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QUANTITATIVE SUBSTRUCTURE AND
TENSILE PROPERTY INVESTIGATIONS
OF NICKEL ALLOYS

FOURTEENTH TECHNICAL REPORT

BY

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INTRODUCTION

The processes by which metals deform are still poorly understood, even though there has been a concentrated research effort in this field during the past fifteen years. Single crystals and individual grains in polycrystalline aggregates often develop a subgrain structure when deformed. Substructure formation occurs as a result of local plastic bending and its development is augmented by annealing at moderate temperatures⁽¹⁾. Investigations made to date, largely qualitative in nature, have shown that the substructure is composed of a network of low angle boundaries. The purpose of the present paper is to present some quantitative data on substructure (obtained by x-ray techniques) and to correlate the results with the strength of the metal in which the substructure existed. Nickel and some of its alloys were used for the experiments. The presence of certain kinds of foreign atoms greatly alters the formation and effect of the substructure.

Experimental Method

Polycrystalline tensile specimens were prepared from high purity nickel, two nickel-titanium alloys, and two nickel-cobalt alloys; the composition of these materials is given in Table I. Tensile specimens having a reduced section one-quarter inch in diameter by three inches long were machined from materials which were refined, cast, forged and swaged in the authors' laboratories. The specimens were annealed for one-half hour at 1150° C., a treatment which produced grains having a diameter of approximately 0.5 mm. This rather large grain size was used to facilitate later examination of the substructure. Duplicate specimens

of pure nickel and of each alloy were strained various controlled amounts in tension at room temperature and recovered for one hour at 800° C. (This strain induced prior to recovery will hereafter be referred to as the "prestrain".) One of each of the recovered specimens was pulled in tension at room temperature to obtain the stress-strain curves. The other bar was cross-sectioned in the gage length, carefully polished and deeply etched to remove all cold worked material. This specimen was then examined by the x-ray microscopic technique⁽²⁾ to determine the nature and magnitude of the substructure. The use of duplicate specimens permitted a direct correlation between tensile properties and substructure characteristics.

X-ray microscopic studies were made with an apparatus constructed to have an average resolving power of about 4 microns. Fine grain photographic plates were used for this work and these were magnified 100 times for examination; this permitted ready detection of angular differences between crystallites which differed in orientation by as little as ten minutes. A very large number of grains (usually over 100) for each specimen was examined in this way and the number of substructure boundaries per grain was tabulated.

Analysis of the Substructure Data

When the number of substructure boundaries per grain was plotted against the frequency, skew (asymmetrical) distribution curves of the type shown in Fig. 1 were obtained. These are smoothed curves for the one-atomic-percent titanium in nickel alloy subjected to various prestrains and are representative of the curves obtained with all the metals. Since skew distribution curves cannot be analyzed by elementary statistical methods, detailed investigations were undertaken in order to (1) test the consistency of the data and (2) obtain parameters suitable for describing the distribution curves and for comparing data

obtained from different specimens.

A modification of the Wilcoxon Test* was used to verify or reject the hypothesis that two sample distributions came from the same general distribution. This test was used to show not only that various samples from one specimen all came from the same distribution, but also that samples from different alloys or those having different prestrain histories did not come from the same distribution. In every case significant differences in the distribution of the number of substructure boundaries per grain were shown to be present among the various specimens having different amounts of prestrain or different alloy content. Only rarely were there indications that various samples from one specimen did not come from the same distribution. The inconsistent data represented only a small part of the total and was discarded.

In order to analyze and compare data from different specimens, it was necessary to transform the skew distribution curves into normal (Gaussian) ones. The cumulative frequency plot (Fig. 2b) corresponding to a representative skew frequency distribution (Fig. 2a) is a curve. By using a suitable transformation function, such a skew distribution may be made normal as illustrated by Fig. 2c and its cumulative frequency is then represented by a straight line (Fig. 2d). Suitable parameters for describing the original distribution may be easily obtained from the latter plot. The derivation of these parameters is given in Appendix B; they are defined as follows: (cf. Fig. 2a and 2c)

Mode: the number of subgrains per grain which occurs most frequently in the distribution, (x_1).

Median: half the grains have this number of subgrains or more, (x_2).

Mean: the average number of subgrains per grain, (x_3).

* See Appendix A for details.

Experimental Results

Results obtained from the statistical analysis of the substructure determinations are summarized in Table I while the cumulative frequency curves from which these data were derived are contained in Appendix C. All three parameters, mode, median and mean vary in the same general manner with prestrain and alloy content. The median and mean, as indices of low angle boundary density, are plotted against the amount of prestrain in Fig. 3 for pure nickel and the four alloys. A similar plot of mode, as substructure parameter, versus prestrain is shown in Fig. 4. The substructure formation behaviours of the nickel-titanium and nickel-cobalt systems are most graphically revealed by using the mode as the index of low angle boundary density.

Room temperature tensile properties for pure nickel and for each alloy, with various amounts of prestrain, are shown in Figs. 5 through 9. The flow stress at a given strain increases regularly with increasing prestrain for all of the alloys tested. This is shown more clearly by Fig. 10, in which the flow stress at 0.25% strain is plotted against the amount of prestrain. The effects of increasing concentrations of foreign atoms on the substructure density and on the strength of nickel are shown in Fig. 11. The flow stress increased linearly with both titanium and cobalt content; the amount of substructure in the titanium alloys increased in a comparable manner. The substructure density in the cobalt alloys, however, rapidly approached a maximum. This evidence made it clear that a simple correlation between substructure density and strength is not generally valid.

Table I

Tabulation of the substructure parameters.

<u>Prestrain</u>	<u>Alloy</u>	<u>Substructure Parameters</u>		
		<u>Mode</u>	<u>Median</u>	<u>Mean</u>
0%	99.95% Ni	0.16	0.34	1.18
	1 at.% Ti in Ni	0.54	0.94	1.78
	2 at.% Ti in Ni	0.92	1.51	2.62
	5 at.% Co in Ni	0.70	1.36	2.24
	20 at.% Co in Ni	0.79	1.78	2.85
1%	99.95% Ni	0.59	1.91	3.42
	1 at.% Ti in Ni	0.87	2.52	4.01
	2 at.% Ti in Ni	1.07	2.99	4.51
	5 at.% Co in Ni	1.05	2.56	3.96
	20 at.% Co in Ni	1.55	3.42	4.79
2%	99.95% Ni	1.15	3.03	4.73
	1 at.% Ti in Ni	1.59	3.84	5.75
	2 at.% Ti in Ni	1.69	4.07	5.90
4%	1 at.% Ti in Ni	2.45	4.80	6.25
	2 at.% Ti in Ni	2.77	5.63	7.70
	5 at.% Co in Ni	1.27	3.65	5.44
	20 at.% Co in Ni	1.59	4.80	6.93

Discussion

Several mechanisms of work hardening operate to increase the strength of a metal subjected to plastic deformation at low temperatures. Before the effects of the individual factors can be isolated, measured quantitatively and understood, the metals to be used in the investigation must be put into comparable initial states. The same strain history associated with the preparation of the specimens, plus the same original annealing treatment (1150°C. for $\frac{1}{2}$ hour), yielded a different density of substructure for each alloy. This (variable) small angle boundary density and the corresponding strengths of the alloys as compared with that of the base metal may be attributed to the presence of the foreign atoms (cf. Fig. 11). However, the nature of the subboundaries after an 1150° anneal is not the same as after an 800°C. treatment (cf. Fig. 12). Consequently, all specimens were strained one percent and heated to 800°C. to produce a base state in which the subboundary structure was similar for each specimen. The number of subboundaries was, of course, different for each metal but this was of minor importance in the room temperature deformation studies because comparison between substructure and strength were made on the basis of differences produced by additional increments of strain beyond the initial one percent.

Substructure boundaries are introduced by local plastic bending. Regions of a crystal which have been bent with a uniform curvature require heating to develop a substructure⁽¹⁾ but other regions may form substructure boundaries even when the deformation is induced at a temperature as low as that of liquid nitrogen. The condition necessary for low temperature boundary formation is that there must be a place in the crystallite where the shear stress decreases abruptly to a stress

lower than that required to keep dislocations moving. In a polycrystalline aggregate, the topology is such that there are many regions where the shear stress does decrease abruptly; substructure boundaries form in these regions during the deformation. In other regions where the stress distribution is such that bending causes a region in the crystallite to assume a uniform curvature, the dislocations remain distributed. These dislocations are stopped in the crystallites wherever they encounter internal barriers; such barriers have been shown to be effective impediments to slip⁽³⁾. Sessile dislocations are one type of internal barriers; they may be formed by the interaction of moving dislocations on two active slip systems. Quantitative data showing the hardening caused by the interaction between dislocations on two slip systems has recently been obtained for zinc single crystals tested in simple shear⁽⁴⁾.

In the present investigation, it was observed that the quantity of substructure boundaries developed by room temperature deformation, i.e. with no recovery, was about the same as the number observed after an 800° C. treatment. However, the nature of the small angle boundary network is dependent upon the recovery temperature. The higher the temperatures the sharper the subboundaries as is illustrated by the x-ray micrographs of Fig. 12. An example of the very sharp subboundaries formed by an 1150° C. anneal is shown in Fig. 12a. Similarly, the diffraction pattern from a grain in a specimen deformed two percent at room temperature and recovered at 800° C. is pictured in Fig. 12b. In contrast, the substructure developed in a duplicate specimen which was also strained two percent but not recovered has diffuse subboundaries as may be seen in Fig. 12c. In this case, there are three major subgrains separated by regions of severely deformed matrix; the matrix, however, is not so distorted as to prevent some of

it from supporting x-ray diffraction. This type of diffraction pattern indicates that the matrix contains some general lattice bending. During recovery the distributed dislocations collect in subboundaries, a configuration which minimizes the free energy of the system.

In a current investigation Washburn⁽⁵⁾ has shown that the interaction of dislocations and low angle boundaries in nearly perfect single crystals is such that unrecovered boundaries, comparable to those yielding the reflections shown in Fig. 12c, are much less effective in strengthening the metal than the clearly defined ones produced by a high temperature recovery treatment. Thus there are two general mechanisms of hardening which operate during straining or during recovery. When plastic flow is taking place, dislocations are impeded in their motion through the lattice by sessile dislocations and by other internal barriers. Such hardening is very effective, e.g. when pure nickel is strained one percent (Fig. 5), the strength is increased by about 6500 psi. Recovery, on the other hand, causes many of the internal barriers to disappear⁽⁴⁾, thus weakening the metal. During this softening process, however, the boundaries become sharper and hence more effective as barriers to moving dislocations⁽⁵⁾. Thus after deformation and recovery the metal was harder than it was in its initial state (in our example by approximately 3000 psi) because of the presence of many sharp substructure boundaries. However, the metal was softer than it was after the deformation and before recovery (by approximately 3500 psi) because annealing removed many distributed dislocations trapped at internal barriers.

The present investigation was concerned with determining the effect of boundaries (developed during recovery) on the yield strength. As shown in Fig. 13, the correlation between strength and substructure density is

similar for pure nickel and the nickel-titanium alloys but the nickel-cobalt alloys behave in an entirely different manner. The strength of the cobalt alloys is markedly increased by prestraining and recovery but its substructure density remains substantially unchanged. It must be concluded, therefore, that the density of sharp subboundaries is not the only factor in establishing the strength of a metal.

The increase in strength in the cobalt alloys was obviously not due to an increase in the number of subboundaries. What, then, caused the strengthening? In order to understand the behavior of the nickel-cobalt alloys, the nature of the dislocations in the deformed metals must be considered in detail. Dislocations in the close packed crystals are dissociated because the free energy is thereby lowered. Each dislocation becomes a pair of parallel Shockley partials⁽⁶⁾. The pair separate because their elastic stress fields are mutually repulsive. Between them is a strip of fault which is hexagonal in the face-centered cubic lattice. The generation of this faulted surface requires energy and so the two partials cannot move apart indefinitely. The equilibrium distance between partials in an extended dislocation is determined by a balance of the force of repulsion between the partials and the surface tension of the connecting fault. The width of an extended dislocation in nickel is probably about 20 atom distances. The interaction of these extended dislocations and the solute atoms will now be considered.

The Suzuki⁽⁷⁾ mechanism of solid solution hardening⁽⁸⁾ is based on the heterogeneous distribution of solute atoms in lattices containing extended dislocations. Suzuki⁽⁷⁾ has shown by thermodynamic calculations that the concentration of solute atoms in the faulted regions, in certain cases, is higher than in the rest of the lattice. Since the cobalt atom

is very nearly the same size as the nickel atom and prefers the hexagonal configuration, it is probable that at equilibrium the concentration of cobalt in the extended dislocations is much higher than in the rest of the lattice. The faulted regions, with their high concentration of foreign atoms, act as barriers to impinging dislocations and therefore produce hardening. Partial dislocations must be forced free from their pinning atmospheres before plastic flow can proceed and this requires high shear stresses. The free energy of extended dislocations is so lowered by the collection of certain foreign atoms that even at high temperatures such dislocations are not readily removed from the lattice, nor can they migrate easily to form small angle boundaries. The distributed dislocations would, therefore, tend to remain distributed during the recovery treatment, and would not collect to form new boundaries. Deformation always increases the dislocation density but it does not necessarily always increase the substructure density; hence a metal may gain in strength because of dislocations distributed through the volume of a crystal which are anchored by the Suzuki mechanism even though there is no appreciable change in the density of sharp subboundaries (Fig. 13).

Partial dislocations are never pure edge type, nevertheless they are always associated with compressed and expanded regions of a lattice. The presence of a titanium atom in a nickel lattice causes high hydrostatic stresses because it is about 17% larger than the nickel. Since an accumulation of the titanium atoms in the expanded portions of the partials will reduce these stresses, it seems likely that Cottrell⁽⁹⁾ atmospheres form in the nickel-titanium alloys. However, the number of atoms involved is necessarily much smaller than that required for a Suzuki atmosphere.

Thus it is expected that the partials in the alloys containing titanium could be freed much more readily from their atmospheres than the partials in the nickel-cobalt alloys. The atmospheres in the titanium bearing alloys are apparently no more effective in hindering dislocation migration into boundaries than are the impurity atoms in the "pure" nickel. This is evident from the fact that the pure nickel and nickel-titanium alloys have the same strength versus small angle boundary density characteristics (Fig. 13).

Summary and Conclusions

1. Increasing amounts of deformation followed by recovery caused a regular gain in the strength of pure nickel, nickel-titanium and nickel-cobalt alloys. The recovered metals were not as hard as the unrecovered ones because many internal barriers to slip were annihilated during recovery. In contrast to this softening effect, the small angle boundaries introduced by the room temperature deformation become sharper during recovery and thus become much more effective in hardening the metal.
2. The strength of the annealed alloys increased linearly with increasing alloy content.
3. The substructure density of the alloys rose with increasing titanium content but approached a maximum with increasing quantities of cobalt. For pure nickel and for the nickel-titanium alloys, the small angle boundary density also increased linearly with increasing amounts of room temperature deformation. For the cobalt alloys, however, the substructure density approached a maximum. Therefore, substructure density is not the only factor in establishing the strength.

4. The number of small angle boundaries developed by room temperature deformation is substantially unchanged by heating up to temperatures as high as 800° C. The nature of the network, however, is dependent on the recovery temperature; the higher the temperatures the sharper the subboundaries.

5. Dislocation density may increase without a corresponding increase in substructure density. This was found to be the case for the nickel-cobalt alloys. Hardening in these alloys may involve the Suzuki mechanism, with the extended dislocations being immobilized by a cobalt atom atmosphere even at high temperatures. In such a case subboundaries would not be developed in proportion to the increase in dislocation density.

6. The Cottrell hardening mechanism may be operative in the nickel-titanium alloys, since the titanium atom is nearly one-fifth larger than the nickel atom. If it is, the titanium atmosphere is relatively ineffective in altering the recovery process since both the pure nickel (containing minor impurities) and the titanium alloys behaved the same during recovery.

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FIG 1 SKEW DISTRIBUTION CURVES FOR 1 at.% Ti
IN Ni WITH VARYING PRESTRAINS

FIG 2 A REPRESENTATIVE SKEW FREQUENCY DISTRIBUTION
TRANSFORMED INTO THE CORRESPONDING NORMAL
DISTRIBUTION

FIG. 3 THE RELATIONSHIP BETWEEN PRESTRAIN AND DENSITY OF SUBSTRUCTURE BOUNDARIES AS INDICATED BY
(a) MEDIAN, & (b) THE MEAN

FIG. 4 THE RELATIONSHIP BETWEEN PRESTRAIN AND DENSITY OF SUBSTRUCTURE BOUNDARIES AS INDICATED BY THE MODE

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FIG 5 EFFECT OF PRESTRAIN AND 800° C
RECOVERY TREATMENT ON TENSILE
PROPERTIES OF PURE NICKEL

FIG 6 EFFECT OF PRESTRAIN AND 800°C RECOVERY TREATMENT ON TENSILE PROPERTIES OF NICKEL-TITANIUM ALLOY

FIG 7 EFFECT OF PRESTRAIN AND 800°C RECOVERY TREATMENT ON TENSILE PROPERTIES OF NICKEL-TITANIUM ALLOY

FIG 8 EFFECT OF PRESTRAIN AND 800°C RECOVERY TREATMENT ON TENSILE PROPERTIES OF NICKEL-COBALT ALLOY

FIG 9 EFFECT OF PRESTRAIN AND 800°C RECOVERY TREATMENT ON TENSILE PROPERTIES OF NICKEL-COBALT ALLOY

FIG.10 EFFECT OF INCREASING AMOUNTS OF PRESTRAIN ON THE FLOW STRESS OF NICKEL ALLOYS

FIG. II EFFECT OF ALLOY CONTENT ON SUBSTRUCTURE DENSITY AND FLOW STRESS

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(a) 1150 °C

(b) 800 °C

(c) ROOM TEMPERATURE

FIG. 12 X-RAY MICROGRAPHS OF SUBBOUNDARIES IN
A PURE NICKEL GRAIN FOLLOWING DEFORMATION
AND RECOVERY AT VARIOUS TEMPERATURES

FIG.13 THE RELATIONSHIP BETWEEN STRENGTH & SUBSTRUCTURE FOR VARIOUS ALLOYS OF COMPARABLE INITIAL STATE

The Wilcoxon Test

Its application is testing the hypothesis that two samples come from the same distribution.

If $X_1, \dots, X_n$ is a sample from a distribution F and $Y_1, \dots, Y_m$ is a sample from a distribution G , the hypothesis to be tested is that $F = G$. Since there are ties present, i.e. X_i may equal X_j for $i \neq j$ and X_i may equal Y_k for any i and k , a modification of the usual Wilcoxon test was devised.

The test procedure is as follows: The two samples are combined into one which is arranged in increasing magnitude. Let r_i , $i = 1, \dots, n$, be the rank of the i^{th} smallest X in the combined ordered sample and similarly let the s_j , $j = 1, \dots, m$, be the ranks of the Y 's. The

sum of the ranks of the Y 's is computed $S = \sum_{j=1}^m s_j$. As a check the sum of the ranks of the X 's are also computed $R = \sum_{i=1}^n r_i$, since $S + R$ is the sum of all the ranks, i.e. $S + R = 1 + 2 + \dots + (m + n) = \frac{(m + n)(m + n + 1)}{2}$. In any case where ties occur the contribution to

the sum of the ranks rather than the individual rank is computed as follows:

If there are iX 's and jY 's tied for a position in the combined order sample, then the starting position for these will be $1 + v$ when v equals the total number of X 's and Y 's which precede the iX 's and jY 's. The last position is $v + i + j$, and therefore the midrank is $\frac{2v + i + j + 1}{2}$.

Then the contribution to the sum of the ranks of the X's is $C_x =$

$\frac{i(2v + i + j + 1)}{2}$ and the contribution to the sum of the ranks of the

Y's is $C_y = \frac{j(2v + i + j + 1)}{2}$.

Next the expected value of S , $E\{S\}$, and the variance of S , σ_s^2 , are computed. When the hypothesis is true

$$E\{S\} = \frac{m}{2} (m + n + 1)$$

and

$$\sigma_s^2 = \frac{m \cdot n}{12} (m + n + 1).$$

Now let $T = \frac{S - E\{S\}}{\sigma_s}$; then if m and n are moderately large (say > 10) and if the hypothesis is true, T is approximately normally distributed with mean zero and variance one. Hence we should expect T to fall well within the normal range most of the time.

Assume the value of T computed to T_0 . In order to compute the probability of finding a value of T as large in absolute value as T_0 , one finds (from the normal tables) the value of $p = 1 - \int_{-|T_0|}^{|T_0|} \exp(-u^2/2) du$.

If this probability p is small (e.g. $< .05$), the hypothesis would be rejected and an error in doing so would be made only on the average of 5% of the time. If the probability is large (e.g. $> .05$), the hypothesis is accepted.

As an example the following test is included here:

Hypothesis: the substructure boundary distribution curves for pure Ni and 1 at.% Ti in Ni at $\xi = 0\%$ come from the same sample.

X_i (Ni)	Y_j (1 at.%Ti in Ni)	v	$(2v+j+i+1)/2$	C_x	³⁰ C_y
87	93	0	90.5	7873.5	8416.5
13	53	180	213.5	2775.5	11315.5
1	17	246	255.5	255.5	4343.5
1	7	264	268.5	268.5	1879.5
	3	272	274.0		822.0
n = 102	1	275	276.0		276.0
	0	276	276.5		0.0
	1	276	277.0		277.0

$$m = 175$$

$$R = 11173.0 \quad S = 27330.0$$

$$R + S = 38503.0$$

$$\frac{(n+m)(n+m+1)}{2} = \frac{(102+175)(102+175+1)}{2} = 38503.0$$

$$E\{S\} = \frac{175}{2} (278) = 24325;$$

$$\bar{S} = \frac{(102)(175)}{12} (278) = 413525$$

$$\sigma_s = 643$$

$$T = \frac{27330 - 24325}{643} = 4.67$$

$$p = 1 - .999^* = 0$$

Hence the probability of obtaining a value of $|T|$ as large as $|4.67|$ is very nearly zero; therefore the X_i and Y_j come from different distributions.

Transformation of Skew Distribution CurvesInto Normal Ones

If the equation of distribution is $u = f(x)$ where x is the substructure boundary variable, then the theoretical cumulative frequency or probability that x will occur is given by

$$P\{x\} = \Phi(f(x)).$$

The derivative of this function

$$dP\{x\} = p\{x\} dx = \phi(f(x)) df(x)$$

or

$$p\{x\} = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(f(x))^2}{2}\right) f'(x)$$

shows that $f(x)$ is normally distributed rather than the variable x itself as in a normal distribution.

If a function $g(x)$ can be found of the type $f(x) = \frac{g(x) - g(\xi)}{\sigma}$ where $g(x)$ includes no unknowns, then $g(x)$ is normally distributed about the constant $g(\xi)$ with the standard deviation σ . A function $g(x)$ is suitable as a transformation function if the transformed observations versus the cumulative frequency plotted on probability graph paper yield a random scatter about the straight line

$$u = \frac{g(x) - g(\xi)}{\sigma} \quad (\text{cf. Fig. 2d}).$$

It was found that the skew distributions of the substructure boundaries per grain for various strains and alloys could be transformed into normal distributions by the use of the fourth root

function. Then $f(x) = \frac{x^{1/4} - \xi^{1/4}}{\sigma}$

$$\text{and } p\{x\} = \frac{1}{4\sigma x^{3/4}} \phi\left(\frac{x^{1/4} - \xi^{1/4}}{\sigma}\right).$$

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By definition $\xi^{\frac{1}{4}}$ is the median of the distribution $x^{\frac{1}{4}}$ and therefore ξ is the median of the distribution x . Since probability paper is constructed such that the median of the distribution is the ordinate at the abscissa $P = 50\%$, the median may be obtained directly from the transformed cumulative frequency plot (cf. Fig. 2d). The mean, of course, may be found by the usual methods of averaging the original data.

To find the mode of the distribution the distribution function is differentiated:

$$p'\{x\} = \frac{3}{-16 \sigma x^{1/4}} \Phi(u) + \frac{1}{4 \sigma x^{3/4}} \frac{d \Phi(u)}{du} \frac{du}{dx}$$

and since $\frac{d \Phi(u)}{du} = -u \Phi(u)$,

$$p'\{x\} = \frac{\Phi(u)}{-16 \sigma x^{3/2}} \left(\frac{3}{x^{1/4}} + \frac{u}{\sigma} \right).$$

The mode is then found by solving for the maximum:

$$\frac{3}{x^{1/4}} + \frac{\frac{1}{x^2} - \xi^{\frac{1}{4}}}{\sigma^2} = 0$$

and
$$x^{\frac{1}{2}} = \xi^{\frac{1}{4}} - 3\sigma^2.$$

Since another property of probability graph paper is that at $P = 15.9\%$, $x^{\frac{1}{4}} = \xi^{\frac{1}{4}} - \sigma$ and at $P = 84.1\%$, $x^{\frac{1}{4}} = \xi^{\frac{1}{4}} + \sigma$, the standard deviation σ of the transformed distribution may also be readily obtained from the graph* (cf. Fig. 2d).

In summary, the characteristics of the original distribution are:

- (1) the mode $= (\xi^{1/4} - 3\sigma^2)^4 = x_1$,
- (2) the median $= (\xi^{1/4})^4 = x_2$,
- (3) the mean $=$ the average $= x_3$,

* In general at $P = 15.9\%$, $\sigma = \xi - x$, but here we are concerned with the transformed distribution.

Transformed Cumulative Frequency Curves

FIG. C1 TRANSFORMED CUMULATIVE FREQUENCY PLOT FOR PURE Ni AT VARIOUS PRESTRAINS

FIG. C2 TRANSFORMED CUMULATIVE FREQUENCY PLOT FOR 1 ATOMIC PERCENT Ti IN Ni FOR VARIOUS PRESTRAINS

FIG. C3 TRANSFORMED CUMULATIVE FREQUENCY PLOT
FOR 2 ATOMIC PERCENT Ti IN Ni FOR VARIOUS
PRESTRAINS

FIG. C4 TRANSFORMED CUMULATIVE FREQUENCY PLOT
FOR 5 ATOMIC PERCENT Co IN Ni FOR VARIOUS
PRESTRAINS

FIG. C5 TRANSFORMED CUMULATIVE FREQUENCY PLOT FOR 20 ATOMIC PERCENT Co IN Ni FOR VARIOUS PRESTRAINS

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