206

1

REACTOR PHYSICS
QUARTERLY PROGRESS REPORT
MAY - JULY, 1953

~~Wholesale Price \$ 3.75
Microfilm Price \$ 4.00
Available from
Office of Technical Services
Department of Commerce
Washington 25, D. C.~~

EDITED BY:
R. A. LAUBENSTEIN

CLASSIFICATION CANCELLED
DATE 12-5-55
For The Atomic Energy Commission
H. F. Canale
Chief, Declassification Branch

ATOMIC ENERGY RESEARCH DEPARTMENT
NORTH AMERICAN AVIATION, INC.
P. O. BOX 309 DOWNEY, CALIFORNIA

SUBMITTED: OCTOBER 2, 1953

ISSUE DATE

MARCH 1, 1954

CT AT 11-1-GEN-8

940-002

1

5103

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

3

TABLE OF CONTENTS

	Page No.
Abstract	5
I. Exponential Experiments.	7
A. Two-Group Lattice Theory and Cadmium Ratio	7
B. Reflection of Neutrons by a Semi-Infinite, Isotropic Scattering, Absorbing Medium	10
II. Water Boiler Neutron Source	16
A. Danger Coefficient Measurements	16
B. Radioactivity of the Gases Contained in the Water Boiler Neutron Source	18
C. Effectiveness of Control Rods in the L-3 Water Boiler Reactor	19
III. Numerical Analysis	20
A. Nordsieck Computer	20
B. Long Term Irradiation of Nuclear Fuels	20
References	22

940-003

LIST OF FIGURES

Page. No.

1. Thermal to Epithermal Flux Ratio Measurements in 32 Uranium Rod Lattices	9
2. Reflected Neutron Flux Distribution, $\Sigma_s / \Sigma_T = 1.0$	13
3. Reflected Neutron Flux Distribution, $\Sigma_s / \Sigma_T = 0.5$	13
4. Fraction of Incident Neutrons Which are Reflected from a Semi-Infinite, Isotropic Scattering, Absorbing Medium	14

ABSTRACT

Exponential Project

The exponential project facilities were used during the quarter for experimental measurements relating to the flux distributions to be expected in a proposed uranium converter reactor. Theoretical analysis of data obtained on the exponential experiments with uranium-heavy water lattices has continued. Cadmium ratio data for 32 uranium-D₂O lattices have been compiled and agreement found with theoretical calculations. Calculations have been made for the reflection of neutrons from concrete shielding around the exponential assembly.

Water Boiler Neutron Source

Measurements with the danger coefficient facility have been simplified by the photographic recording of data. Preliminary work indicates that danger coefficient measurements can be made with an apparent uncertainty of less than 10^{-3} cm^2 .

Analysis of the atmosphere in the WBNS gas handling system indicates that the only detectable radioactive isotopes are the xenon and krypton, together with their daughter products. No iodine activity was found.

Numerical Analysis

A Nordsieck computer has been received and placed in operation. This machine is a mechanical type of differential analyzer.

A problem on the long term irradiation of nuclear fuels has been developed in a form convenient for calculation on an IBM card programmed computer and calculations are now in progress. (O. H. E.)

I. EXPONENTIAL EXPERIMENTS

The program of exponential experiments of uranium rod lattices in heavy water was temporarily discontinued during this quarter in order to use the experimental facilities for a series of measurements of the intra-cell neutron flux distribution in the proposed Uranium Converter Reactor. Some theoretical effort in connection with the exponential project has continued and is reported below.

A. Two-Group Lattice Theory and Cadmium Ratio (F. B. Estabrook)

In connection with a continuing analysis of the experimentally obtained bucklings and thermal utilizations of a series of uranium rod D_2O lattices, a subsidiary analysis has been made of the cadmium ratios obtained in the D_2O at the cell centers. These indium foil cadmium ratio measurements were made during the buckling measurements. Data were obtained on 22 natural uranium lattices, seven depleted uranium lattices (0.49 per cent U^{235}), and three enriched uranium lattices (0.90 per cent U^{235}).

In a "homogenized" two-group treatment of these lattices, it can be shown that

$$\bar{\Sigma}_a^0 \bar{f} \frac{1 + \mathcal{L}^2 L^2}{\xi \bar{\Sigma}_s} = \left\{ \int_{E_c}^{\infty} \sigma_{In} \frac{dE}{E} / \sigma_{In}^0 \right\}_{eff}^{-1} \cdot R^{-1} \quad \dots (1)$$

$\bar{\Sigma}_a^0$ is the 2200 m/sec value of the average macroscopic absorption cross section of the lattice, with contributions from moderator and uranium properly flux-weighted according to our experimental data.

$\bar{f}$ is the usual correction factor for non-reciprocal- v absorption.

$\mathcal{L}^2$ is the measured buckling.

L is the thermal diffusion length of the lattice ($L^{-2} = 3 \bar{\Sigma}_a^0 \bar{f} \frac{2}{\sqrt{\pi}} \sqrt{\frac{T}{T_0}} \bar{\Sigma}_{tr}$).

We have assumed it proper to calculate $\bar{\Sigma}_{tr}$ also by a flux-weighting procedure.

$\xi \bar{\Sigma}_s$ is the averaged slowing-down power in the indium resonance region.

$\left\{ \int_{E_c}^{\infty} \sigma_{\text{In}} (dE/E) / \sigma_{\text{In}}^0 \right\}$ is the ratio of the indium resonance integral

from cadmium cut-off to the 2200 m/sec absorption value. $\{\}_{\text{eff}}$ is the same observationally but labeled "effective" to allow for the use of indium foils so thick as to have appreciable self-shielding at indium resonance.

R is the flux ratio measured with 95 mg/cm² indium foils. R is obtained from the saturated activities, A, using the equation

$$R = \frac{A_{\text{Bare}} - 1.072 A_{\text{Cd Cov}}}{1.072 A_{\text{Cd Cov}}} \quad \dots (2)$$

The factor 1.072 applies to the 0.020-inch thick cadmium boxes which were used.¹

The linear relation, Eq. (1), between the quantity to the left of the equal sign and R^{-1} has been confirmed for the 32 lattices so far investigated, with values of R^{-1} ranging from 0.02 to 0.37 (see Fig. 1). The value of the slope, within say ± 3 per cent, gives

$$\{\}_{\text{eff}} = 3.70 \quad \dots (3)$$

(95 mg/cm² foils)

From other data taken at this laboratory, we have

$$\{\} / \{\}_{\text{eff}} = 3.90 \quad \dots (4)$$

So, combining these we obtain

$$\left\{ \int_{E_c}^{\infty} \sigma_{\text{In}} \frac{dE}{E} / \sigma_{\text{In}}^0 \right\} = 14.4 \quad \dots (5)$$

Fig. 1. Thermal to Epithermal Flux Ratio Measurements in 32 Uranium Rod Lattices

This number, which was computed to be 13.9 in CRP-459² and was given the older value of 16.5 in CP-3202,³ has also been independently confirmed at this laboratory.⁴

The ± 3 per cent scatter in the values of R , however, completely precludes any use by us of Eq. (1) in obtaining experimental values of L^2 . If the assumption were made that the 2200 m/sec diffusion constant D^0 for a lattice is D_{mod}^0 (contrasted to our assumption that $1/D^0 = 3 \bar{\Sigma}_{\text{tr}}$), then Eq. (1) would appear as almost a direct proportionality between L^2 and R^{-1} . However, Eq. (1) would then be superfluous, as L^2 would result immediately from D^0 and $\bar{\Sigma}_a^0$.

B. Reflection of Neutrons by a Semi-Infinite, Isotropic Scattering, Absorbing Medium (S. W. Kash)

Calculations relating to the number of neutrons which might be scattered into the exponential assembly from surrounding structures have led to an exact solution to the following problem: "A parallel beam of neutrons is incident at some arbitrary angle upon the surface of a semi-infinite, isotropic scattering, absorbing medium. What is the distribution of the reflected neutrons and what is the total fraction of neutrons reflected?" The results are based on distribution formulae given by S. Chandrasekhar.⁵

Let N = intensity of the incident beam = number of neutrons per second crossing a unit area normal to the incident beam direction

$\cos^{-1} \mu_0$ = angle of incidence,

c = ratio of scattering cross section to total cross section of the semi-infinite medium ($= \bar{w}_0$ in the notation of Chandrasekhar), and

$\cos^{-1} \mu$ = angle between the direction vector and the outer normal to the semi-infinite medium.

Then the reflected (i. e. emergent) neutron flux distribution is

$$F(\mu, \mu_0, c) = N \frac{c}{4\pi} \frac{\mu_0}{\mu + \mu_0} H(\mu_0, c) H(\mu, c) \quad \dots (6)$$

The dimensionless function $H(\mu, c)$ is tabulated by Chandrasekhar for various values of μ and c . In general, $H(\mu, c)$ increases as both μ and c increase and $H(0, c)$ is unity for all c . It is convenient to normalize the reflected neutron flux distribution $F(\mu, \mu_0, c)$ to the neutron current incident on the surface of the semi-infinite medium. Thus, the reflected neutron flux distribution per unit current density incident on the semi-infinite medium is

$$\frac{F(\mu, \mu_0, c)}{N \mu_0} = \frac{c}{4\pi} \frac{1}{\mu + \mu_0} H(\mu_0, c) H(\mu, c) \quad \dots (7)$$

Representative plots of this function are shown in Figs. 2 and 3.

The total fraction $f(\mu_0, c)$ of neutrons reflected by the semi-infinite medium is obtained by multiplying Eq. (7) by μ and integrating over all outward directions. The result is

$$f(\mu_0, c) = 1 - (1-c)^{1/2} H(\mu_0, c) \quad \dots (8)$$

As might be expected, Eq. (8) shows that when the absorption is zero ($c = 1$) all the neutrons are reflected; and that as the angle of incidence increases the number of neutrons reflected increases. At grazing incidence the fraction of neutrons reflected is simply,

$$f(0, c) = 1 - (1-c)^{1/2} \quad \dots (9)$$

The fraction $f(\mu, c)$ is graphically presented in Fig. 4.

For $\mu_0 = 0.05$ the reflected neutron distribution (see Figs. 2 and 3) has a large maximum in the direction along the surface of the semi-infinite medium. To understand this, consider the more extreme case of grazing incidence. Let $N \mu_0$, the incident current per unit surface of the semi-infinite medium, remain constant as μ_0 approaches zero. Then at grazing incidence

$$\frac{F(\mu, 0, c)}{N\mu_0} = \frac{c}{4\pi} \frac{H(\mu, c)}{\mu} \quad \dots(10)$$

Now at grazing incidence all the incident neutrons have their first scattering collision at the surface of the semi-infinite medium. Hence, Eq. (10) gives the distribution of the completely equivalent problem of an isotropic plane source (of strength c) on the surface of a semi-infinite, isotropic scattering, absorbing medium. Imagine viewing such a plane isotropic source through a long tube of unit area. Since presumably each unit of surface of the source emits isotropically, the intensity perceived through the tube will depend upon the amount of source surface visible. This will be greater the more oblique the view, and in fact will be proportional to $1/\mu$. Thus, the large magnitude of the distribution for small μ is due to the incident neutrons which are scattered at the surface, that is, are scattered only once. These neutrons contribute to the distribution in Eq. (10) an amount $c/4\pi\mu$. The neutrons that are scattered more than once contribute the amount $(c/4\pi\mu) \cdot [H(\mu, c) - 1]$. In general, for any angle of incidence the reflected neutron flux distribution, Eq. (7), can be separated into a part,

$$\frac{c}{4\pi} \frac{1}{\mu + \mu_0} \quad ,$$

due to neutrons scattered only once by the semi-infinite medium, and a part,

$$\frac{c}{4\pi} \frac{1}{\mu + \mu_0} [H(\mu_0, c) H(\mu, c) - 1] \quad ,$$

due to neutrons scattered more than once. It is primarily the first part that determines the large values of the distribution function for μ near zero. Similarly the contribution to $f(c, \mu_0)$ in Eq. (8) by neutrons scattered only once by the medium is

Fig. 2. Reflected Neutron Flux Distribution, $\Sigma_s/\Sigma_T = 1.0$

Fig. 3. Reflected Neutron Flux Distribution, $\Sigma_s/\Sigma_T = 0.5$

Fig. 4. Fraction of Incident Neutrons Which are Reflected from a Semi-Infinite, Isotropic Scattering, Absorbing Medium

$$f_1(\mu_o, c) = \frac{c}{2} \left[1 - \mu_o \ln \frac{1 + \mu_o}{\mu_o} \right], \quad \dots (11)$$

and the contribution of neutrons scattered more than once is the difference between f and f_1 . The function f_1 has been plotted (dotted curves) in Fig. 4 for $c = 1.0, 0.5$ and 0.1 . Comparing $f_1(\mu_o, c)$ with the total fraction reflected $f(\mu_o, c)$, we see that the increase in the number of neutrons reflected as the angle of incidence is increased is completely due to the increase in the reflection of neutrons scattered only once by the medium. For highly absorbing media (compare the curves for $c = 0.1$), the reflection of neutrons is almost completely attributable to the neutrons suffering only a single scattering collision in the medium.

II. WATER BOILER NEUTRON SOURCE

The water boiler neutron source has been operated for a total of approximately 200 watt hours during the report period. The use of the reactor has been divided among operation as a neutron source for the UCR experimental program, service irradiations, and nuclear measurements with the danger coefficient technique. Approximately 40 watt hours of operation were devoted to service irradiations, some of which were done concurrently with other experimental programs.

A. Danger Coefficient Measurements (M. E. Remley, D. C. Woods)

The instrumentation for the danger coefficient measurements has been improved to facilitate the gathering of data. A large BF_3 ionization chamber is located between the shield and reflector of the reactor. The signal from the ion chamber is fed to a sensitive galvanometer (10^{-10} amp/mm), which is used to monitor the power level of the reactor. The major portion of the current from the ionization chamber is balanced by current from a standard source consisting of a potentiometer and a set of precision resistors. The drift in the power level of the reactor is given by the change in the deflection of the galvanometer. A continuous recording of the deflection is made on photographic paper mounted on a rotating drum which is located one meter from the galvanometer.

The procedure for determining the critical position of the control rod consists of setting the control rod to give a slightly rising period, then a slightly falling period, and finally back to a rising period. For each control rod setting a photographic record of galvanometer deflection as a function of time is obtained and from this record the rate of drift of the reactor power level is computed. When the reactor power level is changing slowly as in the above cases, it is given as a function of time by

$$P = P_0 e^{t/T} , \quad \dots (12)$$

where:

P = power level at any time t ,

P_0 = initial power level,

T = period of the reactor (positive for rising period and negative for falling period).

Then:

$$1/T = (1/P) \frac{dP}{dt} \quad . . . (13)$$

The reciprocal period is found for the three positions of the control rod and is plotted as a function of the control rod position. The position of the rod for an exact critical condition of the reactor is then found from this plot.

The rod control is located in the graphite reflector and has about 80 centimeters of travel. The reactivity control is a non-linear function of the rod position. Standard neutron absorbing samples have been used to partially calibrate the control rod. Measurements with boron samples show that the effectiveness of the control rod is equivalent to $5.91 \times 10^{-4} \text{ cm}^2$ of absorption cross section per mil of travel when the rod is removed 26 centimeters from the completely inserted position. In this region, it is found that the critical position of the control rod can be determined to within ± 0.002 inch, so that the uncertainty in cross section for a given critical determination is $1.2 \times 10^{-3} \text{ cm}^2$. Remembering that two determinations are required for a cross section measurement, the uncertainty resulting from the control rod settings alone would be about $2 \times 10^{-3} \text{ cm}^2$. Improved accuracy can be obtained in the cross section measurements by using the control rod in a region where it is less effective. Preliminary data taken with gold foils at a control rod setting of 65 centimeters indicate that the effectiveness of the control rod in this region is equivalent to $6.1 \times 10^{-5} \text{ cm}^2$ of cross section per mil of rod. The uncertainty in determining the critical position in this region is about ± 0.008 inch, so that the uncertainty in a cross section measurement would be less than 10^{-3} cm^2 .

The great advantage in sensitivity of the WBNS for measurements using the danger coefficient technique is shown when one determines the effect on reactivity for a given cross section from the above data. This constant for samples placed in the central exposure facility of the WBNS is found to be $4.19 \times 10^{-2} \text{ cm}^2/\text{inh}$. This is to be compared to the constant obtained for the Argonne graphite reactor⁶ of $2.05 \text{ cm}^2/\text{inh}$.

One of the problems associated with a static measurement of this type is that of maintaining minimum extraneous reactivity changes in the reactor during the period required for the measurement. This has been partially surmounted by operating the WBNS at a very low power level, approximately 0.2 watt, so that changes resulting from temperature effects, power coefficients, and other

variations in reactivity associated with high power operation are minimized. In addition, the determinations of the critical positions necessary to obtain the effect of a given cross section are made cyclically, so that slow drifts in the reactivity can be removed by averaging the cyclic measurements.

B. Radioactivity of the Gases Contained in the Water Boiler Neutron Source
(M. E. Remley)

The question of the type and amount of radioactivity in the atmosphere above the fuel solution in the WBNS has been investigated. Using the methods of Thornton and Houghton,⁷ the growth of β -activity resulting from possible volatile fission product elements has been calculated. The elements Kr and Xe, which are always in a gaseous state, plus Br, Te, and I have been considered as contributing to the gaseous radioactivity. This calculation indicates that the equilibrium activity level, from these five elements, which would be reached at the end of about 2×10^5 minutes of operation, is approximately 1.1 curies per watt. These calculations do not include the daughters of the above mentioned elements, so the calculated gaseous activity level may be somewhat low. However, of probably greater importance is the question of how much of the activity from the elements other than Kr and Xe actually gets into the gaseous state.

This latter problem has been investigated by removing a sample of the gas from the WBNS immediately after an operating period of about two hours and analyzing the sample for radio iodine by radio chemical methods. The amount of iodine activity to be expected in the gas was calculated from the operational history of the WBNS prior to the removal of the gas with the assumption that all the iodine activity was gaseous. The results of the analysis indicate that no more than about 10^{-5} of the iodine actually stays as gas in the reactor gas system.

The decay of the total activity in the gas has been followed for a period of approximately two weeks. This was done by placing a sample of approximately 400 cm^3 of the gas (NTP) in a completely closed container in which a thin window geiger counter was inserted. An analysis of the decay curve shows activities of three different half-lives, those corresponding to 2.8 hour Kr^{88} , 5.27 day Xe^{133} , and 9.2 hour Xe^{135} . Since only the characteristic decay of the above three isotopes is observed, this appears to corroborate the finding of no iodine in the sample from the WBNS gas. The daughters of the above isotopes do not complicate the analysis of the decay curve, as Kr^{88} decays to Rb^{88} and then to

Sc^{88} . Rb^{88} has a half-life of only eighteen minutes and hence will decay with the 2.8 hour half-life of the parent Kr^{88} , and Sr^{88} is stable. Xe^{133} decays to stable Cs^{133} , while Xe^{135} decays to the 3×10^6 year Cs^{135} .

If iodine were present in any appreciable quantity in the gas as it is removed from the system, the I^{135} , which has a half-life of 6.7 hours, would decay to Xe^{135} . Similarly I^{133} with a half-life of 21 hours decays to Xe^{133} . The contributions to the gross activity from the iodine decay plus the growth of Xenon daughters would then result in a gross decay curve for the gaseous activity which would differ markedly from the curve obtained for gaseous activity in which no iodine was present. Thus the experimental decay curve, which shows only the half-lives corresponding to Kr^{88} , Xe^{133} and Xe^{135} , gives further evidence to support the lack of iodine in the reactor gas system.

It must be noted that this analysis was done after operation at approximately one watt and does not necessarily give information applicable to operation at higher powers.

C. Effectiveness of Control Rods in the L-3 Water Boiler Reactor

(D. L. Hetrick)

Calculations have been made giving the absolute effectiveness of unshadowed control rods in a proposed water boiler reactor (designated L-3) by integrating the distribution of thermal neutron statistical weight over the surfaces of the control rods. The rods are taken as thin flat strips of black absorber. The statistical weight distribution is given an absolute normalization by performing the corresponding calculation for the NAA-WBNS reactor and comparing the result with the measured reactivity controlled by each rod. The shapes of the distributions for the two reactors differ slightly and the correction factor for this difference is simply the ratio of the integrals of statistical weight over the respective reactor volumes.

Reactivity controlled by a rod as a function of rod position is proportional to the surface integral of statistical weight over the rod where the lengthwise integration is calculated as a function of the upper limit (the lower limit being the co-ordinate of the tip of the rod when full inserted).

The distributions used were obtained by two-group calculations. For comparison with experiment, the computed value of relative effectiveness for the two WBNS safety rods is 0.85 while the measured ratio is 0.81.

III. NUMERICAL ANALYSIS

A. Nordsieck Computer (R. J. Graydon)

A Nordsieck Computer has been acquired by the Atomic Energy Research group at North American. This differential analyzer has, as its primary function, the evaluation of numerical solutions of equations having no known formal solution. It evaluates only particular numerical solutions of differential equations in which all coefficients have numerical values and for which initial conditions are known.

The Nordsieck Computer is a mechanical type of differential analyzer, all the components of which drive, or are driven by, synchro notors. Its basic unit consists of a wheel and disk integrator. The present machine contains twelve integrators, twelve multipliers, eight adders and four plotting tables, one of which is usually used to plot the desired answer function. The machine also contains constant multiplier motors and adding motors. Although the present model embodies several design changes, a general design description of this equipment is contained in UCRL 1717.

B. Long Term Irradiation of Nuclear Fuels (E. R. Cohen)

Long term irradiation of nuclear fuels will become increasingly important with the application of nuclear reactors primarily to power production. At the suggestion of H. C. Ott of the Division of Reactor Development, USAEC, calculations of the effects of long irradiations are being set up in a form suitable for an IBM card programmed calculator. Exposures of as much as 15,000 MWD/T, which corresponds to almost complete burnout of the original U^{235} content of a natural uranium rod, are to be investigated.

The usual formulae of radioactive build-up and decay involve combinations of exponentials. Where there is feedback in the systems of equations, the decay constants involved are roots of algebraic equations rather than simple elements of the equations themselves. Furthermore, where several elements are involved,

the entire analytic solution is quite complicated and difficult to handle. For these reasons the present calculation is set up in terms of power series for which numerical results can be conveniently obtained on an IBM card programmed calculator.

These calculations will be carried out for initial conversion ratios ranging from 0.6 to 1.0. Enrichments will vary from natural uranium to 2 per cent. Initial concentrations of Pu^{239} and Pu^{240} will be used which correspond to expected concentrations following chemical processing of irradiated fuel elements for removal of fission products and re-enrichment of the fuel. Details of the method of calculation and the numerical results will be presented in a separate report.

940-020

REFERENCES

1. Kunstadter, J. W., Phys. Rev. 78, 484 (1950).
2. Sargent, B. W. and J. C. Duckworth, "Cadmium Ratio with Thin and Thick Indium Foils in the NRX Pile and in Other Lattices," CRP-459, Sept. 12, 1950.
3. Friedman, F. L. and C. Redman, "The Cd Ratio for In Foils and the Slowing Down Power of D_2O and Graphite," CP-3202, Aug. 9, 1945.
4. D. H. Martin, Private Communication.
5. Chandrasekhar, S., "Radiative Transfer," Oxford (1950); Chap. III and IV.
6. Anderson, H. L., E. Fermi, A. Wattenberg, G. L. Weil and W. H. Zinn, "Method of Measuring Neutron Absorption Cross Sections by the Effect on Reactivity of a Chain Reacting Pile," Phys. Rev. 72, 16 (1947).
7. Thornton, J. K., and W. J. Houghton, "An Influence Function for the Beta Activity of Uranium (235) Fission Products," NAA-SR-45, Sept. 1, 1950.