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A MEASUREMENT OF THE TRANSFER FUNCTION
OF A FAST CRITICAL ASSEMBLY

by

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ABSTRACT

The zero power transfer function of the ZPR-III mockup assembly of EBR-I, Mark III was obtained over a frequency range from 0.01 to 10 cycles per second. Various oscillator rod materials and shapes were investigated in order to determine the type of oscillator rod that would give the desired amplitude and wave shape for use in future measurements in the EBR-I, Mark III. A null balance system was used to obtain the amplitude and phase data to a reasonably high degree of accuracy.

INTRODUCTION

Experiments planned for EBR-I, Mark III require that the response of the reactor to sinusoidal oscillations of reactivity, at various power levels and frequencies, be determined to a high degree of accuracy in order to obtain information on the power coefficient of reactivity. A method for analyzing experimental frequency response data to deduce the response of reactivity to power changes has been given by Dr. Bethe.⁽¹⁾

The ZPR-III mockup assembly of EBR-I, Mark III offered a facility to measure the zero power transfer function of a fast critical assembly and to determine what types of oscillator rod and measurement techniques could be used to advantage on the EBR-I, Mark III. A review of available information on past reactor oscillation experiments indicated that two methods of measurement of the reactor transfer function were capable of providing data with a reasonably high degree of accuracy.

One method, originally developed at Harwell,⁽²⁾ consists of performing an electromechanical Fourier analysis of the current from an ion chamber by means of sine and cosine integrators. This method requires considerable special equipment and is subject to errors caused by drifts in electronic equipment.

The second method of measurement, developed and used by Stone, Frost, Schemel and others at KAPL,⁽³⁾ consists of passing an ion chamber current through a sine potentiometer and decade resistor in such a manner

as to obtain zero voltage at the wiper of the sine potentiometer when a variable phase coupler (explained in apparatus section) and the decade resistor are properly adjusted. This method, as developed at KAPL, requires a comparatively small amount of special equipment and is free from error caused by drifts in electronic equipment. However, it requires higher ion chamber currents than are readily available at power levels normally attainable with the ZPR-III assembly. The null principle used in the KAPL method was used as a basis for developing the method described below.

Several oscillator rods were designed and statically calibrated over a 360-degree rotation by the period method. This gave the total rod worth information required for obtaining the experimental transfer function and the input reactivity wave form.

Several types of neutron detectors were used to determine the effects of detector type and location on the amplitude or phase data.

THEORY

Equations relating the reactivity to the stable reactor period and the solutions to the kinetic equations for small sinusoidal inputs in reactivity have been published.⁽⁴⁾⁽⁵⁾⁽⁶⁾ In a form suitable for an analysis pertaining to a fast critical assembly, they are:

$$\rho = \frac{\ell}{T k_{\text{eff}}} + \beta \sum_{i=1}^6 \frac{a_i \gamma_i}{(\lambda_i T + 1)} \quad (1)$$

$$\frac{\Delta N}{N_0 \rho} = \frac{1}{s \left[\ell + \beta \sum_{i=1}^6 \frac{a_i \gamma_i}{(s + \lambda_i)} \right]} \quad (2)$$

where

- ρ = reactivity
- ℓ = prompt neutron lifetime
- T = stable reactor period
- β = total delayed neutron fraction
- a_i = fraction of delayed neutrons in i 'th group
- γ_i = effectiveness factor of the i 'th group
- λ_i = decay constant of i 'th group
- ΔN = time-dependent neutron density term
- N_0 = steady state neutron density term
- $s = j\omega$
- $j = \sqrt{-1}$

If the following assumptions are made, Equations (1) and (2) may be simplified and terms containing (ℓ) neglected:

1. Prompt neutron lifetime $\sim 5 \times 10^{-8}$ sec
2. $k_{\text{eff}} \sim 1$
3. ρ is of the order of 10^{-4}
4. The effectiveness of each of the six groups of delayed neutrons is approximately the same, since their energies are nearly the same, and has an average value γ .
5. The maximum value of ω is less than 1,000 radians/second.

The following equations were obtained using the above assumptions:

$$\rho \sim \beta \gamma \sum_{i=1}^6 \frac{a_i}{(\lambda_i T + 1)} \quad (3)$$

$$\frac{\Delta N}{N_0 \rho} \sim \frac{1}{s \beta \gamma \sum_{i=1}^6 \frac{a_i}{(s + \lambda_i)}} \quad (4)$$

Examination of Equations (3) and (4) shows that the experimentally determined transfer function, obtained by measuring $\frac{\Delta N}{N_0 \rho}$, should agree with the calculated transfer function regardless of the values assigned to β and γ . Therefore, the relative abundance, decay constant, and effectiveness factors for each group of delayed neutrons are the only variables which can effect the shapes of the amplitude of phase-shift response curves. The curves have been obtained from calculations made using both the Hughes and Keepin delayed neutron data.⁽⁷⁾ The differences obtained, over the frequency range from 0.01 to 10 cycles per second, were less than the expected accuracy of our measurements. The calculated curves shown in all figures were obtained using the Hughes data.

APPARATUS AND METHODS

The wave shape and worth of the oscillator rods were determined by measuring the reactor period at a number of angular positions of the oscillator rod. A selsyn system was used to indicate the position of the control rod to an accuracy of ± 1 degree. A control rod was moved to reduce the power level of the assembly at the end of each run and was returned to a position fixed by a limit stop prior to each run. The accuracy of the rod calibration was limited by the accuracy of the period measurements and by reactivity changes caused by expansion and contraction of the assembly with room temperature fluctuations. Figure 1 shows the shapes and dimensions of the oscillator rods which were calibrated. The oscillator rod was located

at the inner edge of the blanket four inches from the radial center of the core and was centered about the midplane of the core as shown in Figure 9.

Figure 2 shows a block diagram of the null balance system used to measure the amplitude of $\Delta N/N_0$ and the phase lag of ΔN with respect to the oscillator rod. A sine potentiometer was connected to the oscillator rod drive shaft through a variable phase coupler. The phase coupler consisted of a selsyn generator, revolution counter, and gear train in the control room, which drove a selsyn motor, gear train and differential on the oscillator drive assembly. The phase angle of the wiper on the sine potentiometer was initially made zero with respect to the oscillator rod by adjusting the potentiometer output voltage to zero when the oscillator rod was at a mid-value position, as estimated from the rod calibration data. After the initial alignment, the position of the wiper on the sine potentiometer with respect to the oscillator rod was adjusted by means of the selsyn system. The revolution counter on the control panel indicated this position to an accuracy of ± 0.2 degree. The detector outputs were connected to the input of a fast response direct-coupled amplifier with negative feedback. Also connected to the input of this amplifier, through a resistance at least 10,000 times the effective amplifier input resistance, was a signal from a variable direct current bucking supply in series with a voltage developed across the sine potentiometer. The output of the amplifier was connected to a direct current meter and to the input of an adjustable band pass filter. The output of the filter was connected to a Brush amplifier and recorder. The meter was used as a null detector for the direct current component of the amplifier output. The filter attenuated noise and blocked the direct component of the amplifier output. The Brush recorder and amplifier were used as a high gain null detector for the fundamental frequency component of the amplifier output.

The voltage appearing at the output of the amplifier (E_0) is given by the following equation:

$$E_0 = AR_{in} \left[I_0 + I_1 \cos (\omega T - \phi) - \frac{V_2}{R} - \frac{V_1}{2R} \cos (\omega T - \theta) \right] \quad (5)$$

where:

A = Amplifier gain

I_0 = Direct component of detector current

I_1 = Amplitude of the time-dependent component of detector current

ϕ = Angle between oscillator rod and I_1

θ = Angle between oscillator rod and wiper on sine potentiometer

V_2 = Direct current bucking voltage

V_1 = Voltage across sine potentiometer.

When E_0 is equal to zero

$$I_0 + I_1 \cos (\omega T - \phi) = \frac{V_2}{R} + \frac{V_1}{2R} \cos (\omega T - \theta). \quad (6)$$

For Equation (6) to be true, I_0 must equal $\frac{V_2}{R}$, I_1 must equal $\frac{V_1}{2R}$, and ϕ must equal θ . Since I_0 is proportional to N_0 , I_1 is proportional to ΔN and ϕ is the phase lag between ρ and ΔN ; we may write the value of the reactor transfer function, at any given frequency, in the following form:

$$\frac{\Delta N}{N_0 \rho} e^{j\phi} = \frac{\frac{V_1}{2R}}{\frac{V_2}{R}} e^{j\theta} = \frac{V_1}{2V_2} e^{j\theta} \quad (7)$$

The following procedure was used to determine the values for V_1 , V_2 and θ to satisfy Equation (6):

1. The phase coupler was adjusted to produce the phase lag calculated for the frequency of oscillation.
2. Voltage V_2 was adjusted to make the average reading of the direct current meter equal to zero. This could be determined to an accuracy of 0.5 per cent.
3. Voltage V_1 was adjusted until the recorder pen deflection was a minimum.
4. The phase coupler was set at several values and recorder pen deflection readings obtained for each setting. Pen deflection was plotted as a function of phase coupler setting and the null value for phase lag read at the intersection of the best straight lines drawn through a series of points on each side of the null.
5. With the phase coupler set at the null value obtained in (4), readings of pen deflection were obtained for several values of voltage V_1 . This information was plotted to obtain a null value for V_1 in the same manner as described for phase lag in (4).
6. Voltages V_1 and V_2 were read with a precision potentiometer for the null settings of the voltage adjusting potentiometers.

The following detector types and locations were used to determine if the data were a strong function of detector type or location:

- a. The first detectors used were BF_3 chambers 10 inches long by 4 inches in diameter with an active volume of one liter and a gas pressure of one atmosphere. One of these detectors was located 20 inches from the radial center line of the core. The other was located 32 inches from the radial center of the core. The center of both detectors was 5 inches off the midplane of the core. Both detectors were covered with 4 inches of polyethylene.
- b. A Dumont Type 6291 photomultiplier tube with a 1.5-inch diameter plastic scintillator was located 2 inches from the radial center line of the core and 12 inches off the midplane of the core.

- c. Two chambers, each 10 inches long by 2 inches in diameter, containing boron-coated parallel plates and BF_3 gas were located 20 inches from the radial center line of the core and centered 5 inches off the midplane of the core. These chambers were covered with 4 inches of polyethylene.

RESULTS AND DISCUSSION

Figure 3 shows the calibration data for the boron-loaded eccentric oscillator rod which was used to obtain the transfer function data. It can be seen that it is not a perfect sine wave; therefore, the angular position of the rod when the fundamental component of the reactivity wave passes through zero is not known to a high degree of accuracy. Figure 4 shows the calibration curves for the other rod shapes used. The steel rod with an eccentric hole gave the same wave shape and was worth approximately half as much as the same rod with the hole loaded with boron. The steel eccentric rod did not give quite as good a wave shape as the boron-loaded rod but was considerably better than the steel half rod.

The large BF_3 -filled detectors used initially resulted in data that caused the transfer function amplitude to be consistently 20 per cent lower than that predicted from calculations. They also caused considerable phase shift at frequencies above 0.5 cycle per second. Figure 5 shows the phase data obtained between 0.1 and 10 cycles per second. Tests are in progress to determine the dynamic response of this type of detector but results are not presently available.

Data obtained using the small B^{10} -coated, BF_3 -filled chambers in the same location as that used for a large BF_3 chamber were in agreement with data obtained using a scintillation detector near the radial center of the core. Figure 6 shows the amplitude and phase shift response data obtained using the three types of detectors. Table 1 gives these data in tabular form.

The amplitude data obtained with the large BF_3 chambers were normalized to the other data at 0.14 cycle per second to correct for the detector characteristics. The other amplitude points are shown unnormalized. An average curve determined from the experimental data is parallel to and 3 per cent below the calculated curve. This constant difference is less than the probable error in the measurement of total oscillator rod worth.

The phase shift data have been normalized to the calculated phase shift curve at 0.14 cycle per second by subtracting 4 degrees from the observed phase shift data. This difference is attributed to the inaccuracies inherent in determining the angular position of the oscillator rod which corresponds to the midordinate point of the fundamental component of the reactivity wave.

The ability of the system to reproduce data was checked by repeating measurements at a frequency of 0.5 cycle per second twice on three successive days. The maximum deviation of any amplitude response point from the average of the six measurements was 1.8%. The maximum deviation of any phase shift point from the average of the six measurements was 0.4 degree.

Figure 7 shows a plot of the error signal amplitude as a function of voltage V_1 . It can be seen that the points used to plot the null are scattered, but the intersection of any straight lines bounded by the points on either side of the null would result in an error of less than one per cent.

Figure 8 shows a plot of the error signal amplitude as a function of phase shift θ . Again the points are scattered, but the intersection of any straight lines bounded by the points on either side of the null would intersect within one degree of the point of intersection shown.

The signal to noise ratio became much smaller at frequencies above one cycle per second, therefore, the error in the data obtained at the higher frequencies is two or three times greater than that at frequencies below one cycle per second.

CONCLUSIONS

The following conclusions may be drawn from the results of the experiment:

1. The agreement between the calculated transfer function and the experimental transfer function of the EBR-I, Mark III assembly, obtained with the null balance method, is satisfactory over the frequency range investigated.
2. Detector location does not measurably affect the data over the frequency range investigated.
3. A steel rod containing an eccentric hole filled with boron is a satisfactory reactivity oscillator for use in a fast reactor.

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Table 1

TRANSFER FUNCTION DATA

<u>Frequency, Cycles/Second</u>	<u>Amplitude</u>	<u>Phase Shift</u>	<u>Detector Type</u>
0.017	289	-38.4	1
0.028	228		1
0.045	197	-28.5	1
0.059		-25.1	1
0.061	182	-26.0	1
0.081	170		1
0.093	163	-20.0	1
0.104	168	-19.6	1
0.132	151	-15.8	1
0.145	150	-15.8	2
0.148	150	-15.5	1
0.205	143	-13.3	1
0.295		-10.5	1
0.301	142	-10.7	1
0.498	134	- 8.0	1
0.499	134	- 8.6	1
0.50	132	- 8.3	1
0.525		- 7.1	2
0.525	134	- 6.8	3
0.554		- 7.4	1
0.562	134		1
0.562	136	- 6.7	2
0.562		- 6.6	2
0.562	134		1
0.81	131	- 6.6	1
0.83	131	- 5.2	3
0.89	123	- 5.8	2
0.90	124	- 5.8	1
1.31	128	- 5.9	1
2.30	129	- 2.4	3
2.79	125	- 7.8	1
4.32		- 3.7	3
5.2	126		3
8.3	131	- 2.7	3
10.3	130	+ .7	2

Detector TypeDescription

1

Large BF₃ chamber

2

Plastic scintillator

3

Small B¹⁰-coated BF₃-filled chamber

POISON ROD ECCENTRIC ROD

SECTION B-B

HALF ROD

SECTION C-C

FIGURE 1

OSCILLATING RODS

FIG. 2
BLOCK DIAGRAM OF NULL BALANCE SYSTEM

FIGURE 3
REACTIVITY VS. ANGULAR POSITION FOR STEEL
ROD WITH ECCENTRIC HOLE FILLED WITH B^{10}

FIGURE 4
REACTIVITY VS. ANGULAR POSITION

FIGURE 5
PHASE SHIFT RESPONSE VS. FREQUENCY
USING 4 IN. DIA. BF_3 CHAMBER

FIGURE 6
RESPONSE OF EBR I, MARK III ASSEMBLY
IN ZPR III AS FUNCTION OF FREQUENCY

FIGURE 7
ERROR SIGNAL AS FUNCTION OF VOLTAGE
 V_1 WITH PHASE θ CONSTANT

FIGURE 8
ERROR SIGNAL AS FUNCTION OF PHASE
 θ WITH VOLTAGE V_1 CONSTANT

FIGURE 9