Higgs Type Lagrangian and Dual Strings*

M. Arik and K. Tanaka
Department of Physics
The Ohio State University
Columbus, Ohio 43210

Abstract

A nonlinear Born Infeld type of Lagrangian which yields the Maxwell and Higgs Lagrangian in the weak limit and the dual string model in the strong limit is studied. In particular, it is argued that the limit in which the string-like solution is obtained is a super quantum-mechanical limit. The effect of the contributions which become zero in this limit is discussed and an approximate vortex line solution is obtained.

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1. INTRODUCTION

There is presently a lot of interest in the possibility that the underlying objects of hadronic matter are string-like structures.¹ Nielsen and Olesen² noticed a close relation between the Landau-Ginsburg Lagrangian which has vortex line solutions in a type II superconductor and the Higgs Lagrangian, and found vortex line or string-like solutions to the Higgs Lagrangian with equations of motion of the Nambu dual string,³ thus connecting the Higgs type of Lagrangian with dual strings.

In a separate paper, Nielsen and Olesen⁴ developed a classical local field theory which contains as a special solution the classical dual string discussed by Goddard, Goldstone, Rebbi and Thorn.⁵

Tze⁶ has suggested from a different point of view a close relationship between the Nambu string and the square root Born-Infeld Lagrangian, a limiting case of which is the Nielsen-Olesen theory of dual strings. Specifically, consider the following non-linear Lagrangian

$$\mathcal{L} = -\Lambda^{-2} \left[(1 + 2\Lambda^2 J)^{1/2} - 1 \right] , \quad (1.1)$$

where

$$J = \frac{1}{4} F_{\mu\nu}^2 , \quad (1.2)$$

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu} , \quad (1.3)$$

$$\Lambda = 2\pi\hbar c^2 \alpha', \quad (1.4)$$

where $F_{\mu\nu}$ is the electromagnetic tensor, A_μ is the electromagnetic potential and α' is the universal slope of the Regge trajectories.

When $\Lambda^2 J \ll 1$, one obtains

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2, \quad (1.5)$$

the Maxwell theory, and when $\Lambda^2 J \gg 1$

$$\mathcal{L} = -\sqrt{2} \Lambda^{-1} J^{1/2} = -\sqrt{2} \Lambda^{-1} \left(\frac{1}{4} F_{\mu\nu}^2 \right)^{1/2}, \quad (1.6)$$

the Nielsen-Olesen theory of dual strings.⁴

There are two ways to look at the strong field limit $\Lambda^2 J \gg 1$. It can be regarded as the limit in space time region of intense fields

$J = \frac{1}{4} F_{\mu\nu}^2 \gg \Lambda^{-2}$ or the limit in which $\Lambda = 2\pi\hbar c^2 \alpha' \rightarrow \infty$. It is noted by Blohincev⁷ that in the solution of non-linear systems

$$\mathcal{L} = \mathcal{L}(K) = \mathcal{L}\left(\frac{1}{2} p^2 - U(q)\right) \quad (1.7)$$

where p is the velocity operator and $U(q)$ is the potential energy operator,

$$[p, q] = i\hbar^* = i\hbar/M(K),$$

(1.8)

where

$$M(K) = \frac{\partial \mathcal{L}}{\partial K}$$

(1.9)

plays the role of variable mass. When $M(K) \rightarrow \infty$, $\hbar^* \rightarrow 0$ and one obtains the classical case, whereas when $M(K) \rightarrow 0$, $\hbar^* \rightarrow \infty$ and one obtains the super-quantum mechanical limit in which there is a strong fluctuation in the velocity of the signal propagation.

Let us consider a model given by Eq.(1.1) in which J is generalized to

$$\tilde{J} = \frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} |(\partial_\mu - ieA_\mu)\phi|^2 - c_2 |\phi|^2 + c_4 |\phi|^4,$$

(1.10)

where ϕ is a scalar field and the metric is $g_{00} = -g_{ii} = 1$. In analogy with Eq.(1.9) we define

$$M(J) = -\frac{\partial \mathcal{L}}{\partial J} = (1 + 2\Lambda^2 J)^{1/2}.$$

(1.11)

The conjugate momentum is

$$\pi^i = \frac{\partial \mathcal{L}}{\partial \dot{A}_i} = - \frac{F^{0i}}{(1+2\Lambda^2 J)^{1/2}} \quad , \quad (1.12)$$

and the canonical equal time commutator is

$$\left[\frac{F^{0i}(x)}{(1+2\Lambda^2 J(x))^{1/2}} , A^k(y) \right]_{x_0=y_0} = -i\hbar \delta^{ik} \delta^3(\underline{x}-\underline{y}) \quad . \quad (1.13)$$

In the approximation where the variable "mass" $M(J)$ commutes with the potential A^i we have

$$\left[F^{0i}(x) , A^k(y) \right]_{x_0=y_0} = -i\hbar^* \delta^{ik} \delta^3(\underline{x}-\underline{y}) \quad , \quad (1.14)$$

where

$$\hbar^* = \hbar / M(J) \quad . \quad (1.15)$$

From the point of view of the nonlinear Born-Infeld Lagrangian (1.1), the string limit corresponds precisely to the case in which there are strong fluctuations in the velocity of signal propagation along the world sheet of the string. This illustration suggests that $\hbar$ in the parameter Λ is effectively replaced by $\hbar^* = \hbar / M$ (M is dimensionless), so one may consider the strong limit as $\Lambda^2 \gg J^{-1}$ ($\hbar^* \rightarrow \infty$) or $J \gg \Lambda^{-2}$ ($F^2 \rightarrow \infty$) which are equivalent.

When $\Lambda^2 J \ll 1$ in Eq.(1.1), one obtains the Higgs Lagrangian

$$\mathcal{L} = -J, \quad (1.16)$$

where J is given by (1.10). A static vortex line solution of (1.16) in the cylindrically symmetric case

$$A_0 = A_z = A_r = 0,$$

$$\underline{A}(r, z, \omega) = A(r) \hat{z} \times \hat{r} = A(r) \hat{\omega},$$

(1.17)

was found in Ref.2.

On the other hand, when $\Lambda^2 J \gg 1$ is satisfied, we find from (1.1) and (1.10)

$$\mathcal{L} = - \frac{\sqrt{2}}{\Lambda} \left[\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} |(\partial_\mu - ie A_\mu) \phi|^2 - C_2 |\phi|^2 + C_4 |\phi|^4 \right]^{\frac{1}{2}}.$$

(1.18)

When $|\phi| = 0$, $\mathcal{L}$ given by (1.18) leads to the following Nambu-Goto action³ integral when the field $F_{\mu\nu}(x)$ is singular on some path in space and time⁴ $z_\mu(\sigma, \tau)$, where σ and τ are the intrinsic parameters that describe the world sheet swept out by the string in space and time;

$$S = - \int d^4x \mathcal{L} = \int d^2\sigma \frac{1}{\Lambda} \left[\left(\frac{\partial \bar{z}}{\partial \sigma} \cdot \frac{\partial z}{\partial \tau} \right)^2 - \left(\frac{\partial \bar{z}}{\partial \sigma} \right)^2 \left(\frac{\partial z}{\partial \tau} \right)^2 \right]^{1/2}.$$

(1.19)

The action (1.19) is proportional to the Minkowski area of a two-dimensional surface spanned between initial and final positions of the string and is invariant under the reparametrization $\sigma \rightarrow \sigma'(\sigma, \tau)$ $\tau \rightarrow \tau'(\sigma, \tau)$.

We regard the non-linear Lagrangian (1.1) as a generalization of the Maxwell and Higgs type Lagrangians and dual string models. In order to recover the standard former Lagrangians, the weak limit is taken as $\Lambda \rightarrow 0$ or $\hbar^* \rightarrow 0$, whereas the strong limit is taken as $\Lambda \rightarrow \infty$ or $\hbar^* \rightarrow \infty$ which yields the dual string models that are consequences of strong interactions.

The strong limit $\Lambda \rightarrow \infty$ is studied in the region in which $|\phi|$ can be treated as a small constant in Sec.2, and the static cylindrical case (1.17) in Sec.3 and remarks are given in Sec.4.

2. NON-LINEAR HIGGS LAGRANGIAN

We wish to study the Lagrangian (1.18) in the case $|\phi|$ can be treated like a small constant in which case

$$\mathcal{L} = - \frac{\sqrt{2}}{\Lambda} \left[\frac{1}{4} F_{\mu\nu}^2 - \frac{1}{2} e^2 |\phi|^2 A_\mu^2 \right]^{1/2}. \quad (2.1)$$

This is a region in which there is no Higgs mechanism. We note that if the variable mass M is proportional to the mass term $e|\phi|$ then the strong

limit $\Lambda = 2\pi c^2 \hbar^* \alpha' \rightarrow \infty$ implies $M \rightarrow 0$. The field $F_{\mu\nu}$ is taken to be a delta function on the string

$$F_{\mu\nu}(x) = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \int d^2\sigma \delta^4(x - z(\sigma, \tau)) f^{\rho\sigma} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} F^{\alpha\beta} \quad (2.2)$$

where

$$f^{\rho\sigma} = \frac{\partial z^\rho}{\partial \sigma} \frac{\partial z^\sigma}{\partial \tau} - \frac{\partial z^\rho}{\partial \tau} \frac{\partial z^\sigma}{\partial \sigma} \quad (2.3)$$

and $d^2\sigma = d\sigma d\tau$. The antisymmetric tensor $\epsilon_{\mu\nu\rho\sigma}$ is introduced so that the magnetic field $\underline{H}$ goes in the direction of the string. The $z_\mu(\sigma, \tau)$ describes the space time path of the string singularity, and the δ function is to be interpreted as smeared out in the transverse direction to the string.

Among the various possible forms of potential A_μ that satisfy (1.3) and (2.2), we choose

$$A_\mu(x) = \frac{1}{2} \epsilon_{\mu\nu\kappa\lambda} \int d^2\sigma f^{\nu\kappa} \frac{\partial D_R(x-z)}{\partial x_\lambda}, \quad (2.4)$$

where D_R is the retarded Green's function and

$$\square D_R(x-z) = \delta^4(x-z). \quad (2.5)$$

We obtain from (2.2) and (2.4), with the choice $z^0 = \tau$

$$\frac{1}{4} F_{\mu\nu}^2 = -\frac{1}{4} F_{\mu\nu}^{*2}$$

$$= -\frac{1}{4} f_{\mu\nu}^2(x) \left[\int d^2\sigma \delta^4(x-z) \right]^2,$$

(2.6)

$$A(x) = - \int d^2\sigma \nabla D_R \times \frac{\partial \underline{z}}{\partial \sigma} - \int d^2\sigma \frac{\partial \underline{z}}{\partial \sigma} \times \frac{\partial \underline{z}}{\partial \tau} \frac{\partial D_R(x-z)}{\partial x_0},$$

(2.7)

$$A_0(x) = \int d^2\sigma \frac{\partial \underline{z}}{\partial \sigma} \times \frac{\partial \underline{z}}{\partial \tau} \cdot \nabla D_R(x-z),$$

(2.8)

where

$$\frac{\partial D_R(x-z)}{\partial x_0} = -\frac{\Theta(\tau)}{4\pi r} \left[\frac{\partial \delta(x-\tau)}{\partial x_0} - \frac{\partial \delta(x+\tau)}{\partial x_0} \right],$$

(2.9)

$$\nabla D_R(x-z) = \Theta(\tau) \cdot \frac{r}{r} \frac{\partial}{\partial r} \left[\frac{\delta(r-\tau)}{4\pi r} - \frac{\delta(r+\tau)}{4\pi r} \right]$$

(2.10)

$$T = t - z^0 = t - \tau, \quad r = |\underline{x} - \underline{z}(\sigma, \tau)|, \quad r = |\underline{r}|.$$

(2.11)

We consider the action of $\mathcal{L}$ given in Eq.(2.1) in two regions near and away from the string.

Near the string the $F_{\mu\nu}^2$ term dominates due to the fact that $F_{\mu\nu}$ is singular on the string, provided $|\phi|^2$ is sufficiently small, so that with the aid of (2.6)

$$\begin{aligned} S &= - \int d^4x \mathcal{L} = \frac{1}{\sqrt{2}\Lambda} \int d^4x (F_{\mu\nu}^2)^{1/2} \\ &= \frac{1}{\sqrt{2}\Lambda} \int d^2\sigma (-f_{\mu\nu}^2(z))^{1/2} = \frac{1}{\Lambda} \int d^2\sigma \left[\left(\frac{\partial z}{\partial \sigma} \cdot \frac{\partial \bar{z}}{\partial \sigma} \right)^2 - \left(\frac{\partial z}{\partial \sigma} \right)^2 \left(\frac{\partial \bar{z}}{\partial \sigma} \right)^2 \right]^{1/2} \end{aligned} \quad (2.12)$$

Away from the string the A^2 term dominates so that

$$S = - \int d^4x \mathcal{L}(x) = \frac{e|\phi|}{\Lambda} \int d^4x \sqrt{-A_\mu^2} = - \int dt L \quad (2.13)$$

In order to simplify $A_\mu(x)$, consider z^0 at a fixed retarded time so that

$$\frac{\partial \underline{z}}{\partial z} = 0, \quad (2.14)$$

$$\underline{A}(x) = \frac{1}{4\pi} \int d\sigma \frac{(\underline{x} - \underline{z}(\sigma))}{|\underline{x} - \underline{z}|^3} \times \frac{\partial \underline{z}}{\partial \sigma}, \quad A_0(x) = 0 \quad (2.15)$$

Then, one obtains

$$\begin{aligned} -A_\mu A^\mu = \underline{A}^2 &= \frac{1}{16\pi^2} \int \frac{d\sigma d\sigma'}{|\underline{r}(\sigma)|^3 |\underline{r}(\sigma')|^3} \left[-\underline{r}(\sigma) \cdot \frac{\partial \underline{z}(\sigma')}{\partial \sigma'} \underline{r}(\sigma') \cdot \frac{\partial \underline{z}(\sigma)}{\partial \sigma} \right. \\ &\quad \left. + \frac{\partial \underline{z}(\sigma)}{\partial \sigma} \cdot \frac{\partial \underline{z}(\sigma')}{\partial \sigma'} \underline{r}(\sigma) \cdot \underline{r}(\sigma') \right]. \end{aligned} \quad (2.16)$$

If we further make the simplification

$$z_1(\sigma) = \sigma, \quad z_2(\sigma) = z_3(\sigma) = 0,$$

and substitute Eq. (2.16) into Eq. (2.13), we obtain at a fixed retarded time and for $0 \leq \sigma \leq \pi$,

$$\begin{aligned} L &= \int d^3x \mathcal{L}(x) = \frac{e|\phi|}{4\pi\Lambda} \int d^3x \int_0^\pi d\sigma (\rho^2 - x_1^2)^{1/2} (\sigma^2 - 2\sigma x_1 + \rho^2)^{-3/2} \\ &= \frac{e|\phi|}{4\pi\Lambda} \int d^3x \frac{1}{\sqrt{\rho^2 - x_1^2}} \left[\frac{\pi - x_1}{\sqrt{\rho^2 - 2\pi x_1 + \pi^2}} + \frac{x_1}{\rho} \right], \end{aligned} \quad (2.17)$$

where

$$\rho = (x_1^2 + x_2^2 + x_3^2)^{1/2}.$$

When the limit $\rho \gg x_1$ is taken in (2.17), we obtain

$$L = \frac{\pi e|\phi|}{\Lambda} \int d\rho. \quad (2.18)$$

The electromagnetic potential A behaves asymptotically (away from the string) like the potential field of a magnetic dipole. The second term A_μ^2 of (2.1) breaks the invariance under reparametrization.

3. VORTEX LINE SOLUTION

We carry out a study of the Lagrangian (1.18) with the aid of the cylindrically symmetric potential (1.17) together with

$$\phi(x) = |\phi(r)| e^{i\omega}, \quad (3.1)$$

and we obtain the Lagrangian

$$\begin{aligned} \mathcal{L} &= -\frac{\sqrt{2}}{\Lambda} \left[\frac{1}{2} \left(\frac{1}{r} \frac{d}{dr} rA \right)^2 + \frac{1}{2} \left(\frac{1}{r} - eA \right)^2 |\phi|^2 + \frac{1}{2} \left(\frac{d|\phi|}{dr} \right)^2 - c_2 |\phi|^2 + c_4 |\phi|^4 \right]^{1/2} \\ &= -\frac{\sqrt{2}}{\Lambda} J^{1/2}, \end{aligned} \quad (3.2)$$

and the equations of motion of $|\phi|$ and $A(r)$

$$\left[-\frac{1}{r} \frac{d}{dr} r \frac{d|\phi|}{dr} + \left(\frac{1}{r} - eA \right)^2 |\phi| - 2c_2 |\phi| + 4c_4 |\phi|^3 + \frac{1}{2} \frac{d|\phi|}{dr} \frac{dJ}{dr} \frac{1}{J} \right] / J^{1/2} = 0, \quad (3.3)$$

$$\left[\frac{d}{dr} \frac{1}{r} \frac{d}{dr} rA(r) + e \left(\frac{1}{r} - eA \right) |\phi|^2 - \frac{1}{2} \frac{1}{J} \frac{dJ}{dr} \frac{1}{r} \frac{d}{dr} rA \right] / J^{1/2} = 0. \quad (3.4)$$

An exact solution of the highly non-linear Eqs. (3.3) and (3.4) has not been obtained. Let us consider a solution in a region where $|\phi|$ is a constant. Then we obtain from (3.3)

$$e^2 \left(\frac{1}{er} - A \right)^2 - 2C_2 + 4C_4 |\phi|^2 = 0 ,$$

or

$$A(r) = \frac{1}{er} + K(r) , \quad (3.5)$$

where K is a function that decreases faster than r^{-1} as $r \rightarrow \infty$ and

$$C_2 - 2C_4 |\phi|^2 \approx 0 . \quad (3.6)$$

We substitute (3.5) in (3.4) and find

$$z^2 \frac{d^2 K}{dz^2} + z \frac{dK}{dz} - (z^2 + 1) K = R(z) , \quad (3.7)$$

where

$$\begin{aligned} R(z) &= \frac{z}{2} \left(\frac{d}{dz} zK \right) \frac{dJ}{dz} \frac{1}{J} \\ &\approx -\frac{e^2}{C_2} \left(\frac{d}{dz} zK \right) \left[\left(\frac{d}{dz} zK \right) \left(\frac{d}{dz} \frac{1}{z} \frac{d}{dz} zK \right) + zK \frac{dK}{dz} \right] , \end{aligned} \quad (3.8)$$

$$z = e|\phi|r . \quad (3.9)$$

It is not possible to obtain a closed solution of Eq.(3.7), but when $K \ll z^{-1}$, $R(z)$ is the smallest term in Eq.(3.7). Therefore, the modified Bessel function $K_1(z)$ which is a solution of (3.7) with $R(z) = 0$ is an

asymptotic solution of (3.7), where

$$\lim_{z \rightarrow \infty} K_1(z) = \sqrt{\frac{\pi}{2z}} e^{-z} \left(1 + \frac{3}{8z} + \dots \right) . \quad (3.10)$$

The magnetic field H_z is with the aid of (3.5) and (3.10)

$$H_z = \frac{1}{r} \frac{d}{dr} nA$$

$$\approx e|\phi| K_0(e|\phi|r) . \quad (3.11)$$

The dual string may be regarded as an idealization of the static closed magnetic flux line that has a characteristic length $(e|\phi|)^{-1}$.

It is gratifying that the equation of motion of A resulting from the Lagrangian (3.2) has essentially the same solution as that of (1.16), provided we seek a solution of K that decreases faster than z^{-1} .

4. REMARKS

A Born Infeld type of Lagrangian which yields the Maxwell and Higgs Lagrangian in the weak limit is studied in the strong limit. This model can be extended to include gauge theories that combine weak and electromagnetic interactions.²

If we arbitrarily regard the variable mass M as proportional to $e|\phi|$, the mass term of A_μ in (2.1) or characteristic length in (3.11), the weak limit corresponds to the limit $M = e|\phi| \rightarrow \infty$ where the Higgs mechanism is effective, whereas the strong limit corresponds to $M \rightarrow 0$ where one obtains the string model.

It is hoped that studies on the Nielsen-Olesen type models would suggest some possible way of synthesizing the elementary particle interactions.

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