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TRANSIENT CONDUCTION IN NUCLEAR REACTOR FUEL PLATES

by

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Abstract

The problem of transient conduction in an unclad fuel plate with negligible surface resistance was solved analytically for the case wherein simultaneous step changes are made in the volume distributed heat generation rate and the surface temperatures. The solutions for the central (maximum) fuel temperature and the space-mean temperature are presented in graphs. A solution for the case of an arbitrary number of successive step changes was obtained by superposition (example), and an approximate solution for the case of arbitrarily and continuously varying heat release rate and surface temperatures coupled to the reactivity was discussed.

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INTRODUCTION

The problem of transient conduction of heat in solid bodies having internally-distributed volume heat sources has become important with the advent of nuclear reactors employing solid fuel elements. In the general case, the rate of heat release G expressed in heat units per unit time per unit volume and the boundary temperatures T_b vary arbitrarily with time. In principle, the problem can be solved for any case howsoever complicated by numerical methods employing difference equations. In practice, however, it often becomes necessary, in order to satisfy the convergence requirements for the difference equations, to select time increments so short that an inconveniently large number of calculations must be made in order to obtain a solution. It is therefore of interest to develop a semi-analytical method free from the convergence requirement. The problem selected is the restricted case of transient conduction following step changes in boundary conditions in unclad fuel plates and rods immersed in a coolant providing little resistance to heat transfer. After a solution for the case of a single set of simultaneous step changes in G and T_b is obtained, the solution for an arbitrary number of successive step changes is readily obtained by superposition. Finally, the solution for the case of continuously varying G and T_b may be obtained, free from any convergence requirement on the time intervals chosen, by representing the continuous changes as step changes at arbitrarily selected instants of time. The series so obtained may be reduced to integrals if desired.

FUEL PLATE IN THE STEADY STATE

An unclad fuel plate operating in the steady state with uniformly distributed volume heat source G and uniformly distributed surface temperature

$T_{b,0}$. At time t_1 the generation rate and surface temperature are changed instantaneously to G_1 and $T_{b,1}$. It is required to express the fuel temperature distribution as a function of time t , position x , heat release rates G_1 and G_0 , surface temperatures $T_{b,1}$ and $T_{b,0}$, plate half-width b , fuel density ρ , specific heat c , and thermal conductivity k . The plate is cooled equally on both sides so that the surface temperatures are equal and all temperature distributions are symmetrical about the midplane of the plate. Let the origin of coordinates be located at the midplane, with the x -direction taken normal to the surfaces and the y - and z -directions taken parallel. Assume that temperature gradients in the y - and z -directions are negligibly small and that heat-flow in the x -direction only need be considered.

The general equation for unsteady heat transfer is

$$\frac{\partial T(x,t)}{\partial t} = \frac{G(x,t)}{\rho c} + \alpha \nabla^2 T(x,t) \quad 1$$

where $T(x,t)$ denotes the temperature as a function of position (x) and time (t), $G(x,t)$ is the local rate of heat release due to the nuclear reaction in heat units per unit time per unit volume, ρ denotes the density, c the specific heat, and α the thermal diffusivity of the fuel. The physical properties are assumed uniform in space and constant in time.

Let the system be in a stationary state corresponding to some steady rate of heat release $G(x, t < t_1) = G_0$ which is uniform in space, and to some steady coolant temperature $T(b, t < t_1) = T_{b,0}$. Equation 1 thereupon takes the form

$$G_0 / \rho c + \alpha \nabla^2 T_0(x) = 0 \quad 2$$

where, for brevity

$$T_0(x) = T(x, t < t_1) \quad 3$$

The solution is

$$T_0(x) - T_{b,0} = \frac{G_0}{2k} (b^2 - x^2) \quad 4$$

where k denotes the thermal conductivity of the fuel, and b is the half-width of the fuel plate.

FUEL PLATE IN TRANSIENT STATE

Consider now the problem of finding the temperature distribution in a fuel plate that has experienced a number of finite step changes in the generation rate G and the surface temperature T_b , as illustrated in Table I.

TABLE I.
Schedule of Step Changes

t	$t < t_1$	$t_1 < t < t_2$	$t_2 < t < t_3$		$t_p < t < t_{p+1}$
T_b	$T_{b,0}$	$T_{b,1}$	$T_{b,2}$		$T_{b,p}$
G	G_0	G_1	G_2		G_p

The problem is to find the temperature distribution $T(x, t_p < t < t_{p+1})$ in terms of convenient functions of $G_0, G_1, \dots, G_p, T_{b,0}, T_{b,1}, \dots, T_{b,p}, x, t$, physical properties, and plate width b .

First, the temperature is expanded into a series of p terms corresponding to the p intervals of Table I.

$$T(x, t) = \sum_{m=0}^m T_m(x, t) \quad . \quad . \quad . \quad 5$$

where

$$T_m(x, t) \begin{cases} = 0; & t < t_m \\ = f_m(x, t); & t > t_m \end{cases} \quad 6$$

and where the $f_m(x, t)$ are functions whose forms are to be determined.

Clearly, in time interval $t < t_1$

$$T_0(x, t) = T(x, t < t_1) = T_0(x) \quad 7$$

as defined by Equation 4. Now, referring again to Equation 5, we have, following the first set of step changes

$$T(x, t_1 < t < t_2) = T_0(x) + T_1(x, t) \quad 8$$

Putting this into Equation 1 and using Equation 2 gives

$$\frac{\partial T_1(x, t)}{\partial t} = \frac{G_1 - G_0}{\rho c} + a \nabla^2 T_1(x, t) \quad 9$$

Now let $T_1(x, t)$ be split into a time dependent part $T'_1(x, t)$ and a time independent part $T_1(x)$.

$$T_1(x,t) = T_1'(x,t) + T_1(x) \quad 10$$

where $T_1(x)$ is required to satisfy the condition that

$$\alpha \nabla^2 T_1(x) + \frac{G_1 - G_0}{\rho c} = 0 \quad 11$$

By analogy to Equation 4, the solution of Equation 11 is

$$T_1(x) - T_1(b) = \frac{(G_1 - G_0)}{2k} [b^2 - x^2] \quad 12$$

$T_1(b)$ is found by eliminating $T_1(x,t)$ between Equations 10 and 8, setting $x = b$, and arbitrarily taking $T_1'(b,t) = 0$.

$$T_1(b) = T(b, t_1 < t < t_2) - T_0(b) \quad 13$$

But $T(b, t_1 < t < t_2) = T_{b,1}$ by definition and $T_0(b) = T_{b,0}$ by Equation 4. Thus

$$T_1(b) = T_{b,1} - T_{b,0} \quad 14$$

Returning now to Equation 10, we set this into Equation 8 and thence into Equation 1, and use Equation 11 to eliminate $\alpha \nabla^2 T_1(x)$, whence

$$\frac{\partial T_1'(x,t)}{\partial t} = \alpha \nabla^2 T_1'(x,t) \quad 15$$

The boundary conditions are

$$T_1'(b,t) = 0$$

as chosen above, and

$$T_1'(x, \infty) = 0$$

since $T_1'(x,t) \rightarrow T_1'(b,t)$ as $t \rightarrow \infty$. Now $T_1'(x,0)$ is found from Equation 10.

$$T_1'(x, t_1) = T_1(x, t_1) - T_1(x) \quad 16$$

But, by Equation 8, since $T(x, t_1) = T_0(x)$

$$T_1(x, t_1) = 0 \quad 17$$

Hence, by Equations 12 and 14

$$T_1'(x, t_1) = -(T_{b,1} - T_{b,o}) - \frac{(G_1 - G_o)}{2k} (b^2 - x^2) \quad 18$$

Equation 15 is a well-known type. Its solution may be put into the form

$$T_1'(x, t) = \sum_{n=1}^{\infty} B_n \cos \left[(2n-1) \frac{\pi x}{2b} \right] \exp \left[- \frac{(2n-1)^2 \pi^2 a (t - t_1)}{4b^2} \right] \quad 19$$

The functions $\cos(2n-1) \frac{\pi x}{2b}$ are orthogonal. It follows that

$$B_n = \frac{2}{\pi} \int_{x=-b}^{x=b} T_1'(x, t_1) \cos \left[(2n-1) \frac{\pi x}{2b} \right] d\left(\frac{\pi x}{2b}\right) \quad 20$$

Substituting from Equation 18, one finds

$$B_n = -(T_{b,1} - T_{b,o}) \frac{4(-1)^n}{\pi(2n-1)} - \frac{(G_1 - G_o)b^2}{2k} \frac{32(-1)^n}{\pi^3(2n-1)^3} \quad 21$$

Combining Equations 21, 20, 14, 12 and 10 gives

$$T_1(x, t > t_1) = (T_{b,1} - T_{b,o}) \left\{ 1 - f_1 \left[\frac{x}{b}, z_1 \right] \right\} + \frac{(G_1 - G_o)b^2}{2k} \left\{ \left(1 - \frac{x^2}{b^2} \right) - f_2 \left[\frac{x}{b}, z_1 \right] \right\} \quad 22$$

$$\text{where } z_m = \frac{a(t - t_m)}{b^2} \quad 23$$

and

$$f_1 \left[\frac{x}{b}, z_m \right] = - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)} \cos \left[(2n-1) \frac{\pi x}{2b} \right] \exp \left[- \frac{(2n-1)^2 \pi^2 z_m}{4} \right] \quad 24$$

$$f_2 \left[\frac{x}{b}, z_m \right] = - \frac{32}{\pi^3} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)^3} \cos \left[(2n-1) \frac{\pi x}{2b} \right] \exp \left[- \frac{(2n-1)^2 \pi^2 z_m}{4} \right] \quad 25$$

SUPERPOSITION OF SUCCESSIVE STEP CHANGES

By induction from Equation 22

$$T_m(x, t) = (T_{b,m} - T_{b,m-1}) \left[1 - f_1 \left(\frac{x}{b}, z_m \right) \right] + \frac{(G_m - G_{m-1}) b^2}{2k} \left[1 - f_2 \left(\frac{x}{b}, z_m \right) \right] \quad 26$$

Substituting into Equation 5, and cancelling like terms gives the general solution for an arbitrary number of step changes in G and T_b .

$$T(x, t_p < t < t_{p+1}) = T_{b,p} + \frac{G_p b^2}{2k} \left[1 - \left(\frac{x}{b} \right)^2 \right] - \sum_{m=1}^p (T_{b,m} - T_{b,m-1}) f_1 \left(\frac{x}{b}, z_m \right) - \sum_{m=1}^p \frac{(G_m - G_{m-1}) b^2}{2k} f_2 \left(\frac{x}{b}, z_m \right) \quad 27$$

Both f_1 and f_2 approach zero with increasing t , hence, in the limit, if no further changes in G and T_b occur, $T(x, t)$ approaches

$$T_{b,p} + \frac{G_p b^2}{2k} \left[1 - \left(\frac{x}{b} \right)^2 \right], \text{ as required.}$$

The midplane temperature is that one most frequently of interest.

It is obtained by setting $x/b = 0$ in Equation 27. The functions $f_1(0, Z)$ and $f_2(0, Z)$ for plates are graphed in Figure 1.

The maximum thermal stress in fuel plates is proportional to the difference between the maximum temperature and the space-mean temperature $\bar{T}(t)$ defined as follows:

$$\bar{T}(t) = \frac{1}{b} \int_0^b T(x,t) dx \quad 28$$

Substitution from Equation 27 followed by integration gives

$$\begin{aligned} \bar{T}(t_p < t < t_{p+1}) &= T_{b,p} + \frac{G_{pb}^2}{3k} - \sum_{m=1}^{m=p} (T_{b,m} - T_{b,m-1}) \\ \bar{f}_1 \left[\frac{z}{b} \right] &= \sum_{m=1}^{m=p} \frac{(G_m - G_{m-1})b^2}{2k} \bar{f}_2 \left[\frac{z}{b} \right] \end{aligned} \quad 29$$

Where

$$\bar{f}_1(z) = \frac{8}{\pi^2} \sum_{n=1}^{n=\infty} \frac{1}{(2n-1)^2} \exp \left[\frac{-(2n-1)^2 \pi^2 z}{4} \right] \quad 30$$

$$\bar{f}_2(z) = \frac{64}{\pi^4} \sum_{n=1}^{n=\infty} \frac{1}{(2n-1)^4} \exp \left[\frac{-(2n-1)^2 \pi^2 z}{4} \right] \quad 31$$

The functions $\bar{f}_1(z)$ and $\bar{f}_2(z)$ are graphed in Figure 2.

FUEL-PLATE TRANSIENT - AN EXAMPLE

Consider a fuel plate operating in the steady state at an uniform heat generation rate G_0 and with uniform surface temperature $T_{b,0}$. It is desired to increase the generation rate. However, it is also desired that the central fuel temperature shall, in the final steady state, equal that before the changes are made. In addition it is desired that the central fuel temperature shall not at any time exceed a specified maximum value $T_{\max}$. The transition from the old to the new operating conditions is to be effected by lowering, at

time t_1 , the surface temperatures to that required for the final steady operating condition, and later, at time t_2 , increasing G to the new required value.

The problem is to determine the time lag $t_2 - t_1$ required between the first and second changes in order that $T(0,t)$ shall not exceed $T_{\max}$. The schedule of changes is as follows:

TABLE II
Schedule of Step Changes

t	$t < t_1$	$t_1 < t < t_2$	$t_2 < t$
G	G_o	$G_1 = G_o$	$G_2 = KG_o$
T_b	$T_{b,o}$	$T_{b,1} = T_{b,2}$	$T_{b,2}$

The new required surface temperature $T_{b,2}$ is determined by setting $G_2 = KG_o$, $t = \infty$, $x = 0$, and $T(0, \infty) = T(0, t < t_1) = T_{b,o} + \frac{G_o b^2}{2k}$ into

Equation 27.

$$T_{b,o} + \frac{G_o b^2}{2k} = T_{b,2} + \frac{KG_o b^2}{2k} \quad 32$$

whence

$$T_{b,2} = T_{b,o} + (1 - K) \frac{G_o b^2}{2k} \quad 33$$

Using this, Equation 27 takes the form, for $t > t_2$, $x = 0$

$$\begin{aligned} T(0, t > t_2) &= T_{b,o} + (1 - K) \frac{G_o b^2}{2k} + \frac{KG_o b^2}{2k} \\ &- \left[T_{b,o} + (1 - K) \frac{G_o b^2}{2k} - T_{b,o} \right] f_1(0, z_1) \\ &- \left[\frac{(KG_o - G_o) b^2}{2k} \right] f_2(0, z_2) \end{aligned} \quad 34$$

Simplifying, subtracting $T(0, t < t_1)$ from both sides, and rearranging gives

$$\begin{aligned} T(0, t > t_2) - T(0, t < t_1) &= T_{b,o} + \frac{G_o b^2}{2k} - T(0, t < t_1) \\ + (K - 1) \frac{G_o b^2}{2k} f_1(0, Z_1) &= (K - 1) \frac{G_o b^2}{2k} f_2(0, Z_2) \end{aligned} \quad 35$$

But, as required,

$$T(0, t < t_1) = T_{b,o} + \frac{G_o b^2}{2k} \quad 36$$

by Equation 4; it follows that the sum of the first three quantities on the right is zero. Dividing by $\frac{G_o b^2 (K - 1)}{2k}$ one has

$$\frac{T_o(0, t > t_2) - T(0, t < t_1)}{(K - 1) \frac{G_o b^2}{2k}} = \phi(Z_1, Z_2) = f_1(0, Z_1) - f_2(0, Z_2) \quad 37$$

The function $\phi(Z_1, Z_2)$ is graphed in Figure 3 versus Z_1 with $(Z_1 - Z_2) = \frac{a(t_2 - t_1)}{b^2}$ as parameter. Note that $f_2 \left[0, \frac{a(t - t_2)}{b^2} \right] = 1$ for all $t < t_2$. The time lag between step changes in surface temperature and generation rate required to limit the maximum temperature to any specified level is readily computed from Figure 3. Note that the temporal relations here are independent of the magnitude of the increase in G as long as the final steady-state central temperature is required to equal the initial central temperature.

It is seen that the central temperature maximizes at $Z_1 \sim 0.15$ with surprisingly little change as the time lag $t_2 - t_1$ increases. The maximum temperature, however, decreases rapidly until, at a time lag of 0.09 Z units (dimensionless), the maximum temperature does not exceed the original central temperature. This time lag corresponding to this "eversafe" condition was

computed for a fuel plate 0.060 inches thick of three materials: aluminum, zirconium, and stainless steel. The corresponding time lags $t_2 - t_1$ are of the order of 0.5, 5, and 7 milliseconds respectively. These times are short relative to other time lags in reactor cooling systems, and so negligible in regard to scheduling the operations involved in transitions in power level. On the other hand, the temperature history is extremely sensitive to variations in the time lag, and this could have important effects on the power level history if there is a strong coupling between temperature and reactivity, as is sometimes the case.

FUEL ROD IN TRANSIENT STATE

The analysis of the case of transient conduction in an infinitely long fuel rod having uniformly distributed heat liberation rate G and surface temperature T_R proceeds analogously to that for the flat plate. The result may be put into the form:

$$T(r, t_p < t < t_{p+1}) = T_{R,p} + \frac{G_p(R^2 - r^2)}{4k} - \sum_{m=1}^{m=p} (T_{R,m} - T_{R,m-1}) f_1 \left[\frac{r}{R}, z_m \right] - \sum_{m=1}^{m=p} \frac{(G_m - G_{m-1})R^2}{4k} f_2 \left[\frac{r}{R}, z_m \right] \quad 38$$

where R is rod radius, r is radial position, m denotes time interval following with change in conditions, and where

$$z_m = \frac{\alpha(t - t_m)}{R^2} \quad 39$$

$$f_1 \left(\frac{r}{R}, z \right) = \sum_{n=1}^{n=\infty} \frac{2J_0(M_n r/R)}{M_n J_1(M_n)} \exp(-M_n^2 z) \quad 40$$

$$f_2\left(\frac{r}{R}, z\right) = \sum_{n=1}^{\infty} \frac{4J_2(M_n)J_0(M_n r/R)}{M_n^2 J_1^2(M_n)} \exp(-M_n^2 z) \quad 41$$

and where the M_n are roots of J_0 . As with the flat plates, the central temperature is that one of most interest. It is found by setting $r/R = 0$.

The functions $f_1(0, z)$ and $f_2(0, z)$ have been graphed in Figure 4.

The space-mean temperature $\bar{T}$ is found to take the form:

$$\begin{aligned} \bar{T}(t_p < t < t_{p+1}) &= T_{R,p} + \frac{G_p R^2}{8k} - \sum_{m=1}^{m=p} (T_{R,m} - T_{R,m-1}) \\ \bar{f}_1(z_m) &= \sum_{m=1}^{m=p} \frac{(G_m - G_{m-1}) R^2}{4k} \bar{f}_2(z_m) \end{aligned} \quad 42$$

where

$$\bar{f}_1(z) = \sum_{n=1}^{\infty} (4/M_n^2) \exp(-M_n^2 z) \quad 43$$

$$\bar{f}_2(z) = \sum_{n=1}^{\infty} \frac{8J_2(M_n)}{M_n^3 J_1(M_n)} \exp(-M_n^2 z) \quad 44$$

The functions $\bar{f}_1(z)$ and $\bar{f}_2(z)$ are graphed in Figure 5.

TRANSIENT CONDUCTION IN A FUEL ROD--AN EXAMPLE

Consider a case analogous to that treated in the preceding example. Let a reactor employing cylindrical fuel elements be operating in the steady state with heat release rate G_0 uniform over a cross section of fuel rod and with surface temperature $T_{R,0}$ uniform around the periphery. Assume conduction in the axial direction to be negligible so that the preceding equations will apply. It is desired to increase the power level but it is required that the final steady-state central fuel temperature shall equal that of the initial steady state. Suppose the transition is to be accomplished by first decreasing the surface temperature to $T_{R,1}$ at time t_1 (by adjusting coolant

temperature, flow rate, etc.) and then increasing the generation rate by a factor K at time t_2 , i.e., $G_2 = KG_0$. The schedule of changes is the same as in Table II of the previous example. The algebra proceeds analogously and the result takes the corresponding form

$$\phi(Z_1, Z_2) = \frac{T(0, t > t_2) - T(0, t < t_1)}{(K - 1) G_0 R^2 / 4k} = f_1(0, Z_1) - f_2(0, Z_2) \quad 45$$

The function $\phi(Z_1, Z_2)$ for rods is graphed in Figure 6 versus Z_1 with $Z_2 - Z_1 = \alpha(t_2 - t_1) / R^2$ as parameter. The temperature appears to maximize at approximately 0.11 Z units (dimensionless) following the initiation of the transient. A time lag $t_2 - t_1$ between decrease of surface temperature and increase of generation rate corresponding to a value of $Z_1 - Z_2$ of 0.0625 units gives a case where the maximum temperature during the transient just equals the initial and final steady state temperatures. This is the condition desired if the system is operating initially at maximum allowable temperature. For a rod one inch in diameter and with physical properties approximately those of uranium ($\alpha \approx 1 \text{ cm}^2/\text{sec}$), the time lag required to achieve the "eversafe" condition is 1.0 sec. This time is short compared to other time lags encountered in the cooling systems of large, heterogeneous reactors, and so is probably negligible from an operational standpoint.

CONTINUOUS VARIATION OF GENERATION RATE AND SURFACE TEMPERATURE

Consider, for example, the transient behavior following the stepwise introduction of excess reactivity into a flat-plate, light-water moderated and cooled reactor having one group of delayed neutrons. The power level will exhibit the characteristic "prompt" rise, following which it will increase exponentially on a "stable" period until the increase in moderator temperature begins to reduce the reactivity appreciably.

The coolant temperature at any point will behave in a similar way, except there will be no prompt rise. For the case where the surface resistance to heat transfer is negligible ($T_b = T_c$), the solution proceeds as follows: The transient is initiated by introducing at time t_1 a stepwise change (the prompt rise, $G_1 - G_0$) in the power level into Equation 29 and computing the space-mean temperature of the fuel $\bar{T}(t_2)$ at each point in the reactor at the end of a short time interval. The amount of heat stored in the fuel ($2bpc\Delta\bar{T}$) is then computed. The remainder of the heat generated in the time interval $t_2 - t_1$ flows into the coolant. The corresponding changes in point-to-point values of T_b are made, and the resulting change in moderator mean density, reactivity, and power level are calculated in that order. The new G and the new T_b are inserted into Equation 29 to give $\bar{T}(t_3)$, and so on. The temperature history for the entire transient is thus obtained. The history of the fuel centerline temperature $T(0,t)$ at any point may be obtained from Equation 27. However, since both $G(t)$ and $T_b(t)$ are now known functions of time, the summations in Equation 27 are conveniently reduced to integrals by setting

$$T_{b,m} - T_{b,m-1} \approx \frac{dT_b(t_m)}{dt_m} \Delta t_m \quad 46$$

$$G_m - G_{m-1} \approx \frac{dG(t_m)}{dt_m} \Delta t_m \quad 47$$

and allowing Δt_m to approach zero, p to approach infinity, whereupon

$$T(0,t) = T_b(t) + \frac{G(t)b^2}{2k} - \int_{t_m=0}^{t_m=t} \frac{dT_b(t_m)}{dt_m} f_1 \left[0, \frac{a(t-t_m)}{b^2} \right] dt_m$$

$$- \int_{t_m=0}^{t_m=t} \frac{b^2 dG(t_m)}{2k dt_m} f_2 \left[0, \frac{a(t-t_m)}{b^2} \right] dt_m$$

48

The integrals are readily evaluated by numerical methods.

CONCLUSIONS

The general solution for transient, one-dimensional heat conduction in unclad fuel plates and rods with arbitrarily varying surface temperature and rate of internal generation of heat has been obtained in terms of series functions f and their integrals. The functions $f(0,Z)$ and $\bar{f}(Z)$, corresponding to centerline and space-mean temperatures respectively, were computed and presented in graphs. By means of these, tedious numerical calculations are avoided.

TABLE OF SYMBOLSRoman

b	Fuel plate half-width
B_n	nth coefficient of Fourier series, defined by Equation 21.
c	Specific heat of fuel
f	Functions defined by Equations 24, 25, 40, and 41.
$\bar{f}$	Functions defined by Equations 30, 31, 43, and 44.
G	Volume-source heat-liberation rate, heat units per unit volume per unit time.
k	Thermal conductivity of fuel
R	Radius of fuel rod
r	Radial coordinate in cylindrical geometry
t	Time
T	Temperature
x	Lateral coordinate in slab geometry.
Z	Dimensionless time modulus defined by Equations 23 and 39.

Greek

α	Thermal diffusivity of fuel, $\alpha = k/c$
∇^2	Laplacian
ρ	Density
ϕ	Dimensionless temperature function defined by Equations 37 and 45.

Subscripts and Superscripts

- 1,2,...m...p denote successive time intervals
- 0 denotes conditions prior to initiation of transient
- b denotes condition at surface of fuel plate
- R denotes condition at surface of fuel rod
- superbar, denotes space mean of variable on which it operates
- ' prime, denotes time dependent part of the functions $T_m(x,t)$.

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Fig. 1A. Temperature Transient Functions
for Plates.

20

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Fig. 1B. Temperature Transient Functions for Plates.

21

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Fig. 2. Space Mean Temperature Transient Functions for Plates.

Fig. 3. Temperature Transient in Reactor Fuel Plate.

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Fig. 4A. Central-Temperature
Transient Functions for Rods.

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Fig. 4B. Central-Temperature Transient Functions.
for Rods.

25

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Fig. 5. Space - Mean Temperature Transient Functions
for Rods.

26

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Fig. 6. Temperature Transient in Reactor Fuel Rod.

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