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A GRAPHIC SOLUTION FOR GAMMA SHIELDING

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July 24, 1956

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ABSTRACT

An equation for calculating gamma radiation through shielding is derived, which is applicable to practical monitoring and equipment-design problems. A graphic solution of the equation is included, as well as examples of its use and tables of values for specific conditions.

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INTRODUCTION

The need for preparation for control of gamma radiation is common to any installation working with radionuclides. Monitors and design engineers must calculate expected radiation doses to operating personnel and specify shielding requirements to maintain established radiation tolerances. In some cases it is necessary to determine the maximum safe working time under a specific set of conditions.

The people in Health Chemistry at Berkeley are responsible for safe handling of radionuclides. They have found that existing formulas and systems for gamma-radiation determination are either insufficient for an adequate answer, or are quite tedious to handle.

This paper presents a graphic solution to the problem that has proved to be a valuable short cut for monitors and design engineers at the University of California Radiation Laboratory.

In the hope that those who may use this method will first satisfy themselves as to the validity of the system, and further, be in a position to correct the graph as better experimental data are available, a derivation of the formula and calculated factors from which the graph was prepared are included.

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† Now with Atomic Power Equipment Dept., General Electric Co., San Jose, California.

DERIVATION OF FORMULA

Consider a point source in air of a single radioactive substance that emits gamma photons of only one energy. The intensity rate of radiation, I_n , at a given distance n from this source then depends on the rate of emission of photons, the energy of the photons, the absorption of photon energy by air, and the reduction in intensity due to distance only. We have

$$I_n \propto \frac{CPE\mu_a}{4\pi n^2}, \quad (1)$$

where C is the true disintegration rate of the substance,

P is the ratio of photons to total disintegrations,

E is the photon energy, and

μ_a is the linear coefficient of absorption by air, corrected for the fraction of initial energy carried off by photons undergoing Compton scattering.

If a shield is interposed between the source and the point of intensity measurement (receiver), then the receiver intensity rate R is given by

$$R = I_n B e^{-\mu_o t}, \quad (2)$$

where B is the dose build-up factor, dependent upon shield material and thickness, and on photon energy. The total linear coefficient μ_o , described below, is based on a narrow, linear beam, in which case scattered photons are assumed lost from the beam. In practical problems, a broad beam and shield must be considered. Therefore there is a build-up in beam intensity, measured at any specific point, from photons deflected to that point.

μ_o is the linear coefficient for the shielding material, and

t is the shield thickness.

For simpler manipulation, one can set

$$10^{-t/s} = e^{-\mu_o t}, \quad (3)$$

in which case s is that thickness of shielding which will reduce the source intensity by a factor of ten:

$$s = \frac{1}{\mu_o \log e} = \frac{1}{(\frac{\mu_o}{\rho}) \rho \log e},$$

where $\frac{\mu_o}{\rho}$ is the mass coefficient of absorption of the shielding material,

and ρ is the density of the shielding material.

Combining Eqs. (1), (2), and (3), we get

$$R \propto \frac{CPE \mu_a B}{4\pi n^2 10^{t/s}}, \quad \text{or}$$

$$R = \frac{kCPE \mu_a B}{n^2 \cdot 10^{t/s}}, \quad (4)$$

where k is the constant of proportionality, n is the total distance between source and receiver, and t is the shield thickness.

If R is in milliroentgens per hour (mr/hr),

C is the true curies of material,

E is in million electron volts (Mev),

μ_a is per centimeter of distance for air at 20°C,

n is in feet,

t and s are in inches,

and $1 \text{ Mev} = 1.6 \cdot 10^{-6} \text{ erg}$,

$1 \text{ roentgen} = 83.8 \text{ ergs/g of air}$,

density of air at 20°C = $1.205 \cdot 10^{-3} \text{ g/cm}^3$,

$1 \text{ curie} = 3.7 \cdot 10^{10} \text{ dis/sec}$,

then $k = 1.81 \cdot 10^8$.

In the more practical case where the source emits gamma photons of various energies, the total receiver intensity rate is than the summation of the receiver intensity rates of the photons of each energy.

For preparation of the graph, Eq. (4) may be rearranged in the form

$$\frac{n^2 R}{PC} = \frac{kEB \mu_a}{10^{t/s}}, \quad (5)$$

in which the left-hand components are quite readily available from the conditions of the problem. The right-hand components are functions of shielding material, shield thickness, and photon energy.

Tables I and II contain factors computed to prepare the graphic solution for lead shielding for energies from 0.5 to 3.0 Mev. If shielding other than lead is considered, a different set of factors is involved. However, an approximate solution can be obtained from this graph by replacing the actual thickness of shielding material by an equivalent thickness of lead. The equivalent thickness t_e may be taken as

$$t_e = t_a \frac{\rho_x}{\rho_L}, \text{ where}$$

t_a is the actual thickness of shielding material,
 ρ_x is the density of this material, and
 ρ_L is the density of lead.

PROCEDURE FOR GRAPHIC SOLUTION

Referring to Eq. 5 and the graph (Fig. 1), we see that a solution consists of determining the single unknown factor among E, t, and F, F being a combination of four independent variables.

To use the graph:

1. List the five known factors among E, t, n, R, P, and C in their proper units. If the nuclide is known, E and P can be found in various charts and tables of nuclides.

2. Set out to determine F from $F = \frac{n^2 R}{PC}$;

(a) if all factors of F are known, the intersection of F with E or t on the graph is the solution for the unknown factor;

(b) if F is not completely known, the intersection of E and t on the graph determines the value for F, and the unknown factor in F can then be computed.

In most problems requiring a solution for t, t should be taken as the next higher whole inch above the intersection of F and E to provide a margin of safety, since the graphic solution does not contain a safety factor. If a more accurate value for t is desired, interpolation between t lines must be made horizontally, on a line of constant energy.

Fig. 1. Graphic solution of $R = \frac{kEB\mu_aCP}{n^2 10^{t/s}}$ for lead shielding in the form

$$F = \frac{n^2 R}{PC} = \frac{kEB\mu_a}{10^{t/s}}$$

EXAMPLES

1. Problem: Find R if C = 40 millicuries, E = 2 Mev, P = 60%, n = 2 feet, and t = 2 inches of lead.

Solution: All given factors are in their proper units except C.
40 millicuries = 0.04 curie,

$$F = \frac{n^2 R}{PC} = \frac{2^2 \times R}{0.6 \times 0.04},$$

$$R = 0.006 F.$$

In the upper section of the graph, F = 1500 at the intersection of E = 2 and t = 2. Thus

$$R = 0.006 \times 1500 = 9 \text{ mr/hr.}$$

2. Problem: A mixture of 3 radionuclides is to be stored in a lead container. R must not exceed 6 mr/hr at 1 foot. The mixture contains:

$$C_1 = 10 \text{ curies, } E_1 = 2.5 \text{ Mev, } P_1 = 30\%,$$

$$C_2 = 10 \text{ curies, } E_2 = 1 \text{ Mev, } P_2 = 100\%,$$

$$C_3 = 100 \text{ curies, } E_3 = 0.5 \text{ Mev, } P_3 = 100\%;$$

find t.

Solution; In addition to the C, E, and P values above,

$$n = 1 \text{ foot, and}$$

$$R = 6 \text{ mr/hr.}$$

Although 6 mr/hr is the limit imposed on the radiation from all three nuclides, first examine to see what thickness of lead will reduce the radiation from each nuclide to less than 6 mr/hr. If one nuclide requires appreciably more shielding than the other two, a lead container designed to adequately shield this nuclide will then be sufficient to shield the mixture. The solution for t should be checked to make sure the summation of radiation from each nuclide does not exceed 6 mr/hr.

$$F_1 = \frac{1^2 \times 6}{0.3 \times 10} = 2,$$

$$F_2 = \frac{1^2 \times 6}{1 \times 10} = 0.6,$$

$$F_3 = \frac{1^2 \times 6}{1 \times 100} = 0.06;$$

t_1 and t_2 can be found from the intersection of F and E on the upper graph, t_3 from the lower graph.

$$8 < t_1 < 9,$$

$$5 < t_2 < 6,$$

$$2 < t_3 < 3.$$

Since t_1 is appreciably larger than either t_2 or t_3 , try $t = 9$ inches as the solution. From the graph for $t = 9$,

$$F'_1 = 1.05,$$

$$F'_2 = 3.5 \cdot 10^{-4},$$

$$F'_3 = \text{less than } 10^{-6}.$$

From these values of F and $R = \frac{FPC}{n^2}$ we have

$$R'_1 = \frac{1.05 \times 0.3 \times 10}{1^2} = 3.15 \text{ mr/hr},$$

$$R'_2 = \frac{3.5 \times 10^{-4} \times 1 \times 10}{1^2} = 0.0035 \text{ mr/hr},$$

$$R'_3 < \frac{10 \times 1 \times 100}{1^2} = 0.0001 \text{ mr/hr}.$$

Total	$R \approx$	3.15 mr/hr
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A similar examination shows that a system with 8 inches of lead would exceed the total radiation of 6 mr/hr. Therefore 9 inches is necessary.

3. Problem: If R must not exceed 6 mr/hr at 1 foot, determine the thickness of lead required to shield the following mixture.

$$C_1 = 10 \text{ curies, } E_1 = 2.7 \text{ Mev, } P_1 = 46\%,$$

$$C_2 = 10 \text{ curies, } E_2 = 2.0 \text{ Mev, } P_2 = 100\%.$$

Solution:

$$F_1 = \frac{1}{0.46 \times 10} = 1.3,$$

$$F_2 = \frac{1}{1 \times 10} = 0.6$$

From the graph we find for these values of F and E

$$t_1 = 9,$$

$$8 < t_2 < 9.$$

Since the first nuclide alone requires at least 9 inches and the second nuclide requires nearly as much, the mixture will probably require 9.5 or 10 inches of lead. Let us first examine the radiation of the second nuclide through 9 inches of lead. From the graph we read

$$F'_2 = 0.43 \text{ and}$$

$$R'_2 = 0.43 \times 1 \times 10 = 4.3 \text{ mr/hr.}$$

Thus at 9 inches, the total radiation would be $6 + 4.3 = 10.3 \text{ mr/hr}$, which is too high. Let

$$t = 9.5 \text{ inches,}$$

$$F''_1 = 0.76,$$

$$F''_2 = 0.235, \text{ and}$$

$$R''_1 = 0.76 \times 0.46 \times 10 = 3.5 \text{ mr/hr,}$$

$$R''_2 = 0.235 \times 1 \times 10 = 2.4 \text{ mr/hr,}$$

$$\text{Total } R = 5.9 \text{ mr/hr.}$$

There, 9.5 inches of lead is acceptable, but if cost will permit, 10 inches would be preferred.

4. Problem: 2000 curies of Co^{60} are to be shipped in a 4-inch-thick uranium container. The container will be placed in a wooden box, four feet cubed. The surface radiation must not exceed 0.1 r/hr . How much lead should be added to the uranium container?

Solution:

$$C = 2000 \text{ curies,}$$

$$\left. \begin{array}{l} E_1 = 1.3 \text{ Mev, } P_1 = 100\% \\ E_2 = 1.1 \text{ Mev, } P_2 = 100\% \end{array} \right\} \text{ from nuclear data for } \text{Co}^{60}.$$

Since E_1 and E_2 are nearly equal, we may assume all photons are 1.3 Mev. Then

$$P = 200\%,$$

$$n = 2 \text{ feet,}$$

$$R = 0.1 \text{ r/hr} = 100 \text{ mr/hr.}$$

We can neglect any shielding effect by the wood.

$$F = \frac{2^2 \times 100}{2 \times 2000} = 0.1.$$

From the graph for $F = 0.1$ and $E = 1.3$, $7 < t < 8$. By interpolation t is 7.6 inches.

If $t_a = 4$ inches, the actual thickness of uranium, then, t_e , the equivalent lead thickness, is

$$t_e = t_a \frac{\rho_U}{\rho_{Pb}},$$

where $\rho_U = 18.7$, the density of uranium, and $\rho_{Pb} = 11.3$, the density of lead;

$$t_e = 4 \times \frac{18.7}{11.3} = 6.6.$$

If we accept the total shielding thickness as 7.6 inches of lead, then the lead to be added is $7.6 - 6.6 = 1$ inch.

Let us check the answer, using the actual Co^{60} photon energies:

$$F'_1 = \frac{2^2 \times R'_1}{1 \times 2000} = 0.002 R'_1,$$

$$F'_2 = \frac{2^2 \times R'_2}{1 \times 2000} = 0.002 R'_2.$$

From the graph, taking $t = 7.6$, $E_1 = 1.3$ and $E_2 = 1.1$,

$$F'_1 = 0.10,$$

$$F'_2 = 0.02; \text{ then}$$

$$R'_1 = 50 \text{ mr/hr},$$

$$R'_2 = 10 \text{ mr/hr},$$

Total $R = 60 \text{ mr/hr}$. Therefore the additional 1 inch of lead makes the container safe.

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Table I

Factors for graph that are dependent only on energy and shielding material

$$\text{Basic equation: } R = \frac{kEB\mu_a CP}{n^2 10^{t/s}}$$

Where: R = received gamma intensity rate through lead shielding (mr/hr)

E = photon energy (Mev)

B = build-up factor for lead

μ_a = linear coefficient of absorption by air (Cm^{-1})

C^a = true curies of specific nuclide

P = ratio of specific photon to total disintegrations

n = total distance between source and receiver (ft)

t = thickness of lead shielding (in.)

s = that thickness of lead which will reduce the gamma intensity rate by a factor of ten (in.)

$\frac{\mu_o}{\rho}$ = total mass absorption coefficient for lead (Cm^2/g)

ρ = density of lead (g/Cm^3)

E	μ_a^*	$\frac{\mu_o^*}{\rho}$	$s = \frac{1}{2.54 \cdot 11.34 \log e \left(\frac{\mu_o}{\rho} \right)}$
0.5	$3.51 \cdot 10^{-5}$	0.153	0.523
0.6	3.49	0.119	0.672
0.7	3.46	0.100	0.799
0.8	3.41	0.0863	0.926
1.0	3.33	0.0708	1.13
1.25	3.19	0.0598	1.34
1.5	3.04	0.0527	1.52
1.75	2.95	0.0481	1.66
2.0	2.85	0.0455	1.76
2.25	2.78	0.0439	1.82
2.5	2.69	0.0428	1.87
3.0	2.57	0.0414	1.93

* μ_o and $\frac{\mu_o}{\rho}$ are taken from Radiological Health Handbook -

Kinsman (1955).

Table II

Factors for graph that are dependent on energy shielding material and thickness

$$R = \frac{kEB\mu_a GP}{n^2 10^{t/s}} \text{ can be rearranged to } F = \frac{n^2 R}{PC} = \frac{kEB\mu_a}{10^{t/s}}$$

Values of B computed from data in Radiological Health Handbook - Kinsman (1955). (Definitions and units of the factors are as given in Table I)

E	Et	$\mu_x - \frac{t}{s \log e}$	B	$10^{t/s}$	$F = \frac{kEB\mu_a}{10^{t/s}}$
0.5	1	4.41	1.73	82.0	67.0
	2	8.82	2.16	$6.75 \cdot 10^3$	1.02
	3	13.2	2.53	$5.55 \cdot 10^5$	$1.45 \cdot 10^{-2}$
	4	17.6	2.72	$4.57 \cdot 10^7$	$1.93 \cdot 10^{-4}$
	5	22.1	2.72	$3.75 \cdot 10^9$	$2.30 \cdot 10^{-6}$
0.6	1	3.42	1.80	30.8	221.
	2	6.86	2.34	951.	9.32
	3	10.3	2.80	$2.93 \cdot 10^4$	0.362
	4	13.7	3.24	$9.06 \cdot 10^5$	$1.36 \cdot 10^{-2}$
	5	17.1	3.56	$2.79 \cdot 10^7$	$4.83 \cdot 10^{-4}$
	6	20.6	4.07	$8.61 \cdot 10^8$	$1.79 \cdot 10^{-5}$
0.7	1	2.88	1.78	17.8	437.
	2	5.76	2.37	318.	32.7
	3	8.64	2.87	$5.68 \cdot 10^3$	2.22
	4	11.5	3.32	$1.01 \cdot 10^5$	0.143
	5	14.4	3.77	$1.81 \cdot 10^6$	$9.12 \cdot 10^{-3}$
	6	17.3	4.20	$3.24 \cdot 10^7$	$5.67 \cdot 10^{-4}$
	7	20.2	4.62	$5.75 \cdot 10^8$	$3.52 \cdot 10^{-5}$
0.8	1	2.49	1.75	12.0	718.
	2	4.97	2.34	145.	79.9
	3	7.46	2.85	$1.74 \cdot 10^3$	8.09
	4	9.95	3.32	$2.09 \cdot 10^4$	0.784
	5	12.4	3.76	$2.51 \cdot 10^5$	$7.39 \cdot 10^{-2}$
	6	14.9	4.20	$3.02 \cdot 10^6$	$6.86 \cdot 10^{-3}$
	7	17.4	4.63	$3.63 \cdot 10^7$	$6.29 \cdot 10^{-4}$
	8	19.9	5.05	$4.37 \cdot 10^8$	$5.71 \cdot 10^{-5}$
1.0	1	2.04	1.70	7.69	1330.
	2	4.08	2.28	59.2	232.
	3	6.12	2.80	455.	37.1
	4	8.16	3.31	$3.50 \cdot 10^3$	5.70
	5	10.2	3.78	$2.69 \cdot 10^4$	0.846
	6	12.2	4.21	$2.07 \cdot 10^5$	0.123
	7	14.3	4.66	$1.59 \cdot 10^6$	$1.80 \cdot 10^{-2}$
	8	16.3	5.09	$1.22 \cdot 10^7$	$2.51 \cdot 10^{-3}$
	9	18.4	5.53	$9.40 \cdot 10^8$	$3.54 \cdot 10^{-4}$
	10	20.4	5.94	$4.37 \cdot 10^9$	$4.95 \cdot 10^{-5}$

Table II(cont'd)

E	t	$\mu_x = \frac{t}{s \log e}$	B	$10^{t/s}$	$F = \frac{kEB\mu_a}{10^{t/s}}$
1.25	1	1.72	1.65	5.60	2120.
	2	3.44	2.20	31.1	506.
	3	5.17	2.73	173.	113.
	4	6.89	3.23	984.	23.7
	5	8.62	3.71	$5.52 \cdot 10^3$	4.85
	6	10.3	4.17	$3.09 \cdot 10^4$	0.973
	7	12.1	4.66	$1.73 \cdot 10^5$	0.194
	8	13.8	5.11	$9.71 \cdot 10^5$	$3.80 \cdot 10^{-2}$
	9	15.5	5.55	$5.43 \cdot 10^6$	$7.37 \cdot 10^{-3}$
	10	17.2	5.96	$3.05 \cdot 10^7$	$1.41 \cdot 10^{-3}$
1.5	1	1.52	1.58	4.57	2850.
	2	3.04	2.13	20.8	843.
	3	4.56	2.65	95.1	230.
	4	6.07	3.13	435.	59.4
	5	7.59	3.61	$1.99 \cdot 10^3$	15.0
	6	9.11	4.10	$9.08 \cdot 10^3$	3.73
	7	10.6	4.58	$4.13 \cdot 10^4$	0.915
	8	12.1	5.06	$1.89 \cdot 10^5$	0.221
	9	13.7	5.56	$8.59 \cdot 10^5$	$5.34 \cdot 10^{-2}$
	10	15.2	6.02	$3.94 \cdot 10^6$	$1.26 \cdot 10^{-2}$
1.75	1	1.39	1.53	4.00	3570.
	2	2.77	2.04	16.0	1190.
	3	4.16	2.55	63.8	373.
	4	5.54	3.06	255.	112.
	5	6.93	3.57	$1.02 \cdot 10^3$	33.6
	6	8.32	4.05	$4.09 \cdot 10^3$	9.24
	7	9.70	4.54	$1.63 \cdot 10^4$	2.60
	8	11.1	5.05	$6.53 \cdot 10^4$	0.722
	9	12.5	5.54	$2.61 \cdot 10^5$	0.198
	10	13.9	6.04	$1.05 \cdot 10^6$	$5.40 \cdot 10^{-2}$
2.0	1	1.31	1.50	3.71	4170.
	2	2.62	1.99	13.8	1490.
	3	3.93	2.48	51.1	501.
	4	5.24	2.98	190.	162.
	5	6.56	3.49	703.	51.2
	6	7.87	4.00	$2.61 \cdot 10^3$	15.8
	7	9.18	4.51	$9.68 \cdot 10^3$	4.80
	8	10.5	5.04	$3.59 \cdot 10^4$	1.45
	9	11.8	5.56	$1.33 \cdot 10^5$	0.431
	10	13.1	6.08	$4.94 \cdot 10^5$	0.127

Table II (cont'd)

E	t	$\mu_x = \frac{t}{s \log e}$	B	$10^{t/s}$	$F = \frac{kEB\mu_a}{10^{t/s}}$
2.25	1	1.27	1.46	3.54	4660.
	2	2.53	1.93	12.5	1740.
	3	3.80	2.42	44.5	616.
	4	5.06	2.91	158.	209.
	5	6.33	3.43	560.	69.3
	6	7.59	3.96	$1.98 \cdot 10^3$	22.6
	7	8.86	4.50	$7.02 \cdot 10^3$	7.26
	8	10.1	5.04	$2.48 \cdot 10^4$	2.30
	9	11.4	5.62	$8.79 \cdot 10^4$	0.723
	10	12.7	6.24	$3.12 \cdot 10^5$	0.226
2.5	1	1.23	1.47	3.43	5210.
	2	2.47	1.93	11.8	1990.
	3	3.70	2.39	40.4	720.
	4	4.93	2.85	139.	250.
	5	6.17	3.37	476.	86.0
	6	7.40	3.90	$1.64 \cdot 10^3$	29.0
	7	8.63	4.47	$5.61 \cdot 10^3$	9.69
	8	9.87	5.06	$1.93 \cdot 10^4$	3.19
	9	11.1	5.68	$6.59 \cdot 10^4$	1.05
	10	12.3	6.30	$2.27 \cdot 10^5$	0.338
3.0	1	1.19	1.40	3.30	5920.
	2	2.39	1.82	10.9	2340.
	3	3.58	2.27	35.8	884.
	4	4.77	2.77	118.	327.
	5	5.97	3.28	389.	118.
	6	7.16	3.82	$1.29 \cdot 10^3$	41.4
	7	8.35	4.40	$4.23 \cdot 10^3$	14.5
	8	9.54	5.04	$1.40 \cdot 10^4$	5.03
	9	10.7	5.70	$4.59 \cdot 10^4$	1.73
	10	11.9	6.40	$1.52 \cdot 10^5$	0.588
0.5	0	0	1.00	1.00	3170.
0.6					3790.
0.7					4380.
0.8					4930.
1.0					6020.
1.25					7210.
1.5					8250.
1.75					9340.
2.0					10,300.
2.25					11,300.
2.5					12,200.
3.0					13,900.