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HIGHER HARMONICS OF RADIAL VIBRATIONS IN SHORT
HOLLOW CYLINDERS OF BARIUM TITANATE

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ABSTRACT

This paper discusses the forbidden and allowed harmonics of a hollow cylinder of barium titanate vibrating in the radial mode.

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307 - 1

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2

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HIGHER HARMONICS OF RADIAL VIBRATIONS IN SHORT HOLLOW CYLINDERS OF BARIUM TITANATE

In a previous memorandum¹, it was noted that the resonance condition for radial vibrations in short hollow cylinders of barium titanate was given by the following relation:

$$\frac{\left(\frac{\omega a}{v}\right) K_0\left(\frac{\omega a}{v}\right) - (1 - \sigma) K_1\left(\frac{\omega a}{v}\right)}{\left(\frac{\omega a}{v}\right) J_0\left(\frac{\omega a}{v}\right) - (1 - \sigma) J_1\left(\frac{\omega a}{v}\right)} = \frac{\left(\frac{\omega b}{v}\right) K_0\left(\frac{\omega b}{v}\right) - (1 - \sigma) K_1\left(\frac{\omega b}{v}\right)}{\left(\frac{\omega b}{v}\right) J_0\left(\frac{\omega b}{v}\right) - (1 - \sigma) J_1\left(\frac{\omega b}{v}\right)}$$

where a and b are outer and inner radii, σ is Poisson ratio and J_0 , J_1 , K_0 , K_1 are Bessel functions of the zeroth and first order and first and second kind. From this it was concluded that the function

$$f\left(\frac{\omega r}{v}\right) = \frac{\left(\frac{\omega r}{v}\right) K_0\left(\frac{\omega r}{v}\right) - (1 - \sigma) K_1\left(\frac{\omega r}{v}\right)}{\left(\frac{\omega r}{v}\right) J_0\left(\frac{\omega r}{v}\right) - (1 - \sigma) J_1\left(\frac{\omega r}{v}\right)}$$

must be a recurring function in the sense that there must be at least two roots for every value of the function for the resonant conditions to be satisfied. A plot of this function is shown in Figure 1. The first U-shaped part of this curve corresponds to the first harmonic and it is seen that there is a first harmonic resonance for every possible value of the ratio of the radii a/b . The other portions of the curve correspond to higher harmonics. It was stated in the previous memo that almost all of the higher harmonics are forbidden for any particular ratio a/b . This is not strictly correct and needs some explanation.

Considering the next section of the curve, and regarding it as representing the second harmonic, resonant conditions will be satisfied by the two points B_1 and A_1 and also by the points B_2 and A_2 . The ratio of these pairs of points A_1/B_1 and A_2/B_2 is just the ratio a/b for which the points satisfy resonant conditions. It is seen that the limits of the ratio a/b for which resonant conditions can be satisfied for the second harmonic are $\infty > a/b > 2.63$, i.e. $5.39/2.05$. This means that for any element which has a ratio a/b greater than 2.63, the second harmonic is permissible.

¹ Technical Memorandum 267-54-51, Radial Vibrations in Short Hollow Cylinders of Barium Titanate.

4

FIGURE 1- PLOT OF FUNCTION $f\left(\frac{\omega r}{v}\right)$ WHICH OCCURS IN RESONANCE CONDITION.

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FIGURE 2 — ALLOWED VALUES OF a/b FOR THE FIRST SIX HARMONICS

6

Regarding the next section of the curve as representing the third harmonic, we see that resonant conditions are satisfied by such pairs of points as B_3 and A_3 and B_4 and A_4 . An examination shows that the ratios of a to b for which the resonant condition can be satisfied for the third harmonic are $\infty > a/b > 4.18$ and $2.63 > a/b > 1.59$.

The above data and similar data for higher harmonics is represented in Figure 2, in which the shaded areas represent the values of a/b for which resonant conditions can be satisfied.

Experimentally these results can be tested and have been tested out to and including the third harmonic for the three ratios of a/b 8.30, 2.85 and 2.13. These are the three elements mentioned in the previous memo. Returning to Figure 1, if one pairs off the corresponding values of B and A one then determines not only the ratios a/b but also the resonant frequencies since $B = \frac{\omega b}{v}$ and $A = \frac{\omega a}{v}$. A detailed examination of the chart then allows one to predict not only the existence of higher harmonics but also allows one to calculate the frequency of this harmonic. For the ratio 8.30, one predicts that all of the first three harmonics exist. For the ratio 2.85, one predicts the existence of the first and second harmonics but not the third. For the ratio 2.13, one predicts the existence of the first and third harmonic but not the second. The results of measurement are shown in Table I.

TABLE I

Comparison of measured and predicted resonant frequencies for the first three harmonics of the elements with a/b ratios 8.30, 2.85 and 2.13

a/b	1st Harmonic		2nd Harmonic		3rd Harmonic	
	Measured	Calculated	Measured	Calculated	Measured	Calculated
8.30	105,220	106,100	264,120	266,000	420,720	430,000
2.85	83,380	82,400	282,810	280,000	None	- - - -
2.13	73,840	73,300	None	- - - -	329,660	332,000

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307 S

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