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CALCULATION OF THE TEMPERATURE DEPENDENCE
OF $\text{Pu}^{239}/\text{U}^{235}$ FISSION RATIO FOR
A GRAPHITE- U^{235} SYSTEM

by

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CALCULATION OF THE TEMPERATURE DEPENDENCE OF $\text{Pu}^{239}/\text{U}^{235}$ FISSION RATIO FOR A GRAPHITE- U^{235} SYSTEM

D. Meneghetti and K. E. Phillips

Introduction:

The fission response per atom for a bare Pu^{239} detector divided by the fission response per atom for a bare U^{235} detector as a function of moderator temperature has been calculated for a graphite- U^{235} homogeneous reactor having for the ratio of atomic densities $N_C/N_{\text{U}^{235}} = 12,500$. The dependence of the fission ratio upon moderator temperature depends upon the fact that, as the thermal distribution of the neutrons shifts toward higher energies with increasing core temperatures, the Pu^{239} neutron resonance at 0.297 ev increasingly contributes to the over-all fission response. The neutron spectrum corresponding to a given moderator temperature was estimated by the gaseous heavy moderator method described by Cohen.⁽¹⁾ The effective mass of the carbon atom was assumed to be independent of temperature and equal to 11.9 neutron mass units. In the quantity⁽¹⁾

$$\Delta = 2m \Sigma_a / \Sigma_s \quad ,$$

where m is now the mass of the carbon atom in neutron mass units, the macroscopic absorption and scattering cross sections, Σ_a and Σ_s , respectively, for the core at the energy kT were computed for various moderator temperatures assuming a $1/\sqrt{T}$ dependence of the absorption cross section of carbon and a $g(T)/\sqrt{T}$ dependence of the absorption cross section for U^{235} , where $g(T)$ are the Westcott non- $1/v$ factors.⁽²⁾ (It should be noted that Σ_a is not an average value over a spectrum, but corresponds to the absorption at energy kT for the case of a $1/v$ absorber. The $g(T)$ factor has been used here to approximate the corresponding cross section for the non- $1/v$ U^{235} .) The asymptotic neutron distribution c/x^2 , where c is a function of Δ and x is the velocity variable as defined by Cohen, was connected with the modified Maxwell distribution by extrapolation of c/x^2 to lower energies, followed by a simple smoothing out at the cross-over energy.

Procedure:

The ratio of the fission responses per atom of thin, bare Pu^{239} and U^{235} detectors is

$$R(T) = \frac{\int_0^{\infty} \sigma_f^{\text{Pu}^{239}}(E) \phi(E, T) dE}{\int_0^{\infty} \sigma_f^{\text{U}^{235}}(E) \phi(E, T) dE}$$

where $\phi(E, T)$ is the flux distribution corresponding to moderator temperature T and σ_f is the microscopic cross section for fission.

As values for the integrals over the $1/E$ region of the flux have been determined⁽²⁾ and are independent of temperature, the ratio can be expressed as

$$R(T) = \frac{\int_0^{2 \text{ ev}} \sigma_f^{\text{Pu}^{239}}(E) \phi(E, T) dE + \int_{2 \text{ ev}}^{\infty} \sigma_f^{\text{Pu}^{239}}(E) \frac{dE}{E}}{\int_0^{0.4 \text{ ev}} \sigma_f^{\text{U}^{235}}(E) \phi(E, T) dE + \int_{0.4 \text{ ev}}^{\infty} \sigma_f^{\text{U}^{235}}(E) \frac{dE}{E}}$$

where $\phi(E, T)$ is normalized in a manner such that $\phi(E, T) = 1/E$ in the $1/E$ region of the flux. Values of the respective integrals in this region are given⁽²⁾ for Pu^{239} as 250 barns for the range 2 ev to infinity and for U^{235} as 271 barns for the range 0.4 ev to 2 Mev. (The contribution above 2 Mev in the latter case may be neglected in the present calculation.) The integrals involving lower energies were determined by a graphical integration of the product of the room-temperature fission cross sections⁽³⁾ and the flux determined by the gaseous heavy moderator theory⁽¹⁾ for moderator temperatures between 300°K and 700°K. (The Doppler effect of the first Pu^{239} resonance due to temperature increase of the foil was estimated to have negligible effect on the integrated fission response.) The analysis neglected the effects of detector foil self-shielding and flux depression at the foil due to the presence of the absorber foil. Estimates of possible errors introduced by these factors have been made, assuming one-mg per cm^2 foils. The errors were found to be negligible and are discussed in the Appendix.

Discussion and Results:

The fission cross sections, calculated neutron spectra and fission responses as functions of neutron energy are shown in Figs. 1 to 12 for the thermal regions for both detectors and at various moderator temperatures. A summary of the contributions of partial integrated fission responses are given in Table I. The resulting ratio of the integrated total fission responses as a function of moderator temperature is given in Fig. 13. It is noted that the ratio is quite sensitive to moderator temperatures above about 400°F. Between 300 and 400°K any increased fission

FIG. 1

FISSION RESPONSE OF Pu^{239} AT MODERATOR TEMPERATURE OF 300°K

FIG. 2

FISSION RESPONSE OF U^{235} AT MODERATOR TEMPERATURE OF 300°K

FIG. 3

FISSION RESPONSE OF Pu^{239} AT MODERATOR TEMPERATURE OF 350°K

FIG. 4

FISSION RESPONSE OF U^{235} AT MODERATOR TEMPERATURE OF 350°K

FIG. 5

FISSION RESPONSE AT Pu^{239} AT MODERATOR TEMPERATURE OF 400°K

FIG. 6

FISSION RESPONSE OF U^{235} AT MODERATOR TEMPERATURE OF 400°K

FIG. 7

FISSION RESPONSE OF Pu^{239} AT MODERATOR TEMPERATURE OF 500°K

FIG. 8

FISSION RESPONSE OF U^{235} AT MODERATOR TEMPERATURE OF 500°K

FIG. 9

FISSION RESPONSE OF Pu^{239} AT MODERATOR TEMPERATURE OF 600°K

FIG. 10

FISSION RESPONSE OF U^{235} AT MODERATOR TEMPERATURE OF 600°K

FIG. 11

FISSION RESPONSE OF Pu^{239} AT MODERATOR TEMPERATURE OF $700^\circ K$

FIG. 12

FISSION RESPONSE OF U^{235} AT MODERATOR TEMPERATURE OF $700^\circ K$

FIG. 13
FISSION RESPONSE RATIO OF $\text{Pu}^{239}/\text{U}^{235}$ VERSUS MODERATOR TEMPERATURE FOR
A CORE WITH $N_c/N_{\text{U}^{235}} = 12,500$

Table I

Integrated Fission Responses for Various Energy Intervals
Comprising the Thermal Region

Material	Temperature of Moderator	Group I 0.0001-0.001 ev	Group II 0.001-0.01 ev	Group III 0.01-0.1 ev	Group IV 0.1-1.0 ev	Group V 1.0-2.0 ev
Pu^{239}	300°K	42	1,136	7,370	2,699	17
	350°K	35	943	7,287	3,250	17
	400°K	31	866	7,395	3,942	16
	500°K	22	625	7,141	6,363	16
	600°K	18	530	6,781	9,130	15
	700°K	15	438	6,669	13,962	16
Material	Temperature	Group I 0.0001-0.001 ev	Group II 0.001-0.01 ev	Group III 0.01-0.1 ev	Group IV 0.1-0.4 ev	
U^{235}	300°K	33	908	5,189	567	
	350°K	28	787	5,347	775	
	400°K	24	679	5,502	1,023	
	500°K	18	535	5,146	1,510*	
	600°K	14	426	4,720	2,088*	
	700°K	12	364	4,606	2,639*	

*In the few cases where the $1/E$ flux begins at energies greater than 0.4 ev, in the case of U^{235} , the assumption of $1/E$ flux at these limits has negligible effect on the over-all fission response ratios.

response due to the proximity of the Pu^{239} resonance is in this temperature region only large enough to compensate for the decreased lower energy response due to the $1/v$ cross-section dependence at the lower energies. A resonance detector having a somewhat lower energy resonance than that of Pu^{239} would be required for detection of moderator temperatures lower than about 400°K .

It would be of interest to determine both the moderator temperature and the detector ratio experimentally for various temperatures and to compare the results with the calculated values of the fractional changes in detector ratio with temperature as well as the detector ratios at the various temperatures. Such experiments would test the utility of such methods of characterizing spectra and/or spectral changes and suggest refinements in the calculations.

It should be noted that the results presented assume carbon to be a heavy gaseous moderator in the manner of the Cohen approximation and that the neutron leakage from the system is zero. One must be careful in applying the calculations in a transition region such as in the neighborhood of a core-reflector interface. The varying ratio of epithermal to thermal flux due to neutron diffusion will probably significantly affect the Pu^{239} resonance response.

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Appendix

Self-Shielding and Flux Depression

Possible errors introduced by neglecting self-shielding and flux depression have been estimated for the case of one-centimeter-diameter foils of thickness one milligram per square centimeter.

Assuming isotropic incident flux and an extremely pessimistic value of about 5,000 barns for the effective absorption cross section of Pu^{239} , which corresponds to the value at the peak of the resonance, it is calculated that only 2% of the incident flux is absorbed by the foil. Insofar as the integrated fission response over all energies is concerned, the effective percentage of the flux absorbed is considerably less than 2% and may be neglected. The self-shielding in U^{235} is similarly negligible.

The depression of the thermal flux in the vicinity of the absorbing foil was estimated both by assuming infinite slab foils by P_3 cell calculations and by using the modified flux depression formulas of Bothe⁽⁴⁾ for one-centimeter disk foils. The flux depression calculations for the Pu foil were also repeated assuming that the absorption cross section of the foil was equal to that of the peak value of the resonance and half the peak value to obtain estimates of the upper limit of resonance flux depression. As expected, the modified Bothe formula,⁽⁴⁾ for the case of a disk-shaped foil of radius appreciably greater than the transport mean free path of the surrounding medium, gave much smaller flux depressions than the infinite slab P_3 calculations. It was found, however, by evaluating the Bothe formula in the limit of infinite disk radius that, except for the cases of very large absorption such as found in the Pu resonance, the resulting depression factors were comparable to the P_3 results. The results of the flux-depression calculations for the infinite slab and finite disk are given in Table II for both the Pu^{239} and U^{235} foils.

Table II
Comparison of Flux-depression Correction Factors

T, °K	Foil (σ_a)	Bothe Formula for 1-cm disk	Bothe Formula for infinite disk	P_3 (infinite slab)
300	U (thermal)	1.00017	1.011	1.014
300	Pu (thermal)	1.00026	1.017	1.02
700	U (thermal)	1.00012	1.0093	1.012
700	Pu (thermal)	1.00017	1.013	1.017
300	Pu (5,000 b)	1.0014	1.092	1.12
700	Pu (5,000 b)	1.0014	1.11	1.15
300	Pu (2,500 b)	1.00074	1.047	-
700	Pu (2,500 b)	1.00074	1.058	1.08

In the case of the thermal flux depression, the results of application of the Bothe formula to the finite disk indicates that the depression is very small. In addition, the thermal flux depressions of both the Pu and U foils are comparable and would tend to cancel in the response ratio.

The P_3 calculation (for example at 300°K) of the Pu resonance flux depression, assuming the pessimistic peak cross-section value of the resonance and infinite slab geometry, gives about 12% depression. The infinite slab extrapolation of the Bothe formula gives about 9% depression. The Bothe formula for the one-centimeter-diameter disk gives 0.14%, which is much smaller and negligible in the final integrated response ratio even if the estimated 0.14% depression is in error by a few factors.

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