ANALYTICAL STUDY OF CURRENT-SHARING AT POWER FREQUENCY BETWEEN A TYPE II
SUPERCONDUCTING COMPOSITE WITH A NORMAL METAL BACKING*

G. Morgan[†]

ABSTRACT

Current sharing in a two-layer composite is studied analytically by numerically solving the 1-dimensional diffusion equation in the normal metal and the phenomenological critical state model with field independent critical current density J_c (T) in the superconductor, with boundary conditions appropriate to a transmission line at each surface and the interface. It is found that with a sufficiently low resistivity (ρ) backing, and reasonable assumptions as to heat transfer, an appreciable fraction of a current density of several thousand amps per cm can be carried in the backing with the balance in a superconductor with high T_c such as Nb_3Sn . The dependence of the dissipation on such parameters as thickness, J_c , ρ and frequency at constant temperature is given, and typical loss voltage waveforms presented.

I. INTRODUCTION

In a superconducting ac power transmission line, a normal metal backing must be provided for the superconductor to handle fault currents of greater magnitude than can be handled by the superconductor alone. The question then arises as to whether the current will divide itself between the superconductor and the backing. If so, the dissipation will be less than is the case if the backing carries the entire current. A complete theoretical answer to this question requires a simultaneous solution of the heat flow equation and the magnetic diffusion equation, $\nabla^2 H = (\mu_0/\rho) \partial H/\partial t$. A partial theoretical attack on the problem was undertaken; in the first section the 1-dimensional problem is solved numerically under the assumption of constant temperature. Even this simplified problem is difficult to attack analytically because the superconductor does not obey the diffusion equation in the low frequencies under study. The appropriate description of the superconductor is the critical state model which states that the current density is either zero or $\pm J_c$ (B, T). To further simplify the analysis, J_c is assumed independent of B; this is thought to affect the results in only a minor way.

In the second section, the temperature is allowed to vary, but the temperature T is everywhere the same, and given by the first order differential equation $(C_s D + C_n d) dT/dt = q - h(T - T_0)$ where C_s and C_n are the superconductor and normal metal volume heat capacities, D and d are the respective thicknesses, q is the instantaneous rate of heat generation per unit area, h is the heat transfer coefficient and T_0 the coolant temperature. This simplified approach to the temperature dependent problem is justified by the very short time required for thermal equilibrium in the laminate. Assuming no thermal barrier between layers, the equilibrium time constant for an aluminum- Nb_3Sn (200 μm , 10 μm) laminate is essentially that

Manuscript received September 30, 1974

*Work performed under the auspices of the U. S. Atomic Energy Commission and the National Science Foundation.

[†]Brookhaven National Laboratory, operated by Associated Universities, Inc., under contract with the United States Atomic Energy Commission.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

of the Nb_3Sn alone, 1.6 $\mu sec.$, whereas the time constant for the composite to come to equilibrium with the helium is about 0.6 msec with $h = 0.1 w/cm^2 K$. These numbers indicate a flat temperature profile within the composite with most of the ΔT in the coolant boundary layer. In the present study, h is assumed constant for lack of better knowledge, though it is known that h may be both time dependent because of transient mass flow effects and ΔT dependent, especially if the coolant is near critical temperature pressure.

II. DETAILS OF THE COMPUTATIONS

The superconductor is assumed to have been cooled in zero field, and the current always begins at zero. No cooling runs were made (a high initial temperature followed by low dissipation, such as might follow a current surge). Since the surface field $H_s = H_m \sin \omega t$ is greater than the penetration field $H_c = J_c D$ for part of the cycle, there is at most one region of persistent current in the superconductor at any time. Unlike the ordinary case of $H_c < J_c$, the persistent current condition does not begin immediately when the field passes its peak value, because doing so would violate continuity of the electric field.

There are two branches in the program corresponding to the two possible conditions of the superconductor. In the first condition, termed the "driven" condition, the current in the superconductor is everywhere the same sign and in the second condition, termed the "coasting" condition, there is a position in the superconductor at which the current changes sign, corresponding to a maximum or minimum in H . The distance to the current reversal point from the interface is termed x_r , as shown in Fig. 1. At x_r , $E=0$. If E_s , and H_s are the surface electric and magnetic fields, and E_i and H_i the interface electric and

Fig. 1 - Magnetic field and current distributions in a type II superconductor with normal metal backing during the "coasting" condition.

magnetic fields, then during the driven condition continuity of E and H requires $\frac{dH_i}{dt} = \frac{dH_s}{dt}$ and $E_s = E_i - \mu_0 \frac{dH_s}{dt}$. During the coasting condition,

$$x_r = E_i / \mu_0 \frac{dH_i}{dt} \quad (1)$$

$$E_s = E_i \mu_0 D \frac{dH_s}{dt} - \mu_0 x_r \left(\frac{dH_i}{dt} - \frac{dH_s}{dt} \right), \text{ and}$$

$$H_i = H_s \pm J_c (D - 2x_r) \quad (2)$$

where the sign of J_c is the same as that of $\frac{dH_s}{dt}$. H_s , of course, is given by the front surface boundary condition $H_s = H \sin \omega t$. The rear surface boundary condition is $H = 0$. Use of cylindrical coordinates (instead of Cartesian) with no θ dependence would have satisfied this condition automatically. The rear surface electric field is in general not zero and is given by $E_d = \rho \partial H / \partial x$. The interface electric field E_i is given by the same expression, but to improve the accuracy, it is computed numerically from

$$E_i = -\mu_0 \int_0^D \frac{\partial H}{\partial x} dx + E_d.$$

During the coasting condition, H_i depends on x_r , as shown in Eq. (2), but by Eq. (1), x_r depends on E_i , which is not known until H_i and hence dH_i/dt are known. In the program the simultaneous solution of these equations is obtained by an iterative procedure: at each increment in time, two values of H_i differing by an amount ΔH_i and centered on ΔH_i (old) +

$\Delta t \frac{dH_i}{dt}$ (old) are selected. Using each of these boundary values the diffusion equation is solved using the Crank-Nicholson method and the Thomas Algorithm¹ and for each value of H_i , E_i , x_r and $F = H_s - H_i \pm J_c (D - 2x_r)$ are computed. Since the correct value of H_i corresponds to $F = 0$ (Eq. 2), H_i is then approximated by

$$H_i = H_{i,2} + F_2 \frac{\Delta H_i}{\Delta F}.$$

A single iteration suffices.

III. CONSTANT TEMPERATURE RESULTS

In the first part of the present study, a superconductor is assumed with a J_c such that the surface resistance is 4×10^{-11} ohm with a surface current density $\sigma = 500$ A/cm rms. The surface resistance is related to power loss per unit area by $r = \rho/\sigma^2$, hence this corresponds to $10 \mu\text{W}/\text{cm}^2$ loss. The surface resistance in the constant J_c , critical state model¹ is $r = 4 \mu_0 f H_m / 3 J_c$. This gives $J_c = 1.78 \times 10^{11}$ A/m². Five parameters: D , d , H_m , frequency f and backing resistivity ρ are allowed to vary around nominal values of $D = 7.96 \times 10^{-7}$ m, $H_m = 282,800$ A/m (twice the penetration field H_p) for which $\sigma = 2000$ A/cm rms, $f = 60$ Hz, $\rho = 1.6 \times 10^{-10}$ ohm-m (100 resistivity ratio in copper), and $d/\delta = 3.0$, where $\delta = \sqrt{2\rho/\mu\omega}$ is the skin depth.

In the cases of variable resistivity and frequency, the results are simply stated: the loss varies as $\sqrt{\rho f}$ and at the nominal values, 39.2% of the time-averaged loss of $0.441 \text{ W}/\text{cm}^2$ occurs in the backing with the balance in the superconductor. This simple result holds as long as the surface field is about 1% or more greater than the penetration field.

The variation with peak surface field is more complicated. At fields slightly over the penetration field H_c , the loss starts off at the hysteretic value, $2 \mu_0 f H_c^3 / 3 J_c$ or $80 \mu\text{W}/\text{cm}^2$ and increases rapidly. The surface resistance increases approximately linearly from the hysteretic value at the penetration field,

$r = 4 \mu_0 f D / 3 = 8.0 \times 10^{-11}$ ohm, up to about $H_m = 1.5 H_c$, then gradually levels off, becoming almost constant at $10 H_c$, and approaching asymptotically the value for the pure metal $r = \rho/\delta = 1.947 \times 10^{-7}$ ohm. The variation of r with surface field is shown in Fig. 2.

Fig. 2 - Effective surface resistance for surface fields well above the penetration field.

Fig. 3 is a plot of surface magnetic field H_s , surface electric field E_s , interface magnetic field H_i and temperature T , as a function of time over a whole cycle.

Fig. 3 - Field and temperature waveforms with variable temperature. As the mean temperature increases, the duration of coasting (the period during which E_s appears to be zero) decreases.

The material parameters are as described in section IV, and the temperature variation is discussed there; only those features which would occur with constant temperatures are discussed here. It is interesting to note that for a short time, from 41.7 msec to 42.5 msec, where H_i is positive and H_s negative, the current in the normal metal is in the opposite direction to that of the total current. The same phenomenon occurs as a function of depth in a pure metal, and is responsible for the minimum in surface resistance at a metal thickness of about 1.7δ . The duration of

the E_s spike depends on H_m/H_c , being zero at $H_m = H_c$ and approaching a half-cycle at $H_m \gg H_c$. Thus the smaller $(H_m/H_c) - 1$ is, the greater is the proportion of harmonics of f which appear.

One effect of the harmonics is the decrease in the value of d/δ at which the minimum in r appears. This is shown in Fig. 4, which consists of plots of r vs. d/δ for two thicknesses D of superconductor, with H_m the same in both. Thus $(H_m/H_c) - 1$ is different for the two curves because H_c varies. Also shown is r vs. d/δ for the base metal alone. The curves have been

Fig. 4 - Surface resistance as a function of backing thickness. The curves are normalized to the value of 1. at three skin depths.

normalized to a value of 1 at three d/δ . The actual values of r at three d/δ are 2.05×10^{-8} , 1.10×10^{-7} , and 1.95×10^{-7} ohm, for $D = 1.39 \mu\text{m}$, $0.80 \mu\text{m}$ and zero, respectively.

IV. VARIABLE TEMPERATURE

If the temperature of the composite is allowed to vary, several phenomena occur. The backing resistivity increases with both temperature and magnetic field but in the present study, for the sake of simplicity and computational speed, ρ is assumed constant. This is justifiable because the computation stops when the temperature exceeds T_c (measured to be 16.6 K) and ρ , in 2500 RR aluminum, increases only 25% at this temperature according to Fickett.² Magnetoresistance becomes significant when most of the current is in the backing, if the backing is copper, but as will be seen, current sharing does not occur significantly in modestly pure (100 RR) copper any-

way. Because of magnetoresistivity effects however, very high purity copper would probably not be as suitable as high-purity aluminum. The specific heat of most materials is a strong function of temperature in this range, typically as the 2.5 to 3 power of T , and the present program incorporates this. Most important, J_c goes to zero at T_c . The dependence of J_c of Nb_3Sn on temperature has until recently been thought to be linear, but recent measurements³ using a new technique give more nearly a quadratic dependence,

$$J_c(1 - (T/T_c)^2), \quad (3)$$

and this is used in the present program.

If the dependence of heat capacity, resistivity and power loss on temperature are all ignored, a simple analytical treatment is possible which is helpful in interpreting the numerical results. The frequency dependence of the dissipation is assumed to be that of the fundamental: $q = q_0 \sin^2 \omega t$. With this, the differential equation for the temperature is

$$q_0 \sin^2 \omega t - h(T - T_0) - C dT/dt = 0$$

of which the solution is:

$$T = T_0 + q_m/hA^2 + \frac{q_m}{\omega CA} \sin \omega t \sin(\omega t - \phi) + Ke^{-ht/C} \quad (4)$$

where K is a constant which depends on initial conditions, $\phi = \tan(\omega C/h)$ and $A = [1 + (h/2 \omega C)^2]^{1/2}$. The peak dissipation q_0 is twice the average dissipation q_m , which can be estimated for any composite using figure 2 if $H_m/H_c \geq 1.01$. The second term in eq. (4) is the mean temperature rise, which approaches q_m/h as $h/2 \omega C$ becomes small. The coefficient of the third term is the oscillation amplitude, which approaches $q_m/\omega C$ as $h/2 \omega C$ becomes small. (The high frequency, high heat capacity limit may be contrary to the fundamental assumption, in the numerical computations, of uniform temperature in the composite.)

For the variable temperature computations, the parameters differ from those of the constant temperature calculations and are chosen to simulate a typical single-layer commercial Nb_3Sn tape. The current density J is $7.48 \times 10^{10} \text{ A/m}^2$ (equivalent to $7.00 \times 10^{10} \text{ A/m}^2$ at 4.2 K) and the thickness is $5.5 \mu\text{m}$, giving $H_c = 3.85 \times 10^5 \text{ A/m}$ or $c = 2720 \text{ A/cm rms}$ at 4.2 K. Computations were made with two values of resistivity: $1.6 \times 10^{-10} \Omega\text{m}$ and $1.0 \times 10^{-11} \Omega\text{m}$, corresponding to 100 RR copper and 2500 RR aluminum; each one or three skin depths thick. Four values of heat transfer coefficient were used, 2000, 1000, 500 and $250 \text{ W/m}^2\text{K}$, which were chosen to cover the range to be expected in a supercritical He cooled transmission line with normal operating current and steady-state conditions. A base temperature of 8 K is assumed.

The results given in Fig. 3 are for $H_m/H_c = 1.25$ with aluminum backing, and with $h = 500 \text{ W/m}^2\text{K}$. In this particular case, the mean temperature is continuing to rise (after three cycles), and T_c would eventually be reached.

Fig. 5 shows the envelope of the oscillating temperature in another case in which the calculation was allowed to continue until T_c was reached. Here, the mean temperature, after the initial transient, rises at first rather slowly and the usual 4 to 6 cycles of computations left the outcome in doubt. This was computed for a copper backing, with $H_m/H_c = 1.03125$ and a heat transfer coefficient of $1000 \text{ W/m}^2\text{K}$. The copper backing has a greater heat capacity than the aluminum, since one skin depth is 4 times as thick.

Fig. 5 - Temperature oscillation envelope from start to quench with copper backing, $H_m/H_c = 1.03125$, and $h = 1000 \text{ w/m}^2 \text{ K}$.

The greater heat capacity increases the mean temperature, and reduces the oscillation amplitude. The net effect of the increased heat capacity is beneficial. This is shown in Table I which summarizes the approximate stability limits computed, and gives the peak temperature at the highest field which computations showed to be stable. The table shows that increasing the thickness from 16 to 36, which essentially leaves the dissipation unchanged, increases the maximum stable field ratio, from somewhat over 1.25 to about 1.5 at $h = 1000 \text{ w/m}^2 \text{ K}$. The table also shows that the maximum stable field ratio with copper backing was 1.016. The computations begin to be sensitive to the detailed model for the superconductor at this field ratio and below, as the peak dissipation increases very rapidly with field at field levels slightly above the penetration field. Consequently, this apparent stability of current sharing with a copper backing should not be taken too seriously. With aluminum 36 thick, and an h of $1000 \text{ w/m}^2 \text{ K}$ however, an H_m/H_c of 1.5 appears possible with $H_c = 315851 \text{ A/m at } 8 \text{ K}$. The dissipation is 4252 w/m^2 , 56% of which is in the aluminum. From Figure 2, this percentage of the pure metal resistivity occurs at an H/H_c of about 3.2, from which one can compute using eq. 3 an effective average temperature of 13.3 K. From the numerical results, this is also the median temperature.

Using the preceding procedure, an estimate can be made of the stability of current sharing of a composite, knowing only the critical current as a function of temperature, the heat transfer coefficient and the backing resistivity. Essentially, one plots dissipation as a function of temperature, using the known temperature dependence of J_c to obtain H/H_c , then using Figure 2 to obtain the surface resistance (Figure 2 is based on a resistivity of $1.6 \times 10^{-10} \Omega \text{ m}$ and scales as $\sqrt{\rho}$). The dissipation is then plotted on a graph with a straight line for heat transfer based on the known heat transfer coefficient. If the heat transfer line lies entirely above the dissipation, the composite should be definitely stable, and if entirely below, definitely unstable; if the curves cross, then it may be stable, depending on oscillation amplitude and temperature at crossing. Figure 6 shows such a plot for all the combinations computed. The agreement with the numerically computed results of Table I is good.

TABLE I
Min. and Peak Oscillation Temperatures

h	250	500	1000	2000	1000	1000
d/6	3	3	3	3	1	1
Mat'l	H _m /H _c	Al	Al	Al	Al	Cu
	1.016	U				8.18
	1.031					8.81
	1.125	10.21				U
		12.24				
	1.25	U	8.40	8.01	8.17	
			11.29	9.85	13.97	
	1.5		11.47	8.08	U	
			15.07	11.95		
	2.0		U	U

Fig. 6 - Heat generated (heavy curves) computed from Fig. 3, and heat transfer with constant h (thin lines). The dashed curves are for copper and the heavy solid ones for aluminum.

In summary, calculations indicate that with good heat transfer, stable current sharing should occur between Nb_3Sn and a pure aluminum backing, with 12% or more excess current. An effective resistivity curve is given which enables one to predict the loss and current sharing under other conditions of resistivity or heat transfer.

REFERENCES

1. D. U. von Rosenberg, "Methods for the Numerical Solution of Partial Differential Equations," American Elsevier, N.Y. 1969.
2. F. R. Fickett, "Aluminum 1.," Cryogenics **11**, No. 5, 349 (Oct. 1971).
3. M. Garber, S. Shen, J. Bussière and G. Morgan, "Critical Currents of Commercial Nb_3Sn Tapes," paper N-2 this session.