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SUBJECT: "Analysis of Borax Experiments" by M. C. Edlund and L. C. Noderer

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FROM: P. R. Kasten

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Introduction

Historically the runaway problem for enriched, heterogeneous, water moderated reactors had its origin in the development of the Materials Testing Reactor. At that time A. M. Weinberg pointed out that there was no real knowledge of the maximum reactivity that could be compensated by the inherent shut-down mechanism of the reactor without damage to the fuel elements. Since the fuel plates were thin, it was felt that sufficient energy could be transferred to the water coolant, produce steam and shut off the reactor for power transients having very short periods. The time required for a heat pulse to travel through the cladding of an MTR fuel plate is approximately 1 millisecond. Hence, it was clear that a runaway on a period of about a millisecond would in all probability melt the fuel elements.

An analysis of this problem, first attempted by E. Greuling and M. C. Edlund\(^1\), indicated that the MTR fuel elements would probably not melt for a reactor period of 10 milliseconds. Later investigations\(^2\) led to substantially the same result. In this early work it was pointed out that information on transient heat transfer from the plate to the water and steam production was inadequate for a completely satisfactory theoretical solution of this problem.

With the development of swimming pool reactors for universities and increased interest in heterogeneous power reactors, it became clear that an experimental investigation of this hazards problem was urgently needed. In 1953 the Argonne National Laboratory\(^3\) ran a series of tests, known as the Borax Experiments, in which a core of MTR fuel elements were placed in an open tank of water and subjected to a series of reactivity additions. The total energy of the resulting power


\(^2\) Ibid. Claiborne and Poppendiek, pp. 31-42.

excursion, fuel plate temperatures and heat content of the fuel plates were measured as a function of the initial reactor period.

These tests showed that power excursions of periods as short as 5 milliseconds with the water initially at saturation temperature were shut down by steam production without melting the fuel plates. (maximum temperature 338°C). The pressure rise in the coolant, however, did reverse the curvature of some of the fuel plates.

In this paper a phenomenological theory of runaway behavior in reactors using MTR-type fuel elements is developed using data from the Borax experiments. The theory is then used to assess the relative safety merits of several research reactors which differ in neutron generation time, steam void coefficients and hydrodynamic characteristics.

II. Analysis of Self-Limitation of Power Excursions

Dynamics

Data from a typical excursion is represented in Figure 1. A very important result, noted by Zinn et al, is that the total energy transferred to the water at the time the initial reactivity has just been compensated far exceeds the energy required to produce sufficient steam to shut off the reaction. In this particular case the reactor would be shut down by the production of about 3 liters of steam which requires only .004 Mw-sec of energy for the phase change and the mechanical work required to displace 3 liters of water. Examination of the data, however, shows that several hundred times as much energy (about 1 Mw-sec) was transferred to the water.

At this point we can examine two possible explanations why this energy is retained as internal energy of the liquid phase of the water: (1) The time scale for vapor bubble formation is on the order of the time of excursion resulting in large superheating of the water and (2) the increase in pressure due to production of steam increases the saturation temperature of the water and reduces
the rate of steam production.

Experiments of Rosenthal at ORNL show that the time interval between the introduction of heat and the appearance of vapor bubbles in saturated water is less than a millisecond. In these experiments a flat platinum filament was heated with a source increasing exponentially in time. High speed motion pictures of the vapor formation were taken and the filament surface temperature was determined. Within the limits of the temperature determination (± 6°F), no superheat was detected in the water adjacent to the filament.

Consequently for reactor periods of a few milliseconds or greater, the so-called "bubble delay time", and hence superheat of the water, can be neglected. We feel that the phenomenon is described by the second "explanation", i.e., that the large energy transfer to the water is due to the increase in the saturation temperature of the water resulting from the rise in pressure during the excursion.

At the time the power is a maximum t\textsubscript{max}, the initial reactivity δK, is just compensated by the various reactivity coefficients, i.e.,

$$\omega_s \left(\frac{\partial K}{\partial v_s}\right) v_s + \omega_w \left(\frac{\partial K}{\partial T_w}\right) AT_w + \omega_p \left(\frac{\partial K}{\partial T_p}\right) AT_p = \delta K$$

(1)

where \(v_s\) is the fraction of core water replaced by steam, \(AT_w\) the change in average water temperature and \(AT_p\) the change in average plate temperature. The partial derivatives are the corresponding uniform reactivity coefficients and the \(\omega\)'s the statistical weights which take into account the non-uniform spatial distribution of steam and temperature changes. Assuming the power to rise exponentially with period \(1/m\), \(v_s\), \(AT_w\), and \(AT_p\) can be estimated as a function of \(P_0 \int_0^{t_{\text{max}}} \epsilon x dt\), which in turn is proportional to the total energy of the excursion. The proportionality factor lies between \(1/2\) and 1, being \(1/2\) for the adiabatic case and about \(3/4\) for Borax.
In the Borax experiments the peak to average power ratio is about 1.8.

Since the heat transfer to the water varies almost as the fourth power of the wall temperature, the small volume of steam required to shut down the reactor for initial reactivities of a few percent are produced in the central "hot" region of the reactor core. In this case, the pressure rise in the water can be written simply as

$$\Delta p = \alpha V_s$$

(2)

where $V_s$ is the steam volume and $\alpha$ is related to the inertia of the fluid.

Estimates of the maximum fluid velocities of interest give turbulent friction losses which are small compared with the inertial term.

We first consider the case where the water and fuel plates are initially at saturation temperature. In order to calculate the volume of steam we try the following simple model. In the rapid excursions we assume that the heat transferred to the water is contained in a layer adjacent to the fuel plate, the thickness of which grows directly as the square root of time. Since the mechanism for sub-cooled nucleate boiling is the production of steam bubbles near the surface and their consequent convection and collapse in the sub-cooled water away from the fuel plate, the diffusivity to be used in determining the growth of the layer is considerably larger than that given by the water conductivity. As the pressure rises, the average temperature of this boundary layer must increase in direct proportion with the saturation temperature of the water. The average temperature of the boundary layer of thickness $\mu \sqrt{t}$, which we assume must follow the saturation temperature as the pressure rises, is then

$$MT_g(t) = \frac{E_w(t)}{C_o \mu \sqrt{t}}$$

(3)
where \( C_0 \) is the heat capacity of a unit volume of water and \( E_w(t) \) is the heat transferred to the water per unit area of fuel plate at time, \( t \). The quantity, \( \mu \), which has the dimensions of the square root of a thermal diffusivity is the only parameter which is to be determined by a comparison with Borax data.

For small changes in saturation temperature, the Clapeyron-Clausius equation can be written

\[
\Delta p = \gamma \Delta T_s
\]  

(4)

Eliminating the pressure rise and \( \Delta T_s \) from equations (2), (3) and (4) gives the steam volume as a function of \( E_w(t) \), i.e.

\[
V_s = \frac{\gamma \mu}{C_0} \frac{E_w(t)}{\sqrt{t}}
\]  

(5)

Since the variation of \( \sqrt{t} \) is slower than the variation of

\[
E_w(t) \int_{t_{max}}^{t} dt' \int_{t_{max}}^{t} dt \frac{E_w(t)}{\sqrt{t}} \approx E_w(t_{max})/m^{3/2}.
\]

Absorbing the proportionality factor in \( \mu \), the steam volume is

\[
V_s(t_{max}) = \frac{\gamma \mu}{C_0} \frac{E_w(t_{max})}{m^{3/2}}
\]  

(6)

The water and fuel plate temperature coefficients account for less than 10% of the reactivity compensation and can be ignored. Thus, equations (1) and (6) give the basic energy relation for the power excursion,

\[
E_w = \frac{\gamma C_0 \mu V_w}{\gamma} m^{3/2} \frac{\partial K}{\partial V_s}
\]  

(7)
where $V_w$ is the total volume of core water.

The total energy of the excursion is

$$E_T = \frac{4}{3} \frac{\rho L}{\bar{E} \bar{\phi}} \frac{V_w}{\bar{E}} m^{3/2} \frac{E}{m s} \left( \frac{dK}{dV_p} \right)$$

(8)

As discussed before, $E(t_{max})$ is about $3/4$ of the total energy of the excursion per unit area of fuel plate, $E$ is the peak to average power ratio and $\phi$ is the fraction of the energy of the excursion at time, $t_{max}$, which is transferred to the water.

The average fuel plate temperature rise at time, $t_{max}$, at the hottest part of the reactor is then

$$\Delta T_p = \frac{(1 - \phi) E}{C_1 b \bar{\phi}}$$

(9)

where $C_1$ is the heat capacity of a unit volume and $b$ the half thickness of the fuel plate.

The pressure rise given by equation (2) is, of course, not the total pressure rise in an excursion. After the reactivity has been reduced to zero, the energy stored in the fuel plates will generate additional steam.

Temperature Distribution

To obtain the relative distribution of energy between the fuel plates and water at time, $t_{max}$, and hence $\Delta T_w$ and $\Delta T_p$, we consider the following heat conduction problem.
uniform heat source, $P_0 e^{mt}$, in $0 \leq x \leq a$ and boundary condition

$$-k \frac{dT(b,t)}{dx} = H[T(b,t) - T_s(t)]^{3.67}$$

which is obtained from steady state surface boiling data. $T_s$ is the saturation temperature of the water.

Since almost all of the energy is released during the two periods preceding $t_{max}$, we shall approximate the solution of the above non-linear problem by (a) linearizing the boundary condition and (b) selecting a fictitious heat transfer coefficient by requiring that the heat transferred to the water as calculated by the linear approximation is the same as that given by using the non-linear boundary condition.

The temperature distribution for the fuel plates initially at the saturation temperature of water is,
\[ 0 \leq x \leq a \]

\[ T(x,t) = \frac{P_0}{m c_1} \ e^{mt} \ (1 - \beta \cosh \sqrt{\lambda} \ x) \]

\[ - \frac{P_0}{m c_1} \ \sum_{n=1}^{\infty} \ \int_{n}^{\infty} e^{-\frac{\nu_n^2}{2} x} \ \frac{\cos \nu_n x}{\cos \nu_n b} + T_s \]  \hspace{1cm} (10)

\[ a \leq x \leq b \]

\[ T(x,t) = \frac{P_0}{m c_1} \ e^{mt} \ \left[ \mathcal{N} \cosh \sqrt{\lambda} \ x - \sinh \sqrt{\lambda} \ a \ \sinh \sqrt{\lambda} \ b \right] \]

\[ - \frac{P_0}{m c_1} \ \sum_{n=1}^{\infty} \ \int_{n}^{\infty} e^{-\frac{\nu_n^2}{2} x} \ \frac{\cos \nu_n x}{\cos \nu_n b} + T_s \]  \hspace{1cm} (11)

where:

\[ \beta = \cosh \sqrt{\lambda} \ a - \mathcal{N} \]  \hspace{1cm} (12)

\[ \mathcal{N} = \sinh \sqrt{\lambda} \ a \ \frac{h \ tanh \sqrt{\lambda} \ b + \sqrt{\lambda}}{\sqrt{\lambda} \ tanh \sqrt{\lambda} \ b + h} \]  \hspace{1cm} (13)

\[ \mathcal{N} = \frac{4 \ sin \ \nu_n \ a \ \cos \ \nu_n \ b \ (b^2 + \nu_n^2) \ \ m \ c_1}{\nu_n \ \left[ b (h^2 + \nu_n^2) + h \right] \ (\nu_n^2 + \lambda) \ k} \]  \hspace{1cm} (14)

\( \nu_n \) are the roots of the equation,

\[ \nu_n \ \tan \ \nu_n \ b = h \]  \hspace{1cm} (15)
To obtain $h$ we match the total heat transferred in the simple calculation with surface boiling data, i.e.:

$$hk \int_0^t dT_b \ dt = H \int_0^t dT_b^{3.67} \ dt$$

(16)

where $dT_b = T(b,t) - T_s$.

Data from the Borax experiments indicate that $dT_b$ is nearly linear for the time interval $t_{max}$ to $t_{max}-2/m$. Hence, equation (16) gives approximately,

$$h = \frac{2H}{4.67k} dT_b^{2.67}$$

(17)

On the other hand,

$$dT_b = \frac{E}{a c_1} f(a,h)$$

(18)

where $E$ is the total energy of the excursion at time $t_{max}$ per unit area of fuel plate and $f(a,h) = \sum_{n=1}^{\infty} \Gamma_n e^{-2(1 + \xi^2 v_n^2/m)}$

(19)
The effect of transient terms in the temperature distribution given in equation (19) are calculated by assuming a time interval of 2 periods since this is the interval during which nearly all of the energy transfer takes place.

The fraction of energy transferred to the water at time $t_{\text{max}}$, is then

$$\Phi = \frac{k}{a \sqrt{\lambda}} - \frac{h}{a} \sum_{n=1}^{\infty} f_n \left( \frac{1 - e^{-2 \xi v_n^2/m}}{e^2 - 1} \right) \frac{1}{v_n}$$

(20)

where $k = \sinh \sqrt{\lambda} \cdot a \cosh \sqrt{\lambda} \cdot b = -\int \sinh \sqrt{\lambda} \cdot b$

$\Phi(h,a)$ is given in Figure 2 for MTR fuel elements.

Combining equations (17) and (18) we obtain

$$E = \frac{a \cdot c_1}{f(w,h)} \left( \frac{4.67 \text{ kh}}{2 H} \right) 0.374$$

(21)

To facilitate calculations, $E_w = \Phi E(h,a)$ is plotted in Figure 3. The calculational procedure then reduces to computing $E_w$ from equation (7), finding $h$ from Figure 3 and finally $\Phi$ from Figure 2.

Comparison of Theory with Borax Experiments

The Borax data for the water initially at saturation temperature giving the total energy of the excursion as a function of the period is reproduced very well by choosing $\mu = 1.21 \text{ cm/seg}^{1/2}$ (Figure 4). This value of $\mu$ corresponds to a thermal diffusivity of the order of $1 \text{ cm}^2/\text{sec}$ which is several hundred times larger than the thermal diffusivity for pure conduction in water.

An independent check on the approximate solution of the heat conduction problem discussed above is given by the agreement between the calculated average plate temperature rise at the hottest part of the reactor and the experimental data (Figure 5).
There is, however, some uncertainty in the calculation of the material pressure coefficient, $\alpha$. The maximum value of $\alpha$ is given by assuming that all the water directly above the core moves upward as a slug and that there is no motion of coolant water down out of the core and up along the side of the core. On the other hand, the minimum $\alpha$ is obtained by assuming the core water to be forced out equally in both directions and having three degrees of freedom for motion outside the core. Motion pictures of the top of the reactor during an excursion indicate that initially the water directly above the core moves upwards more or less as a slug. Furthermore, calculations show that there is also appreciable pressure relief due to motion of the fluid down out of the core and up along the sides. In view of these considerations, we have assumed that half of the water moves upwards and that the water directly above the core moves as a slug. An elementary calculation then gives,

$$\alpha = \frac{\rho}{\bar{\alpha}} \left( \frac{L}{2A_1} + \frac{h}{A_o} \right)$$

(22)

where $\rho$ - density of water

$L$ - height of core

$A_1$ - cross sectional area of coolant water in core

$A_o$ - cross sectional area of core

$h$ - height of water above core

The total energy of the excursion and the maximum plate temperature are then given by,

$$E_T = \frac{1}{3} \frac{C_o \mu \rho}{\gamma \epsilon} \left( \frac{L^2}{4} + \frac{LIA_1}{2A_o} \right) \frac{u_{3/2}}{\bar{\alpha}} \frac{6K}{\epsilon}$$

(23)

$$\Delta T_p = \frac{L^2}{4} \frac{C_o \mu \rho}{\gamma \epsilon} \left( \frac{L^2}{4} + \frac{LIA_1}{2A_o} \right) \frac{u_{3/2}}{\bar{\alpha}} \frac{4K}{\epsilon}$$

(24)
A list of definitions and data used in the calculations for Borax is given below:

\[ a = 0.0267 \text{ cm} = \text{half thickness of fuel bearing part of fuel plate} \]

\[ b = 0.0762 \text{ cm} = \text{half thickness of fuel plate} \]

\[ d = 0.150 \text{ cm} = \text{half thickness of water coolant channel} \]

\[ L = 60 \text{ cm} = \text{height of core} \]

\[ h = 90 \text{ cm} = \text{height of water above core} \]

\[ A_1 = 947 \text{ cm}^2 = \text{cross sectional area of coolant water in core} \]

\[ A_0 = 1600 \text{ cm}^2 = \text{cross sectional area of core} \]

\[ \xi = 0.67 \text{ cm}^2/\text{sec} = \text{thermal diffusivity of Al} \]

\[ C_\text{i} = 2.48 \text{ watt-sec/cm}^3 - \text{°C} = \text{heat capacity of unit volume of Al} \]

\[ C_\text{o} = 1.185 \text{ watt-sec/cm}^3 - \text{°C} = \text{heat capacity of unit volume of H}_2\text{O} \]

\[ \gamma = 5.66 \times 10^4 \text{ dyne/cm}^2 - \text{°C} = \text{linear expansion coefficient of the Clepeyron-Clausius relation} \]

\[ k = 1.67 \text{ watt-sec/cm-sec-°C} = \text{thermal conductivity of Al} \]

\[ H = 1.26 \times 10^{-3} \text{ watts/cm}^2 - (\text{°C})^{3.67} \]

\[ \omega = 1.8 = \text{peak to average power} \]

\[ \omega_b = 3.25 = \text{statistical weight for steam production at center of reactor} \]

\[ \frac{\partial x}{\partial \gamma} = 23.4\%/\text{fraction steam in water} = \text{steam reactivity coefficient} \]

\[ \lambda = 6.5 \times 10^{-5} \text{ sec} = \text{prompt neutron generation time} \]

The pressure rises computed for Borax at the time, \( t_{\text{max}} \), are given in Table 1.

<table>
<thead>
<tr>
<th>( \mu \text{(sec}^{-1}) )</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta p \text{(atm)} )</td>
<td>.097</td>
<td>.510</td>
<td>1.57</td>
<td>2.92</td>
</tr>
</tbody>
</table>

**Limiting Case for Thin Fuel Elements**

In the limit of infinitesimal fuel plate thickness \( (1 - \frac{1}{b})/b \) becomes \( \lambda/h \). The plate temperature rise for a given excursion is then a minimum. If the thickness of the fuel plates in Borax had been infinitesimal the plate temperature...
rise would have been only 40% and 15% of the actual plate temperature rise for the 20 and 5 millisecond period cases respectively.

The Effect of Subcooling on the Energy of Excursion

Since research reactors operate at temperatures substantially below the saturation temperature of the water in the reactor, the total excursion energy and fuel plate temperature rise is considerably larger than those obtained in cases for which the coolant water is initially at saturation temperature. A relatively simple estimate of the effect of subcooling on excursion energy and fuel plate temperature rise which agrees fairly well with the rather meager Borax data can be made by assuming that all the water in the coolant channels at the center of the reactor must be heated to saturation temperature.

Since the water at the center of the reactor boils before there is a large change in the bulk temperature of the coolant water, the compensation of the initial reactivity is still due mainly to production of steam. The energy transfer to the water per unit area of fuel plate for the reactor initially at temperature, \( T_0 \), is increased by \( C_0 \Delta(T_s - T_0) \) and equation (7) is then replaced by

\[
E_v = \frac{\xi C_0 \mu V_w}{\gamma} \omega^3/2 \frac{\partial \xi}{\partial \omega} \Delta(T_s - T_0) + C_0 \Delta(T_s - T_0)
\]

A comparison with Borax data (Figure 6) shows that the calculated rate of increase of the total energy of the excursion with subcooling agrees fairly well with the experimental data for the .013 second period excursion. On the other hand, data for the .022 second period excursion shows a smaller rise in the total energy with subcooling than predicted.
As the fuel plate spacing is increased, there will be a limiting layer of water adjacent to the plate which is heated to saturation temperature. Further experimental work would be very useful in determining the thickness of this layer. For our purposes, however, the assumption that all the water is heated to saturation temperature is conservative and, furthermore, seems to be about right for MTR fuel elements.

III. Comparison of Light and Heavy Water Research Reactors

As pointed out by M. Benedict, a heavy water reactor of the CP-5 type may, under some circumstances, have a marked advantage in safety over the light water, "swimming pool" or the Oak Ridge Research Reactor (ORR), now under construction, because of the longer neutron lifetime in the heavy water reactors. It is clear that a given reactivity increase will result in a considerably smaller fuel plate temperature rise in the heavy water reactor only if its thermal, geometric, nuclear and hydrodynamic characteristics are about the same as in the light water reactors. To evaluate the relative importance of these parameters, we have considered in some detail two reactors, one of which is representative of CP-5 and the other of ORR.

For purposes of comparison, the reactors are represented as reflected cylinders with an axial buckling to account for the end reflectors in a two-group model. The usual fluxes and adjoints are calculated in order to obtain lifetime and reactivity coefficients by means of first order perturbation theory for disturbances assumed to be uniform over the core.

The two-group constants for various combinations of light or heavy water and aluminum as a function of temperature are given by the following semi-empirical formulas which are valid for the range of parameters usually associated with MTR-type fuel elements but are not necessarily valid for mixtures of light and heavy water.
\[ \frac{1}{\bar{v}} = 0.176 \frac{V_{H}}{V_{H}^{*}} + 0.057 \frac{V_{Al}}{V_{Al}^{*}} + 0.0913 \frac{R_{D}}{V_{D}} \]

\[ \frac{1}{D_{0}} = 0.8724 \frac{R_{H}}{V_{H}^{*}} + 0.6027 \frac{V_{Al}}{V_{Al}^{*}} + 0.885 \frac{R_{D}}{V_{D}} \]

\[ \frac{1}{D_{1}} = 6.311 \frac{R_{H}}{S_{H}} \frac{V_{H}^{*}}{V_{Al}} + 1.667 \frac{V_{Al}}{V_{Al}^{*}} + 1.192 \frac{R_{D}}{S_{D}} \frac{V_{D}}{V_{D}^{*}} \]

Where

- \( \tau \) = age, cm²
- \( D_{0} \) = fast diffusion coefficient, cm
- \( D_{1} \) = thermal diffusion coefficient, cm
- \( V_{H}, V_{Al}, V_{D} \) = fraction of core volume occupied by H₂O, aluminum and uranium, and D₂O respectively
- \( R_{H}, R_{D} \) = relative density of H₂O or D₂O
- \( S_{H}, S_{D} \) = ratio of averaged thermal transport cross section at temperature \( T \) to that at 20°C

The averaged absorption cross sections used are:

- \( \Sigma_{H_{2}O} = 0.0295 \text{ cm}^{-1} \)
- \( \Sigma_{D_{2}O} = 7.67 \times 10^{-5} \text{ cm}^{-1} \)
- \( \Sigma_{Al} = 0.013 \text{ cm}^{-1} \)
- \( \Sigma_{25} = 604.6 \text{ barns} \)

The two group constants used in reflectors are:

<table>
<thead>
<tr>
<th>Group</th>
<th>( D_{20} )</th>
<th>( E_{20} )</th>
<th>( B_{s} )</th>
<th>( 2% \text{ H}_{2}O )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau )</td>
<td>120</td>
<td>33</td>
<td>89</td>
<td></td>
</tr>
<tr>
<td>( D_{0} )</td>
<td>1.13</td>
<td>1.19</td>
<td>1.601</td>
<td></td>
</tr>
<tr>
<td>( L^{2} )</td>
<td>11,000</td>
<td>8.12</td>
<td>532</td>
<td></td>
</tr>
<tr>
<td>( D_{1} )</td>
<td>.839</td>
<td>.16</td>
<td>.851</td>
<td></td>
</tr>
</tbody>
</table>

The average neutron velocities are:

- \( \bar{v} = 2.482 \times 10^{5} \text{ cm sec}^{-1} \) for the thermal group and
- \( \bar{v}_{0} = 5.94 \times 10^{6} \text{ cm sec}^{-1} \) for the fast group.
The neutron lifetime was calculated using equation (26) and the reciprocal period, \( m \), determined from the usual inhour relation, equation (27). Introducing,

- \( \phi_0, \phi_1 \) are the fast and thermal neutron fluxes
- \( \phi_0^*, \phi_1^* \) are the fast and thermal adjoint functions
- \( \langle \cdot \rangle_c \) signifies integration over the core volume
- \( \langle \cdot \rangle_T \) signifies integration over the total reactor volume
- \( R_0 \) is the core radius
- \( \nabla \) is the gradient in an outward direction
- \( \beta_i \) is the number of delayed neutrons of the \( i \)th group per prompt fission neutron
- \( \lambda_1 \) is the decay constant for the \( i \)th delayed neutron group

we obtain

\[
\ell = \frac{1}{\eta \Sigma_U \langle \phi_0^* \phi_1 \rangle_c} \left[ \frac{\langle \phi_1^* \phi_1 \rangle_T}{v_1} + \frac{\langle \phi_0^* \phi_0 \rangle_T}{v_0} \right] \tag{26}
\]

\[
\xi_K = \frac{1}{1 + m \ell} \left( m \ell + \frac{m \beta_1}{m + \lambda_1} \right) \tag{27}
\]

The reactivity coefficients are calculated by first obtaining the small changes in the two group macroscopic constants and then determining \( \xi_K \) by means of the following reactivity equation which assumes uniform changes in the core of the reactor only.
The important calculated parameters are given in Table 2. The maximum fuel plate temperature rise for the reactors initially at saturation temperature calculated as a function of initial reactivity (Figure 7) shows that the CP-5 will have a smaller plate temperature rise for a given reactivity than any of the light water reactors considered. The maximum plate temperature rise from equations (7) and (9) is given by

\[
\Delta T_p = \frac{1 - \Phi}{\Phi} \frac{\lambda c_o \mu v_w}{\gamma c_1 b} \frac{m^{3/2} \delta \kappa}{\omega_3 \left( \frac{\delta \kappa}{\delta \phi} \right)}
\]  

From Table 2, we note that for a given \( \delta \kappa \), the \( \Delta T_p \) for ORR and CP-5 are almost in the direct ratio of \( (1 - \Phi) m^{3/2} \Phi \). Since the neutron lifetime for CP-5 is some six to eight times longer than for ORR, the smaller \( m \) for CP-5 determines the relative \( \Delta T_p \).

ORR-1 and ORR-2, which differ only by increased reflector thickness in the latter case, show that there is relatively little advantage to be gained by increasing the neutron lifetime merely by adding more reflector. Although the lifetime may be increased, the steam reactivity coefficient is reduced.
<table>
<thead>
<tr>
<th></th>
<th>HORAI</th>
<th>ORR-1</th>
<th>ORR-2</th>
<th>CP-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core Radius, cm</td>
<td>22.5</td>
<td>22.5</td>
<td>22.5</td>
<td>30.9</td>
</tr>
<tr>
<td>Volume Al</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volume Water</td>
<td>.686</td>
<td>.581</td>
<td>.581</td>
<td>.084</td>
</tr>
<tr>
<td>Refl. material</td>
<td>H2O</td>
<td>Be</td>
<td>Be</td>
<td>D2O</td>
</tr>
<tr>
<td>Refl. Thickness, cm</td>
<td>30</td>
<td>16</td>
<td>32</td>
<td>60</td>
</tr>
<tr>
<td>$\frac{\partial K}{\partial V_e}$</td>
<td>.234</td>
<td>.0865</td>
<td>.0486</td>
<td>.558</td>
</tr>
<tr>
<td>$\omega \frac{\partial K}{\partial V_e}$</td>
<td>.760</td>
<td>.371</td>
<td>.208</td>
<td>.857</td>
</tr>
<tr>
<td>$\frac{\partial K}{\partial T∕P}$</td>
<td>1.56x10^-4</td>
<td>1.08x10^-4</td>
<td>9.23x10^-5</td>
<td>4.89x10^-4</td>
</tr>
<tr>
<td>$\frac{K}{T∕P}$</td>
<td>9.63x10^-6</td>
<td>2.77x10^-6</td>
<td>1.51x10^-6</td>
<td>2.83x10^-6</td>
</tr>
<tr>
<td>$\ell$ sec</td>
<td>6.50x10^-5</td>
<td>8.83x10^-5</td>
<td>1.15x10^-4</td>
<td>6.82x10^-4</td>
</tr>
<tr>
<td>$V_w$, liters</td>
<td>56.8</td>
<td>60.6</td>
<td>60.6</td>
<td>166</td>
</tr>
<tr>
<td>$\frac{\varphi}{cm^2}$</td>
<td>h.40x10^-2</td>
<td>h.30x10^-2</td>
<td>h.30x10^-2</td>
<td>3.91x10^-2</td>
</tr>
</tbody>
</table>
In fact, a reactor could possibly be made less safe by increasing the albedo of the reflector. In such cases, there is an optimum reflector thickness which will minimize the maximum temperature rise resulting from a power excursion.

The maximum amount of reactivity which can be self-limited without melting the fuel plates, by itself, does not give a complete picture of the relative safety merits of research reactors. If reactors are to be used for in-pile corrosion testing of fuel solutions, for example, they should be compared on the basis of equivalent accidents; in this case the sudden addition of a given amount of fissionable material. Other accidents to experimental apparatus could be represented by the sudden filling of a void volume or removal of a poison. These considerations are illustrated by a calculation of $\Delta T_p$ as a function of U-235 added uniformly and of void volume filled (Figures 8 and 9). In these cases the relative advantage of CP-5 over ORR has been reduced because of its smaller critical mass and larger void coefficient.

The effect of subcooling on the $\Delta T_p$ was calculated using the conservative assumption that all the water in the central coolant channels must be heated to boiling (Figure 10). This may prejudice somewhat the CP-5 results, since the fuel plate spacing is considerably larger than in ORR.

All of the results are summarized in Table 3, which lists the "safe" upper limits for reactivity, U-235 addition and void removal based on a melting temperature of 660°C for aluminum.
### Table 3

**MAXIMUM "SAFE" REACTIVITY CHANGES**

<table>
<thead>
<tr>
<th>SUBCOOLING °C</th>
<th>BORAX</th>
<th>ORR-1</th>
<th>ORR-2</th>
<th>CP-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.2</td>
<td>2.15</td>
<td>2.6</td>
<td>9.5</td>
</tr>
<tr>
<td>30</td>
<td>2.1</td>
<td>2.35</td>
<td>2.5</td>
<td>7.75</td>
</tr>
<tr>
<td>60</td>
<td>1.95</td>
<td>2.2</td>
<td>2.35</td>
<td>6.95</td>
</tr>
<tr>
<td>0</td>
<td>220</td>
<td>139</td>
<td>166</td>
<td>223</td>
</tr>
<tr>
<td>30</td>
<td>210</td>
<td>133</td>
<td>160</td>
<td>182</td>
</tr>
<tr>
<td>60</td>
<td>195</td>
<td>125</td>
<td>1150</td>
<td>163</td>
</tr>
<tr>
<td>0</td>
<td>5.3</td>
<td>17</td>
<td>32</td>
<td>28</td>
</tr>
<tr>
<td>30</td>
<td>5.1</td>
<td>16</td>
<td>31</td>
<td>23</td>
</tr>
<tr>
<td>60</td>
<td>4.7</td>
<td>15</td>
<td>29</td>
<td>21</td>
</tr>
</tbody>
</table>

- **BORAX**
- **ORR-1**
- **ORR-2**
- **CP-5**

- U-235 Grams
- Liters of Void

*vh*
Fig. 1. Power Excursion, Borax Experiments
Fig. 2. Fraction of Energy Transferred to Water
Fig. 3. Energy Transferred to Water per Unit Area of Fuel Plate
Fig 4 Total Energy of Excursion, Borax Experiment
Fig 5 Plate Temperature Rise, Borax Experiment

- Maximum Plate Temperature Rise (°C)
- Reciprocal Period (sec⁻¹)

O Representative Borax Data
- Theoretical

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Fig. 6. Effect of Subcooling on Total Energy of Excursion–Borax Excursion
Fig 7. Plate Temperature Rises
Fig 8. Plate Temperature Rises
Fig 9 Plate Temperature Rises

MAXIMUM PLATE TEMPERATURE RISE (°C)

LITERS OF VOID IN CORE

BORAX
ORR-1
CP-5
ORR-2

577 032
Fig. 10. Effect of Subcooling on Maximum Temperature Rises