2

This is an informal report intended primarily for internal or limited external distribution. (The opinions and conclusions stated are those of the author and may or may not be those of the laboratory.) This report is not to be given additional external distribution or cited in external documents without the consent of the author or LLL Technical Information Department.

UCID - 16418

24,999

LAWRENCE LIVERMORE LABORATORY

University of California/Livermore, California

THE "THETA FIELD LINER" CONCEPT

James W. Shearer

December, 1973

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

THE "THETA FIELD LINER" CONCEPT

James W. Shearer

Lawrence Livermore Laboratory, University of California
Livermore, California 94550

December, 1973

ABSTRACT

The implosion of a metallic liner is analyzed algebraically, computed numerically, and discussed qualitatively -- from the point of view of comparing an azimuthal B_θ magnetic field driver with the previously proposed axial B_z magnetic field driver. From all points of view, reasons are found for preferring the former (theta-field) concept.

I. INTRODUCTION

The implosion of metallic cylindrical shells ("liners") by means of capacitor-bank-produced high magnetic fields is well known.^{1,2} The concept has been expanded into a proposal to use a liquid lithium liner to compress and heat a deuterium-tritium plasma to fusion temperature.³ Calculations of this proposal have recently been done at NRL, using theta pinch (axial B_z field) geometry.⁴ The NRL group has coined the name "LINUS" for their system.

In the present report, we discuss the possibilities of a theta field liner, modeled after the capacitor bank experiments at Novosibirsk,⁵ where the driving magnetic field is in the z-pinch (azimuthal B_θ field) configuration. We shall compare the effects of the two magnetic fields (B_z and B_θ), and offer reasons why we believe that the B_θ field would be superior in a practical liner device.

The complications introduced by rotating the liner^{3,4} will be neglected in this report. We shall also not consider the recently proposed "cusp" geometry configuration.⁶

II. ANALYTICAL COMPARISONS

Consider a cylindrical volume whose outer radius R is assumed to be the inner surface of an external "coil" circuit (See Fig. 1). The outer surface of the liner, initially at radius r_0 , will move inward to radius r_1 at some later time, due to the magnetic forces. We shall consider here only the magnetic field circuit; we do not need the fluid dynamics of the shell.⁷ Also, we neglect skin depths (resistive effects) throughout this simplified analysis.

For the B_z case, we have

$$B_z = \mu \frac{I}{z} = 4\pi \frac{I}{z} \quad (1)$$

where I is the total current summed over all the coil turns, z is the length of the cylinder, $\mu = 4\pi$, and end effects are neglected. For the B_θ cases, we have:

$$B_\theta = \frac{\mu}{2\pi} \frac{I}{r} = 2 \frac{I}{r} \quad (2)$$

for the same conditions.

Next, we write down the total magnetic energy E'_m per unit length in the cylinder.

$$E'_m = \int_r^R (B^2/2\mu) 2\pi r dr = \frac{1}{4} \int_r^R B^2 r dr \quad (3)$$

For the axial B_z field case, the magnetic field is not a function of r (see Eq. 1) and we find:

$$E'_{mz} = \left(B_z^2 / 8 \right) (R^2 - r^2) \quad (4)$$

For the azimuthal B_θ field case, we substitute from Eq. 2, and obtain:

$$E'_{m\theta} = I^2 \ln(R/r) \quad (5)$$

Pushing the outer radius of the liner from the radius r_0 to the radius r_1 implies that work was done on the liner by the magnetic field. Let W' represent this work, in units of energy per unit length:

$$W' = - \int_{r_0}^{r_1} (B^2 / 2\mu) (2\pi r) dr = \frac{1}{4} \int_{r_1}^{r_0} B^2 r dr \quad (6)$$

Let us simplify our analysis by considering first the special case of a constant current I .⁸ In order to compare the two magnetic fields, we assume that the field value at the coil radius R is the same:

$$B_z(R) = B_\theta(R) \equiv B_R \quad (7)$$

A practical value for B_R might be 200 kilogauss for large pulsed coils.

In the B_z case, Eqs. (4) and (6) then lead to the following result:

$$\Delta E'_{mz} = W'_z = \left(B_R^2 / 8 \right) (r_0^2 - r_1^2) \quad (8)$$

The work done on the liner is exactly equal to the additional magnetic energy which fills in the space left behind. The work, which appears as kinetic energy of the liner, represents the useful part of the energy. The efficiency

ϵ can then be defined as,

$$\epsilon_z \equiv \frac{W'_z}{E'_{mz} + W'_z} = \frac{r_0^2 - r_1^2}{R^2 + r^2 - 2r_1^2} \quad (9)$$

In the B_θ case, Eqs. (5) and (6) lead to the following result.

$$\Delta E'_{m\theta} = W'_\theta = \frac{RB_R^2}{2} \ln(r_0/r_1) = I^2 \ln(r_0/r_1) \quad (10)$$

The same relationships between work and energy hold, so the efficiency ϵ becomes:

$$\epsilon_\theta \equiv \frac{W'_\theta}{E'_{m\theta} + W'_\theta} = \frac{\ln(r_0/r_1)}{\ln(Rr_0/r_1^2)} \quad (11)$$

If $r_0 = R$, then both efficiencies ϵ_z and ϵ_θ reduce to 0.50. However, in practical cases, the liner must be mounted inside the coil so that we have $r_0 < R$. A reasonable comparison can be made for this constant current approximation, by setting $r_0 = (3/4)R$. Figure 2 is a plot of the resultant efficiencies $\epsilon_z, \epsilon_\theta$ as a function of the ratio r_1/R . One can see from this plot and from an analysis of Eqs. (9) and (11) that the B_θ case is more efficient because ϵ_θ always approaches the value 0.5 as the radius r_1 shrinks to zero, whereas ϵ_z approaches the values $1/(1 + [R/r_0]^2)$, which takes the value 0.36 in this example.

It should be pointed out that the constant current approximation is not an equal energy comparison between the two magnetic fields. A comparison of Eqs. (8) and (10) shows that $W'_\theta > W'_z$ and similarly $E'_\theta > E'_z$. The physical reason for this is that the inner magnetic field $B(r)$ is always greater in the B_θ case. Therefore, we shall now make an equal energy comparison.

Assume a total stored energy E'_0 (entirely in the magnetic field), at the time when $r = r_0$. Then, at later times, the movement of the liner will increase its kinetic energy W' at the expense of the initial stored magnetic energy. Using circuit concepts,⁹ we can calculate the magnetic energy E' .

$$E' = E'_0 - W' = (1/2) L' I^2 = (1/2) \frac{\phi^2}{L'} \quad (12)$$

where L is the inductance per unit length and ϕ is the flux. Since flux ϕ is conserved, we can unite

$$W' = E'_0 - E' = E'_0 (1 - L'_0/L') \quad (13)$$

In the axial field (B_z) case, we use well-known inductance formulæ to obtain the constant energy efficiency η_z .

$$\eta_z \equiv \frac{W'_z}{E'_0} = \frac{r_0^2 - r_1^2}{R^2 - r_1^2} \quad (14)$$

Using the inductance formulæ for the azimuthal (B_θ) case, we find the efficiency η_θ at constant energy.

$$\eta_\theta \equiv \frac{W'_\theta}{E'_0} = \frac{\ln(r_0/r_1)}{\ln(R/R_1)} \quad (15)$$

Again, a reasonable comparison can be made by setting $r_0 = (3/4)R$ for the initial condition whence the energy E'_0 is entirely magnetic. Figure 3 is a plot of the resultant efficiencies η_z, η_θ as a function of the ratio r_1/R .

Once again the calculations show that the azimuthal (B_θ) field is more efficient than the axial (B_z) field. From the circuit analysis standpoint one says that the reason for this is that as $r_1 \rightarrow 0$, $L' \rightarrow \infty$ as in the B_θ case, whereas L' remains finite in the B_z case. Physically, what is happening in the B_θ case is that the magnetic energy is concentrated near

r_1 , whereas in the B_z case it is uniformly distributed throughout the volume. The local magnetic field at radius r_1 is, therefore, higher in the B_θ case. Thus, the energy is being used more efficiently.

These two simple circuit and energy considerations have demonstrated the qualitative improvement of efficiency one obtains by using a B_θ liner. However, for a more realistic calculation involving external circuits, resistive losses, etc., we shall forsake analytic methods and turn to the computer runs described in the next section.

III. COMPUTATIONAL COMPARISONS

The computer code that was used is "MAGPIE", written by R. E. Kidder and Alex Cecil.^{10,11} The equations in this code for lithium metal are similar to those previously used for copper,¹⁰ but with constants appropriate for lithium. The code is a magnetohydrodynamic calculation in cylindrical geometry, which can compute either a B_z - J_θ geometry, or a B_θ - J_z geometry, where J is the current density.

The initial geometry of these runs is based on early calculations of the NRL group,⁴ and is shown in Fig. 4. There is no initial magnetic field, the energy E_T for the problem is stored in a capacitor circuit which is connected to the insulating void space between the lithium liner and the container mockup. The value of the capacitor C per unit length of the cylinder was varied from run to run according to the equation:

$$C = \frac{1}{L_0} \frac{2}{\pi} T_{1/4}^2 \quad (16)$$

where L_0 is the initial inductance per unit length of the insulating void, and where $T_{1/4}$ is the quarter period of the initial circuit. It is of

interest to find out how slow a quarter period can be used with good efficiency. Then for each run the corresponding voltage V_c and charge Q_c on the capacitor are:

$$V_c = \sqrt{\frac{2E_T}{c}} = \frac{\pi}{2T_{1/4}} \sqrt{2L_0 E_T} \quad (17)$$

$$Q_c = \sqrt{2CE_T} = \frac{2T_{1/4}}{\pi} \sqrt{2 \frac{E_T}{L_0}} \quad (18)$$

The circuit is automatically crowbarred when the capacitor is discharged, thus trapping the energy E_T in the problem.

For the first series of runs, the total energy per cm E_T was chosen so that the maximum magnetic field at the container mockup would not exceed 200 kilogauss in any run. This value was chosen as a conceivable practical limit to the pulsed field such a container could survive without damage. The maximum field is approached most closely for fast circuits (low values of $T_{1/4}$), where all of the energy is injected before appreciable liner motion can start.

$$E_T \approx \left(\frac{B^2}{2\mu} \right) \pi (R^2 - r_0^2) = \frac{(2 \times 10^5)^2}{8\pi} \pi (50^2 - 46^2) \quad (19)$$

$$\approx 1.9 \times 10^{12} \frac{\text{ergs}}{\text{cm}} = 19 \frac{\text{Megajoules}}{\text{meter}}$$

Table I and Figs. 5-6 show some results from this first series of runs, plus a later series run at doubled E_T . The following conclusions have been drawn from these results.

First, in every case the B_θ field couples more of the capacitor bank energy into the lithium liner than the B_z field does, for equal energy and time constant.

Second, in every case the B_θ field produces a lower value for the maximum magnetic field at the coil than the B_z field does. This implies that if the code runs had been adjusted to produce equal magnetic fields at the coil, the margin of superiority of the B_θ field would have been even greater.

Third, for the cases of longer time constants, it is possible to couple more energy into the liner without exceeding the allowable magnetic field value at the coil. No detailed search for the optimum energy was conducted, but it appears likely that one can find parameters which will allow the coupling of over 50 megajoules/meter into the lithium liner without exceeding the 200 kilogauss limitation at the coil.

Figures 7 and 8 present detailed time histories of one particular MAGPIE code run. The liner thickens as it converges, and then comes apart after turnaround, due to the particular equation of state that was used. The magnetic field is influenced both by the amount of magnetic energy available and by the radius. Note how much higher the field is at the liner than at the container for this theta field example. The capacitor energy curve does not reach zero at the initial quarter period $T_{1/4}$ because the motion of the liner has increased the inductance. The maximum magnetic field energy is quite small compared to the maximum kinetic energy; the magnetic field can be described as a "coupling fluid" which links the capacitor bank energy with the moving liner. Similar graphs were made for the other code runs, but are not shown here.

TABLE I
Summary of Computational Runs
on System of Figure 4

Input Parameters		Maximum Kinetic Energy in Lithium Liner Megajoules/Meter		Maximum Magnetic Field at 50 cm Radius Outer Coil Kilogauss	
Capacitor Energy E_T Megajoules/Meter	Initial Quarter Period $T_{1/4}$ μsec	B_z Field	B_θ Field	B_z Field	B_θ Field
19.0	32	15.0	17.2	180	170
19.0	100	14.8	17.3	138	128
19.0	316	10.6	12.7	86	80
38.0	100	28.9	34.4	174	160
38.0	178	24.4	29.1	136	126
38.0	316	16.9	19.9	106	96

IV. SOME QUALITATIVE COMMENTS ON ENGINEERING PROBLEMS

Figure 9 is a schematic illustration of the way the voltage might be applied to the container by the power supplies. One finds that the container for the B_θ case is continuous around its perimeter; therefore, it should be possible to manufacture it to withstand a greater hoop stress than for the B_z case.

In both cases, it will probably be convenient to connect several power supplies in series to the container, in order to spread the electric fields out over several gaps. For example, for the B_θ problem shown in Figs. 7 and 8, the maximum electric field applied (at $t = 0$), was approximately 628 volts/cm, as measured in the axial direction. If the slots shown in Fig. 9b were spaced 50 cm apart, then each shot would have to sustain approximately 31.4 kV maximum. For the corresponding B_z problem we find

that the maximum electric field is approximately 100 volts/cm, as measured around the perimeter. If a single slot were used (instead of the double slot shown in Fig. 9a), one would then have 31.4 kV maximum voltage at the slot, similar to the B_θ case.

The time constant of 178 μ sec is more than long enough for capacitor bank circuits, but it is still too short for cheaper power supplies such as homopolar generators. A longer time constant could be obtained by scaling up to a larger size, or using a heavier liner.

V. CONCLUSIONS AND SUGGESTIONS

This document has demonstrated the advantages of a B_θ driving field for a heavy liner for confinement of hot plasmas for fusion. Many other topics remain to be explored and discussed. For the liner, one needs to study other materials, instabilities, and methods of creating the initial liner shell. In addition, methods of creating a hot, high density plasma inside the liner need much study, particularly by means of such recently developed technologies as electron beam accelerators and efficient long pulse CO_2 lasers.

In short, the B_θ -liner is superior to the B_z -liner because it can more efficiently couple greater energy into the liner due to the higher magnetic field at the liner and because it should permit greater allowable hoop stress in the outer container.

VI. REFERENCES

1. E. C. Gnare, J. Appl. Phys. 37, 3812 (1966).
2. D. Kachilla, F. Herlach, and T. Erber, Rev. of Scientific Instr. 41, 1, (1970).

3. E. Velikhov, Proceedings of the Fifth European Conference on Controlled Fusion and Plasma Physics (Grenoble, August, 1972).
4. Jay P. Boris, Ramy A. Shanny, and N. K. Winsor, "Generation of a Fusion Plasma in the LINUS 0-Pinch", Bulletin of the American Physical Society, 17, No. 11, 1029 (November, 1972).
5. S. G. Alikhanov, V. G. Belan, A. I. Ivanchenko, V. N. Karasjunk and G. N. Kichigin, Journal of Sci. Instr. (J. of Phys. E) 1, 543 (1968).
6. J. P. Boris, A. F. Robson, R. A. Shanny, P. J. Turchi, and W. K. Winsor, "Imploding Liner Fusion Device -- LINUS Flying Cusp", Bulletin of the American Physical Society, 18, No. 11, 1299 (1973).
7. Heinz Knoepfel, Pulsed Magnetic Fields (American Elsevier Publishing Co., New York, 1970), Chapter 9.
8. See Ref. 7, pages 128-129.
9. See Ref. 7, Chapter 8.
10. R. E. Kidder, "Compression of Magnetic Field Inside a Hollow Explosive-Driven Cylindrical Conductor", in Proceedings of the Conference on Mega-gauss Magnetic Field Generation by Explosives and Related Experiments, H. Knoepfel and F. Herlach, Eds. (EURATOM, Brussels, 1966) p. 37 ff.
11. Daniel Steinberg, R. E. Kidder, Alex B. Cecil, A One-Dimensional Magnetohydrodynamics Code, Lawrence Livermore Laboratory Report UCRL-14931 (1966).

Figure Captions

Figure 1: Definitions of radii used in analytical comparisons. The fixed inner radius of the container is at R . The outer surface of the moving liner is initially at r_0 , and is pushed inward to r_1 at a later time t .

Figure 2: Efficiencies ϵ_z and ϵ_θ of energy transfer to the liner in the constant current approximation. The initial radius ratio is $(r_0/R) = 0.75$.

Figure 3: Efficiencies η_z and η_θ of energy transfer to liner in constant energy approximation. The initial radius ratio $(r_0/R) = 0.75$.

Figure 4: Initial geometry of MAGPIE code computer problems, approximately to scale.

Figure 5: Results from computer runs: Maximum kinetic energy imparted to liner as a function of the initial time constant of the circuit. Curves are labeled with two different initial energies stored in the capacitor bank.

Figure 6: Results from computer runs: Maximum magnetic field at the fixed outer wall plotted as for Fig. 5.

Figure 7: Time history of a typical computation: Magnetic fields and radii versus time for a circuit where the initial capacitor energy was 38 megajoules/meter, and the initial quarter period was 178 μ sec.

Figure 8: Energy history versus time for the same computation as for Fig. 7, where E_c is the capacitor energy, E_k is the liner kinetic energy, and E_m is the magnetic field energy.

Figure 9: Schematic circuit connections to cylindrical container for the two cases of (a) B_z driving field, and (b) B_θ driving field.

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

(a) B_z - driven liner

(b) B_θ - driven liner

Figure 9

DISTRIBUTION

External

Dr. J. P. Boris
Dr. R. A. Shanny
Naval Research Laboratory
Washington, D. C. 20390

Prof. John Dawson,
Dept. of Physics
University of California at Los Angeles
Los Angeles, CA 90024

Dr. C. M. Fowler,
Los Alamos Scientific Laboratory
Los Alamos, NM 87544

Prof. Abraham Herzberg
Prof. George C. Vlases
University of Washington
Seattle, Washington, 98105

Dr. Kenneth Moses
Dr. A. Trivelpiece
U. S. Atomic Energy Commission
Division of Controlled Thermonuclear Research
Washington, D. C. 20545

Internal

D. S. Bailey	L-81	H. L. Sahlin	L-71
S. E. Bodner	L-81	E. Teller	L-0
R. J. Briggs	L-382	H. W. Van Ness	L-156
T. J. Burgess	L-156	J. G. Vidal	L-80
W. C. Condit	L-80	T. Wainwright	L-71
G. E. Cooper	L-71	J. R. Wilson	L-24
T. K. Fowler	L-382	J. W. Shearer (10)	L-80
E. Garelis	L-80	TID Files (5)	L-9
C. W. Hartman	L-387		
A. C. Haussmann	L-212		
R. N. Keeler	L-71		
R. E. Kidder	L-218		
W. L. Kruer	L-32		
W. A. Lokke	L-33		
J. S. Luce	L-504		
G. Maenchen	L-80		
D. A. Nowak	L-80		
J. H. Nuckolls	L-32		
J. S. Pettibone	L-80		
H. L. Reynolds	L-21		

NOTICE

"This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately-owned rights."