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ORNL
CENTRAL FILES NUMBER

56-4-189

DATE: April 19, 1956

SUBJECT: Thermal Expansion Stress in
Clad Spherical Shells

TO: W. R. Gall

FROM: R. F. Hughes

COPY NO. 476

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A spherical shell clad with a dissimilar material is stressed as it is heated (or cooled) because of a difference in coefficient of thermal expansion. These stresses in the circumferential direction are given by:

$$(1) * \sigma_L = \frac{(P - P_N)}{2} \frac{a}{t_L} ; \quad \sigma_s = \frac{P_N}{2} \left[\frac{a}{t_s} + \frac{t_L}{t_s} \right]$$

Where, σ_L = liner stress

σ_s = shell stress

a = sphere internal radius

t_L = liner thickness

t_s = shell thickness

P = sphere internal pressure

P_N = pressure at boundary of dissimilar materials

From an examination of equations (1), it is clear that the liner is in compression or tension depending upon whether the pressure at the boundary of the materials is greater or less than the internal pressure of the sphere; on the other hand the shell always experiences tension.

The following analysis is concerned with the derivation of an expression for P_N whereby the sphere thermal expansion stresses can then be determined.

If the materials do not separate at the boundary under stress, the total circumferential strain of liner and shell must be equal where they contact.

* $a \gg t_s, t_L$

This strain is the increment in boundary radius under stress divided by the original boundary radius length, i.e. $\Delta a_B / a_B$.

An expression relating stress to strain for shell and liner may be written which states that the strain due to pressure is equal to the total strain minus thermal strain.

$$(2) \quad E_s (\Delta a_B / a_B - \alpha_s \Delta T) = \frac{P_N}{2} \left[\frac{a}{t_s} + \frac{t_L}{t_s} \right] (1 - \nu)$$

$$(3) \quad K_L (\Delta a_B / a_B - \alpha_L \Delta T) = \left[\frac{P - P_N}{2} \right] \frac{a}{t_L} (1 - \nu)$$

Where, α_s = shell coefficient thermal expansion

α_L = liner coefficient thermal expansion

ν = poisson's ratio

ΔT = temperature change from stress-free condition

E_s = shell elasticity modulus

K_L = ratio of liner stress to strain = σ_L / ϵ_L

Since the sphere wall stress is equal in two principal coordinate directions, the strain of equations (2) and (3) has been adjusted by the factor $1 - \nu$.

If the liner stress is within its elastic limit, K_L of equation (3) is the liner elasticity modulus. It is more generally defined as the ratio of stress to strain for simple one-dimensional loading. Since the relative thickness of clad material to shell is usually small, the liner may easily be stressed beyond

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its yield point; K_L must then be determined from a stress-strain diagram of the liner material.

The boundary pressure is determined by subtracting equation (3) from (2) and solving for P_N .

$$[\alpha_L - \alpha_s] \Delta T = \frac{P_N}{2} (1 - \nu) \left[\frac{a}{t_L} \frac{1}{K_L} + \frac{a}{t_s} \frac{1}{E_s} + \frac{t_L}{t_s} \frac{1}{E_s} \right] - \frac{P a}{2 t_L K_L} (1 - \nu)$$

$$(4) \quad P_N = \left[\frac{2 E_s (\alpha_L - \alpha_s) \Delta T}{1 - \nu} \frac{t_s}{a} + \frac{P E_s t_s}{K_L t_L} \right] \frac{1}{\left[1 + \frac{t_L}{a} + \frac{E_s t_s}{K_L t_L} \right]}$$

Normal procedure for using equation (4) is to first make an estimate of liner strain ϵ_L , determine K_L from stress-strain diagram, solve for P_N and liner stress σ_L , and then make a check of the strain estimate.

Graphical representation of equation (4) is presented in an attached figure for a carbon steel shell clad with a stainless steel liner, temperature equal 600°F, and internal pressure equal 2000 psia.

In addition to the tangential stress given by equation (1) the shell is subjected to a radial compression stress which is a maximum at the boundary and equal to $-P_N$. The maximum shear stress is given by:

$$\sigma_s (\text{SHEAR}) = \left[\sigma_s (\text{TANGENT}) - \sigma_s (\text{RADIAL}) \right] \frac{1}{2} = \frac{P_N}{4} \left[2 + \frac{a}{t_s} + \frac{t_L}{t_s} \right]$$

The corresponding shear stress for the liner is given by:

$$\sigma_L (\text{SHEAR}) = \frac{1}{2} \left[\frac{(P - P_N)}{2} \frac{a}{t_L} + P_N \right]$$

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SHELL
LINER
T
P
 α_L
 α_S

A-212 CARBON STEEL
STAINLESS 347
600°F
2000 PSI
 $9.82 \times 10^{-6} \text{ } 1/^{\circ}\text{F}$
 $7.23 \times 10^{-6} \text{ } 1/^{\circ}\text{F}$

EFFECT OF DIFFERENCE IN THERMAL EXPANSION
STAINLESS LINER IN CARBON STEEL SPHERICAL SHELL

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