Radial coordinate
μ	Viscosity of coolant
$\bar{\rho}$	Mean electrical resistivity of metal in coils

Fig. 1. Internally Cooled Magnetic Coil.

Fig. 2. Section Through Copper Turn.

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