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Odd-Even Features in $^3\text{He} + ^3\text{H}$ Scattering

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Abstract: The single-channel resonating-group method is used to study effects of the Pauli principle on $^3\text{He} + ^3\text{H}$ scattering. Comparison is made with previous similar calculations for $d + ^3\text{He}$ scattering, and it is found that the Pauli principle affects the $s = 0$ state of the $^3\text{He} + ^3\text{H}$ system rather similarly to the way it affects the $s = 1$ state, whereas the Pauli principle affects the $s = 3/2$ state of the $d + ^3\text{He}$ system quite differently from the way it affects the $s = 1/2$ state. Mention is made of the possibility of observing similar effects of the Pauli principle in other nuclear systems.

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1. Introduction

In a recent resonating-group study of the $d + {}^3\text{He}$ and $d + {}^3\text{H}$ systems¹⁾, each system was found to exhibit a behavior in the channel-spin state $s = 3/2$ which is strikingly different from that exhibited in the channel-spin state $s = 1/2$. This behavior is a result of the Pauli exclusion principle, which is accounted for in resonating-group calculations through the use of a completely antisymmetrized wave function. Thus, in the $s = 3/2$ state it is difficult for either of the nucleons in the deuteron to approach closely to the mass-3 nucleus, which causes the ${}^3\text{He}$ or ${}^3\text{H}$ to act toward the deuteron as a spin-isospin saturated substructure. Therefore, in the $s = 3/2$ state the systems behave similarly to the $d + \alpha$ system. In the $s = 1/2$ state, however, it is much more likely for one of the nucleons in the deuteron to approach the mass-3 nucleus to form a cluster similar to a diffuse α particle. Thus, in the $s = 1/2$ state these systems have a similarity to the $p + \alpha$ system and even more so to the more diffuse ${}^3\text{He} + \alpha$ system.

In the present work we apply similar considerations to the ${}^3\text{He} + {}^3\text{H}$ interaction. A simple consideration of the effect of the Pauli principle on this system indicates that in both possible channel-spin states, $s = 0$ and $s = 1$, a nucleon from one of the mass-3 clusters is allowed to approach closely to the other cluster. Therefore it is expected not only that the ${}^3\text{He} + {}^3\text{H}$ interaction should be rather insensitive to channel spin²⁾, but also that the system should behave similarly to the $d + \alpha$ system and to the $d + {}^3\text{He}$ system in its $s = 3/2$ state. One of the methods used in ref.¹⁾ to compare the characteristics of cluster-cluster interactions from one system to another or between different channel-spin states of the same system was to compare the odd-even effects³⁻⁵⁾ exhibited by the calculated phase shifts. This method of comparison will also be used here.

In sect. 2 a brief description of the formulation of the resonating-group calculation for the ${}^3\text{He} + {}^3\text{H}$ system is presented. A phenomenological imaginary potential is included in the calculation in order to account approximately for open reaction channels. The parameters of the imaginary potential were not varied, but were simply taken from a similar calculation for the ${}^3\text{He} + {}^3\text{He}$ system⁶⁾, which reproduced ${}^3\text{He} + {}^3\text{He}$ elastic scattering differential cross sections quite well. Sect. 3 contains a comparison of the calculated ${}^3\text{He} + {}^3\text{H}$ elastic scattering differential cross sections with experimental data^{7,8)}. In sect. 4 the calculated phase shifts are presented along with a discussion of the odd-even effect and a comparison with the $d + {}^3\text{He}$ system. Finally, sect. 5 contains conclusions from the present study and mention of the possibility of observing in other nuclear systems effects similar to those discussed here.

2. Brief description of the formulation

A rather detailed formulation of the single-channel resonating-group treatment of the ${}^3\text{He} + {}^3\text{H}$ system has been given in ref. 2); however, the present calculation differs in some respects from that described there. Here we use the nucleon-nucleon potential of refs. 1,9). This potential is somewhat different from that of ref. 2), and therefore the forms we use for the potentials V_D and V_C and for the kernel k_{ls} are not the same as given there. However, the required modifications of these quantities are not major and can be performed in a manner similar to that indicated in the appendix of ref. 9). The exchange constant u in the nucleon-nucleon potential is given the value 1.04 as it was in the ${}^3\text{He} + {}^3\text{He}$ analysis of ref. 6). The imaginary potential we employ here is that given in eqs. (1) and (2) of ref. 6), and the parameters of this potential are also taken from this reference. Thus, the radius and diffuseness of the potential are set equal to 3.3 fm and 0.5 fm, respectively,

and the strength U_0 and exchange parameter C_I of the potential are interpolated from tables 1-3 of ref. ⁶⁾. The actual values used for these parameters in the present calculation are given in table 1.

3. Cross sections

In fig. 1 we show, at three c.m. energies, comparisons of the calculated $^3\text{He} + ^3\text{H}$ elastic scattering differential cross sections with experimental data ^{7,8)}. There is reasonable agreement between the calculation and experiment, both in the general shape of the cross section and in the average magnitude. This is rather satisfying because, as we have mentioned, the parameters of the calculation are taken from a previous analysis ⁶⁾ of $^3\text{He} + ^3\text{He}$ scattering, and, therefore, no parameter adjustment has been carried out in the present calculation. There are, however, some differences in detail between the calculation and experiment. These differences can arise from several omissions in the calculation, such as specific distortion[†], Coulomb exchange^{††}, and spin-orbit interaction[‡]. These differences are unimportant for our present purpose, which is mainly to discuss effects produced by the Pauli exclusion principle.

[†]Inclusion of specific distortion improves the wave function in the region of strong cluster-cluster interaction. In ref. ¹⁰⁾ distortion in the $d + \alpha$ system is discussed, and in ref. ¹¹⁾ distortion in the $\alpha + \alpha$ system is discussed.

^{††}Coulomb-exchange effects have been studied in resonating-group calculations for the $d + ^3\text{He}$ and $d + ^3\text{H}$ systems ¹⁾, for the $p + \alpha$ system ⁹⁾, and for the $^3\text{He} + ^4\text{He}$ and $^3\text{H} + ^4\text{He}$ systems ¹²⁾.

[‡]A nucleon-nucleon spin-orbit potential has been included in resonating-group studies of the $p + \alpha$ and $n + \alpha$ systems ^{9, 13)} and of the $^3\text{He} + ^4\text{He}$ system ¹⁴⁾.

4. Phase shifts and odd-even behavior

In table 1 we present, along with the parameters of the imaginary potential and the calculated total reaction cross section σ_R , the calculated complex phase shifts $^{2s+1}\delta_\ell$ at the three c.m. energies of fig. 1. Although the present calculation does not yield complete agreement with experiment, we feel that these phases should be useful as starting values in phase-shift searches and would help to avoid ambiguities inherent in such searches. In particular, the calculated complex phases exhibit a property which might be especially useful; namely, the property that the odd- ℓ phases in the spin-singlet state are very nearly equal to the odd- ℓ phases in the spin-triplet state. This property is a consequence of the use of a near Serber nucleon-nucleon potential in this system and can be deduced from an examination of table 1 of ref. 2).

In fig. 2 we compare[†], at 50 MeV (c.m.), phase shifts calculated here for $^3\text{He} + ^3\text{H}$ scattering with previously calculated¹⁾ phase shifts for $d + ^3\text{He}$ scattering. The odd-even structure of the phases, which is quite apparent in fig. 2, is a manifestation of the Pauli principle. The odd-even structure of the $s = 0$ state of the $^3\text{He} + ^3\text{H}$ system is similar to both that of the $s = 1$ state of the $^3\text{He} + ^3\text{H}$ system and that of the $s = 3/2$ state of the $d + ^3\text{He}$ system. For these three cases the structure also resembles that of the $d + \alpha$ system³⁾ and indicates that the even- ℓ effective interaction is more attractive than the odd- ℓ effective interaction. These characteristics give quantitative support to the qualitative picture of the $^3\text{He} + ^3\text{H}$ interaction presented in

[†]For convenience, no imaginary potential was included in these calculations. The absorptive strengths needed for the two systems of fig. 2 would not appreciably affect the values of the real parts of the phases.

sect. 1.

We conclude this section with some general remarks about the odd-even effect in cluster-cluster effective interactions. It is possible to discuss the nature of this effect in the framework of the harmonic oscillator cluster model¹⁵⁾, or, equivalently¹⁶⁾, in terms of the harmonic oscillator shell model. This possibility exists because resonating-group calculations¹⁷⁾ have shown that, in the region of configuration space where the clusters strongly penetrate each other, the wave function over a wide range of energies closely resembles that of the oscillator model. It is perhaps clearest to continue the discussion in terms of a specific example, for which purpose we consider the $^3\text{He} + \alpha$ system. In the oscillator cluster model the low-energy states of ^7Be are described by relative oscillations between the unexcited ^3He and α clusters. The smallest number n of relative oscillation motion quanta allowed by the Pauli principle for this type of oscillation is three, and this implies that the lowest-energy states of ^7Be are represented by $^3\text{He} + \alpha$ relative oscillations having relative orbital angular momenta ℓ equal to the odd values 1 and 3.[†] For the unexcited clusters to undergo even- ℓ relative oscillations, n must be equal to at least four^{††}, and therefore these even- ℓ states lie higher in energy than the odd- ℓ states having $n = 3$. Consequently, the $^3\text{He} + \alpha$ effective interaction is more attractive in odd- ℓ states than in even- ℓ states. The evident generalization is that the number n for the lowest-energy states of the oscillator cluster model determines the strength of the even- ℓ relative to the odd- ℓ cluster-cluster effective interaction. Thus, when n is even, the effective interaction in even- ℓ states is more attractive than that in

[†]The same statement in shell-model language is that the states of highest spatial symmetry contained in the lowest-energy configuration $(1s)^4(1p)^3$ have total orbital angular momenta 1 and 3.

^{††}In general, ℓ and n are related by $\ell = n, n-2, \dots, 1$ or 0 .

odd- ℓ states, and, when n is odd, the opposite relation holds.

Application of this idea to light nuclear systems shows that the systems $d + {}^3\text{He}$ ($s = 3/2$), $d + \alpha$, ${}^3\text{He} + {}^3\text{He}$, and ${}^3\text{He} + {}^3\text{H}$ have a more attractive interaction in even- ℓ states than in odd- ℓ states and that the systems $d + {}^3\text{He}$ ($s = \frac{p + {}^3\text{He}}{1/2}$), $p + \alpha$, and ${}^3\text{He} + \alpha$ have a more attractive interaction in odd- ℓ states than in even- ℓ states. This statement is consistent with phase-shift behavior found from resonating-group calculations for all these systems.

5. Conclusion

Reasonable reproductions of experimental ${}^3\text{He} + {}^3\text{H}$ elastic scattering differential cross sections have been obtained through a single-channel resonating-group calculation in which the parameters of an added phenomenological imaginary potential were taken from a previous analysis of ${}^3\text{He} + {}^3\text{He}$ scattering. Through a study of the odd-even structure of calculated phase shifts, effects of the Pauli principle on ${}^3\text{He} + {}^3\text{H}$ scattering have been investigated and have been compared with such effects on $d + {}^3\text{He}$ scattering. For ${}^3\text{He} + {}^3\text{H}$ scattering the Pauli principle has an influence on the $s = 0$ state similar to its influence on the $s = 1$ state, whereas for $d + {}^3\text{He}$ scattering the Pauli principle affects the $s = 3/2$ state quite differently from the way it affects the $s = 1/2$ state. We have also seen how the general nature of the odd-even structure can be obtained through simple cluster-model considerations.

Finally, the general ideas presented here can be applied to other systems. For example, the ${}^3\text{He} + {}^{15}\text{N}$ effective interaction (for both $s = 0$ and $s = 1$) should be more attractive in odd- ℓ states than in even- ℓ states, whereas the opposite behavior is expected for the ${}^3\text{He} + {}^{16}\text{O}$ effective interaction. This particular difference between these systems

should manifest itself through differences in the elastic differential cross sections, especially at backward angles. An experimental test of this prediction and of similar predictions for other nuclear systems should lead to a better understanding of the role played by the Pauli principle in nuclear scattering and reactions.

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TABLE I

Values of U_0 (MeV), C_I , σ_R (mb), and $^{2s+1}\delta_\ell$ (deg) for the $^3\text{He} + ^3\text{H}$ system at c.m. energies E (MeV).

E	3.91	9.955	13.85
U_0	0.15	0.42	1.00
C_I	0	0	0.2
σ_R	68	198	384
$^1\delta_0$	$-71.16 + i0.80$	$-118.83 + i2.58$	$-138.78 + i7.32$
$^1\delta_1$	$12.24 + i1.82$	$-4.62 + i4.79$	$-15.98 + i7.92$
$^1\delta_2$	$-1.14 + i0.37$	$-8.06 + i2.61$	$-15.19 + i8.11$
$^1\delta_3$	$1.91 + i0.08$	$29.54 + i5.02$	$55.06 + i13.86$
$^1\delta_4$	$0.24 + i0.01$	$7.65 + i0.67$	$20.41 + i6.10$
$^1\delta_5$	$0.01 + i0$	$0.83 + i0.06$	$2.72 + i0.44$
$^1\delta_6$		$0.19 + i0.01$	$0.83 + i0.12$
$^1\delta_7$		$0.02 + i0$	$0.12 + i0.01$
$^3\delta_0$	$-55.59 + i0.97$	$-101.64 + i2.82$	$-121.81 + i7.71$
$^3\delta_1$	$12.33 + i1.83$	$-4.47 + i4.79$	$-15.82 + i7.92$
$^3\delta_2$	$3.02 + i0.43$	$3.91 + i3.13$	$-1.68 + i9.11$
$^3\delta_3$	$1.91 + i0.08$	$29.63 + i5.04$	$55.25 + i13.89$
$^3\delta_4$	$0.24 + i0.01$	$8.23 + i0.74$	$23.06 + i7.20$
$^3\delta_5$	$0.01 + i0$	$0.82 + i0.05$	$2.71 + i0.44$
$^3\delta_6$		$0.18 + i0.01$	$0.79 + i0.12$
$^3\delta_7$		$0.02 + i0$	$0.12 + i0.01$

Figure Captions

Fig. 1. ${}^3\text{He} + {}^3\text{H}$ elastic scattering differential cross sections at c.m. energies of 3.91, 9.955, and 13.85 MeV. The curves show the results of the present calculation and the points show the data of refs. 7, 8).

Fig. 2. Phase shifts at a c.m. energy of 50 MeV vs the orbital-angular momentum l and the channel spin s . No imaginary potential was used in the calculations. The phases for the $d + {}^3\text{He}$ system are from ref. 1), and those for the ${}^3\text{H} + {}^3\text{He}$ system are from the present work.

Fig. 1

Fig. 2