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Final Report
No. F-A1971ANALYSIS OF STRESSES AND DEFLECTIONS
IN
TOP SUPPORT GRID, PWR REACTORby
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ABSTRACT

The top grid of the PWR reactor core assembly is treated as a simply supported circular plate. The theory of plate is applied to the grid under mechanical loads. The thermal stress problem is analyzed by treating the plate as under combined action of a laterally distributed load and forces in the middle plane of the plate. The load distribution is calculated from the temperature variation over the grid. The thermal stress problem then is equivalent to two problems: one, of bending of plate; and, the other, a plane stress problem. The theoretical formulation for plates under non-uniform heating is developed by neglecting the effect of uneven expansion in the direction perpendicular to the plane of the plate. In replacing the partial differential equations by difference equations, the latter are modified to take into account the change in thickness and spacing of the grid webs near the boundary. Twenty-three difference equations for the twenty-three stations in one octant of the grid are obtained for each second order partial differential equation. The difference equations are solved by assuming that the twisting moments and shearing stresses in the plane of the grid vanish at the boundary. The stresses and deflections due to mechanical loads and thermal expansion are then superposed. (auth)

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NOTATIONS

A	Cross-sectional area of a web (sq in.)
b	Thickness of web (in.)
D	Flexural rigidity of a plate = $Eh^3/12(1-\nu^2)$
E	Modulus of elasticity
h	Depth of web
M	$= (M_x + M_y)/1 + \nu$
M_r, M_θ	Bending moments per unit width of a section of a plate perpendicular to the radial and tangential directions respectively
$M_{r\theta}$	Twisting moment per unit width of a section of a plate perpendicular to the radial direction
M_x, M_y	Bending moments per unit width of a section of plate perpendicular to x- and y-axes respectively
M_{xy}	Twisting moment per unit width of a section of plate perpendicular to x-axis
m	Bending moment per unit width of the section of a plate as a result of unequal thermal expansion
Q_x, Q_y	Shearing forces parallel to z-axis per unit width of a section of a plate perpendicular to the x- and y-axes respectively
q	Intensity of distributed load
r, θ , z	Cylindrical coordinates
T	Temperature difference* between temperature of the point in question and that of the surrounding medium
T_e	Equivalent temperature
t	Average temperature
V	A potential function
w	Component of displacement parallel to the z-axis
x, y, z	Rectangular coordinates
α, β	Ratio of two web lengths in x- and y-directions respectively
γ	Ratio of flexural rigidities or cross-sectional areas of two webs

* From here on, temperature difference will simply be referred to as temperature.

NOTATIONS (Continued)

δ	Spacing of the grid (in.)
ϵ_x, ϵ_y	Normal components of strain parallel to x- and y-axes
η	Coefficient of expansion
ν	Poisson's ratio
σ	$=(\sigma_x + \sigma_y)/1 + \nu$
σ_x, σ_y	Normal components of stress parallel to x- and y-axes
τ_{xy}	The y-component of the shearing stress of a section of plate perpendicular to the x-axis
ϕ	Stress function

1. INTRODUCTION

The top grid of the PWR reactor is a gridwork designed to carry the mechanical loads and at the same time sustain thermal stresses. While the grid has equal square meshes made up of thin webs of equal cross sections in the central part of the grid, near the circular boundary of the grid, the webs were thickened and shape of the meshes was varied to fit the curved boundary. In addition to the difficulties from lack of uniformity and axisymmetry in distribution of mechanical loads, and structural geometry of the grid, the three-dimensional temperature variation over the grid further complicates the problem.

It would not be possible to expect that a solution to the stress problem of the grid exists in closed form. An approximate solution with numerical analysis, therefore, was attempted. The approximate method consists of treating the grid as a circular plate of equal strength and flexural rigidity. While the theory of plates subject to mechanical loads is well established, a general treatise on the thermal stresses of plates due to non-uniform heating seems to be wanting. Therefore, a theoretical formulation of the thermal stress problem is included in this report. This problem was broken into two parts: first, the bending of a plate with free boundary by the resultant moments of thermal stresses; and secondly, the expansion of the plate with temperature varying over the plate but constant across the thickness of the plate. The partial differential equations for bending, thermally or mechanically, and plane expansion of a plate are ultimately of fourth order. For the purpose of numerical analysis, each fourth order partial differential equation was replaced by two, second order partial differential equations.

The second order partial differential equations can then be replaced by difference equations for each nodal point which was taken as coincident with juncture of two perpendicular webs within the boundary, and juncture of a web and the boundary on the boundary. On account of the octant type of symmetry in structural geometry of the grid, and

thermal and mechanical loading, it is necessary only to take one-eighth of the grid into consideration. With due account taken of the change in span and thickness of the webs, and therefore, flexural rigidity adjacent to the boundary, 23 difference equations, corresponding to each second order difference equation, were thus obtained for the 23 nodal points within the boundary. With appropriate assumptions on boundary conditions, the difference equations can be solved.

The grid is supported by pins spaced along its boundary. This type of support approximates the simple support and it was therefore assumed that the grid is simply supported, i.e., w , σ_r , M_r and $M_{r\theta}$ vanish at boundary. It was further assumed that τ_{xy} and M_{xy} vanish at the boundary, since the torsional rigidity of the web is small compared to flexural rigidity.

The solutions for thermal and mechanical stresses were worked out separately and the results were superposed to obtain the total stresses. In doing so, it was tacitly assumed beforehand that the thermal expansion of the middle surface is small, and that change in curvatures of the grid as a result of thermal expansion of the middle plane of the grid is negligible.

The web sections are quite deep, as compared to the diameter of the grid, and the deflection due to shear might well be expected to be significant. This was also calculated in comparison to that due to bending by mechanical loading.

2. BENDING OF TOP GRID UNDER MECHANICAL LOADING

2.1 Method of Analysis

2.1.1 Bending Stress and Deflection

The differential equation governing the deflection of a plate loaded laterally is:

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} . \quad (1)$$

For numerical solution, Equation (1) is difficult to use, and it is more convenient to use the following equations:

$$\frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2} = -q , \quad (2)$$

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = -\frac{M}{D} , \quad (3)$$

where

$$M = \frac{M_x + M_y}{1+\nu} .$$

Equation (2) relates the moments throughout the plate and the load distribution. Equation (3) relates the deflection of the plate and moment.

To effect a numerical solution of these differential equations, they are replaced by difference equations. For square meshes of uniform flexural rigidity (Fig. 1), these become

$$M_l + M_r + M_a + M_b - 4M_p = -q \delta^2 , \quad (4)$$

and

$$w_l + w_r + w_a + w_b - 4w_p = -\frac{M_p}{D} \delta^2 , \quad (5)$$

where δ is the spacing of the nodal points.

The intersecting points of the webs of the gridwork are taken as nodal points. These points are shown in Figure 2. In Figure 2, point d is so close to the boundary that it is assumed to be actually on the boundary and the diagonal web, shown by dotted lines, which connects Station 23 to the boundary, is replaced by two perpendicular thin webs in the analysis.

Near the boundary of the grid we have unevenly spaced meshes and change of web section. The difference equations for nodal points have to be modified. As shown in Figure 3, the second derivative of a function, $f(x)$, at point p, by using parabolic interpolation, is given approximately by*

$$\frac{d^2 f}{dx^2} \approx \frac{1}{\delta^2} \frac{2}{a(1+a)} [af_l + f_r - (1+a)f_p], \quad (6)$$

where a is a constant. This equation can be written as

$$\frac{d^2 f}{dx^2} \approx \frac{2}{1+a} \frac{1}{\delta} \left[\frac{f_r - f_p}{a\delta} - \frac{f_p - f_l}{\delta} \right]. \quad (6a)$$

The bracketed term represents the difference in approximate slopes to the right and left of point p. In applying Equation (6a) to the differential equation for deflection, w , with segments of different web thickness, we can write

$$\left(D \frac{\partial^2 w}{\partial x^2} \right)_e \approx \frac{1}{\delta} \frac{2}{1+a} \left[\frac{D^e (w_r - w_p)}{a\delta} - \frac{D (w_p - w_l)}{\delta} \right],$$

where $\left(D \frac{\partial^2 w}{\partial x^2} \right)_e$ is the equivalent of the product of flexural rigidity

* See, for example, M. Salvadori: Numerical Methods in Engineering, pp 46-48, Prentice-Hall, Inc., N.Y., 1952.

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and the second partial derivative with respect to x , D' , the flexural rigidity of the web to the right, and D , the flexural rigidity of the web to the left of point p . Let

$$\frac{D'}{D} = \gamma.$$

Then

$$\left(D \frac{\partial^2 w}{\partial x^2}\right)_e \approx \frac{D}{\delta^2} \left[\frac{2\gamma}{\alpha(1+\alpha)} (w_r - w_p) - \frac{2}{1+\alpha} (w_p - w_l) \right]. \quad (7)$$

A similar expression can be obtained for $\left(D \frac{\partial^2 w}{\partial y^2}\right)_e$.

The meshes adjacent to the boundary fall into three categories. By adding the expressions for the product of flexural rigidity and second partial derivative with respect to x and y , the difference equations for the three configurations, as shown in Figure 4, are:

$$\text{Case I: } \frac{2\gamma}{\alpha(1+\alpha)} w_r + \frac{2}{1+\alpha} w_l + \gamma w_a + \gamma w_b - \left[\frac{2}{1+\alpha} \left(\frac{\gamma}{\alpha} + 1 \right) + 2\gamma \right] w_p = - \frac{M_p \delta^2}{D} \quad (8)$$

$$\text{Case II: } \gamma w_r + w_l + w_a + \gamma w_b - 2(1+\gamma) w_p = - \frac{M_p \delta^2}{D} \quad (9)$$

$$\begin{aligned} \text{Case III: } & \frac{2}{\alpha(1+\alpha)} w_r + \frac{2\gamma}{1+\alpha} w_l + \frac{2\gamma}{1+\beta} w_a + \frac{2}{\beta(1+\beta)} w_b \\ & - \left[\frac{2}{1+\alpha} \left(\frac{1}{\alpha} + \gamma \right) + \frac{2}{1+\beta} \left(\frac{1}{\beta} + \gamma \right) \right] w_p = - \frac{M_p \delta^2}{D} \end{aligned} \quad (10)$$

The coefficients for the corresponding w 's are illustrated in Figure 4. When $\alpha=\beta=\gamma=1$, Equations (8), (9) and (10) all reduce to Equation (5), the difference equation for square meshes with uniform web thickness.

To replace the Equation (2) by difference equations for unequally spaced meshes we note that M is independent of flexural rigidity. By putting $\gamma=1$ and replacing w 's by M 's on the left side and M_p/D by q_p on the right side of Equation (10), we have the difference equation for the configuration as shown in Figure 5a.

$$\text{Case I: } \frac{2}{a(1+a)} M_r + \frac{2}{1+a} M_\ell + \frac{2}{1+\beta} M_a + \frac{2}{\beta(1+\beta)} M_b - \frac{2(a+\beta)}{a\beta} M_p = -q_p \delta^2 \quad (11)$$

Let $\beta=1$, Equation (11) reduces to

$$\text{Case II: } \frac{2}{a(1+a)} M_r + \frac{2}{1+a} M_\ell + M_a + M_b - \frac{2(1+a)}{a} M_p = -q_p \delta^2 \quad (12)$$

Equation (12) represents the configuration of Figure 5b. With $a=1$, Equation (12) becomes Equation (4). The coefficients corresponding to the M 's are illustrated in Figure 5, which cover all the cases of interest for this analysis.

By applying Equations (4) to (12) to the 23 nodal points in one octant of the grid, a set of 23 equations in M 's and another set of 23 equations in w 's are obtained. With pertinent boundary conditions, these two sets of equations can be solved numerically.

It will be assumed that the grid is simply supported at the edge, i.e., $w=0$, and $M_r = M_{r\theta} = 0$ at the boundary. It should be noted that the latter boundary conditions cannot be conveniently applied to define M on the boundary. Since

$$M_r = M_x \cos^2 \theta + M_y \sin^2 \theta - 2 M_{xy} \sin \theta \cos \theta ,$$

and

$$M_{r\theta} = M_{xy} (\cos^2 \theta - \sin^2 \theta) + (M_x - M_y) \sin \theta \cos \theta ,$$

where θ is the angle between the positive x -axis and a radius, we take advantage of the fact that the torsional rigidity of the webs is small

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compared to flexural rigidity, consequently the twisting moment M_{xy} transmitted to boundary is of minor importance. By taking $M_{xy} = 0$ at the boundary, we conclude from the above equations that both M_x and M_y would vanish and hence $M=0$ at the boundary.

After the appropriate boundary conditions are incorporated into the difference equations, they can be solved with numerical method. To obtain M_x and M_y , the deflections at nodal points have to be used. Since

$$M_x = -D \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right), \quad (13)$$

the corresponding difference equation for square mesh of uniform web thickness is

$$M_{px} = \frac{D}{\delta^2} [2(1+\nu)w_p - w_\ell - w_r - \nu(w_a + w_b)]. \quad (14)$$

For the cases represented in Figure 4, M_x is obtained by using Equation (7), thus,

$$\text{Case I: } M_{px} = \frac{D}{\delta^2} \left\{ \left[\frac{2(\alpha+\gamma)}{\alpha(1+\alpha)} + 2\nu\gamma \right] w_p - \frac{2\gamma}{\alpha(1+\alpha)} w_r - \frac{2}{(1+\alpha)} w_\ell - \nu\gamma(w_a + w_b) \right\} \quad (15)$$

$$\text{Case II: } M_{px} = \frac{D}{\delta^2} [(1+\nu)(1+\gamma) w_p - \nu w_r - w_\ell - \nu w_a - \nu\gamma w_b] \quad (16)$$

$$\text{Case III: } M_{px} = \frac{D}{\delta^2} \left\{ \left[\left(\frac{1}{\alpha} + \gamma \right) \left(\frac{2}{1+\alpha} \right) + \left(\frac{1}{\beta} + \gamma \right) \left(\frac{2\nu}{1+\beta} \right) \right] w_p - \frac{2}{1+\alpha} \left(\frac{1}{\alpha} w_r + \gamma w_\ell \right) - \frac{2\nu}{1+\beta} \left(\gamma w_a + \frac{1}{\beta} w_b \right) \right\} \quad (17)$$

M_{py} is calculated from

$$M_{py} = (1+\nu) M - M_{px}. \quad (18)$$

The moments, M_{px} and M_{py} , are moments per unit length and must be multiplied by the web spacing to obtain total moment on each web.

The bending stresses are given by

$$\sigma_{px} = \frac{6M_{px}}{\gamma b h^2} \left(\frac{1+\beta}{2} \right) \delta ,$$

and

$$\sigma_{py} = \frac{6M_{py}}{\gamma b h^2} \left(\frac{1+\alpha}{2} \right) \delta ,$$
(19)

where b and h are the width and depth of the web respectively. For evenly spaced meshes of equal web thickness, $\alpha=\beta=\gamma=1$. For meshes near the boundary, corresponding values of α , β and γ should be substituted.

2.1.2 Deflection Due to Shear

The shear force per unit width, Q_x , of the equivalent plate is given by

$$Q_x = -D \frac{\partial}{\partial x} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) = D \frac{\partial}{\partial x} \left(\frac{M}{D} \right).$$

The slope in the x -direction due to shearing stress is

$$\frac{\partial w'}{\partial x} = \frac{k}{AG} (Q_x \delta) ,$$

where w' is the deflection due to shear and k is a constant, taken to be 1.5 for rectangular cross-sections. $Q_x \delta$ is the shear force in one web and δ is the web spacing in y -direction. Integrating with respect to x from nodal point, q , to nodal point, p , it becomes

$$w'_p - w'_q = \frac{k}{G} \int_q^p \frac{\delta D}{A} \frac{\partial}{\partial x} \left(\frac{M}{D} \right) dx$$

$$\approx \frac{k}{G} \sum_q^p \frac{\delta D}{A} \Delta \left(\frac{M}{D} \right) .$$

If we take the increment, $\Delta(M/D)$, as the increase of M/D from one end of the segment of one web to the other, D , then, is constant.

$$w_p^* - w_q^* \cong \frac{k}{G} \sum_q^p \left(\frac{M_{n+1}}{A_{n+1}} - \frac{M_n}{A_n} \right) \delta.$$

Here, A_n , at one end of one web, is equal to A_{n+1} at the other end of the web. Remembering we should use the same $\frac{M_n}{A_n} \delta$ when the nodal point is approached from either side along x-direction to insure the continuity of shearing stress distribution, then all the intermediate terms under the summation sign are cancelled. By taking the nodal point, q , at the boundary, then $w_q^* = M_q = 0$, and

$$w_p^* = \frac{k\delta}{G} \frac{M_p}{A_p}. \quad (20)$$

For points just inside the boundary, Equation (20) must be modified. The quantity, δM_p , is the moment in one web, and M_p is proportional to the sum of M_x and M_y . It is reasonable to replace δ by the average of the spans of the webs in x- and y-directions for nodal points with webs of unequal lengths. Thus, for Stations 7, 13 and 18, Equation (20) becomes

$$w_p^* = \frac{k\delta}{G} \left(\frac{1+\alpha}{2} \right) \frac{M_p}{A_p}. \quad (20a)$$

For Station 23,

$$w_p^* = \frac{k\delta}{2G} \left(\frac{1+\alpha}{2} + \frac{1+\beta}{2} \right) \frac{M_p}{A_p}. \quad (20b)$$

It is interesting to note from these equations that w_p^* is independent of the path taken to sum up the deflections from one nodal point to the next nodal point. This should be the case, since starting from a boundary point, the summation of the deflections of webs along

the x-direction up to point p should be the same as the summation of deflections of webs along the y-direction up to the same point. As the depth of the web is large compared to its thickness, the deflection due to shear may be considerable.

2.2 Numerical Calculations

The difference equations for all of the nodal points, except those near the boundary or near a change of cross-section, can be written by inspection using Equations (4) and (5). For Stations 7, 13 and 18 (Fig. 2), Case I (Fig. 4a) applies for the deflection equations, and Case II (Fig. 5b) applies for the moment equations. For Stations 21 and 22, Case II (Fig. 4b) applies for the deflection equations; since at these stations the meshes are equally spaced, and since the flexural rigidity does not enter into the moment equation, the moment equations for these two stations can be written by inspection, using Equation (4). For Station 23, Case III (Fig. 4c) applies for the deflection equation and Case I (Fig. 5a) applies for the moment equation.

The lengths of the webs near the boundary were calculated from Westinghouse Electric Corporation Dwg. No. 512-F-532, and are noted on Figure 2. The coefficients α and β are calculated by dividing the length of an unevenly spaced web by the length of the evenly spaced webs; for instance,

$$\beta_{23} = \frac{3.594}{6.000} = 0.599$$

The grid was assumed to be supported at a radius of 45 inches, in calculating these coefficients. In Figure 2, the coefficients α and β are shown at the web to which they pertain.

The term, q_n , in the moment equations, is the load intensity, in pounds per square inch. Therefore, the term of $q_n \delta^2$ is a unit load which may be considered concentrated at the nodal point for which the difference equation is written. The load on the webs varies in different

locations over the grid, as is shown in Figure 6. The load intensity for the various regions was calculated from data furnished by Westinghouse; the calculation is shown in detail in Table 1. The loads shown in Figure 6 are for the operating ("hot") condition. In calculating the load at each nodal point, it was assumed that one-quarter of the load of each of the four adjacent assemblies was exerted on that point. For example, at Station 15, the load would be

$$q_{15}\delta^2 = \left(\frac{1765}{4}\right) + 3\left(\frac{1530}{4}\right) = 1589 \text{ lbs.}$$

and, at Station 21, it would be

$$q_{21}\delta^2 = \left(\frac{1143}{4}\right) + 2\left(\frac{1765}{4}\right) = 1168 \text{ lbs.}$$

The loads at the nodal points are listed in Table 2.

In writing the deflection equation for Station 23, account must be taken of the variation in spacing and variation in flexural rigidity in both the x- and y-directions. The procedure for obtaining the difference equations at stations near the boundary is typified by the method used at Station 23.

Applying Case III (Eq. (10) and Fig. 4c) to mesh 23, using notation as in Figure 2,

$$\frac{2}{a(1+a)} w_e + \frac{2\gamma}{1+a} w_{22} + \frac{2\gamma}{1+\beta} w_{21} + \frac{2}{\beta(1+\beta)} w_f - \left[\frac{2}{1+a} \left(\frac{1}{a} + \gamma \right) + \frac{2}{1+\beta} \left(\frac{1}{\beta} + \gamma \right) \right] w_{23} = - \frac{M_{23}\delta^2}{D}.$$

Because Stations e and f are on the boundary, w_e and $w_f = 0$. Also, from the geometry of the grid at this point, $a = 0.5000$, $\beta = 0.5990$, and $\gamma = 1.8333$. Substituting these numerical values into the expressions for the coefficients:

$$\frac{2\gamma}{1+\alpha} = 2.443 ,$$

$$\frac{2\gamma}{1+\beta} = 2.2931 ,$$

and

$$\frac{2}{1+\alpha} \left(\frac{1}{\alpha} + \gamma \right) + \frac{2}{1+\beta} \left(\frac{1}{\beta} + \gamma \right) = 9.4921 .$$

The deflection difference equation at Station 23 is:

$$2.2931 w_{21} + 2.4443 w_{22} - 9.4921 w_{23} = - \frac{M_{23} \delta^2}{D} .$$

The difference equations for the moments in the grid are given in Table 3; for the deflections, in Table 4. The solutions for these two sets of equations are tabulated in Table 5; in Figure 7, the deflections are shown plotted vertically on an isometric of the grid. In computing the deflections, it is necessary to calculate the flexural rigidity (D) of the grid. By definition, for a solid plate,

$$D = \frac{Eh^3}{12(1-\nu^2)} .$$

To take into account the reduction in stiffness of the structure due to the void spaces, the term, D, was multiplied by the ratio of web thickness to web spacing b/δ :

$$D = \frac{b}{\delta} \frac{Eh^3}{1-\nu^2} .$$

This represents the flexural rigidity of the grid in the central part. At the boundary, where the webs are thickened, the flexural rigidity is increased. It is not necessary, however, to calculate another D for this region, since account has been taken of the increase in flexural rigidity in writing the difference equations at the points near the boundary.

* The term ν^2 was retained even though the webs of the grid are only partially restrained laterally. Dropping this term would increase the deflection by 9%.

The following numerical values were used in computing the flexural rigidity:

$$b = 0.2813 \text{ in.}$$

$$\delta = 6 \text{ in.}$$

$$E = 26 \times 10^6 \text{ lb/in}^2$$

$$h = 12 \text{ in.}$$

$$\nu = 0.3$$

Substituting these in the equation for D

$$D = 1.9268 \times 10^8 \text{ lb-in.}$$

It has been pointed out that the quantity, M, in Equations (2) and (3) is a sum of the moments in the x- and y-directions. To calculate the moments in the perpendicular directions, Equations (14) through (18) must be applied. The equations for the moment in the x-direction are given in Table 6. Poisson's ratio was taken as 0.3 in these equations. The moments in the y-direction were calculated by Equation (18). The stresses were calculated by use of Equations (19). In the stress equations, for central part of the grid, $\alpha=\beta=\gamma=1$, the width of the web, $b = 0.2813 \text{ in.}$ and the depth of the web, $h = 12 \text{ in.}$ Substituting the numerical values

$$\sigma_{px} = 0.8889 M_{px} ,$$

and a similar equation exists for σ_{py} . For points near the boundary, appropriate values of α , β and γ must be substituted in Equation (19). The moments and stresses are given in Table 7, and shown in Figure 8 and Figure 9, respectively, on a schematic of the grid. It will be noted that two values for stress are tabulated and illustrated at those meshes where a change of flexural rigidity exists in the direction under consideration.

The deflection due to shear is calculated from Equation (20).
The numerical values for this equation are as follows:

$$k = 1.5$$

$$\delta = 6 \text{ in.}$$

$$G = \frac{E}{2(1+\nu)} = \frac{26 \times 10^6}{2(1+0.3)} = 10 \times 10^6 \text{ lb/in}^2 ;$$

for flexible sections,

$$A_p = 0.2813 \times 12 = 3.375 \text{ in}^2 ;$$

for stiffened sections,

$$A_p = 0.5156 \times 12 = 6.187 \text{ in}^2 .$$

Substituting these numerical values into Equation (20),

$$w_p^f = 2.667 \times 10^{-7} M_p \text{ for the flexible sections;}$$

$$w_p^f = 1.455 \times 10^{-7} M_p \text{ for the stiffened sections.}$$

The deflections due to shear are tabulated in Table 5, along with the total deflections.

3. THERMAL STRESSES AND DEFLECTIONS OF TOP GRID

3.1 Theoretical Formulation - Thermal Stresses and Deflections of a Plate with Free Boundary

The following assumptions are made in addition to those usually associated with the theory of plates:

- (a) The stretch of the middle surface of the plate as a result of thermal expansion is small, to the extent that the effect of normal stresses on the curvatures and bending moments is negligible.

- (b) The thickness of the plate is small compared with other dimensions. As a result of this assumption, the stresses caused by the expansion in the direction perpendicular to the plane of the plate are negligible.
- (c) Lines on any vertical cross-section that are originally normal to the middle surface of the plate before bending, remain straight and normal to the middle surface.
- (d) Any mechanical load distributed on the surface of the plate has a negligible effect on the expansion of the plate in its own plane.

While the second and last assumptions eliminate the necessity of taking into consideration the thermal expansion in the third dimension, the last imposes a more severe restriction that is usually not encountered in practical cases unless the intensity of load is small. The third assumption is approximately true for points away from the boundary. At the free boundary, points of higher temperature expand more in the direction normal to the boundary and the boundary surface is thus distorted.

Consider an infinitesimal rectangular block cut out of a plate by planes perpendicular to x - and y -axes, and take the middle plane of the plate as xy -plane, as shown in Figure 10. If this block is to expand freely, then

$$\epsilon_x = \epsilon_y = \eta T,$$

where $T = T(x, y, z)$, and η is the coefficient of expansion.

In order to suppress these strains, stresses,

$$\sigma_x^* = \sigma_y^* = - \frac{E\eta T}{1-\nu}, \quad (21)$$

must be applied on the side surfaces of the block. The application of these stresses is equivalent to applying a uniform stress,

$$\sigma_a = - \frac{\eta E t}{1-\nu} , \quad (22)$$

where

$$t = \frac{1}{h} \int_{-\frac{h}{2}}^{\frac{h}{2}} T dz ,$$

and a bending moment,

$$m = - \frac{\eta E}{1-\nu} \int_{-\frac{h}{2}}^{\frac{h}{2}} T z dz . \quad (23)$$

At the boundary, surface forces must be applied to give stresses, σ_x^i and σ_y^i , determined by the temperature at the boundary. If, at every point of the plate, the thermal strains are suppressed by applying stresses defined by Equation (21), then the plate would remain flat and retain its original shape and size.

If bending and twisting moments, defined by

$$\left. \begin{aligned} \sigma_x'' &= \frac{12 M_x z}{h^3} , \\ \sigma_y'' &= \frac{12 M_y z}{h^3} , \\ \tau_{xy}'' &= - \frac{12 M_{xy} z}{h^3} , \end{aligned} \right\} \quad (24)$$

and

are applied on the side surfaces of the elementary block, the side surfaces would assume the shape of a trapezoid and the plate would be bent. The resultant moments on the surface become $M_x^i = M_x + m$, $M_y^i = M_y + m$, and M_{xy}^i . These moments must satisfy the equation of equilibrium,

$$\frac{\partial^2 M_x^i}{\partial x^2} + \frac{\partial^2 M_y^i}{\partial y^2} - 2 \frac{\partial^2 M_{xy}^i}{\partial x \partial y} = 0 .$$

After substitution, this becomes

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_y}{\partial y^2} - 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} = -\left(\frac{\partial^2 m}{\partial x^2} + \frac{\partial^2 m}{\partial y^2}\right) . \quad (25)$$

As m produces no curvature, the plate is bent by M_x , M_y and

M_{xy} :

$$\left. \begin{aligned} M_x &= -D \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) , \\ M_y &= -D \left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right) , \\ M_{xy} &= D(1-\nu) \frac{\partial^2 w}{\partial x \partial y} ; \end{aligned} \right\} \quad (26)$$

and

from which we have

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = -\frac{M}{D} , \quad (27)$$

where

$$M = \frac{M_x + M_y}{1+\nu} .$$

After substituting from Equations (26) and (27), Equation (25) becomes

$$\frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2} = -\left(\frac{\partial^2 m}{\partial x^2} + \frac{\partial^2 m}{\partial y^2}\right) . \quad (28)$$

By comparing Equations (27) and (28) with differential equations for bending of a plate due to mechanical load, we see that

$$\frac{\partial^2 m}{\partial x^2} + \frac{\partial^2 m}{\partial y^2}$$

is equivalent to load intensity, q .

Following the procedure outlined above, we are left with $M_x + m$ and $M_y + m$ at the boundary, where m is determined by the temperature variation at the boundary. M_x and M_y can be adjusted so that the boundary conditions are satisfied. To be consistent with the assumptions made in the analysis for mechanical loading, we assume here

$$M_x + m = 0 ,$$

and

$$M_y + m = 0$$

at the boundary. Then,

$$M = - \frac{2m}{1+\nu} . \quad (29)$$

In addition to this, we have

$$w = 0 . \quad (30)$$

From the above analysis, the solution of the problem of bending due to non-uniform heating consists of solving Equations (27) and (28) with the boundary conditions given by Equations (29) and (30). The bending moments and bending stresses are then obtained from Equations (26) and (24).

In addition to bending, the plate is further stressed as a result of non-uniform expansion in its own plane. Based on the assumptions made, we can treat the plate as under plane stress, and t , defined by Equation (22), as a potential function.

Let

$$\left. \begin{aligned} \sigma_x - V &= \frac{\partial^2 \phi}{\partial y^2} , \\ \sigma_y - V &= \frac{\partial^2 \phi}{\partial x^2} , \\ \tau_{xy} &= - \frac{\partial^2 \phi}{\partial x \partial y} , \end{aligned} \right\} \quad (31)$$

and

where V is a potential function, and ϕ is the Airy stress function.
Adding the first two equations of Equations (31) yields:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = (1+\nu) \sigma - 2V, \quad (32)$$

where

$$\sigma = \frac{\sigma_x + \sigma_y}{1+\nu}.$$

Therefore,

$$(1+\nu) \left(\frac{\partial^2 \sigma}{\partial x^2} + \frac{\partial^2 \sigma}{\partial y^2} \right) - 2 \left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right).$$

Since the compatibility equation for plane stress problems is*

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) = -(1-\nu) \left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right),$$

we have

$$\frac{\partial^2 \sigma}{\partial x^2} + \frac{\partial^2 \sigma}{\partial y^2} = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}. \quad (33)$$

For the problems of thermal stresses,

$$V = \frac{\eta E t}{1-\nu},$$

where t is defined by Equations (22) in our case. Equations (32) and (33) then become

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = (1+\nu) \sigma - \frac{2\eta E t}{1-\nu}, \quad (34)$$

and

$$\frac{\partial^2 \sigma}{\partial x^2} + \frac{\partial^2 \sigma}{\partial y^2} = \frac{\eta E}{1-\nu} \left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} \right). \quad (35)$$

* S. Timoshenko and J.N. Goodier, Theory of Elasticity, 2nd ed., N.Y.: McGraw-Hill Book Company, 1951, pp 23-27.

The boundary conditions that solutions of Equations (34) and (35) should satisfy are:

$$\sigma_x \cos\theta + \tau_{xy} \sin\theta = \frac{\eta Et}{1-\nu} \cos\theta \quad (36)$$

and

$$\sigma_y \sin\theta + \tau_{xy} \cos\theta = \frac{\eta Et}{1-\nu} \sin\theta$$

where θ is the angle between the normal to the boundary and the x-axis (Fig. 11). In terms of stress function, ϕ , Equations (36) are:

$$\frac{\partial^2 \phi}{\partial y^2} \cos\theta - \frac{\partial^2 \phi}{\partial x \partial y} \sin\theta = 0 ,$$

and

$$\frac{\partial^2 \phi}{\partial x^2} \sin\theta - \frac{\partial^2 \phi}{\partial x \partial y} \cos\theta = 0 .$$

Since $\cos\theta = \frac{dy}{ds}$ and $\sin\theta = -\frac{dx}{ds}$, these equations can be written as

$$\frac{d}{ds} \left(\frac{\partial \phi}{\partial y} \right) = 0 ,$$

and

$$\frac{d}{ds} \left(\frac{\partial \phi}{\partial x} \right) = 0 .$$

Thus, $\partial\phi/\partial x$ and $\partial\phi/\partial y$ must be constant along the boundary. Moreover, ϕ must be of the form $Ax + By + C$, where A , B and C are constants. Since the stress components depend only on the second derivatives of ϕ , we can take

$$\phi = 0 \quad (37)$$

on the boundary.

We assume $\tau_{xy} = 0$ at the boundary, in applying the theory of plate to the gridwork, since the webs of the grid are relatively weak in transmitting shear to the boundary. Then, Equations (36) reduce to

$$\sigma_x = \sigma_y = \frac{\eta E t}{1-\nu},$$

and

$$\sigma = \frac{2\eta E t}{1-\nu^2} \quad (38)$$

The task of investigation of the stresses of a plate due to thermal expansion in its own plane is to solve Equations (34) and (35) together with the boundary conditions, Equations (37) and (38). The stress components can then be evaluated from Equations (31) with stress function, ϕ , obtained. These stresses should be superposed on the bending stresses and the compressive stresses given in Equation (21) to obtain total thermal stresses.

It should be observed, after the superposition of stresses, the surface force and bending moment are zero at the boundary, but stresses at any point on the boundary surface do not necessarily vanish.

3.2 Temperature Variation

Because of the heat generated at the seed elements and local heating at the junctions of the webs, the temperature variation over the top grid of the core assembly of the PWR reactor presents a rather complex picture. For the purpose of analyzing the thermal stresses, the temperature variation was expressed approximately by analytical expressions defining the peak temperatures at the junctions of the webs relative to the temperature at the centers of the meshes, where the seed elements are located, and the temperature at any point of the web relative to the temperature at the junction of the web.

The temperature variation over one-half of a web of the grid is shown in Figure 12a. The curves, representing the temperature variation at three different locations of the web, are shown in the same plane in Figure 12b. At a section, at a distance more than "a" from the intersection of two perpendicular webs, the temperature variation is represented by curve C_1 and

$$T = T_m(x,y) f(z') ,$$

where

$$f(z') \begin{cases} = 1.32 e^{-0.266 z'} + 0.198 z'^3 - 0.575 z'^2 \\ \quad + 0.757 z' - 0.420 & \text{for } 0 \leq z' \leq 1.189 \\ = 1.32 e^{-0.266 z'} & \text{for } 1.189 \leq z' \leq 3.271 \\ = 1.05 e^{-0.196 z'} & \text{for } 3.271 \leq z' \leq 12 \end{cases} \quad (39)$$

and z' is measured upwards from the bottom of the web. At a distance less than "a" from the intersection, the temperature is given by

$$T_i = [T_m(x,y) + \tau_m(x,y)] f(z') = T_n(x,y) f(z') ,$$

and

$$\tau_i = \tau_m(x,y) f(z') .$$

Along a vertical line in this region, T_i is represented by curve C_2 . Curve C_3 represents T_i at the line of intersection of two perpendicular webs. The subscript, n , is used to refer to the numbers assigned to the nodal points,

$$T_n = T_m(x,y) + \tau_n(x,y) ,$$

where T_n and τ_n are the maxima of T_i and τ_i respectively. τ_n and τ_i are related by

$$\tau_i = \tau_n \left(1 - \frac{x'}{a}\right) \text{ or } \tau_n \left(1 - \frac{y'}{a}\right) ,$$

where x' and y' are measured from the intersection of the webs, and are used to distinguish the coordinates of τ_i from the coordinates, x and y , measured from center of grid as the origin.

It is advantageous to have the calculations based on temperatures at the intersection of the webs. We express T_m in terms of T_n :

$$\tau_n = \left(\frac{45-32}{32}\right) T_m = 0.40625 T_m, \quad (40)$$

and

$$T_n = T_m + \tau_n,$$

Therefore,

$$T_m = 0.71111 T_n. \quad (41)$$

T_n is defined by

$$T_n = 15(2 - \cos \frac{\pi r}{r_s}), \quad (42)$$

where

$$r_s = \frac{19.6068 \cos \theta + 5.5472 \sqrt{2.50118 \cos^2 \theta - 1}}{\cos^2 \theta + 0.100081} \quad (43)$$

The term, r_s , is the radial distance from the origin (the center of the grid) to the point of maximum temperature. Originally, this point was assumed to lie on the centerline of the seed elements. From Figure 13, it can be seen that this is not a smooth curve. Physically, the maximum temperature would be expected to lie on a smooth curve. Also, if the maximum temperature does not lie on a smooth curve, the derivatives of temperature distribution function are discontinuous at certain points. Therefore, the point of maximum temperature was assumed to lie on four, tangential, elliptical arcs as shown in Figure 13, and the equation for r_s was formulated accordingly. The radial distance from the origin to the nodal point under consideration, along the radial line r_s , is r . The angle between the radial line r_s and the x-axis is θ . Equations (39) to (43) completely define the temperature at any point in the grid.

The temperatures (T_n) at the nodal points were calculated using Equation (42) and are tabulated in Table 8. To account for the temperature increase at the junctions of the webs, note that the average temperature increase over the distance "a" is $\tau_n/2$. Then, the total expansion of a horizontal strip of the web of length δ , at maximum temperature, is

$$\eta T_m \frac{\delta}{2} + \eta \tau_n \frac{a}{2} = \eta T_e \frac{\delta}{2} .$$

Therefore, define an equivalent temperature, T_e , such that

$$T_e = T_m + \frac{a}{\delta} \tau_n^* . \quad (44)$$

The equivalent temperature of any other horizontal strip, then, is $T_e f(z')$.

To express the equivalent temperature, T_e , as a fraction of the maximum temperature, substitute Equations (40) and (41) into Equation (44). Then, with $a = 0.500$ in. and $\delta = 6.00$ in.,

$$T_e = 0.73518 T_n . \quad (45)$$

These equivalent temperatures are also tabulated in Table 8.

3.3 Bending of the Top Grid Due to Non-Uniform Heating

The bending moment, m , is defined by Equation (23). In applying this equation to the grid problem, it is obviously more realistic to consider the expansion of the web in the plane of the grid as one-dimensional and the quantity, $1-v$, should be replaced by unity. Observing

* Equation (44) should be modified for points just inside and on the boundary to account for the change of web span. Since $a\tau_n/\delta$ is small compared to T_m , the effect of this modification is negligible. Equation (44) is used for all nodal points.

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that m is the moment per unit width of the equivalent plate, and thickness of web, b , corresponds to a width, δ , of the plate, we have

$$m = -E\eta \frac{b}{\delta} \int_{-\frac{h}{2}}^{\frac{h}{2}} T_e f(z) z dz, \quad (46)$$

where z is measured downward from the middle plane of the grid. Note that in Equation (39), which represents the data originally supplied for temperature variation across the depth of the web, z' is measured upward from the bottom of the web. Figure (14) is a graphical representation of $f(z)$ having the origin, O , at the mid-line of the web and O' at the bottom. The coordinate, z , is measured downward from O . The change is necessary to maintain consistency in coordinates with those commonly used in theory of plates.

After substituting appropriate values, we have

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} f(z) z dz = 15.2713.$$

Note that T_e is a function of x and y only. Let

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} T_e f(z) z dz = n; \quad (47)$$

then

$$n = 15.2713 T_e. \quad (48)$$

Values of n for each nodal point are tabulated in Table 8.

Equation (46) may be written as:

$$m = -E\eta \frac{b}{\delta} n. \quad (49)$$

Then, from Equation (28)

$$\frac{\partial^2 m}{\partial x^2} + \frac{\partial^2 m}{\partial y^2} = -E\eta \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left(\frac{b}{\delta} n \right) . \quad (50)$$

In replacing this equation by difference equations, the variation of web thickness and spacing of nodal points must be taken into account.

The difference equations for the Laplacian can be written by the procedures developed in the analysis of the grid under mechanical loading (See Section 2.1). For square meshes of uniform flexural rigidity (b and δ constant),

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left(\frac{b}{\delta} n \right) \approx \frac{b}{\delta} \cdot \frac{1}{\delta^2} (n_a + n_b + n_\ell + n_r - 4n_p) , \quad (51)$$

where the subscripts indicate the relative positions of nodal points as shown in Figure 1. The nodal points with non-uniform spacing and web thickness fall into three cases, as shown in Figure 15. For Case I,

$$\frac{\partial^2}{\partial x^2} \left(\frac{b}{\delta} n \right) \approx \frac{1}{(\delta)_x} \cdot \frac{2}{1+a} \left[\frac{n_r - n_p}{(a\delta)_x} \left(\frac{b'}{\delta} \right)_y - \frac{n_p - n_\ell}{(\delta)_x} \left(\frac{b}{\delta} \right)_y \right] .$$

The subscripts, x and y , are used to indicate that the dimensions within the parentheses are measured in the x - and y -directions respectively. Let

$$b' = \gamma b .$$

Then, (a),

$$\frac{\partial^2}{\partial x^2} \left(\frac{b}{\delta} n \right) \approx \frac{b}{\delta^3} \left[\frac{2}{1+a} n_\ell + \frac{2\gamma}{a(1+a)} n_r - \frac{2}{1+a} \left(1 + \frac{\gamma}{a} \right) n_p \right] .$$

Similarly, (b),

$$\begin{aligned} \frac{\partial^2}{\partial y^2} \left(\frac{b}{\delta} n \right) &\approx \frac{1}{(\delta)_y} \left[\frac{n_a - n_p}{(\delta)_y} \cdot \frac{b'}{\left\{ \frac{1}{2}(1+\alpha)\delta \right\}_x} - \frac{n_p - n_b}{(\delta)_y} \cdot \frac{b'}{\left\{ \frac{1}{2}(1+\alpha)\delta \right\}_x} \right] \\ &= \frac{b}{\delta^3} \left[\frac{2\gamma}{1+\alpha} n_a + \frac{2\gamma}{1+\alpha} n_b - \frac{2}{1+\alpha} (2\gamma) n_p \right] \end{aligned}$$

Adding the above equations, (a) and (b), yields, for Case I:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left(\frac{b}{\delta} n \right) \approx \frac{b}{\delta^3} \left[\frac{2\gamma}{1+\alpha} n_a + \frac{2}{1+\alpha} n_\ell + \frac{2\gamma}{\alpha(1+\alpha)} n_r + \frac{2\gamma}{1+\alpha} n_b - \frac{2}{1+\alpha} \left(\frac{\gamma}{\alpha} + 2\gamma + 1 \right) n_p \right]. \quad (52)$$

Similarly, for Case II:

$$\frac{\partial^2}{\partial x^2} \left(\frac{b}{\delta} n \right) \approx \frac{b}{\delta^3} \left[n_a + n_\ell + \gamma n_r + \gamma n_b - 2(1+\gamma) n_p \right]. \quad (53)$$

For Case III:

$$\begin{aligned} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left(\frac{b}{\delta} n \right) &\approx \frac{b}{\delta^3} \cdot \frac{4}{(1+\alpha)(1+\beta)} \left[\gamma n_a + \gamma n_\ell + \frac{1}{\alpha} n_r + \frac{1}{\beta} n_b \right. \\ &\quad \left. - \left(\frac{1}{\alpha} + \frac{1}{\beta} + 2\gamma \right) n_p \right]. \quad (54) \end{aligned}$$

The "operating molecules" for obtaining the Laplacian of $\frac{b}{\delta} n$ are shown in Figure 15. Define the quantity,

$$\Delta^2 n = A n_a + I n_\ell + B n_b + R n_r - P n_p,$$

where the constants depend on the web spacing and thickness. Then, by Equation (50),

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left(\frac{b}{\delta} n \right) \approx -E\eta \frac{b}{\delta^3} \Delta^2 n.$$

The quantity, $\Delta^2 n$, is also tabulated in Table 8; since $\nabla^2 m$ is analogous to a mechanical load, $\Delta^2 n$ is proportional to this fictitious load.

The difference equations for the bending moment can be written in the same manner as those for the bending moments for mechanical loading of the plate. This is explained in detail in Section 2.1. For equally spaced meshes,

$$\frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2} \approx \frac{1}{\delta^2} (M_a + M_\ell + M_b + M_r - 4M_p) . \quad (55)$$

Using Equations (51) and (55), Equation (30) can be written as

$$M_a + M_\ell + M_r + M_b - 4M_p = E\eta \frac{b}{\delta} (n_a + n_\ell + n_r + n_b - 4n_p) . \quad (56)$$

The derivation of difference equations for points near the boundary is somewhat more complicated. According to Equations (29) and (49),

$$M = E\eta \frac{b}{\delta} \cdot \frac{2}{1+\nu} n \quad (57)$$

on the boundary. As an example, the difference equation for the moment at Station 7 will be derived. From the "operating molecules", presented in Figure 5, for Station 7,

$$\frac{2}{1+\alpha} M_6 + \frac{2}{\alpha(1+\alpha)} M_a + M_{13} - \frac{2(1+\alpha)}{\alpha} M_7 = E\eta \frac{b}{\delta} \Delta^2 n_7 .$$

For Station 7, $\alpha = 0.9833$,

$$1.008 M_6 + 1.025 M_a + M_{13} - 3.034 M_7 = E\eta \frac{b}{\delta} \Delta^2 n_7 .$$

From Table 1,

$$\Delta^2 n_7 = -51.350 ,$$

$$n_a = 181.807 .$$

Also, from Equation (57),

$$M_a = E\eta \frac{b}{\delta} \cdot \frac{2}{1+\nu} n_a ,$$

$$1.008 M_6 + M_{13} - 3.034 M_7 = E\eta \frac{b}{\delta} (\Delta^2 n_7 - 1.025 \frac{2}{1+\nu} n_a) .$$

Using $\nu = 0.3$,

$$\Delta^2 n_7 - 1.025 \frac{2}{1+\nu} n_a = -338.053 .$$

The difference equation for moments at Station 7 is

$$1.008 M_6 + M_{13} - 3.034 M_7 = -338.053 E\eta \frac{b}{\delta} .$$

The writing of the difference equations for the other stations near the boundary is accomplished in a similar manner. These equations appear in Table 9.

For the problem of thermal stress, the difference equations for deflections are written in the same manner as those for the problem of mechanical loading. The coefficients are exactly the same as those given in Section 2.1. These equations are given in Table 4.

The solutions of the moment equations and the deflection equations are tabulated in Table 10. In the calculations, the following numerical values were used:

(1), For 340 stainless steel,

$$E = 26 \times 10^6 \text{ lb/in.}^2$$

$$\eta = 10 \times 10^{-6} \text{ in./in.}^\circ\text{F}$$

$$\nu = 0.3$$

$$\begin{aligned}
 (2), \quad & b = 0.28125 \text{ in.} \\
 & \delta = 6.00 \text{ in.} \\
 & h = 12 \text{ in.} \\
 & E\eta = 260 \text{ lb/in.}^2 \cdot \text{F} \\
 & \frac{b}{\delta} = 0.046875 \\
 & E\eta \frac{b}{\delta} = 12.1745 \\
 & D = 1.9268 \times 10^8 \text{ lb-in.}
 \end{aligned}$$

Difference equations, corresponding to Equations (26), are used to calculate the moments in the x- and y-directions. These equations are given in Table 11. The moments in the perpendicular directions, M_x and M_y , are listed in Table 12.

The bending stresses at any nodal point are given by Equation (19). The bending stresses, σ_x'' and σ_y'' , in the perpendicular directions, are also listed in Table 12.

3.4 Plane Expansion of Top Grid

It remains to determine the stresses produced by the non-uniform expansion of the grid in its own plane. As was done in the case of the bending of the grid, in Equation (22), T is replaced by $T_e f(z)$, T_e being a function only of x and y . Then,

$$t = T_e \frac{1}{h} \int_{-\frac{h}{2}}^{\frac{h}{2}} f(z) dz .$$

The integral can be evaluated by using Equations (39), due notice being taken of the difference of the origins, O and O' , (Fig. 14).

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} f(z) dz = 5.00771 ,$$

and, with $h = 12$,

$$t = 0.41730 T_e .$$

Values of t are tabulated for each nodal point and points on the boundary in Table 13.

The finite differences for the Laplacian of t can be written by use of the methods discussed previously. Note that t is not dependent on thickness of cross-section, so that the quantity γ will not enter into the difference equations. For equally spaced meshes,

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} \approx \frac{1}{\delta^2} (t_a + t_l + t_r + t_b - 4t_p) .$$

For unequally spaced meshes, the "operating molecules" of Figure 5 of Section 2.1 can be used. The numerical values for $\delta^2 \nabla^2 t$ are given in Table 13.

The writing of the difference equations, corresponding to Equation (35), is similar to the procedure for writing the moment equations for the bending problem. Note that the $(1-\nu)$ term is replaced by unity for the same reason as in the bending problem. Since σ is not a function of the web thickness, the coefficients for these difference equations will be the same as those for the moment equations.

Equation (38) can be written as

$$\sigma = \frac{E \eta}{1-\nu} \left(\frac{2}{1+\nu} \right) t .$$

Replacing $(1-\nu)$ by unity, as before, and using $(1+\nu) = 1.30$, yields

$$\sigma = 1.5385 E \eta t$$

on the boundary. The derivation of the difference equation for σ , for points near the boundary, is completely analogous to the derivation of

the moment equations. This was done in detail for Station 7 in the section on the bending of the grid due to non-uniform heating (Section 3.3), and will not be repeated here. The difference equations, corresponding to Equation (35), are listed in Table 14.

The differential equation for the Airy stress function, ϕ , is given by Equation (34). The stress function, ϕ , must be modified to take into account the change in cross-sectional area of the webs. The function, ϕ , has the dimensions of a force. At any nodal point where the web thickness changes, in either of the perpendicular directions, the forces in the webs must vary continuously, regardless of their cross-sectional areas. Define another function, ψ , such that

$$\phi = A\psi .$$

Then, after replacing the quantity, $(1-\nu)$, by unity, as before, Equation (34) becomes

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(A\psi) = (1+\nu)\sigma - 2E\eta t . \quad (58)$$

The derivatives of ϕ will be replaced by finite differences of $A\psi$, to account for the change in cross-sectional areas. It should be noted, that the quantity, $A\psi$, is exactly analogous to the quantity, Dw , in the problem of bending of the grid under thermal or mechanical loading. Therefore, the same coefficients will appear on the left-hand side of the respective difference equations. The right-hand side of each equation contains the quantities, σ and $2t$, at the nodal point under consideration. The quantity, $2t$, is given in Table 13. The difference equations, corresponding to Equation (58) are given in Table 15.

The solutions of the difference equations for σ and ψ are given in Table 16. The quantity $A\psi/E\eta\delta^2$, is tabulated instead of ψ , as a matter of convenience. The stresses in the perpendicular directions are determined by Equations (31):

$$\sigma_y''' = \frac{\partial^2 \phi}{\partial x^2} + E\eta t, \quad (59)$$

$$\sigma = \frac{\sigma_x''' + \sigma_y'''}{1+\nu},$$

and
$$\sigma_x''' = (1+\nu)\sigma - \sigma_y'''. \quad (60)$$

Remembering that $\phi = A\Psi$, the differentiation in Equations (59) can be carried out numerically. For stations of equal spacing and equal web cross-section,

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{\partial^2}{\partial x^2} (\Psi A) \approx \frac{A}{\delta^2} (\Psi_r + \Psi_\ell - 2\Psi_p).$$

Since only differentiation with respect to x is necessary to obtain σ_y'' , and σ_x''' is calculated from Equation (60), there are but two cases of unequal spacing and cross-sectional web area. These are shown in Figure 16, in which the coefficients for the finite differences of $\frac{\partial^2}{\partial x^2} (A\Psi)$ are derived in a similar manner to that for Equation (7) of Section 2.1. For Case I,

$$\frac{\partial^2}{\partial x^2} (A\Psi) \approx \frac{A}{\delta^2} \left[\frac{2}{1+\alpha} \Psi_\ell + \frac{2\gamma}{\alpha(1+\alpha)} \Psi_r - \frac{2}{1+\alpha} \left(1 + \frac{\gamma}{\alpha} \right) \Psi_p \right],$$

and for Case II,

$$\frac{\partial^2}{\partial x^2} (A\Psi) \approx \frac{A}{\delta^2} \left[\frac{2\gamma}{1+\alpha} \Psi_\ell + \frac{2}{\alpha(1+\alpha)} \Psi_r - \frac{2}{1+\alpha} \left(\frac{1}{\alpha} + \gamma \right) \Psi_p \right].$$

Case I is applicable to Stations 7, 13, 18, 21 and 22; Case II is applicable to Station 23. The finite differences for $\frac{\partial^2}{\partial x^2} (\Psi A)$ are given in Table 17. In Table 18, the numerical values of

$$\frac{1}{E\eta} \frac{\partial^2}{\partial x^2} (\Psi A)$$

are tabulated, along with the calculated values for σ_x''' and σ_y''' .

3.5 Resultant Thermal Stresses and Deflections

There are three stress components that make up the normal stress in x- or y-direction at any point in the grid:

(a) The compressive stress applied to suppress the thermal strain. This is given by Equation (21). Neglecting the lateral restraint by replacing the $(1-\nu)$ term in this equation with unity, we get

$$\sigma_x^t = \sigma_y^t = -E\eta T$$

The distribution of this stress across the web is proportional to $f(z)$, as shown in Figure 14. As a compressive stress, $E\eta T_e f(z^t)$, was actually applied, the equivalent temperature, T_e , will be used in calculating these stresses for consistency. Therefore,

$$\sigma_x^t = \sigma_y^t = -E\eta T_e f(z^t)$$

On the top of the grid ($z^t = 12$), $f(z^t) = 0.1002$; and on the bottom of the grid ($z^t = 0$), $f(z^t) = 0.900$. Therefore, on the top of the grid,

$$\sigma_{px}^t = \sigma_{py}^t = -0.1002 E\eta T_e$$

and, on the bottom of the grid,

$$\sigma_{px}^t = \sigma_{py}^t = -0.900 E\eta T_e$$

These stresses are listed in Table 19 for each nodal point at the top and bottom of the grid.

(b) There are the maximum bending stresses as given by Equations (24) and tabulated in Table 12. These stresses are linearly distributed across the grid and are maximum in compression on the top of the grid, and in tension at the bottom.

(c) There are the plane stress components, σ_{px}''' and σ_{py}''' , as given in Table 18. These stresses are constant across the depth of the grid.

The results of the superposition of all three stresses are given in Table 19. It will be noted that the maximum stresses, both tensile and compressive, occur at Station 21. A plot of the stress distribution at this nodal point is shown in Figure 17. The maximum compressive stress at this point does not occur in the bottom fiber, but at a distance one-half inch up from it. Unlike the case of mechanical loading, the highly stressed points are far away from the center of the grid, as would be expected from a configuration defined by the seed element locations. The resultant thermal stresses in top and bottom fibers of the webs are shown in Figure 18.

The deflections due to non-uniform expansion of the grid are caused by bending moments only, and no superposition is necessary as far as the thermal stress is concerned. These deflections (see Table 10) are shown isometrically in Figure 19.

4. SUPERPOSITION OF STRESSES AND DEFLECTIONS

To obtain the total stresses, the bending stresses due to mechanical loads (see Table 7), and thermal stresses (see Table 19), are superposed. It should be noted that the mechanical load causes upward deflection of the grid, and, hence, tensile stress in the upper fibers and compressive stress in the lower fibers. In the process of superposition, due attention must be paid to the signs on the stresses. In Table 20, the results of the superposition are tabulated. In Figure 20, the total stresses at each nodal point are shown on a schematic grid. By comparing this figure with Figure 9, it is seen that the magnitude of the stresses near the center of the grid due to mechanical loads are not materially changed by non-uniform heating of the grid. The increase in compressive stresses due to non-uniform heating near the meshes where the seed elements are located are considerable, but here the bending stresses due to mechanical loads are well below the maximum. Thus, the combined action of mechanical loading and thermal expansion

results in maximum tensile and compressive stresses at points near the center of the grid.

The results of superposition of deflections due to bending by mechanical and thermal expansion appear in Table 21. The mechanical load causes an upward deflection of the grid. The temperature gradient in the z-direction is such that the bottom of the grid is at a higher temperature than the top. Under the thermal load only, the grid will tend to deflect downward. Therefore, the mechanical deflection and the thermal deflection are subtractive. The data in Table 21 were used to plot the deflection surface of the grid in Figure 21.

5. CONCLUSIONS

The partial differential equation governing the bending or plane stress problem of a plate is ultimately of fourth order. Equation (1) defines the deflection of a plate subject to lateral load. For bending by thermal expansion, Equations (27) and (28) represent one equation:

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D} \left(\frac{\partial^2 m}{\partial x^2} + \frac{\partial^2 m}{\partial y^2} \right).$$

For the plane stress problem, we can combine Equations (34) and (35) to give:

$$\frac{\partial^4 \phi}{\partial x^4} + 2 \frac{\partial^4 \phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \phi}{\partial y^4} = -\eta E \left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} \right).$$

In analyzing the stresses of the grid with the method of finite differences, the legitimate procedure would be to replace the fourth order partial differential equations by difference equations. However, in doing so, we would find it difficult to obtain the difference equations for nodal points just inside the curved boundary, since it is usually necessary to use fictitious nodal points outside the boundary, and it is

generally not possible to calculate the values of the variable in the difference equations for these fictitious points with the given boundary conditions of a curved boundary. By using two, second order partial differential equations instead of one of the fourth order, this difficulty is circumvented and the corresponding difference equations, with appropriate boundary conditions, can be readily solved by relaxation or iteration method.

The advantage of taking the intersection of the webs as the nodal points is obvious, and spacing of these points is thus prescribed by the structural geometry of the grid. For large mesh size, the departure of the grid from the equivalent plate becomes evident, and the numerical solution may not give a good approximation to that of the actual problem.

In view of the fact that the width is small compared to the depth of the web, the shearing force in the plane of the grid, and twisting moment transmitted to the boundary must be small compared to other force and moment components. The actual bending moment at the edge of the grid cannot be conveniently determined because of the manner of support, i.e., the grid is supported at the edge by 28 shear pins. By assuming the grid to be simply supported, and neglecting M_{xy} at the boundary in the bending problem, and τ_{xy} at the boundary in the plane stress problem, the boundary conditions can be easily incorporated into the difference equations. This is equivalent to assuming that the boundary is a polygon that has as many sides as the number of webs running into the boundary*.

For the equivalent plate, which has the same flexural rigidity as the grid, the effect of shearing stress is probably negligible, since the thickness of the plate is considerably smaller than the depth of the grid web. For the grid itself, it is seen that the contribution of

* See, for example, S. Timoshenko, Theory of Plates and Shells, N.Y.: McGraw-Hill Book Co., 1940, p 100.

shearing stress to the deflection of the grid is considerable, i.e., maximum deflection due to shear is 14% of that due to bending by mechanical loads.

The problems of thermal stresses are intrinsically more difficult than the ones in which only mechanical loads are involved. Though much attention has been directed to this kind of problem as a result of advances made in the fields of nuclear and aeronautical engineering, only a few simple cases of thermal stress problems have been solved. The temperature variations over the PWR reactor is three-dimensional and cannot be expressed in terms of any simple function. With the added non-uniform and non-axisymmetrical structural features of the grid, the problem defies any rigorous solution.

By applying the theory of plate to the grid, the effect of variation of expansion over the plane of the grid is ignored. The points of higher temperature tend to expand more, and any expansion in excess of that of neighboring points produces a normal tensile stress in directions perpendicular to the direction of expansion. Thus, the vertical excessive expansion of the grid, at and near the meshes where the seed elements are located, relieves, somewhat, the high compression in the plane of the grid in the neighboring webs. The temperature rise at the intersection of two webs gives rise to further complication to the stress pattern. However, it is believed that the uneven expansion in the direction perpendicular to the plane of the grid, due to both a sinusoidal temperature variation along any diameter of the grid and temperature rises at intersections of webs, would not materially affect the stress distribution of the grid as a whole. The change of temperature gradient is slow along the diameter, and temperature rise at the intersection of two webs should change, only locally, the stress distribution near the intersection. It is possible to investigate the stresses as a result of this uneven expansion, by treating the webs on one straight line as a long strip, but the problem would become too involved. It is beyond the scope of this analysis to undertake such a task.

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The maximum deflection (Station 1, Fig. 2) is only 34.7% of that due to mechanical loading. This assures a small increase in the diameter of the grid due to thermal expansion. This fact is borne out by calculating the expansion of the middle plane from Station 1, near the center, to the boundary point, Station a, of the same illustration. Since the application of the stress, $-E\eta t$, suppressed the expansion of the middle plane completely, the strain at the middle section of a web is caused by σ_x''' or σ_y''' only. Thus,

$$\epsilon_x = \frac{\sigma_x'''}{E}$$

The change of length of the strip, 1-a, due to thermal stresses, is then

$$\Delta l_{1a} = \int_0^l \frac{\sigma_x'''}{E} dx .$$

The integral is evaluated numerically with Simpson's One-Third Rule,

$$\Delta l_{1a} = 0.00322 \text{ in. ,}$$

which is negligibly small. As a result, the changes in curvatures of the grid due to thermal stresses are negligible, and it is legitimate to superpose the thermal stresses on that caused by mechanical loading.

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Based on the steady-state thermal stresses calculated in this report, it seems that buckling of the webs is quite unlikely to occur. The scope of this contract does not include the analyses of thermal stresses induced by system transients.

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Laboratories for Research
and Development

EQUALLY SPACED NODAL POINTS

PARABOLIC INTERPOLATION

COEFFICIENTS FOR SECOND FINITE DIFFERENCES OF DEFLECTIONS (W)

COEFFICIENTS FOR SECOND FINITE DIFFERENCES OF MOMENTS (M)

DISTRIBUTION OF MECHANICAL LOADS

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FIGURE 8

095 055

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FIGURE 9

895 056

(I) COORDINATES FOR A PLATE

(II) BOUNDARY CONDITIONS FOR THE PLANE STRESS PROBLEM

TANGENTIAL ELLIPTIC ARCS REPRESENTING POINTS
OF MAXIMUM TEMPERATURE

GRAPHIC REPRESENTATION OF $f(Z)$

COEFFICIENTS OF FINITE DIFFERENCES FOR $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \left(\frac{bn}{\delta}\right)$

Case I

Case II

COEFFICIENTS OF FINITE DIFFERENCES FOR $\frac{\partial^2 \theta}{\partial x^2}$ ($A\Psi$)

STRESS DISTRIBUTION OF STATION 21

ISOMETRIC VIEW OF TOTAL DEFLECTION

A P P E N D I X

TABULATED DATA

Table 1. MECHANICAL LOADS ON TOP GRID

Regions 1 and 4:

Hydraulic load	=	413 lb
Spring force (hot)	=	1200
-Weight in water	=	<u>-470</u>
Load	=	1143 lb

Regions 2 and 3:

Hydraulic load	=	1035 lb
Spring force (hot)	=	1200
-Weight in water	=	<u>-470</u>
Load	=	1765 lb

Seed Assembly Region:

Hydraulic load	=	1035 lb
Spring force (hot)	=	820
-Weight in water	=	<u>-325</u>
Load	=	1530 lb

Table 2. MECHANICAL LOAD ON EACH NODAL POINT

<u>p</u>	<u>$q_p \delta^2$</u>	<u>p</u>	<u>$q_p \delta^2$</u>
1	1143	13	572
2	1143	14	1706
3	1454	15	1589
4	1648	16	1648
5	1648	17	1454
6	1454	18	572
7	572	19	1589
8	1299	20	1706
9	1610	21	1168
10	1648	22	1324
11	1648	23	441
12	1454		

Table 3. DIFFERENCE EQUATIONS FOR BENDING MOMENTS
DUE TO MECHANICAL LOADS

Station	
1	$2M_2 - 2M_1 = -1143$
2	$M_1 + M_3 + M_8 - 3M_2 = -1143$
3	$M_2 + M_4 + M_9 - 3M_3 = -1454$
4	$M_3 + M_5 + M_{10} - 3M_4 = -1648$
5	$M_4 + M_6 + M_{11} - 3M_5 = -1648$
6	$M_5 + M_7 + M_{12} - 3M_6 = -1454$
7	$1.008 M_6 + M_{13} - 3.034 M_7 = -572$
8	$2M_2 + 2M_9 - 4M_8 = -1299$
9	$M_3 + M_8 + M_{10} + M_{14} - 4M_9 = -1610$
10	$M_4 + M_9 + M_{11} + M_{15} - 4M_{10} = -1648$
11	$M_5 + M_{10} + M_{12} + M_{16} - 4M_{11} = -1648$
12	$M_6 + M_{11} + M_{13} + M_{17} - 4M_{12} = -1454$
13	$M_7 + 1.082 M_{12} + M_{18} - 4.358 M_{13} = -572$
14	$2M_9 + 2M_{15} - 4M_{14} = -1706$
15	$M_{10} + M_{14} + M_{16} + M_{19} - 4M_{15} = -1589$
16	$M_{11} + M_{15} + M_{17} + M_{20} - 4M_{16} = -1648$
17	$M_{12} + M_{16} + M_{18} + M_{21} - 4M_{17} = -1454$
18	$M_{13} + 1.273 M_{17} - 5.503 M_{18} = -572$
19	$2M_{15} + 2M_{20} - 4M_{19} = -1589$
20	$M_{16} + M_{19} + M_{21} + M_{22} - 4M_{20} = -1706$
21	$M_{17} + M_{20} + M_{23} - 4M_{21} = -1168$
22	$2M_{20} + 2M_{23} - 4M_{22} = -1324$
23	$1.251 M_{21} + 1.333 M_{22} - 7.339 M_{23} = -441$

Table 4. DIFFERENCE EQUATIONS FOR DEFLECTIONS
DUE TO MECHANICAL LOADS

Station	
1	$2w_2 - 2w_1 = -M_1 \delta^2/D$
2	$w_1 + w_3 + w_8 - 3w_2 = -M_2 \delta^2/D$
3	$w_2 + w_4 + w_9 - 3w_3 = -M_3 \delta^2/D$
4	$w_3 + w_5 + w_{10} - 3w_4 = -M_4 \delta^2/D$
5	$w_4 + w_6 + w_{11} - 3w_5 = -M_5 \delta^2/D$
6	$w_5 + w_7 + w_{12} - 3w_6 = -M_6 \delta^2/D$
7	$1.0084 w_6 + 1.8333 w_{13} - 4.7218 w_7 = -M_7 \delta^2/D$
8	$2w_2 + 2w_9 - 4w_8 = -M_8 \delta^2/D$
9	$w_3 + w_8 + w_{10} + w_{14} - 4w_9 = -M_9 \delta^2/D$
10	$w_4 + w_9 + w_{11} + w_{15} - 4w_{10} = -M_{10} \delta^2/D$
11	$w_4 + w_{10} + w_{12} + w_{16} - 4w_{11} = -M_{11} \delta^2/D$
12	$w_6 + w_{11} + w_{13} + w_{17} - 4w_{12} = -M_{12} \delta^2/D$
13	$1.8333 w_7 + 1.0820 w_{13} + 1.8333 w_{18} - 7.0869 w_{13} = -M_{13} \delta^2/D$
14	$2w_9 + 2w_{15} - 4w_{14} = -M_{14} \delta^2/D$
15	$w_{10} + w_{14} + w_{16} + w_{19} - 4w_{15} = -M_{15} \delta^2/D$
16	$w_{11} + w_{15} + w_{17} + w_{20} - 4w_{16} = -M_{16} \delta^2/D$
17	$w_{12} + w_{16} + w_{18} + w_{21} - 4w_{17} = -M_{17} \delta^2/D$
18	$1.8333 w_{13} + 1.2731 w_{17} - 9.0272 w_{18} = -M_{18} \delta^2/D$
19	$2w_{15} + 2w_{20} - 4w_{19} = -M_{19} \delta^2/D$
20	$w_{16} + w_{19} + w_{21} + w_{22} - 4w_{20} = -M_{20} \delta^2/D$
21	$w_{17} + w_{20} + 1.8333 w_{23} - 5.6666 w_{21} = -M_{21} \delta^2/D$
22	$2w_{20} + 3.6666 w_{23} - 5.6666 w_{22} = -M_{22} \delta^2/D$
23	$2.2931 w_{21} + 2.4443 w_{22} - 9.4921 w_{23} = -M_{23} \delta^2/D$

Table 5. MOMENTS AND DEFLECTIONS DUE TO MECHANICAL LOADS

<u>Station (p)</u>	<u>M_p</u>	<u>w_p</u>	<u>w_p[*]</u>	<u>w_p + w_p[*]</u>
1	19568	0.036617	0.005243	0.041860
2	19086	0.034782	0.005092	0.039874
3	17929	0.031188	0.004783	0.035971
4	15926	0.025803	0.004248	0.030051
5	12968	0.019460	0.003459	0.022919
6	9079	0.011982	0.002421	0.014403
7	4450	0.004005	0.000642	0.004647
8	18528	0.032978	0.004942	0.037920
9	17321	0.029445	0.004620	0.034065
10	15234	0.024349	0.004063	0.028412
11	12251	0.017987	0.003268	0.021255
12	8366	0.010787	0.002232	0.013019
13	3777	0.003271	0.000508	0.003779
14	15982	0.026033	0.004263	0.030296
15	13790	0.021130	0.003679	0.024809
16	10787	0.015064	0.002878	0.017942
17	6903	0.008345	0.001841	0.010186
18	2387	0.001891	0.000273	0.002164
19	11569	0.016496	0.003086	0.019582
20	8554	0.010783	0.002282	0.013065
21	4620	0.004349	0.000673	0.005022
22	5534	0.005630	0.000805	0.006435
23	1853	0.002536	0.000358	0.002894

Table 6. DIFFERENCE EQUATIONS FOR COMPONENT MOMENTS
DUE TO MECHANICAL LOADS

$$\begin{aligned}
M_{1x} &= D/\delta^2 (1.3 w_1 - 1.3 w_2) \\
M_{2x} &= D/\delta^2 (2.3 w_2 - w_1 - w_3 - 0.3 w_8) \\
M_{3x} &= D/\delta^2 (2.3 w_3 - w_2 - w_4 - 0.3 w_9) \\
M_{4x} &= D/\delta^2 (2.3 w_4 - w_3 - w_5 - 0.3 w_{10}) \\
M_{5x} &= D/\delta^2 (2.3 w_5 - w_4 - w_6 - 0.3 w_{11}) \\
M_{6x} &= D/\delta^2 (2.3 w_6 - w_5 - w_7 - 0.3 w_{12}) \\
M_{7x} &= D/\delta^2 (3.4385 w_7 - 1.0084 w_6 - 0.5500 w_{13}) \\
M_{8x} &= D/\delta^2 (2.6 w_8 - 1.3 w_2 - 1.3 w_9) \\
M_{9x} &= D/\delta^2 (2.6 w_9 - 0.3 w_2 - w_8 - w_{10} - 0.3 w_{14}) \\
M_{10x} &= D/\delta^2 (2.6 w_{10} - 0.3 w_4 - w_9 - w_{11} - 0.3 w_{15}) \\
M_{11x} &= D/\delta^2 (2.6 w_{11} - 0.3 w_5 - w_{10} - w_{12} - 0.3 w_{16}) \\
M_{12x} &= D/\delta^2 (2.6 w_{12} - 0.3 w_6 - w_{11} - w_{13} - 0.3 w_{17}) \\
M_{13x} &= D/\delta^2 (4.5203 w_{13} - 0.5500 w_7 - 1.0820 w_{12} - 0.5500 w_{18}) \\
M_{14x} &= D/\delta^2 (2.6 w_{14} - 1.3 w_9 - 1.3 w_{15}) \\
M_{15x} &= D/\delta^2 (2.6 w_{15} - 0.3 w_{10} - w_{14} - w_{16} - 0.3 w_{19}) \\
M_{16x} &= D/\delta^2 (2.6 w_{16} - 0.3 w_{11} - w_{15} - w_{17} - 0.3 w_{20}) \\
M_{17x} &= D/\delta^2 (2.6 w_{17} - 0.3 w_{12} - w_{16} - w_{18} - 0.3 w_{21}) \\
M_{18x} &= D/\delta^2 (6.4606 w_{18} - 0.5500 w_{13} - 1.2731 w_{17}) \\
M_{19x} &= D/\delta^2 (2.6 w_{19} - 1.3 w_{15} - 1.3 w_{20}) \\
M_{20x} &= D/\delta^2 (2.6 w_{20} - 0.3 w_{16} - w_{19} - w_{21} - 0.3 w_{22}) \\
M_{21x} &= D/\delta^2 (3.6833 w_{21} - 0.3 w_{17} - w_{20} - 0.5500 w_{23}) \\
M_{22x} &= D/\delta^2 (3.6833 w_{22} - 1.3 w_{20} - 2.3833 w_{23}) \\
M_{23x} &= D/\delta^2 (6.4255 w_{23} - 0.6879 w_{21} - 2.4443 w_{22})
\end{aligned}$$

Table 7. BENDING MOMENTS AND STRESSES
DUE TO MECHANICAL LOADS

Station (p)	M_{px}	M_{py}	σ_{px}	σ_{py}
1	+12778	+12778	+11358	+11358
2	+12322	+12489	+10953	+11101
3	+11394	+11914	+10128	+10590
4	+9772	+10932	+8686	+9717
5	+7444	+9415	+6617	+8369
6	+4597	+7206	+4086	+6405
7	-593	+6378	-527 -288	+3066
8	+12043	+12043	+10705	+10705
9	+10982	+11535	+9762	+10253
10	+9315	+10489	+8280	+9324
11	+6817	+9109	+6060	+8097
12	+3692	+7184	+3282	+6386
13	-673	+5583	-598 -326	+2501
14	+10388	+10388	+9234	+9234
15	+8490	+9437	+7547	+8388
16	+5693	+8329	+5060	+7404
17	+1075	+7899	+956	+7021
18	-1117	+4221	-993 -542	+1609
19	+7520	+7520	+6684	+6684
20	+5277	+5843	+4691	+5194
21	+7149	-1144	+6355 +3466	-990 -555
22	+3598	+3598	+3198 +1744	+3198 +1744
23	-2426	+4835	-940 -1723	+1758 +3224

Table 8. TEMPERATURES (T_n) AND FINITE DIFFERENCES FOR n

Station (p)	T_n	T_e	n	Δ^2_n
1	16.7000	12.2776	187.498	+177.102
2	24.587	18.0759	276.049	+77.942
3	35.931	26.416	403.414	-19.774
4	43.914	32.285	493.042	-86.125
5	43.814	32.212	491.919	-94.915
6	35.655	26.213	400.315	-45.169
7	24.190	17.7841	271.592	-51.350
8	28.072	20.638	315.177	+134.144
9	37.531	27.592	421.377	-7.868
10	44.326	32.588	497.668	-84.723
11	43.419	31.921	487.485	-91.887
12	34.938	25.686	392.265	-41.228
13	23.595	17.3467	264.912	-63.910
14	41.094	30.211	461.381	+5.320
15	44.894	33.006	504.045	-79.849
16	42.414	31.182	476.201	-86.272
17	33.411	24.563	375.120	-33.355
18	22.427	16.4879	251.798	-75.779
19	44.630	32.811	501.081	-92.470
20	40.248	29.590	451.882	-83.070
21	30.894	22.713	346.861	-153.940
22	35.655	26.213	400.315	-248.101
23	27.123	19.9404	304.522	+89.239
a	16.1931	11.9049	181.807	
b	16.6216	12.2199	186.618	
c	17.6582	12.9820	198.257	
d	20.711	15.2263	232.532	
e	22.510	16.5489	252.730	
f	24.152	17.7562	271.165	

Table 9. DIFFERENCE EQUATIONS FOR MOMENTS
DUE TO THERMAL LOADINGStation

1	$2M_2 - 2M_1 = 177.102 E\eta \frac{b}{\delta}$
2	$M_1 + M_3 + M_8 - 3M_2 = 77.942 E\eta \frac{b}{\delta}$
3	$M_2 + M_4 + M_9 - 3M_3 = -19.774 E\eta \frac{b}{\delta}$
4	$M_3 + M_5 + M_{10} - 3M_4 = -86.125 E\eta \frac{b}{\delta}$
5	$M_4 + M_6 + M_{11} - 3M_5 = -94.915 E\eta \frac{b}{\delta}$
6	$M_5 + M_7 + M_{12} - 3M_6 = -45.169 E\eta \frac{b}{\delta}$
7	$1.008 M_6 + M_{13} - 3.034 M_7 = -338.053 E\eta \frac{b}{\delta}$
8	$2M_2 + 2M_9 - 4M_8 = 134.144 E\eta \frac{b}{\delta}$
9	$M_3 + M_8 + M_{10} + M_{14} - 4M_9 = -7.868 E\eta \frac{b}{\delta}$
10	$M_4 + M_9 + M_{11} + M_{15} - 4M_{10} = -84.723 E\eta \frac{b}{\delta}$
11	$M_5 + M_{10} + M_{12} + M_{16} - 4M_{11} = -91.887 E\eta \frac{b}{\delta}$
12	$M_6 + M_{11} + M_{13} + M_{17} - 4M_{12} = -41.228 E\eta \frac{b}{\delta}$
13	$M_7 + 1.082 M_{12} + M_{18} - 4.358 M_{13} = -429.978 E\eta \frac{b}{\delta}$
14	$2M_9 + 2M_{15} - 4M_{14} = +5.320 E\eta \frac{b}{\delta}$
15	$M_{10} + M_{14} + M_{16} + M_{19} - 4M_{15} = -79.849 E\eta \frac{b}{\delta}$
16	$M_{11} + M_{15} + M_{17} + M_{20} - 4M_{16} = -86.272 E\eta \frac{b}{\delta}$
17	$M_{12} + M_{16} + M_{18} + M_{21} - 4M_{17} = -33.355 E\eta \frac{b}{\delta}$
18	$M_{13} + 1.273 M_{17} - 5.503 M_{18} = -1113.414 E\eta \frac{b}{\delta}$
19	$2M_{15} + 2M_{20} - 4M_{19} = -92.470 E\eta \frac{b}{\delta}$
20	$M_{16} + M_{19} + M_{21} + M_{22} - 4M_{20} = -83.070 E\eta \frac{b}{\delta}$
21	$M_{17} + M_{20} + M_{23} - 4M_{21} = -153.940 E\eta \frac{b}{\delta}$
22	$2M_{20} + 2M_{23} - 4M_{22} = -248.101 E\eta \frac{b}{\delta}$
23	$1.251 M_{21} + 1.333 M_{22} - 7.339 M_{23} = -1818.871 E\eta \frac{b}{\delta}$

Table 10. MOMENTS AND DEFLECTIONS DUE TO THERMAL BENDING

<u>Station (p)</u>	<u>M_p</u>	<u>w_p</u>
1	3793.3133	0.014510245
2	4872.5284	0.014156202
3	6430.1076	0.013313934
4	7543.7228	0.011770519
5	7584.0981	0.0093792258
6	6581.0432	0.0061553001
7	5218.3275	0.0022418745
8	5344.0822	0.013734889
9	6633.0758	0.012814795
10	7567.3159	0.011210235
11	7470.7505	0.0087961590
12	6390.2081	0.0056163391
13	5078.6942	0.0018570832
14	7094.9051	0.011761945
15	7589.1536	0.010046904
16	7221.5081	0.0075842951
17	5927.8780	0.0044639750
18	4760.0610	0.0011051292
19	7499.7256	0.0082145008
20	6846.8087	0.0056821239
21	4933.2210	0.0024436012
22	6720.3641	0.0032080269
23	5082.0542	0.0015163612

Table 11. DIFFERENCE EQUATIONS FOR MOMENT COMPONENTS
DUE TO THERMAL BENDING

$$\begin{aligned}
M_{1x} &= D/\delta^2 [1.3 w_1 - 1.3 w_2] \\
M_{2x} &= D/\delta^2 [2.3 w_2 - w_1 - w_3 - 0.3 w_8] \\
M_{3x} &= D/\delta^2 [2.3 w_3 - w_2 - w_4 - 0.3 w_9] \\
M_{4x} &= D/\delta^2 [2.3 w_4 - w_3 - w_5 - 0.3 w_{10}] \\
M_{5x} &= D/\delta^2 [2.3 w_5 - w_4 - w_6 - 0.3 w_{11}] \\
M_{6x} &= D/\delta^2 [2.3 w_6 - w_5 - w_7 - 0.3 w_{12}] \\
M_{7x} &= D/\delta^2 [3.4385 w_7 - 1.0084 w_6 - 0.5500 w_{13}] \\
M_{8x} &= D/\delta^2 [2.6 w_8 - 1.3 w_2 - 1.3 w_9] \\
M_{9x} &= D/\delta^2 [2.6 w_9 - 0.3 w_3 - w_8 - w_{10} - 0.3 w_{14}] \\
M_{10x} &= D/\delta^2 [2.6 w_{10} - 0.3 w_4 - w_9 - w_{11} - 0.3 w_{15}] \\
M_{11x} &= D/\delta^2 [2.6 w_{11} - 0.3 w_5 - w_{10} - w_{12} - 0.3 w_{16}] \\
M_{12x} &= D/\delta^2 [2.6 w_{12} - 0.3 w_6 - w_{11} - w_{13} - 0.3 w_{17}] \\
M_{13x} &= D/\delta^2 [4.5203 w_{13} - 0.5500 w_7 - 1.0820 w_{12} - 0.5500 w_{18}] \\
M_{14x} &= D/\delta^2 [2.6 w_{14} - 1.3 w_9 - 1.3 w_{15}] \\
M_{15x} &= D/\delta^2 [2.6 w_{15} - 0.3 w_{10} - w_{14} - w_{16} - 0.3 w_{14}] \\
M_{16x} &= D/\delta^2 [2.6 w_{16} - 0.3 w_{11} - w_{15} - w_{17} - 0.3 w_{20}] \\
M_{17x} &= D/\delta^2 [2.6 w_{17} - 0.3 w_{12} - w_{16} - w_{18} - 0.3 w_{21}] \\
M_{18x} &= D/\delta^2 [6.4606 w_{18} - 0.5500 w_{13} - 1.2731 w_{17}] \\
M_{19x} &= D/\delta^2 [2.6 w_{19} - 1.3 w_{15} - 1.3 w_{20}] \\
M_{20x} &= D/\delta^2 [2.6 w_{20} - 0.3 w_{16} - w_{19} - w_{21} - 0.3 w_{22}] \\
M_{21x} &= D/\delta^2 [3.6833 w_{21} - 0.3 w_{17} - w_{20} - 0.5500 w_{23}] \\
M_{22x} &= D/\delta^2 [3.6833 w_{22} - 1.3 w_{20} - 2.3833 w_{23}] \\
M_{23x} &= D/\delta^2 [6.4255 w_{23} - 0.68793 w_{21} - 2.4443 w_{22}]
\end{aligned}$$

Table 12. MOMENT AND STRESS COMPONENTS DUE TO THERMAL BENDING

Station (p)	M_{px}	M_{py}	σ''_{px}	σ''_{py}
1	2466	2465	2192	2191
2	3293	3041	2927	2703
3	4558	3801	4052	3379
4	5443	4364	4838	3879
5	5398	4461	4798	3965
6	4560	3995	4053	3551
7	2573	4211	2287 1248	2024
8	3474	3473	3088	3087
9	4557	4066	4051	3614
10	5306	4532	4716	4028
11	5113	4599	4545	4088
12	4090	4217	3636	3748
13	2554	4048	2270 1238	1814
14	4612	4611	4100	4099
15	5082	4784	4517	4252
16	4627	4761	4113	4232
17	2673	5033	2376	4474
18	2332	3856	2073 1131	1470
19	4875	4875	4333	4333
20	4702	4199	4180	3732
21	6135	278	5453 2975	247 135
22	4368	4368	3883 2118	3883 2118
23	1184	5423	459 841	1972 3615

Table 13. AVERAGE TEMPERATURES AND LAPLACIAN
OF AVERAGE TEMPERATURES

Station (p)	t	$\frac{\sigma^2}{V} t$	2t
1	5.1234	+4.8394	10.2468
2	7.5431	+2.1299	15.0862
3	11.0234	-0.5405	22.0468
4	13.4725	-2.3532	26.9450
5	13.4420	-2.5943	26.8840
6	10.9387	-1.2339	21.8774
7	7.4213	+0.8414	14.8426
8	8.6122	+3.6656	17.2244
9	11.5141	-0.2149	23.0282
10	13.5990	-2.3154	27.1980
11	13.3206	-2.5104	26.6412
12	10.7188	-1.1268	21.4376
13	7.2388	+0.8569	14.4776
14	12.6070	-0.1469	25.2140
15	13.7734	-2.1833	27.5468
16	13.0122	-2.3569	26.0244
17	10.2501	-0.9110	20.5002
18	6.8804	+0.8537	13.7608
19	13.6920	-2.5255	27.3840
20	12.3479	-2.2705	24.6958
21	9.4781	-0.6394	18.9562
22	10.9387	-2.4168	21.8774
23	8.3211	-0.7481	16.6422
a	4.9679		
b	5.0993		
c	5.4174		
d	6.3539		
e	6.9058		
f	7.4096		

Table 14. DIFFERENCE EQUATIONS FOR STRESSES

Station

1	$2\sigma_2 - 2\sigma_1 = 4.8394 E\eta$
2	$\sigma_1 + \sigma_3 + \sigma_8 - 3\sigma_2 = 2.1299 E\eta$
3	$\sigma_2 + \sigma_4 + \sigma_9 - 3\sigma_3 = -6.5405 E\eta$
4	$\sigma_3 + \sigma_5 + \sigma_{10} - 3\sigma_4 = -2.3532 E\eta$
5	$\sigma_4 + \sigma_6 + \sigma_{11} - 3\sigma_5 = -2.5943 E\eta$
6	$\sigma_5 + \sigma_7 + \sigma_{12} - 3\sigma_6 = -1.2339 E\eta$
7	$1.008 \sigma_6 + \sigma_{13} - 3.034 \sigma_7 = -6.9928 E\eta$
8	$2\sigma_2 + 2\sigma_9 - 4\sigma_8 = 3.6656 E\eta$
9	$\sigma_3 + \sigma_8 + \sigma_{10} + \sigma_{14} - 4\sigma_9 = -0.2149 E\eta$
10	$\sigma_4 + \sigma_9 + \sigma_{11} + \sigma_{15} - 4\sigma_{10} = -2.3154 E\eta$
11	$\sigma_5 + \sigma_{10} + \sigma_{12} + \sigma_{16} - 4\sigma_{11} = -2.5104 E\eta$
12	$\sigma_6 + \sigma_{11} + \sigma_{13} + \sigma_{17} - 4\sigma_{12} = -1.1268 E\eta$
13	$\sigma_7 + 1.082 \sigma_{12} + \sigma_{18} - 4.358 \sigma_{13} = -9.1458 E\eta$
14	$2\sigma_9 + 2\sigma_{15} - 4\sigma_{14} = -0.1469 E\eta$
15	$\sigma_{10} + \sigma_{14} + \sigma_{16} + \sigma_{19} - 4\sigma_{15} = -2.1833 E\eta$
16	$\sigma_{11} + \sigma_{15} + \sigma_{17} + \sigma_{20} - 4\sigma_{16} = -2.3569 E\eta$
17	$\sigma_{12} + \sigma_{16} + \sigma_{18} + \sigma_{12} - 4\sigma_{17} = -0.9110 E\eta$
18	$\sigma_{13} + 1.273 \sigma_{17} - 5.503 \sigma_{18} = -27.4998 E\eta$
19	$2\sigma_{15} + 2\sigma_{20} - 4\sigma_{19} = -2.5255 E\eta$
20	$\sigma_{16} + \sigma_{19} + \sigma_{21} + \sigma_{22} - 4\sigma_{20} = -2.2705 E\eta$
21	$\sigma_{17} + \sigma_{20} + \sigma_{23} - 4\sigma_{21} = -0.6394 E\eta$
22	$2\sigma_{20} + 2\sigma_{23} - 4\sigma_{22} = -2.4168 E\eta$
23	$1.251 \sigma_{21} + 1.333 \sigma_{22} - 7.339 \sigma_{23} = -52.8872 E\eta$

Table 15. DIFFERENCE EQUATIONS FOR Ψ Station

1	$2\Psi_2 - 2\Psi_1 = [1.30\sigma_1 - 10.2468 E\eta] \delta^2/A$
2	$\Psi_1 + \Psi_3 + \Psi_8 - 3\Psi_2 = [1.30\sigma_2 - 15.0862 E\eta] \delta^2/A$
3	$\Psi_2 + \Psi_4 + \Psi_9 - 3\Psi_3 = [1.30\sigma_3 - 22.0468 E\eta] \delta^2/A$
4	$\Psi_3 + \Psi_5 + \Psi_{10} - 3\Psi_4 = [1.30\sigma_4 - 26.9450 E\eta] \delta^2/A$
5	$\Psi_4 + \Psi_6 + \Psi_{11} - 3\Psi_5 = [1.30\sigma_5 - 26.8840 E\eta] \delta^2/A$
6	$\Psi_5 + \Psi_7 + \Psi_{12} - 3\Psi_6 = [1.30\sigma_6 - 21.8774 E\eta] \delta^2/A$
7	$1.0084 \Psi_6 + 1.8333 \Psi_{13} - 4.7218 \Psi_7 = [1.30 \sigma_7 - 14.8426 E\eta] \delta^2/A$
8	$2\Psi_2 + 2\Psi_9 - 4\Psi_8 = [1.30 \sigma_8 - 17.2244 E\eta] \delta^2/A$
9	$\Psi_3 + \Psi_8 + \Psi_{10} + \Psi_{14} - 4\Psi_9 = [1.30 \sigma_9 - 23.0282 E\eta] \delta^2/A$
10	$\Psi_4 + \Psi_9 + \Psi_{11} + \Psi_{15} - 4\Psi_{10} = [1.30 \sigma_{10} - 27.1980 E\eta] \delta^2/A$
11	$\Psi_5 + \Psi_{10} + \Psi_{12} + \Psi_{16} - 4\Psi_{11} = [1.30 \sigma_{11} - 26.6412 E\eta] \delta^2/A$
12	$\Psi_6 + \Psi_{11} + \Psi_{13} + \Psi_{17} - 4\Psi_{12} = [1.30 \sigma_{12} - 21.4376 E\eta] \delta^2/A$
13	$1.8333 \Psi_7 + 1.0820 \Psi_{12} + 1.8333 \Psi_{18} - 7.0869 \Psi_{13} = [1.30\sigma_{13} - 14.4776 E\eta] \delta^2/A$
14	$2\Psi_9 + 2\Psi_{15} - 4\Psi_{14} = [1.30 \sigma_{14} - 25.2140 E\eta] \delta^2/A$
15	$\Psi_{10} + \Psi_{14} + \Psi_{16} + \Psi_{19} - 4\Psi_{14} = [1.30 \sigma_{15} - 27.5468 E\eta] \delta^2/A$
16	$\Psi_{11} + \Psi_{15} + \Psi_{17} + \Psi_{20} - 4\Psi_{16} = [1.30 \sigma_{16} - 26.0244 E\eta] \delta^2/A$
17	$\Psi_{12} + \Psi_{16} + \Psi_{18} + \Psi_{21} - 4\Psi_{17} = [1.30 \sigma_{17} - 20.5002 E\eta] \delta^2/A$
18	$1.8333 \Psi_{13} + 1.2731 \Psi_{17} - 9.0272 \Psi_{18} = [1.30 \sigma_{18} - 13.7608 E\eta] \delta^2/A$
19	$2\Psi_{15} + 2\Psi_{20} - 4\Psi_{19} = [1.30 \sigma_{19} - 27.3840 E\eta] \delta^2/A$
20	$\Psi_{16} + \Psi_{19} + \Psi_{21} + \Psi_{22} - 4\Psi_{20} = [1.30 \sigma_{20} - 24.6958 E\eta] \delta^2/A$
21	$\Psi_{17} + \Psi_{20} + 1.8333 \Psi_{23} - 5.6666 \Psi_{21} = [1.30 \sigma_{21} - 18.9562 E\eta] \delta^2/A$
22	$2\Psi_{20} + 3.6666 \Psi_{23} - 5.6666 \Psi_{22} = [1.30 \sigma_{22} - 21.8774 E\eta] \delta^2/A$
23	$2.2931 \Psi_{21} + 2.4443 \Psi_{22} - 9.4921 \Psi_{23} = [1.30 \sigma_{23} - 16.6422 E\eta] \delta^2/A$

Table 16. VALUES OF σ AND Ψ

Station (p)	σ_p	$\frac{A}{Ea\delta^2} \Psi_p$
1	1767.818000	65.20299148
2	2396.940000	64.49913647
3	3303.560517	61.92182913
4	3947.153592	55.62346990
5	3963.595332	44.34670614
6	3367.501220	28.39904060
7	2543.920249	9.62217052
8	2673.215483	63.27108881
9	3426.057968	60.11388118
10	3962.472920	53.39264439
11	3901.613191	41.95158795
12	3274.174082	26.18835512
13	2505.684802	8.00380309
14	3709.108952	55.97205240
15	3973.062938	48.49599857
16	3753.506429	36.74551620
17	3028.929119	21.33225891
18	2455.289762	4.79836727
19	3899.505459	40.19229150
20	3497.626480	27.94534941
21	2395.886199	12.24124707
22	3351.277836	15.39467291
23	2890.745195	7.15206041

Table 17. FINITE DIFFERENCES FOR $\frac{\partial^2}{\partial x^2} (\Psi A)$

Station	
1	$\frac{\partial^2}{\partial x^2} (\Psi A)_1 = A/\delta^2 (\Psi_2 - \Psi_1)$
2	$\frac{\partial^2}{\partial x^2} (\Psi A)_2 = A/\delta^2 (\Psi_1 + \Psi_3 - 3\Psi_2)$
3	$\frac{\partial^2}{\partial x^2} (\Psi A)_3 = A/\delta^2 (\Psi_2 + \Psi_4 - 2\Psi_3)$
4	$\frac{\partial^2}{\partial x^2} (\Psi A)_4 = A/\delta^2 (\Psi_3 + \Psi_5 - 2\Psi_4)$
5	$\frac{\partial^2}{\partial x^2} (\Psi A)_5 = A/\delta^2 (\Psi_4 + \Psi_6 - 2\Psi_5)$
6	$\frac{\partial^2}{\partial x^2} (\Psi A)_6 = A/\delta^2 (\Psi_5 + \Psi_7 - 2\Psi_6)$
7	$\frac{\partial^2}{\partial x^2} (\Psi A)_7 = A/\delta^2 (1.0084 \Psi_6 - 2.8885 \Psi_7)$
8	$\frac{\partial^2}{\partial x^2} (\Psi A)_8 = A/\delta^2 (\Psi_2 + \Psi_9 - 2\Psi_8)$
9	$\frac{\partial^2}{\partial x^2} (\Psi A)_9 = A/\delta^2 (\Psi_8 + \Psi_{10} - 2\Psi_9)$
10	$\frac{\partial^2}{\partial x^2} (\Psi A)_{10} = A/\delta^2 (\Psi_9 + \Psi_{11} - 2\Psi_{10})$
11	$\frac{\partial^2}{\partial x^2} (\Psi A)_{11} = A/\delta^2 (\Psi_{10} + \Psi_{12} - 2\Psi_{11})$
12	$\frac{\partial^2}{\partial x^2} (\Psi A)_{12} = A/\delta^2 (\Psi_{11} + \Psi_{13} - 2\Psi_{12})$
13	$\frac{\partial^2}{\partial x^2} (\Psi A)_{13} = A/\delta^2 (1.0821 \Psi_{12} - 3.4206 \Psi_{13})$
14	$\frac{\partial^2}{\partial x^2} (\Psi A)_{14} = A/\delta^2 (\Psi_9 + \Psi_{15} - 2\Psi_{14})$
15	$\frac{\partial^2}{\partial x^2} (\Psi A)_{15} = A/\delta^2 (\Psi_{14} + \Psi_{16} - 2\Psi_{15})$
16	$\frac{\partial^2}{\partial x^2} (\Psi A)_{16} = A/\delta^2 (\Psi_{15} + \Psi_{17} - 2\Psi_{16})$
17	$\frac{\partial^2}{\partial x^2} (\Psi A)_{17} = A/\delta^2 (\Psi_{16} + \Psi_{18} - 2\Psi_{17})$
18	$\frac{\partial^2}{\partial x^2} (\Psi A)_{18} = A/\delta^2 (1.2723 \Psi_{17} - 5.3501 \Psi_{18})$
19	$\frac{\partial^2}{\partial x^2} (\Psi A)_{19} = A/\delta^2 (\Psi_{15} + \Psi_{20} - 2\Psi_{19})$
20	$\frac{\partial^2}{\partial x^2} (\Psi A)_{20} = A/\delta^2 (\Psi_{19} + \Psi_{21} - 2\Psi_{20})$
21	$\frac{\partial^2}{\partial x^2} (\Psi A)_{21} = A/\delta^2 (\Psi_{20} - 2.8333 \Psi_{21})$
22	$\frac{\partial^2}{\partial x^2} (\Psi A)_{22} = A/\delta^2 (\Psi_{20} + 1.8333 \Psi_{23} - 2.8333 \Psi_{22})$
23	$\frac{\partial^2}{\partial x^2} (\Psi A)_{23} = A/\delta^2 (2.4443 \Psi_{22} - 5.1110 \Psi_{23})$

Table 18. VALUES OF Ψ_A AND STRESS COMPONENTS

Station (p)	$\frac{1}{E\eta} \frac{\partial^2}{\partial x^2} (\Psi_A)_p$	σ_{yp}	σ_{xp}
1	-0.7039	1149.1	1149.1
2	-1.8734	1474.1	1641.9
3	-3.7210	1898.6	2396.0
4	-4.9785	2208.4	2922.9
5	-4.6709	2280.5	2872.2
6	-2.8291	2108.5	2269.3
7	+0.8438	2148.9	1158.2
8	-1.9292	1737.6	1737.6
9	-3.5641	2067.0	2386.9
10	-4.7197	2308.6	2842.6
11	-4.3222	2339.6	2732.5
12	-2.4214	2157.3	2099.1
13	+0.9607	2131.9	1125.5
14	-3.3343	2410.9	2410.9
15	-4.2753	2469.5	2695.5
16	-3.6627	2430.9	2448.7
17	-1.1207	2373.6	1564.0
18	+1.4692	2170.9	1021.0
19	-3.9433	2534.7	2534.7
20	-3.4571	2311.6	2235.3
21	-6.7377	712.5	2402.2
22	-2.5606	2178.3	2178.3
23	+1.0749	2443.0	1315.0

Table 19

A. THERMAL STRESSES IN TOP FIBRES

Station (p)	$\sigma_x^t =$ -E ϵ_T	σ_x^n Bending	σ_x^m Plane Stress	σ_x	$\sigma_y^t =$ -E ϵ_T	σ_y^n Bending	σ_y^m Plane Stress	σ_y
1	-320	-2192	+1149	-1363	-320	-2191	+1149	-1363
2	-471	-2927	+1642	-1756	-471	-2703	+1474	-1700
3	-688	-4052	+2396	-2344	-688	-3379	+1899	-2168
4	-841	-4838	+2923	-2756	-841	-3879	+2208	-2512
5	-839	-4798	+2872	-2765	-839	-3965	+2280	-2524
6	-683	-4053	+2269	-2467	-683	-3551	+2108	-2126
7	-463	-2287 -1248	+1158	-1592 -553	-463	-2024	+2149	-338
8	-538	-3088	+1738	-1888	-538	-3087	+1738	-1887
9	-719	-4051	+2387	-2383	-719	-3614	+2067	-2266
10	-849	-4716	+2843	-2722	-849	-4028	+2309	-2568
11	-832	-4545	+2732	-2645	-832	-4088	+2340	-2580
12	-669	-3636	+2099	-2206	-669	-3748	+2157	-2260
13	-452	-2270 -1238	+1126	-1596 -564	-452	-1814	+2132	-134
14	-787	-4100	+2411	-2476	-787	-4099	+2411	-2475
15	-860	-4517	+2696	-2681	-860	-4252	+2470	-2642
16	-812	-4113	+2449	-2476	-812	-4232	+2431	-2613
17	-640	-2376	+1564	-1452	-640	-4474	+2374	-2740
18	-430	-2073 -1131	+1021	-1482 -540	-430	-1470	+2171	+271
19	-855	-4333	+2535	-2653	-855	-4333	+2535	-2653
20	-771	-4180	+2235	-2716	-771	-3732	+2312	-2191
21	-592	-5453 -2975	+2402	-3643 -1165	-592	-247 -135	+712	-127 -15
22	-683	-3883 -2118	+2178	-2388 -623	-683	-3883 -2118	+2178	-2388 -623
23	-519	-459 -841	+1315	+337 -45	-519	-1972 -3615	+2443	-48 -1691

B. THERMAL STRESSES IN BOTTOM FIBRES

	$\sigma_x^t =$ -E ϵ_T	σ_x^n Bending	σ_x^m Plane Stress	σ_x	$\sigma_y^t =$ -E ϵ_T	σ_y^n Bending	σ_y^m Plane Stress	σ_y
	-2873	+2192	+1149	+468	-2873	+2191	+1149	+467
	-4230	+2927	+1642	+339	-4230	+2703	+1474	-53
	-6181	+4052	+2396	+267	-6181	+3379	+1899	-903
	-7555	+4838	+2923	+206	-7555	+3879	+2208	-1468
	-7538	+4798	+2872	+132	-7538	+3965	+2280	-1293
	-6134	+4053	+2269	+188	-6134	+3551	+2108	-475
	-4162	+2287 +1248	+1158	-717 -1756	-4162	+2024	+2149	+11
	-4829	+3088	+1738	-3	-4829	+3087	+1738	-4
	-6327	+4051	+2387	+111	-6327	+3614	+2067	-646
	-7626	+4716	+2843	-67	-7626	+4028	+2309	-1289
	-7470	+4545	+2733	-192	-7470	+4088	+2340	-1042
	-6010	+3636	+2099	-275	-6010	+3748	+2157	-105
	-4059	+2270 +1238	+1126	-663 -1695	-4059	+1814	+2132	-113
	-7069	+4100	+2411	-558	-7069	+4099	+2411	-559
	-7723	+4517	+2696	-510	-7723	+4252	+2696	-775
	-7296	+4113	+2449	-734	-7296	+4232	+2449	-615
	-5748	+2376	+1564	-1808	-5748	+4474	+2374	+1100
	-3858	+2073 +1131	+1021	-764 -1706	-3858	+1470	+2171	-217
	-7678	+4333	+2535	-810	-7678	+4333	+2535	-810
	-6924	+4180	+2235	-509	-6924	+3732	+2235	-957
	-5315	+5453 +2975	+2402	+2540 +62	-5315	+247 +135	+712	-4356 -4468
	-6134	+3883 +2118	+2178	-73 -1838	-6134	+3883 +2118	+2178	-73 -1838
	-4666	+459 +841	+1315	-2892 -2510	-4666	+1972 +3615	+2443	-251 +1392

Table 20. TOTAL STRESSES IN TOP AND BOTTOM FIBRES OF GRID

Station	Mechanical		Thermal				Total Stresses			
	$\pm \sigma_x^*$ $\pm \sigma_y^*$		Top Fibre		Bottom Fibre		Top Fibre		Bottom Fibre	
	σ_x	σ_y	σ_x	σ_y	σ_x	σ_y	σ_x	σ_y	σ_x	σ_y
1	+11358	+11358	-1363	-1363	+468	+468	+9995	+9995	-10890	-10890
2	+10953	+11101	-1756	-1700	+339	-53	+9197	+9401	-10614	-11154
3	+10128	+10590	-2344	-2168	+267	-903	+7784	+8422	-9861	-111493
4	+8686	+9717	-2756	-2512	+206	-1468	+5930	+7205	-8480	-11185
5	+6617	+8369	-2765	-2524	+132	-1293	+3852	+5845	-6485	-9662
6	+4086	+6405	-2467	-2126	+188	-475	+1619	+4279	-4618	-5281
7	-527 -288	+3066	-1592 -553	-338	-717 -1756	+11	-2056 -828	+2678	-190 -1468	-3055
8	+10705	-10705	-1888	-1888	-3	-4	+8817	+8817	-10708	-10709
9	+9762	+10253	-2383	-2266	+111	-646	+7379	+7987	-9651	-10899
10	+8280	+9324	-2722	-2568	-67	-1289	+5558	+6756	-8347	-10613
11	+6060	+8097	-2645	-2580	-192	-1042	+3415	+5517	-6252	-9193
12	+3282	+6386	-2206	-2260	-275	-105	+1706	+4126	-3557	-6491
13	-598 -326	+2501	-1596 -564	-134	-663 -1695	-113	-2194 -890	+2367	-65 -1369	-2614
14	+9234	+9234	-2476	-2475	-558	-559	+6758	+6758	-9792	-9792
15	+7547	+8388	-2681	-2642	-510	-775	+4866	+5746	-8057	-9163
16	+5060	+7404	-2476	-2613	-734	-615	+2584	+4791	-5794	-8019
17	+956	+7021	-1452	-2740	-1808	+1100	-496	+4281	-2764	-5921
18	-993 -542	+1609	-1482 -540	+271	-764 -1706	-217	-2475 -1082	+1880	+229 -1164	-1826
19	+6684	+6684	-2653	-2653	-810	-810	+4031	+4031	-7494	-7494
20	+4691	+5194	-2716	-2191	-509	-957	-1975	+3003	-5200	-6151
21	+6355 +3466	-990 -555	-3643 -1165	-127 -15	+2540 +62	-4356 -4468	+2712 +2301	-1117 -570	-3815 -3404	-3366 -3913
22	+3198 +1744	+3198 +1744	-2388 -623	-2388 -623	-73 -1838	-73 -1838	+810 +1121	+810 +1121	-3261 -3582	-3261 -3582
23	-940 -1723	+1758 +3224	+337 -45	-48 -1691	-2892 -2510	-251 +1392	-603 -1768	+1710 +1533	-1952 -787	-2009 -1832

* For top fibre, "+" sign applies; for bottom fibre, "-" sign applies.

Table 21. TOTAL DEFLECTION OF GRID

<u>Station</u>	<u>w_p (mechanical) x 10³</u>	<u>w_p (thermal) x 10³</u>	<u>w_p (total) x 10³</u>
1	41.860	14.510	27.350
2	39.874	14.156	25.718
3	35.971	13.314	22.657
4	30.051	11.770	18.281
5	22.919	9.379	13.540
6	14.403	6.155	8.248
7	4.647	2.242	2.405
8	37.920	13.735	24.185
9	34.065	12.815	21.250
10	28.412	11.210	17.202
11	21.255	8.796	12.459
12	13.019	5.616	7.403
13	3.779	1.857	1.922
14	30.296	11.762	18.534
15	24.809	10.047	14.762
16	17.942	7.584	10.358
17	10.186	4.464	5.722
18	2.164	1.105	1.059
19	19.582	8.215	11.367
20	13.065	5.682	7.383
21	5.022	2.444	2.578
22	6.435	3.208	3.227
23	2.894	1.516	1.378