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DESIGN BASIS ACCIDENT STUDIES

Final Report

Richard B. Nicholson
The Ohio State University

June 22, 1974

Prepared for the U.S. Atomic Energy Commission
Under Contract No. AT(11-1)-2286

MASTER

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Final Report

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Abstract

An extension of the ANL equation of state in the VENUS-II program to include lower pressure conditions and an isentropic expansion calculation shows that the energy available for mechanical damage is strongly dependent upon how far the expansion can proceed. Expansion to one atmosphere in weak accidents can lead to one to two orders of magnitude more energy than R. W. Stratton's eight-fold volume expansion. The modified Bethe-Tait method of Edwards and Mather is approximately reproduced and verified but at high Doppler effect, low excursion energy conditions, the ANL equation of state in VENUS leads to significantly greater energy release. A potentially autocatalytic effect, associated with neutron streaming through voids, (first discussed by Richard Webb), cannot be disregarded in HCDA analysis. More work is required to establish its quantitative significance. Most of the SAS-2A computer program modeling is found to be based on sound principles. We are generally in agreement with ANL on the few weaknesses of the modeling. The DBA concept as a basis for breeder containment design is found to be a sound concept when accompanied by a commitment to provide the highest practicable containment capacity and by a commitment to assure responsibility and visibility in containment design decisions.

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Part One

Introduction and Review of Progress in DBA Analysis

Prepared by:

Richard B. Nicholson

I. INTRODUCTION

This Final Report on "The Methods, Computer Programs and Approximations in Fast Reactor Design Basis Accident Calculations" is presented as a collection of topical parts all related to the project objectives. The six following parts single out the areas where the greatest success has been achieved in our efforts to evaluate and comment upon the state of the art in DBA analysis.

The original project objectives were:

- (1) to review the modeling in three principle DBA computer programs, VENUS-II, SAS-2A, and MELT-2.
- (2) to review the general concept of the DBA as a basis for determining the design of containment related structures.
- (3) To carry out special assignments as requested by the sponsor.

Part Two of the report is an extension of the VENUS-II ANL equation of state that was required to carry out an evaluation of the HCDA studies of Edwards and Mather (1) of the UKAEA as compared to similar studies with VENUS-II. These comparisons are presented in Part Three. Part Two also provides some interesting comparisons with the methods of R. W. Stratton (1). Thus Parts Two and Three relate to all three objectives stated above.

Parts Four and Five cover a feedback effect which may be of crucial importance to HCDA studies of the type VENUS-II was designed to evaluate. This effect, originally discussed by Richard Webb (3), involves the closure of neutron streaming paths and so a positive addition to reactivity near the time of peak power in a power excursion accident.

Part Six is a review of the physical models incorporated in the SAS-2A computer program.

Part Seven is a discussion of the role that HCDA studies should play in determining the basis for the design of containment structures and represents our contribution to objective (2) above.

In this Part One we present a discussion of the way in which the succeeding six parts and other work have contributed to the objectives.

II. THE VENUS-II PROGRAM

The VENUS-II program (4) has been widely used in recent years to study the power excursion and disassembly phase of HCDA events. Having been closely involved in its development and use we are perhaps prejudiced toward it. We feel it is a sound and useful tool for the analysis of severe transients, ones in which the maximum internal energy density reaches say 3 kilojoules per gram or more

relative to a zero in the liquid state at 273°K. When the energy does not reach this level the energy available for destructive effects is small and there is not much argument that it can be contained. Therefore, VENUS-II is useful over much of the range of accidents that are important for public safety evaluations. Nevertheless, it does have some deficiencies.

In large reactors and accidents that involve a large part of the core the destructive potential can conceivably be important at energies too low for VENUS validity. This is especially true if the fuel coolant interaction can be efficient in energy conversion. The most important breakdown of the model under these conditions is the assumption of homogeneity and isotropy. The subassembly structure cannot reasonably be ignored. While Nicholson and Jackson (5) did use VENUS-II to make a rough estimate of the effect of subassembly structure, one should not attach much quantitative significance to such calculations.

A situation wherein VENUS can potentially fail is the one where large material displacements are required to terminate the excursion or where the motions produced can lead to compaction and a second excursion. It is not clear that this can happen. The new program at Los Alamos on an extended motion disassembly code may answer this question (6).

The work in Parts Four and Five suggests that Richard Webb could be right in his assertions (3) that the closure of neutron streaming paths during an excursion can be autocatalytic. It is imperative that this question be quickly resolved. The simple methods used in Parts Four and Five produce violent results even for \$10/sec initiators without requiring unrealistic assumptions about the parameters of the theory. We expect the methods used to give correctly the sign and order of magnitude of the effect. They are not quantitatively reliable, at least not until they can be checked by more nearly exact methods such as Monte Carlo calculations of the reactivity and better models of the spatial feedback distribution in VENUS. This work once again emphasizes the great importance of voids in determining the severity of the power excursion. It is well known that in the absence of voids the energy release will be easily manageable. The most important "parameter" of HCDA analysis is the input void content and distribution. The work of Parts Four and Five suggest that the voids may be even far more important than we had previously thought them to be. On the other hand, we repeat that this work must be rechecked by better methods before one can come to any definite conclusions.

The work on Part Three gives some support to the validity of VENUS in that for moderate to severe accidents it checks

with the work of Edwards and Mather (1) and with the relatively simple Bethe-Tait method presented also in Part Three. In weak accidents VENUS-II gave significantly greater yields than the other two methods. It is not yet clear whether this was due to method differences or due to an incorrect or at least different disassembly feedback function that is input to the Bethe-Tait methods. This can be easily checked but has not been as yet.

III. THE MELT-2 COMPUTER PROGRAM

We failed to make the MELT-2 Computer Program (7) operational on our IBM 370-165 computer soon enough to carry out a proper evaluation of its physical models. We have commented in progress reports that its fuel sweeping model cannot be supported as a conservative or even realistic model for fuel motion. It is clearly non-conservative. Calculations using that model cannot be accepted as a demonstration of safe behavior. This program has been improved and changed since we received it in early 1973. Our comments apply to that version of the code. We are not familiar with the more recent improvements.

IV. THE SAS-2A COMPUTER PROGRAM (8)

The SAS-2A computer program is reviewed in Part Six. Again, the review applies to the program version received

in early 1973 and not to more recent improvements. The review is generally favorable in the sense that the program does everything correctly where there is sufficient understanding of the phenomena to know what is correct. There are still large gaps in our knowledge about sodium voiding and fuel movement after melting. The effects of the fission product pressures are still generally speculative. Some of these effects are being added to form a more advanced SAS code that we have not yet seen.

In these speculative areas we favor the development of SAS models which give the user some flexibility to study variations of the models themselves as opposed to parametric variations. One should not accept the notion that it is possible now or that it will be possible in the near future to actually calculate the course of an HCDA accident from initiation through to disassembly in a single continuous calculation. At the present time we do not even feel that this is an intelligent goal to work toward. The analysis must necessarily be segmented into its logical parts. One can do a continuous calculation up to the point of sodium boiling or fuel failure. From that point on it is necessary to employ a sequence of alternative rational models rather than detailed programming of definite behavior (which is not known). If it is convenient to do this in the form of a master driving program which calls the models, that is

acceptable, provided one can easily substitute alternative models.

V. THE GENERAL DBA CONCEPT

We have attempted to analyze the basic notion that the design of containment related structures should spring from design criteria based upon the results of Design Basis Accident (DBA) calculations. Especially, we have examined the role that the HCDA accident should play in determining the design. This latter question is discussed in detail in Part Seven of this report.

The DBA concept is dependent upon the ability of the safety analyst to bound in some sense the severity of accidents with one or a few calculations which can be defended as conservative evaluations. He must be able to argue that all more severe accidents including potentially some unconceived ones have acceptably low probability of occurrence. For this procedure to be viable all parties including the licensing authorities must accept the premise that it is permissible to rely quite heavily upon some engineering judgments whose validity cannot be absolutely established. Because of this we have recommended in Part Seven that this procedure be accompanied by an additional commitment by the designer to provide the containment structures having "minimum practicable failure probability"

for HCDA type accidents; this, independently of his DBA calculations. This may mean some large expenditures in design and equipment to increase the natural containment capacity several fold.

The acceptance of engineering judgment implies a guarantee of technical competence to carry out the analysis and make the judgments required. A high degree of personal responsibility is also implied. The best way of assuring competence and responsibility is to require that all such judgments be exposed to public view and that individuals be cited as accepting the responsibility.

The work of Parts Four and Five relate in an important way to the viability of the DBA concept. If autocatalytic effects of this type are actually found to be possible or even probable then the demonstration that the HCDA has been reasonably bounded becomes more difficult. Also, the requirement of designing for minimum practicable failure probability becomes more essential. The design of multiple protective systems to assure against accidents increases in importance. Finally, a vigorous experimental program on fast reactor meltdown with new experimental facilities is essential to reduce the extent of the judgment factor in the containment design basis.

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Part Two

Extensions of the UO_2 Equation of State
and Applications to Isentropic Expansions

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I. INTRODUCTION

Most of the calculations done with VENUS-II (1) in recent years have utilized a built-in optional equation of state designated the ANL equation of state. We elected to use this one for most of our calculations because we are familiar with it and because we believe it is very much like the equation of state used by Edwards and Mather. (2) Part III is a comparison of our results with theirs.

The ANL equation of state is based upon the corresponding states calculations of Menzies (3). His calculations in the reference report only go down to 20 atmospheres pressure and in order to compare with Edwards and Mather we needed to extend the ANL equation of state down to atmospheric pressure. The extension also provides some interesting comparisons with the work of Stratton (4). Menzies carried out his calculations for UO_2 using the tables and charts of Hougen, Watson, and Ragatz (5). These are readily available and so it was possible to undertake an extension of his calculations in a relatively consistent manner. There are two problems that arose. First, it became evident that Menzies stopped at 20 atmospheres because the tables only go down to the reduced pressure of .01, which corresponds to 20 atmospheres with the UO_2 critical pressure estimate of 2000 atm. This is not really

too serious because the functions are well behaved at low pressure and extrapolation is therefore reliable enough.

The other problem is that after having used Menzies tables for several years, we now noticed that they contain an inconsistency. At the lower pressures the saturation temperature is different in his Table I, "Equation of State of UO_2 at Constant Volume," as compared to his Table II, "Adiabats of Uranium Dioxide." The ANL equation of state was based upon his Table I and we wished to use and extend his Table II. In using the work of isentropic expansion as a measure of severity of an accident it is important that the equation of state in the power excursion calculation be consistent with that in the isentropic expansion calculation. This is especially true in weak excursions where the available work is a small fraction of the excursion energy. Thus if the core disassembled at 10 atm pressure one certainly does not want the isentrope to start at 100 atm or vice versa.

We did not completely resolve this question. We calculated points at 20 atm and on down to one atm in a way we felt to be consistent with Menzies' Table I. We then joined this relatively smoothly to Menzies' Table II values which we used from 100 atmospheres and up.

There is a certain amount of internal inconsistency in our extrapolation of the Hougen, Watson, and Ragatz tables. For example, the internal energy is sometimes

found to increase as the isentropic expansion extends to very low pressures. This cannot happen physically. There may also be inconsistencies in the Hougen, Watson, and Ragatz tables themselves. An example is seen in Figure 2 where the entropy departure lines at constant temperature cross each other and the saturation line. This is probably not possible physically.

In spite of the several troubles discussed above we believe our extension of Menzies calculations to be useful for our purposes. For the future, it is probably time for someone to do a complete reevaluation of the $\text{UO}_2\text{-PuO}_2$ equation of state which utilizes some of the increasing body of knowledge in high temperature properties of $\text{UO}_2\text{-PuO}_2$ mixtures, including also the fission products. Some of this has been done but it will not be greatly useful for HCDA analysis until the P-v-T-u relationships are evaluated self-consistently over the entire range of conditions needed.

II. THE EXTENSIONS OF THE ANL EQUATION OF STATE TO INCLUDE ISENTROPES TO ONE ATMOSPHERE

The tables and graphs in Hougen, Watson and Ragatz (5) and its counterpart, "Chemical Process Principles Charts," include data for reduced pressures as low as .01. In the case of the internal energy departure $\left(\frac{U^*-U}{T_c}\right)$ the curves

Figure 1 Internal Energy Departure vs. Reduced Pressure

Figure 1-a Internal Energy Extrapolation

Figure 2 Fugacity Coefficient vs. Reduced Pressure

Figure 3 Excess Entropy vs. Reduced Pressure

could be extrapolated in a straightforward manner by plotting the existing high pressure curves in considerable detail and visually extending the saturation line to .0005 reduced pressure (one atmosphere). The saturation line is nearly straight in this range. This extrapolation is shown in Fig. 1.

There were some peculiarities in the entropy departure which led us to proceed differently. The entropy departure was calculated as a function of the fugacity coefficient (f/P) and the enthalpy departure,

$$S^*-S = \left[\frac{H^*-H}{T_c} \cdot \frac{1}{T_r} + R \ln(f/P) \right] \quad (1)$$

The enthalpy departure $\frac{H^*-H}{T_c}$ is, in turn, a function of the internal energy departure and the compressibility factor, Z ,

$$\frac{H^*-H}{T_c} = \frac{U^*-U}{T_c} + (1-Z)R T_r \quad (2)$$

The fugacity coefficient, plotted in Figure 3, is very nearly equal to one for points lying on the saturation line at pressures less than 0.01. The compressibility factor Z , for these low pressures is approximately one for saturated vapor conditions and almost zero for the saturated liquid. Combining equations (1) and (2) and substituting the low pressure values of f/P and Z , yields

$$S^*-S = \left[\frac{U^*-U}{T_c} + (1-Z)R T_r \right] \frac{1}{T_r} + R \ln \frac{f}{P} \quad (3)$$

$$S^*-S \rightarrow \left[\frac{U^*-U}{T_c} \right] \frac{1}{T_r} \quad (\text{saturated vapor}) \quad (4)$$

$$S^*-S \rightarrow \left[\frac{U^*-U}{T_c} + R T_r \right] \frac{1}{T_r} \quad (\text{saturated liquid}) \quad (5)$$

Equations (3)-(5) were used to calculate the low pressure values of entropy departure seen in Figure 2 as a function of the already extrapolated low pressure internal energy departure values.

Using these extrapolations for internal energy and entropy departures it is straightforward to extend Menzies' equation of state and adiabat tables. The calculations were carried out according to the flow chart shown in Figure 4 and the results are shown in Table 1.

Figure 4 Calculation of Data in Table 1

Table 1 (Extension of "Table 2," Menzies)
Adiabats of Uranium Dioxide

Entropy	Pressure (atm)	Temp. (°K)	Specific Volume (cm ³ /mole)	Mole Fraction of Vapor	Internal Energy (kJ/gm)
8	20	7343	30,125	1.0	4.01
	10	6644	54,514	1.0	3.89
	3	5597	153,078	1.0	3.69
	1	4783	392,445	1.0	3.54
6	20	6365	26,112	1.0	3.83
	10	5760	47,261	1.0	3.72
	3	4852	132,757	1.0	3.55
	1	4147	340,261	1.0	3.42
4	20	5518	22,638	1.0	3.68
	10	4992	40,959	1.0	3.58
	3	4208	115,089	1.0	3.43
	1	3600	295,380	1.0	3.32
2	20	4800	19,692	1.0	3.54
	10	4400	35,813	.992	3.41
	3	3920	103,674	.967	3.32
	1	3440	271,244	.961	3.22
0	20	4720	17,608	.923	3.41
	10	4400	33,033	.915	3.33
	3	3920	96,705	.902	3.21
	1	3440	255,156	.904	3.11
-2	20	4720	15,990	.838	3.28
	10	4400	30,253	.838	3.21
	3	3920	89,736	.837	3.10
	1	3440	239,067	.847	3.02
-4	20	4720	14,315	.750	3.14
	10	4400	27,438	.760	3.08
	3	3920	82,768	.772	2.99
	1	3440	222,697	.789	2.91

Table 1 (continued)

Entropy	Pressure (atm)	Temp. (°K)	Specific Volumes (cm ³ /mole)	Mole Fraction of Vapor	Internal Energy (kJ/gm)
-6	20	4720	12,754	.668	3.01
	10	4400	24,441	.677	2.94
	3	3920	75,799	.707	2.87
	1	3440	206,608	.732	2.81
-8	20	4720	11,136	.583	2.88
	10	4400	21,950	.608	2.83
	3	3920	68,830	.642	2.76
	1	3440	190,520	.675	2.70
-10	20	4720	9,518	.498	2.74
	10	4400	19,170	.531	2.68
	3	3920	61,861	.577	2.65
	1	3440	174,432	.618	2.60
-12	20	4720	7,900	.413	2.61
	10	4400	16,390	.454	2.57
	3	3920	54,785	.511	2.53
	1	3440	158,343	.561	2.50
-14	20	4720	6,282	.328	2.47
	10	4400	13,610	.377	2.45
	3	3920	47,817	.446	2.42
	1	3440	142,255	.504	2.40
-16	200		Subcooled Liquid		
	100	5520	391	.083	2.39
	20	4720	4,664	.243	2.34
	10	4400	10,831	.300	2.32
	3	3920	40,848	.381	2.31
	1	3440	126,167	.447	2.29
-18	100		Subcooled Liquid		
	20	4720	3,028	.157	2.20
	10	4400	8,051	.223	2.20
	3	3920	33,879	.316	2.20
	1	3440	109,796	.389	2.19

Table 1 (continued)

Entropy	Pressure (atm)	Temp. (°K)	Specific Volume (cm ³ /mole)	Mole Fraction of Vapor	Internal Energy (kJ/gm)
-20	100		Subcooled Liquid		
	20	4720	1,409	.072	2.07
	10	4400	5,271	.146	2.07
	3	3920	26,910	.251	2.08
	1	3440	93,708	.332	2.09
-22	20		Subcooled Liquid		
	10	4400	2,491	.069	1.94
	3	3920	19,941	.186	1.97
	1	3440	77,619	.275	1.98
-24	10		Subcooled Liquid		
	3	3920	12,973	.121	1.86
	1	3440	61,531	.218	1.88
Condition for Saturated Liquid					
-16	184	5864	---	0	2.41
-18	84	5408	---	0	2.23
-20	38	5000	---	0	2.07
-22	18.6	4680	---	0	1.94
-24	8.6	4408	---	0	1.82

III. THE WORK ENERGY IN AN ISENTROPIC EXPANSION

The work energy in an isentropic expansion has been regarded as a measure of the destructive capacity of an HCDA. If the pressure at the end of the power excursion is above the strength of the structures containing the core, then these structures will be strained, possibly failed, and the materials set in motion. The isentropic expansion energy is then converted to strain energy and kinetic energy. Eventually the materials come to rest with the kinetic energy dissipated in further strain energy and heat.

Clearly an upper limit to the kinetic energy and strain energy production attainable is obtained by following the expansion down to atmospheric pressure where the driving force goes to zero. But the energy conversion process can end sooner than that. The pressure may fall below the strength of some intact vessel. Then the expansion will stop at an elevated pressure. The expansion process may vent into some larger gas space at a certain point in the expansion. Then the energy of the hot gases is dissipated by mixing with the larger volume of cool gases. All such effects are closely related to the detailed mechanical design of the reactor. It is not possible to define a general measure of destructive capacity which accounts for these effects. We therefore prefer to use the expansion to atmospheric pressure and point out that the energy conversion process may stop short of this.

The isentropic expansion energy will depend in general upon three independent thermodynamic variables. The three we find it convenient to work with are

1. The initial internal energy
2. The initial specific volume
3. The final pressure (usually one atmosphere)

In this section we always expand to one atmosphere and therefore the work will depend only upon 1. and 2. We carried out the calculation for initial specific volumes ranging from 40 cm³/gm.mole (6.8 gm/cm³) to 135 cm³/gm mole (2 gm/cm³). We found that over this range of initial densities the available work is independent of initial density within the accuracy of the calculations. Therefore we are able to present the results as a single table of available work energy vs. internal energy. This is given in Table 2. Our internal energy scale is based upon a zero of energy for the liquid state at 273°K. This table should only be used in connection with calculations using the ANL equation of state in VENUS-II. It will be inconsistent with other equations of state.

For internal energies above 3.0 this table of values is fit to good accuracy by the empirical equation

$$y = \frac{7.12}{x^{(0.3)}} + x - 7.74$$

where x is internal energy and y is available work. This fit was made before we did the low energy calculations.

Table 2

Available Work vs. Internal Energy

Internal Energy (kj/gm)	Available Work (kj/gm)	Internal Energy (kj/gm)	Available Work (kj/gm)
6.0	2.5	2.4	.19
5.5	2.1	2.3	.16
5.0	1.7	2.2	.13
4.5	1.3	2.1	.11
4.0	1.0	2.0	.08
3.5	.70	1.9	.06
3.0	.40	1.8	.04
2.5	.18	1.7	.02
		1.6	~0

We have no doubt that adjustments in the constants could be made to extend the fit over the entire range of the table.

We have used this fit to extrapolate to energies above the range of the table. In fact the general form was chosen so that a sensible extrapolation would result. Note that for large x and y $dy/dx = 1$, i.e. at high energy any added internal energy is recoverable as work. This is perhaps not exactly true but is a good approximation. In any case the extrapolation is probably as good as continuing the calculations beyond their range of validity. We have used this extrapolation in Part Four of this report where autocatalytic effects lead to very high energy release.

There are two ways of arriving at numbers for Table 2 from the Menzies tables and their extension in Table 1. of Section II. In an isentropic expansion the work done is equal to the decrease in internal energy. One can simply follow a line of constant entropy through the tables from initial to final state and then subtract to get the change in internal energy. This is the most straightforward and simple procedure and was used for internal energies below 3 kJ/gm. At the lower energies the differences are too small and even sometimes give negative values for the work due to internal inconsistencies. Thus at the lower energies we calculated the work from the integral of pressure with respect to volume along an isentrope. A smooth curve was drawn through the intermediate range connecting the results for high and low energy.

IV. COMPARISON OF OUR WORK ENERGY WITH STRATTON'S KINETIC ENERGY

In W. R. Stratton's calculations using the PAD program, the program outputs the kinetic energy of all the materials in the system. At the time the power excursion ends the materials have little kinetic energy and the excursion energy is in the form of internal energy of the hot core. The PAD program continues the hydrodynamic calculation, following the transformation of internal energy into kinetic

energy as the materials accelerate. The program employs equilibrium thermodynamics with no dissipative effects and so the physical principles require that the kinetic energy generated must equal the loss in internal energy.

As a measure of destructive capacity Stratton calculates the total kinetic energy at the time the core volume has expanded by a factor of eight. With his choice of parameters in his equation of state this measure of damage potential checks reasonably well with some experimental estimates of damage in transients on Godiva, Kiwi, and SNAP (4). We do not share his confidence that this can be extended to the kind of accident conditions we are interested in the LMFBR HCDA studies.

It is important to recognize that Stratton's kinetic energy at eight-fold volume increase is a very different measure of damage potential than our available work in expansion to one atmosphere. There is not simple relationship between the two. In certain instances the two numbers may agree within a factor of two. In other cases they will differ by a factor of 100. One has to look closely to determine whether a large discrepancy between a PAD calculation and a VENUS calculation with the isentropic work evaluated is because there is a basic difference in damage potential or only because the two measures of damage potential are so different.

The reason the numbers can be so different for these two measures is that in weak accidents where only a small part of the excursion energy is available for work most of the available work in our expansion to one atmosphere comes after the core has expanded beyond a factor of eight in volume.

We have calculated as a function of internal energy the work done in an expansion by a factor of eight in volume and in an expansion to one atmosphere. This is given in Table 3. We also show the work in an expansion to a final pressure of 20 atmospheres.

Table 3

Comparison of Our Measure of Damage Potential
and Stratton's Measure

Internal Energy (kj/gm)	Available Work to One Atmosphere <u>kj/gm</u>	Available Work to Twenty Atmospheres <u>kj/gm</u>	Stratton's Kinetic Energy <u>kj/gm</u>
6.0	2.5	2.1	1.1
5.0	1.7	1.4	.7
4.0	1.0	.75	.3
3.0	.40	.25	.07
2.5	.18	.10	~.01
2.2	.13	.03	~.005
2.0	.08	0	~.001

One has a very good argument that in weak accidents the expansion to one atmosphere is a poor measure of damage potential. For example, if the initial internal energy is

2.0 kJ/gm the initial pressure is only 20 atmospheres. This could probably do no mechanical damage at all unless there is a fuel coolant interaction to raise the pressure. But providing the initial pressure can fail the reactor vessel or the plug restraint, then certainly the expanding gases will continue to accelerate the pieces far beyond a factor of eight volume increase.

We do not have the answer to the quest for the best index of damage potential. But as long as there are such diverse ones as Stratton's expansion by a factor of eight in volume and our expansion to one atmosphere, one needs to be very cautious in interpreting the literature on HCDA results.

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Part Three

Comparison of Our Calculations With Those of
Edwards and Mather for a Particular HCDA

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I. INTRODUCTION AND DISCUSSION OF RESULTS

The paper by Edwards and Mather(1) at the 1972 Karlsruhe conference on Engineering of Fast Reactors for Safe and Reliable Operation attracted considerable interest in the USA. Their methods are relatively simple, they presented the results in particularly useful form, and the explosive energy yields calculated appeared to be low enough to be readily accommodated (except at very high reactivity insertion rates) without severe perturbations in a normal reactor design. We were asked by our sponsor to examine this work and determine whether it is consistent with what would be estimated for the same conditions by methods in use in this country, especially the VENUS-II method.

We have endeavored to understand the methods of Edwards and Mather as well as possible in the time available to reduce the risk that chance compensating effects would confuse the issue. To this end we first developed our own modified Bethe-Tait procedure which very closely parallels one of their methods. This work is described in Section II. The agreement using these methods and using their disassembly feedback function is exceedingly good. However, using their pressure-internal energy function and a simple parabolic power distribution and reactivity distribution, with constants based on their distributions, we are unable

to come close to matching their disassembly reactivity feedback function (see Fig. 1 of Reference 1). In fact, we get a good match at energies above 6,000 megajoules but we get one order of magnitude lower at about 4.5 mj and two orders of magnitude lower at about 3.2 mj. We apologize to Edwards and Mather that time ran out on this work before we were able to correspond further with them on this. It seems likely that the difference is an error by either them or us or a simple misunderstanding of their input data. But note that we also get a discrepancy at low yield between our VENUS calculations and their results, which may be related to this. This is discussed in Section III.

Another word of caution is appropriate. Our simple method ignores the delayed neutrons. Edwards and Mather state that they have included delayed neutrons. With only this much information one might jump to the conclusion that the delayed neutrons are unimportant to the excursion. This may not be true.

There is an important role for the delayed neutrons in this kind of excursion. One important effect is to determine the power level at the time the reactivity passes prompt critical. If one ignores them totally then the calculation starts at prompt critical with a specified initial power. But with delayed neutrons included the power rises by about a factor of six (for our particular problem) between delayed critical and prompt critical. The Doppler

feedback then comes in more quickly and terminates the burst with less energy production. If the Doppler effect is strong enough to terminate the excursion before there is pressure produced then the well known Doppler induced power oscillations develop and continue until a final burst raises the pressure high enough for disassembly.

The same general behavior occurs with no delayed neutrons, except that we expect the bursts to be stronger and fewer. The final energy production may be comparable. However, the plot of energy vs. Doppler constant at constant ramp rate can be different with and without delayed neutrons. Part of the effect can be easily accounted for by starting the calculations with no delayed neutrons at a higher power level. However, the minimum power between the peaks is also influenced by the delayed neutrons and this affects the development of the following peak.

Actually, with the parameters and conditions involved in our comparison calculations only one power peak occurs up to Doppler constant of .003 and two peaks between .003 and .007. When there is just one peak the only important effect of the delayed neutrons is to determine the power level at the time prompt critical is reached.

While one expects the detailed behavior to depend upon the delayed neutrons the envelope of the peaks, or the smooth curve which averages the decaying oscillations in the energy yield vs. Doppler constant curves is expected

to be less dependent upon delayed neutrons. Thus we expect our simple Bethe-Tait model in Section II to be a useful one. Its apparent high degree of success in matching the Edwards and Mather calculations is a little surprising.

The second step taken in the comparison of methods was to set up the problem for solution with VENUS-II. This is described in Section III of this Part Three of the report. The agreement is not as good. The main discrepancy is in the Doppler oscillation which is stronger and results in higher energy yield at large Doppler effect.

There are some problems in devising a VENUS model that corresponds closely to the model of Edwards and Mather. This is discussed in Section III. Nevertheless we believe that we have matched models closely enough for meaningful comparisons.

At the time VENUS-II was developed the popular concept of how the thermal energy of a VENUS power excursion is converted to mechanical energy for damage estimates involved the fuel-coolant interaction process. Therefore VENUS-II calculated the energy in molten fuel as a measure of damage potential. Edwards and Mather report the work potential in an expansion of the UO_2 to one atmosphere as the "explosion yield." We have therefore extended the ANL equation of state used in VENUS-II to include a calculation of the work in an isentropic expansion of the core following the power excursion. Because of the press of time we have not integrated

this calculation into VENUS-II. The development of this equation of state extension is described in Part Two of this report.

From the comparisons we have made we are convinced that the relatively simple methods of Edwards and Mather are valid and consistent with our similar method. Further study is required to determine why we disagree on the disassembly reactivity feedback function. We are concerned about the fact that VENUS gives larger energy at high Doppler effect. We have had considerable experience with VENUS and perhaps trust it more than we should. Perhaps the most useful next step toward a resolution of this question is to calculate a disassembly feedback function using the ANL equation of state and the Edwards and Mather power and reactivity distributions, but without the parabolic fits, i.e., do the calculation by numerical integration. This is an easy calculation. The feedback function can then be used in our modified Bethe-Tait program and the energy compared to the VENUS-II results.

We also obtain somewhat higher energy at low Doppler effect but the percentage difference is not really significant. If one compares the curves for work energy vs. total yield in Figure 8 it can be seen that the difference here between them and us tends to compensate for the difference in total yield. We believe this just reflects the difference between their equation of state and the ANL equation of state in VENUS-II.

At low yields the situation is more serious because a small difference in total yield can change the destructive capacity significantly. One must take great pains to assure that the equation of state in the isentropic expansion is consistent with the equation of state used in the excursion calculation. We encountered some difficulty in attaining complete consistency and perhaps they did also. One must attach a greater uncertainty to the work energy than to the total excursion energy. This is especially so at low yields (say when the work energy is only 10% or less of the excursion energy).

One should note that the Edwards and Mather calculations were done for initial fuel temperatures of 800K and 1500K. We also used 1500K for the comparison calculations. These are suitable initial conditions for certain kinds of hypothetical accidents. But there are other accident types in which the fuel is heated to or above the melting point of 3040K before entering the power excursion phase. At the higher temperature the Doppler effect is only about half what it is at 1500 K and so is less effective in limiting the excursion. There are, in fact, many other possible parameter variations to consider and so one should not attempt to conclude from these limited calculations that the HCDA in general can be shown to be a mild event if only one uses these methods.

II. A MODIFIED BETHE-TAIT METHOD FOR COMPARISON WITH EDWARDS AND MATHER

2.1 Basic Method of Analysis

The basic method of analysis was developed by H.A. Bethe and J.H. Tait. The procedure is a simple one, but it is subject to several simplifying assumptions. The two most important assumptions are:

1. The distribution of reactivity worth and also the power distribution are assumed independent of time. This ignores the fact that the core begins to disassemble. Reactivity changes are calculated by first order perturbation theory.
2. In the hydrodynamics equation describing the force balance and the equation of state, the density is assumed constant. Thus the pressure is assumed independent of the expansion required to terminate the excursion, and is calculated from the energy content and density at the beginning of the excursion. This assumption ignores the propagation and reflection of pressure waves.

The assumptions introduce little error for many situations.

2.2 Reactor Kinetics

The reactor kinetics used to describe the power excursions

are:

$$\frac{d^2 Q}{dt^2} = \frac{(k-1-\beta)}{l} \frac{dQ}{dt} + \sum_{i=1}^6 \lambda_i C_i + S \quad (1)$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{l} \frac{dQ}{dt} - \lambda_i C_i \quad (i=1 \neq 6)$$

where $Q(t)$ is the time dependance of the energy generation, $E(r,t)$. The energy generation is assumed separable so that

$$E(r,t) = N(r) Q(t)$$

where $N(r)$ is the spatial power distribution. The quantity $C_i(t)$ is proportional to the concentration of delayed-neutron precursors. The source, S , also contains the proportionality constant relating the power density to the neutron density. 'k' is the neutron multiplication constant, 'l' is the prompt neutron generation time, ' β ' is the delayed neutron fraction, and ' λ ' is the delayed neutron-precursor decay constant.

The neutron multiplication constant is given by

$$k(t) = k_0 + R_0(t) + R_m(t) + R_D(t) \quad (2)$$

where k_0 is the initial multiplication constant. $R_0(t)$ is the reactivity insertion responsible for initiating the excursion. $R_m(t)$ is the reactivity feedback due to material displacement. $R_D(t)$ is the feedback from the Doppler effect which is an instantaneous function of material temperature during the power excursion.

The Doppler reactivity for these calculations was assumed to be of the form

$$T \frac{dk}{dT} = A \quad (3)$$

where T is the temperature in $^{\circ}\text{K}$, and A is a constant in the

usual range of -0.002 to -0.006 . This form was assumed to be typical of UO_2 .

The material displacement reactivity feedback can be written as

$$R_m(t) = \int \underline{u}(\underline{r}, t) \cdot \underline{\text{grad}} D(\underline{r}) d^3 \underline{r} \quad (4)$$

where $-D(\underline{r})$ is the reactivity change that would occur if a unit volume of homogenized core material at position $\underline{r}$ were to be removed from the core. Therefore $\underline{u}(\underline{r}, t) \cdot \underline{\text{grad}} D(\underline{r})$ is the reactivity which occurs when a unit volume of material is displaced from its initial location $\underline{r}$ to the position $\underline{r} + \underline{u}(\underline{r}, t)$.

When the density is assumed constant

$$\nabla^2 \underline{u}(\underline{r}, t) = -\frac{1}{\rho} \underline{\text{grad}} P(\underline{r}, t) \quad (5)$$

$$P(\underline{r}, t) = P(E(\underline{r}, t), \rho) \quad (6)$$

where P is the pressure.

The continuity equation is not needed because the change in density is neglected. It can be used, however, to check the assumption that density changes are indeed minor.

Differentiating equation 4 twice with respect to time and combining with equation 5 gives

$$\frac{d^2 R_m(t)}{dt^2} = -\frac{1}{\rho} \int [\underline{\text{grad}} P(\underline{r}, t)] \cdot [\underline{\text{grad}} D(\underline{r})] d^3 \underline{r} \quad (7)$$

Conditions at the Core Surface

Since the $\text{grad } P(\underline{r}, t) \rightarrow -\infty$ at the edge of the core because the energy density is discontinuous across the core boundary, it is necessary to specify whether the integral in equation 7 goes just up to the inner surface or just beyond it.

This report assumes the tamped core model which gives a higher estimate of the energy release than the free surface model. A discussion of the difference is given by Nicholson⁽²⁾. The effect is to ignore the surface integral and replace $P(\underline{r}, t)$ by $P_i(\underline{r}, t)$ in equation 7.

2.3 Modified Bethe-Tait Method

Nicholson⁽²⁾ developed a modified Bethe-Tait method of analysis. The main difference in the final result being the expression for reactivity changes by the function $D(\underline{r})$ rather than by one energy group perturbation theory. We reproduce that derivation here with the inclusion of the Doppler effect.

Additional Approximations

In order to perform the analysis, several additional assumptions are made. Which are (1) to neglect delayed neutrons, (2) restriction of the analysis to uniform spherical geometries, (3) use of a linear approximation for the relation between pressure and energy, and (4) a division of the excursion into two phases. Restriction 4 will be removed later. Restrictions 2 and 3 can be partially removed if $d^2R_a/dt^2 =$ a function of

energy can be found.

During the first phase the reactivity is assumed to be added at a constant rate; $R_0(t) = \alpha t$. In the modified Bethe-Tait method the reactivity is ignored in the first phase. In the modified method with Doppler feedback, only the reactivity feedback due to material movement is ignored during the first phase. Phase 1 is assumed to start at prompt critical in both cases.

Phase 2 starts just as the material feedback starts to become important. In the modified Bethe-Tait method the additional reactivity insertion during disassembly is ignored. This is acceptable because once the disassembly begins, it proceeds quite rapidly. In the modified Bethe-Tait with Doppler feedback the small additional reactivity insertion from the ramp insertion during disassembly is included.

Modified Bethe-Tait

The energy generation is assumed to be governed by:

$$\frac{d^2 Q}{dt^2} = \frac{(\beta - 1 - \rho)}{\lambda} \frac{dQ}{dt} + S \quad (8)$$

and the source is neglected above prompt critical, the starting point. For a linear approximation to the equation of state

$$P(r,t) = \begin{cases} (\gamma-1) P \{ E(r,t) - Q^* \} & E(r,t) > Q^* \\ = 0 & E(r,t) < Q^* \end{cases} \quad (9)$$

is used. The asymptotic initial conditions for the start of phase 2 are:

$$\frac{dQ(t_1)}{dt} = \frac{\alpha t_1}{l} Q(t_1) \quad (10)$$

and $Q(t_1) = Q^*$

These conditions and equations 7, 8, and 9 define the modified Bethe-Tait problem during phase 2.

Nicholson⁽²⁾ showed that if one assumes a uniform spherical geometry, and a parabolic distribution for $N(r)$ and $D(r)$ as;

$$N(r) = 1 - q \frac{r^2}{a^2} \quad 0 \leq r \leq a$$

$$D(r) = D(0) - \frac{d}{2} \frac{r^2}{a^2} \quad (11)$$

a solution for $d^2 R_m / dt^2$ can be found. The solution is:

$$\frac{d^2 K}{d\tau^2} = -\left(\frac{y+1}{x}\right) \left(\frac{y}{y+1}\right)^{5/2} \quad 0 < y < \frac{q}{1-q} \quad (12)$$

$$\frac{d^2 K}{d\tau^2} = -q^{5/2} \frac{(y+1)}{x} \quad y > \frac{q}{1-q} \quad (13)$$

$$\frac{d^2 y}{d\tau^2} = K \frac{dy}{d\tau} \quad (14)$$

where

$$y = \frac{Q - Q^*}{Q^*} ; \quad K(t) = \frac{k_1(t) - 1 - \beta}{k_1 - 1 - \beta} ; \quad \tau = t \frac{k_1 - 1 - \beta}{l}$$

k_1 is the reactivity at the start of phase 2.

Fig. 1 Energy Release as a Function of 'x' in Modified
Bethe-Tait Calculations for a Tamped Core

The initial conditions are:

$$\begin{aligned} y(0) &= 0 & \frac{dy}{d\tau} &= 1 \\ K(0) &= 0 & \frac{dK}{d\tau} &= 0 \end{aligned}$$

A curve of the calculated value of $y(\infty)$ for the modified Bethe Tait method is plotted in Figure 1 for $q = 0.6$. The results are for a tamped core.

2.3 Modified Bethe-Tait with Doppler Feedback

The Doppler feedback is added as:

$$T \frac{dR_0}{d\tau} = A \quad (15)$$

The Doppler feedback is calculated during the entire transient starting from the initial conditions at prompt critical which are:

$$\begin{aligned} y(0) &= -1 & \frac{dy(0)}{d\tau} &= \frac{P_0 l}{(k_1 - 1 - \beta) Q^*} \\ K(0) &= 0 & \frac{dR_0(0)}{d\tau} &= 0 \end{aligned} \quad (16)$$

In order to use equation 15 the units are converted to the tau, kappa, and y system.

The Doppler effect should be weighted according to a φ^2 distribution. Assume:

$$\varphi(r) = 1 - q \frac{r^2}{a^2} \quad (17)$$

and

$$\varphi^2(r) = 1 - 2q \frac{r^2}{a^2} + q^2 \frac{r^4}{a^4}$$

There are at least two methods of applying the weighting. The first to be presented is what is believed to be the method used by Edwards⁽¹⁾. It is convenient to assume a uniform initial temperature in the reactor. Then

$$\bar{T} \frac{dk_p}{d\bar{T}} = A \quad (18)$$

where

$$\bar{T} = \frac{4\pi \int T(r) N^2(r) r^2 dr}{4\pi \int N^2(r) r^2 dr}$$

In many cases, the flux and power shapes are the same. If this is not the case as in a reactor with varying enrichments, it may be desirable to use a different shape factor for power. In this case assume:

$$N(r) = 1 - q_2 \frac{r^2}{a^2} ; 0 \leq r \leq a \quad (19)$$

Then

$$T(r) = T_0 + \frac{Q N(r)}{C_p} \quad (20)$$

$$\bar{T} = T_0 + \frac{Q}{C_p} \frac{\int N(r) \varphi^2(r) r^2 dr}{\int \varphi^2(r) r^2 dr} \quad (21)$$

Substituting and evaluating gives

$$\bar{T} = T_0 + \frac{Q}{C_p} \frac{(1 - 1.2q - .6q_2 + \frac{3}{4}q^2 + \frac{6}{7}qq_2 + q_2^2/3)}{1 - 1.2q + \frac{3}{4}q^2} \quad (22)$$

or

$$\overline{T} = T_0 + \frac{Qp}{C_p} \quad (23)$$

where

$$p = 1 + \frac{.6 q_2 + \frac{6}{7} q_2^2 - \frac{1}{3} q_2^3}{1 - 1.2 q + \frac{3}{7} q^2}$$

The second weighting method gives different but similar results. Although more difficult to utilize, it is more correct. Assume the contribution of a unit volume at r is

$$T \frac{dk(r)}{dt} = C (N^2(r)) \quad (24)$$

C is evaluated by requiring that if T is constant then

$$\int \frac{dk(r)}{dT} dV = \frac{+A}{T} \quad (25)$$

Then in spherical co-ordinates

$$C \int N^2(r) 4\pi r^2 dr = A$$

or

(26)

$$C = \frac{A}{4\pi \int N^2(r) r^2 dr}$$

Then

$$\int k(r) = \frac{A}{4\pi \int N^2(r) r^2 dr} N^2(r) \ln \left(\frac{T(r)}{T_0} \right) \quad (27)$$

Integrating gives

$$\int k = \frac{A}{4\pi \int N^2(r) r^2 dr} \int \left[N^2(r) \cdot \ln \left(\frac{T(r)}{T_0} \right) \right] dr \quad (28)$$

In either case the modified temperature is $\bar{T} = \frac{PQ}{C_p} + T_0$ and substituting for y gives

$$\bar{T} = \frac{PQ^*}{C_p} (y+1) + T_0 \quad (29)$$

Differentiating gives

$$\frac{dy}{d\bar{T}} = \frac{C_p}{PQ^*} \quad (30)$$

Therefore

$$\frac{dk_0}{d\bar{T}} = \frac{dk_0}{dy} \cdot \frac{dy}{d\bar{T}} = \frac{dk_0}{dy} \cdot \frac{C_p}{PQ^*} \quad (31)$$

Regrouping equation 18 and substituting for T gives

$$\frac{dk_0}{d\bar{T}} = \frac{A}{\frac{PQ^*}{C_p} (y+1) + T_0} \quad (32)$$

and substituting from equation 31 gives

$$\frac{dk_0}{dy} = \frac{A}{y+1 + \frac{T_0 C_p}{PQ^*}} \quad (33)$$

Again substituting

$$\frac{dK_0}{dR} = \frac{1}{R_i - 1 - \beta} \quad (34)$$

gives

$$\frac{dK_D}{dy} = \frac{A/(k_1 - 1 - \beta)}{y + 1 + \frac{T_0 C_p}{P Q^*}} \quad (35)$$

Equation 35 may be separated to give

$$K_D = \frac{A}{k_1 - 1 - \beta} \ln \left(y + 1 + \frac{T_0 C_p}{P Q^*} \right) + D \quad (36)$$

Applying the initial condition that $K_D(0) = 0$ implies that

$$D = -\frac{A}{k_1 - 1 - \beta} \ln \left(y(0) + 1 + \frac{T_0 C_p}{P Q^*} \right) \quad (37)$$

Therefore

$$K_D = \frac{A}{k_1 - 1 - \beta} \ln \left(\frac{y + 1 + \frac{T_0 C_p}{P Q^*}}{y(0) + 1 + \frac{T_0 C_p}{P Q^*}} \right) \quad (38)$$

Equations 12, 13, 14, and 38 are solved by explicitly marching forward in time (τ) in the computer iteration.

2.5 Curve Fit for Disassembly Reactivity

A related but somewhat different approach is to fit a curve to the disassembly characteristic, $\frac{d^2 R_m}{dt^2}$, of the reactor. For some of the cases analyzed here, a curve was fit to the disassembly characteristic derived by Edwards in TRG-Report 2370. He used the diffusion code SCRAMBLE to produce regular and adjoint flux tapes as required as input for EXTRA. The code accommodates two dimensional (RZ) 100 zone geometry and produces tables of the disassembly characteristic and other parameters. The disassembly characteristic was fit for a 670 Mw(th) reactor starting at 1500°K with a fuel density of 2000 Kg/m³. The curve

was fit by the function

$$\frac{d^2 R_m}{dt^2} = C_2 \exp \left(\frac{-2.3 C}{W - W^*} \right) \quad (39)$$

where W is the total energy release in the reactor, and C , C_2 , and W^* are empirical fitting parameters. As can be seen, this curve is a logarithmic hyperbola.

The empirical parameters are shown in Figure 2 which is a plot of the disassembly characteristic. As can be seen C is a shape factor, W^* is an energy below which there is no reactivity feedback from disassembly, and C_2 is the asymptotic value of the disassembly characteristic for the particular shape factor.

This curve may be easily converted to the modified system of units. It can be shown that

$$y = \frac{Q - Q^*}{Q^*} = \frac{W - W^*}{W^*} \quad (40)$$

and

$$\frac{d^2 K_m}{d\tau^2} = \frac{d^2 R_m}{dt^2} \left[\frac{l}{(k_1 - 1 - \beta)} \right]^2 \frac{1}{k_1 - 1 - \beta} \quad (41)$$

Therefore

$$\frac{d^2 K_2}{d\tau^2} = C_2 \frac{l^2}{(k_1 - 1 - \beta)^3} \exp \left(\frac{-2.3 C/W^*}{y} \right) \quad (42)$$

Special Instructions For Figure 2 ~~of~~ of part Three

Figure 2 has a transparency overlaying the figure. The two must be assembled such that the curve on the transparency coincides with the curve labeled $C=12$ on the figure and with the coordinate axes of the transparency parallel to (not coincident with) the axes of the figure. With the figures in the master copy this happens when the upper left hand corners and the left edge of the papers coincide.

Disassembly Characteristic - Edwards

FIG-2 DISASSEMBLY CHARACTERISTIC FITTING CURVE

Disassembly Characteristic - Edwards

2.6 Reactor Model Used by Edwards et. al.

The specific case which was used in order to try to correlate with the modified Bethe-Tait model with Doppler feedback was a 670 Mw(th) cylindrical reactor. According to Edwards⁽¹⁾ the calculations were based on EXTRA using two dimensional reactor models and a standard set of UO₂ data without fission products. The hydrodynamics was treated in the standard Bethe-Tait method.

The calculational model used by Edwards was treated by the diffusion code SCRAMBLE to produce regular and adjoint flux tapes as required as input for EXTRA. The code accomodates a 100 zone two diminsional (RZ) geometry, and produces tables of the functions of disassembly characteristic, the Doppler feedback, and yield as a function of deposited energy. The excursion kinetics modules use these tabular functions to calculate yield.

More specific information was requested from Edwards as to the exact nature of the reactor; which was then supplied. A cross section of the reactor model is shown in Figure 3.

The normalized flux and power shapes are shown in Figures 4 and 5. The enrichment is greater in the outer zone so that the power jumps at the interface by a factor of about 1.34.

Parameters used to Model Reactor

$$k_1 - 1 - \beta = 0.001 \text{ (arbitrary)} \quad l = 0.5 \times 10^{-6} \text{ sec}$$

FIG. 3 REACTOR MODEL CROSS SECTION

FIG-4 NORMALIZED FLUX

FIG. 5 NORMALIZED POWER

$$P_0 = 178 \text{ joule/sec-gram}$$

$$q = 0.2$$

$$q_2 = 0.55$$

$$T_0 = 1500^\circ\text{K}$$

$$M = 3.75 \times 10^6 \text{ gram}$$

$$C = 12$$

$$W^* = 200 \text{ MJ}$$

$$C_2 = -2.5 \times 10^6$$

2.7 Comparision with Edwards Result

The maps of total energy release for the reactor described in the previous section are shown in the following figures. Figure 6 is Edwards work, and Figure 7 is the result of the calculation using the Modified Bethe-Tait with Doppler Feedback model.

There are four noticeable differences in the two plots. The shape of the peak in the center of the figures is different. The peak does not protrude into the lower ramp rates in Edwards map. Note that the values on the high Doppler coefficient side of the peak do agree closely.

In the block bounded by a ramp rate of \$130/sec and a Doppler coefficient of -0.0015, Edwards gets lower values of energy release.

In the right hand section of the map, additional fine structure is shown in Figure 7 which is not shown either way by Edwards.

In the center of the maps the shape of the 3000 contour is different.

Special Instructions For Figures 6 and 7. of Part Three

Figure 6 is a transparency which overlays Figure 7. They must be assembled such that the coordinate axes coincide.

FIG-6 EXCURSION YIELD - EDWARDS WORK (MJ).

FIG-7 MODIFIED BETHE-TAIT WITH DOPPLER FEEDBACK

Fig. 8 Available Work as a Function of Total Yield

To determine the destructive potential of the accident we calculate the isentropic work in an expansion to one atmosphere for each element of volume of the core and then integrate over volume. In our notation the specific internal energy at each point in the reactor is given by

$$u(r) = \frac{w(1-.35r^2/a^2)}{M(1-0.6q_2)} + 0.33 \text{ kj/gm}$$

where the additive constant .33 kj/gm adjusts to the zero of the internal energy scale used in Part Two. In that part also, the work potential is given as a function of the internal energy. This was integrated over the volume. The results are given in Figure 8 and compared there to the corresponding results of Edwards and Mather.

III. COMPARISON OF VENUS RESULTS WITH THE EDWARDS AND MATHER CALCULATIONS

In Section II we found excellent agreement between our calculations and those of Edwards and Mather when we used a method of analysis and equation of state as nearly as possible equivalent to theirs. While those methods are expected to be good for most purposes, VENUS does some

things differently from our modified Bethe-Tait calculation; especially it includes delayed neutrons and it solves the hydrodynamics equations.

It is not possible to set up a model for VENUS-II that is truly identical to the one used by Edwards and Mather. The main problem is that in the interest of conservatism they elect to do most of their calculations with an equation of state (pressure vs. internal energy) for an effective fuel density of 2 gm/cm^3 , regardless of what the actual fuel density is. But their caloric equation of state (temperature vs. energy addition) utilizes the actual density, which in the calculations we are comparing was 2.47 gm/cm^3 . With VENUS we believe the best comparison is obtained by using the 2 gm/cm^3 density and then correcting the calculated energy by multiplying VENUS results by the density ratio 1.23. This is what has been done. All the VENUS results reported below have been corrected by this factor.

The VENUS-II Model

A. G. Edwards and D. J. Mather kindly provided us with sufficient detail on the model and data for their calculations to set up comparable VENUS-II calculations. We were able to use their reactivity worth and power distributions directly. Their geometric model consisted of the entire core and blanket, which was not quite symmetrical about the midplane. The power and reactivity worths were slightly

135.7 *****

0.0 *****

			4	7	1
			6	3	2
0	7				0
•	•		•	•	•
0	6		7	2	0

Fig. 9. Model for Venus Calculations.
Dimensions are in cm.

Table 1

NORMALIZED POWER DENSITIES (RADIAL PROFILES, STARTING AT TOP OF REACTOR)

0.1350-09	0.1340-09	0.1320-09	0.1580-08	0.1490-08	0.1420-08	0.1360-08	0.1300-08	0.1240-08	0.1180-08
0.1120-08	0.1060-08	0.9960-09	0.9270-09	0.8560-09	0.7830-09	0.7070-09	0.6290-09	0.4860-09	0.3090-09
0.2020-09	0.1290-09	0.7460-10							
0.2750-09	0.2730-09	0.2690-09	0.3300-08	0.3140-08	0.3010-08	0.2890-08	0.2780-08	0.2660-08	0.2540-08
0.2420-08	0.2290-08	0.2150-08	0.2010-08	0.1860-08	0.1700-08	0.1540-08	0.1380-08	0.1090-08	0.7030-09
0.4600-09	0.2940-09	0.1690-09							
0.5540-09	0.5500-09	0.5430-09	0.7000-08	0.6690-08	0.6440-08	0.6210-08	0.5980-08	0.5750-08	0.5510-08
0.5260-08	0.4990-09	0.4700-08	0.4390-08	0.4060-08	0.3720-08	0.3370-08	0.3010-08	0.2360-08	0.1540-08
0.1000-08	0.6330-09	0.3610-09							
0.1030-08	0.1030-08	0.1020-08	0.1460-07	0.1400-07	0.1360-07	0.1310-07	0.1270-07	0.1230-07	0.1180-07
0.1130-07	0.1090-07	0.1020-07	0.9530-08	0.8820-08	0.8070-08	0.7280-08	0.6470-08	0.4920-08	0.3190-08
0.2050-08	0.1280-08	0.7210-09							
0.1690-08	0.1680-08	0.1670-09	0.3020-07	0.2940-07	0.2870-07	0.2810-07	0.2750-07	0.2670-07	0.2590-07
0.2500-07	0.2400-07	0.2280-07	0.2140-07	0.1980-07	0.1800-07	0.1610-07	0.1400-07	0.9650-08	0.6110-08
0.3830-08	0.2330-08	0.1300-08							
0.2050-08	0.2040-08	0.2010-08	0.5020-06	0.4920-06	0.4840-06	0.4750-06	0.4650-06	0.4530-06	0.4400-06
0.4240-06	0.4060-06	0.5120-06	0.4810-06	0.4450-06	0.4070-06	0.3660-06	0.3270-06	0.1470-07	0.6830-08
0.5360-08	0.3200-08	0.1760-08							
0.2230-08	0.2220-08	0.2190-08	0.5610-06	0.5530-06	0.5450-06	0.5360-06	0.5250-06	0.5130-06	0.4980-06
0.4810-06	0.4620-06	0.5840-06	0.5460-06	0.5080-06	0.4630-06	0.4160-06	0.3700-06	0.1690-07	0.1000-07
0.6030-08	0.3570-08	0.1960-08							
0.2410-08	0.2400-08	0.2370-08	0.6210-06	0.6130-06	0.6060-06	0.5970-06	0.5860-06	0.5720-06	0.5570-06
0.5380-06	0.5180-06	0.6550-06	0.6160-06	0.5700-06	0.5200-06	0.4670-06	0.4140-06	0.1900-07	0.1120-07
0.6680-08	0.3940-08	0.2160-08							
0.2590-08	0.2580-08	0.2550-08	0.6770-06	0.6700-06	0.6630-06	0.6540-06	0.6420-06	0.6280-06	0.6110-06
0.5920-06	0.5690-06	0.7200-06	0.6780-06	0.6820-06	0.5720-06	0.5130-06	0.4540-06	0.2090-07	0.1220-07
0.7270-09	0.4270-08	0.2330-08							
0.2760-08	0.2750-08	0.2720-08	0.7270-06	0.7200-06	0.7130-06	0.7040-06	0.6910-06	0.6770-06	0.6590-06
0.6380-06	0.6140-06	0.7780-06	0.7320-06	0.6780-06	0.6180-06	0.5540-06	0.4900-06	0.2260-07	0.1310-07
0.7790-08	0.4570-08	0.2490-08							
0.2900-08	0.2890-08	0.2860-08	0.7680-06	0.7620-06	0.7550-06	0.7450-06	0.7330-06	0.7170-06	0.6980-06
0.6770-06	0.6520-06	0.8250-06	0.7770-06	0.7190-06	0.6560-06	0.5880-06	0.5200-06	0.2390-07	0.1390-07
0.8230-08	0.4810-08	0.2620-08							
0.3010-08	0.3000-08	0.2970-08	0.8000-06	0.7950-06	0.7870-06	0.7770-06	0.7640-06	0.7480-06	0.7290-06
0.7070-06	0.6210-06	0.8620-06	0.8110-06	0.7520-06	0.6850-06	0.6140-06	0.5430-06	0.2500-07	0.1450-07
0.8570-08	0.5010-08	0.2730-08							
0.3090-08	0.3080-08	0.3050-08	0.8230-06	0.8170-06	0.8090-06	0.7990-06	0.7860-06	0.7700-06	0.7500-06
0.7270-06	0.7000-06	0.8870-06	0.8350-06	0.7730-06	0.7050-06	0.6320-06	0.5580-06	0.2570-07	0.1490-07
0.8800-08	0.5140-08	0.2800-08							
0.3130-08	0.3120-08	0.3090-08	0.8340-06	0.8280-06	0.8210-06	0.8110-06	0.7970-06	0.7810-06	0.7610-06
0.7370-06	0.7110-06	0.9000-06	0.8470-06	0.7850-06	0.7150-06	0.6410-06	0.5670-06	0.2610-07	0.1510-07
0.8920-08	0.5210-08	0.2830-08							

Table 2

INPUT MATERIAL WORTH FOR REGION 1 (DELTA K/CM**3)

0.206310-09 0.206210-09 0.206050-09
 0.158470-08 0.157410-08 0.155330-08
 0.846050-08 0.841440-08 0.832820-08
 0.369230-07 0.368930-07 0.367180-07
 0.113780-06 0.113360-06 0.112600-06
 0.159810-06 0.162360-06 0.167580-06
 0.136980-06 0.138950-06 0.142960-06
 0.996790-07 0.101010-06 0.103700-06
 0.560680-07 0.570490-07 0.590430-07
 0.120760-07 0.129550-07 0.144530-07

0.280330-07-0.273980-07-0.260550-07

0.610560-07-0.605060-07-0.593590-07

0.846140-07-0.841240-07-0.831000-07

0.970900-07-0.966320-07-0.956710-07

R LATTICE (CM)

0.19200+01 0.38400+01 0.57600+01

Z LATTICE (CM)

0.25390+01 0.76170+01 0.12700+02 0.17770+02 0.22850+02 0.27930+02 0.33010+02 0.38090+02 0.43160+02 0.54710+02
 0.72730+02 0.90750+02 0.10880+03 0.12680+03

INPUT MATERIAL WORTH FOR REGION 2 (DELTA K/CM**3)

0.244790-06 0.250390-06 0.251120-06 0.248430-06 0.243060-06 0.235580-06 0.226380-06 0.215820-06 0.204210-06
 0.262380-06 0.268170-06 0.268810-06 0.266130-06 0.260740-06 0.253140-06 0.243690-06 0.232730-06 0.220480-06
 0.283680-06 0.287050-06 0.287200-06 0.284260-06 0.278660-06 0.270770-06 0.260930-06 0.249420-06 0.236430-06
 0.302120-06 0.305600-06 0.305330-06 0.302100-06 0.296210-06 0.288000-06 0.277740-06 0.265740-06 0.252190-06
 0.320780-06 0.322720-06 0.322180-06 0.318720-06 0.312570-06 0.304040-06 0.293420-06 0.281000-06 0.267030-06
 0.335960-06 0.337590-06 0.336870-06 0.333240-06 0.326880-06 0.318090-06 0.307170-06 0.294430-06 0.280120-06
 0.348040-06 0.349450-06 0.348650-06 0.344900-06 0.338390-06 0.329410-06 0.318270-06 0.305280-06 0.290710-06
 0.356490-06 0.357770-06 0.356920-06 0.353110-06 0.346500-06 0.337400-06 0.326100-06 0.312950-06 0.298200-06
 0.360920-06 0.362140-06 0.361270-06 0.357420-06 0.350770-06 0.341600-06 0.330230-06 0.316990-06 0.302140-06

R LATTICE (CM)

0.98520+01 0.14200+02 0.18540+02 0.22890+02 0.27230+02 0.31570+02 0.35920+02 0.40260+02 0.44610+02

Z LATTICE (CM)

0.25390+01 0.76170+01 0.12700+02 0.17770+02 0.22850+02 0.27930+02 0.33010+02 0.38090+02 0.43160+02

Table 2 (continued)

INPUT MATERIAL WORTH FOR REGION 3 (DELTA K/CM**3)

0.24875D-06	0.22851D-06	0.20428D-06	0.17781D-06	0.15086D-06	0.12519D-06
0.28272D-06	0.26192D-06	0.23702D-06	0.20947D-06	0.18118D-06	0.15395D-06
0.31910D-06	0.29802D-06	0.27213D-06	0.24336D-06	0.21350D-06	0.18436D-06
0.35643D-06	0.33453D-06	0.30758D-06	0.27737D-06	0.24579D-06	0.21453D-06
0.39179D-06	0.36912D-06	0.34107D-06	0.30944D-06	0.27611D-06	0.24279D-06
0.42303D-06	0.39967D-06	0.37060D-06	0.33767D-06	0.30276D-06	0.26759D-06
0.44831D-06	0.42437D-06	0.39448D-06	0.36047D-06	0.32424D-06	0.28758D-06
0.46617D-06	0.44183D-06	0.41133D-06	0.37654D-06	0.33939D-06	0.30166D-06
0.47558D-06	0.45102D-06	0.42020D-06	0.38500D-06	0.34735D-06	0.30906D-06

R LATTICE (CM)

0.4878D+02	0.5341D+02	0.5782D+02	0.6224D+02	0.6666D+02	0.7107D+02
------------	------------	------------	------------	------------	------------

Z LATTICE (CM)

0.2539D+01	0.7617D+01	0.1270D+02	0.1777D+02	0.2285D+02	0.2793D+02	0.3301D+02	0.3809D+02	0.4316D+02
------------	------------	------------	------------	------------	------------	------------	------------	------------

INPUT MATERIAL WORTH FOR REGION 4 (DELTA K/CM**3)

0.25709D-10	0.22490D-10	0.21534D-10	0.21116D-10	0.20814D-10	0.20483D-10	0.20057D-10	0.19503D-10	0.18807D-10	0.17967D-10
0.16997D-10	0.15939D-10	0.14841D-10	0.13757D-10	0.12738D-10					
0.37582D-09	0.34393D-09	0.32557D-09	0.31137D-09	0.29821D-09	0.28467D-09	0.26999D-09	0.25379D-09	0.25395D-09	0.21641D-09
0.19542D-09	0.17367D-09	0.15179D-09	0.13051D-09	0.11049D-09					
0.20304D-08	0.19073D-08	0.18469D-08	0.18023D-08	0.17574D-08	0.17049D-08	0.16408D-08	0.15628D-08	0.14694D-08	0.13597D-08
0.17340D-08	0.16070D-08	0.15397D-08	0.14145D-08	0.12602D-08					
0.79584D-08	0.79736D-08	0.81204D-08	0.82390D-08	0.82913D-08	0.82681D-08	0.81674D-08	0.79843D-08	0.77060D-08	0.73036D-08
0.67538D-08	0.60700D-08	0.52693D-08	0.44635D-08	0.36518D-08					
0.18655D-07	0.23377D-07	0.26436D-07	0.28421D-07	0.29735D-07	0.30647D-07	0.31362D-07	0.32050D-07	0.32872D-07	0.32966D-07
0.31957D-07	0.29554D-07	0.26080D-07	0.21807D-07	0.16955D-07					

R LATTICE (CM)

0.9852D+01	0.1420D+02	0.1854D+02	0.2289D+02	0.2723D+02	0.3157D+02	0.3592D+02	0.4026D+02	0.4461D+02	0.4878D+02
0.5341D+02	0.5782D+02	0.6224D+02	0.6666D+02	0.7107D+02					

Z LATTICE (CM)

0.5471D+02	0.7273D+02	0.9075D+02	0.1088D+03	0.1268D+03
------------	------------	------------	------------	------------

INPUT MATERIAL WORTH FOR REGION 5 (DELTA K/CM**3)

0.49272D-11	0.21398D-11	0.94758D-12	0.43300D-12	0.21887D-12
0.38760D-10	0.16546D-10	0.68780D-11	0.28605D-11	0.12876D-11
0.24012D-09	0.96594D-10	0.37276D-10	0.14206D-10	0.57637D-11
0.13054D-08	0.48787D-09	0.17417D-09	0.60403D-10	0.21169D-10
0.41254D-08	0.21474D-08	0.72160D-09	0.22755D-09	0.64627D-10
0.20119D-07	0.64629D-08	0.19489D-08	0.54975D-09	0.13197D-09
0.26763D-07	0.85296D-08	0.25571D-08	0.70757D-09	0.16201D-09
0.32503D-07	0.10542D-07	0.31440D-08	0.86964D-09	0.19187D-09
0.37559D-07	0.12399D-07	0.37470D-08	0.10261D-08	0.21997D-09
0.41960D-07	0.14042D-07	0.42725D-08	0.11686D-08	0.24501D-09
0.45648D-07	0.15423D-07	0.47189D-08	0.12905D-08	0.26604D-09
0.48538D-07	0.16504D-07	0.50700D-08	0.13866D-08	0.28240D-09
0.50546D-07	0.17254D-07	0.53139D-08	0.14535D-08	0.29368D-09
0.51605D-07	0.17650D-07	0.54426D-08	0.14888D-08	0.29961D-09

R LATTICE (CM)

0.7796D+02	0.8732D+02	0.9668D+02	0.1060D+03	0.1154D+03
------------	------------	------------	------------	------------

Z LATTICE (CM)

0.2539D+01	0.7617D+01	0.1270D+02	0.1777D+02	0.2285D+02	0.2793D+02	0.3301D+02	0.3809D+02	0.4316D+02	0.5471D+02
0.7273D+02	0.9075D+02	0.1088D+03	0.1268D+03						

different in the upper and lower halves, and the densities and reflector thicknesses are a little different. The differences were not great enough to justify doing the complete mock-up on VENUS, with the added computer costs, so we mocked up the upper half of the system. The geometrical configuration for the VENUS calculation is shown in Figure 9. For the hydrodynamics subroutine the density of zone 1 was set at 1.56 gm/cm^3 ; zones 2 and 3, 5.1 gm/cm^3 ; zone 4, 4.44 gm/cm^3 ; and zone 5, 5.0 gm/cm^3 . The fuel density for the equation of state was 2.0 gm/cm^3 . The power distribution is given in Table 1, and the reactivity worth distribution and space lattice are shown in Table 2. The Doppler effect was divided evenly between zones 2 and 3. The ANL equation of state was specified for zones 2 and 3. The other regions were assigned equation of state option 4, which is for compressible material with no heat generation. The prompt neutron generation time was 0.5×10^{-6} sec, the delayed neutron fraction .00312 and the six standard delay groups were used.

We were not sure how the control and reflector regions enter the Edwards and Mather calculations. The treatment of these regions is not critically important so we took some liberties with them. In the initial calculations we specified the material to be steel and assigned no void fraction to the medium. VENUS requires a very small time step to

avoid instability when there is a region with such low compressibility. We therefore increased the steel compressibility to 10^{-11} cm²/dyne and added 30% void space. The net effect was to substantially relieve our time step limitation and increase the energy release in a check calculation at \$100/sec and Doppler constant .003 by about 10%. The increase is apparently due to the internal pressure relief and slight reactivity input permitted by expansion into the control zone. At small Doppler effect our VENUS energy release calculations are a little larger than Edwards and Mather and this may partly explain the difference. This does not explain the greater difference discussed below for calculations at large Doppler effect.

Comparison of Results

For the comparison we carried out a series of calculations at \$100/sec reactivity input and uniform initial temperature of 1500K. We varied the Doppler constant from .001 to .007. Table 3 displays the results for the total excursion energy and gives a comparison with the Edwards and Mather results taken from their Figure 4 for the same conditions. A comparison is also given for the available work energy.

Table 3

Energy Yields (mega-joules) for \$100/sec Ramp

<u>Doppler Constant</u>	<u>Total Energy</u>		<u>Available Energy</u>	
	<u>VENUS</u>	<u>E.&M.</u>	<u>VENUS</u>	<u>E.&M.</u>
.001	10.5	9.0	1.7	1.5
.002	7.4	6.2	.45	.60
.0025	6.1	5.5	.20	.36
.003	4.8	4.2	.04	.10
.0035	5.1	3.5	.06	.03
.004	6.6	3.2	.28	0
.005	6.3	4.1	.24	.09
.006	5.1	3.8	.06	.06
.007	4.6	3.2	.025	0

The agreement is really very satisfactory in the more severe cases at low Doppler effect. In the earlier discussion we pointed out that differences in treatment of the control region and reflectors might possibly explain 10% differences in results. Certainly equation of state uncertainties produce more error than this.

There are very significant differences in the Doppler range .004 to .007. VENUS-II predicts roughly 50% more total excursion energy and significant available energy where Edwards and Mather get almost none. We do not have a definite explanation for this. We mentioned in Section I of this Part Three that we were also unable to check their disassembly reactivity function at low energy. This was a little surprising because we check it correctly at high energy and we are using about the same pressure function. We tentatively suspect an error in their low energy feedback

function but cannot make a definite conclusion. Once again we apologize for letting time run out before this could be settled. It is important to pursue this to a more complete understanding.

REFERENCES

1. A. G. Edwards and D. J. Mather, "The Contribution of Bethe-Tait Analysis to the Assessment of Fast Reactor Safety," TRG Report-2370 (1973).
2. R. B. Nicholson, "Methods for Determining the Energy Release in Hypothetical Fast-Reactor Meltdown Accidents," Nucl. Sci. Eng. 18 207-219 (1964).

Appendix A of Part Three

Description of the Modified Bethe-Tait Program

The equations have been described in the earlier text of the report. The resulting equations were:

$$R_0 = \alpha t \quad \text{or} \quad \alpha \tau \left(\frac{l}{k_1 - 1 - \beta} \right) = K_0 \quad (\text{A-1})$$

$$R_D = A \ln \left(\frac{T}{T_0} \right) \quad \text{or} \quad K_D = \frac{A}{k_1 - 1 - \beta} \ln \left\{ \frac{y + 1 + T_0 c_p / Q^* p}{T_0 c_p / Q^* p} \right\} \quad (\text{A-2})$$

$$\frac{d^2 Y}{d \tau^2} = K \frac{dY}{d \tau} \quad (\text{A-3})$$

$$R_m = C_2 \exp \left(\frac{-2.3 C}{\omega - \omega^*} \right) \quad \text{or} \quad K_m = C_2 \frac{l^3}{(k_1 - 1 - \beta)^3} \exp \left(\frac{-2.3 C}{\omega^* y} \right) \quad (\text{A-4})$$

$$\sum K = \frac{K_0 - 1 - \beta + R_0 + R_D + R_m}{k_1 - 1 - \beta} \quad (\text{A-5})$$

The initial conditions are:

$$y = -1$$

$$\kappa(0) = 0, \text{ implies prompt critical} \quad (\text{A-6})$$

$$P(0) = P_0$$

Inputs

The inputs are entered on three data cards. The format is 5E12.5. Prepare the data as follows:

Card	Column				
	1-12	13-24	25-36	37-48	49-60
1	$k_1 - l - \beta$	1	P_o	α	A
2	q	W*/M	C/W*	C_2	q_2
3	$\Delta 1$	$\Delta 2$	T_o		

$k_1 - l - \beta = 0.001$ (this is arbitrary)

l = neutron lifetime

P_o = specific power in Joule/sec-gram

α = ramp rate in β /sec

A = Doppler coefficient

q = flux shape factor. To model a cylindrical reactor by a sphere use the average between radial and diagonal values.

W*/M = W* divided by the mass of UO_2 in the reactor

C/W* = shape factor divided by W* in gigajoules (the same as used on the fitting curve)

C_2 = asymptotic value of R_m

q_2 = power shape factor. To model a cylindrical reactor as a sphere use an average between the radial and diagonal values.

$\Delta 1$ = tau step before $y = 0$

$\Delta 2$ = tau step after $y = 0$

T_0 = initial temperature

Outputs

Depending on which print option is used the program will print out a time history of tau, y, κ_M , κ_D , and κ_0 , the summation of the kappas, and the modified power, or only the final values. In either case the following labels are used:

TK1MBM = $k_1 - 1 - \beta$

TL = 1

PO = P_0

ALPHA = α

A = A

TLQ = q

WDM = W^*/M

CDW = C/W^*

C2 = C_2

TLQ2 = q_2

DELT1 = $\Delta 1$

DELT2 = $\Delta 2$

TEMPI = T_0

TAU = τ

$$Y = y$$

$$KMAT = R_M(k_1 - 1 - \beta)$$

$$KO = R_O / (k_1 - 1 - \beta)$$

$$KD = R_D / (k_1 - 1 - \beta)$$

$$SUMK = \Sigma \kappa$$

$$POWER = power \cdot 1/Q^* \cdot (k_1 - 1 - \beta) \text{ where } Q^* = WDM(k_1 - 1 - \beta)$$

Sample output is shown on the following pages with a listing of the program.

C THE FOLLOWING STATEMENTS READ IN THE DATA

C

```

1 READ(5,2) TK1MBM,TL,PO,ALPHA,A
2 FORMAT(5E12.5)
  IF(TK1MBM.EQ.999.) GO TO 999
  READ(5,2) TLQ,WDM,CDW,C2,TLQ2
  READ(5,2) DELT1,DELT2,TEMPI
  WRITE(6,40) TK1MBM,TL,PO,ALPHA,A
40 FORMAT(4X,'TK1MBM=',E12.5,' TL=',E12.5,' PO=',E12.5,' ALPHA='
  C ',E12.5,' A=',E12.5)
  WRITE(6,42) TLQ,WDM,CDW,C2,TLQ2
42 FORMAT(4X,'TLQ=',E12.5,' WDM=',E12.5,' CDW=',E12.5,' C2='
  C ',E12.5,' TLQ2=',E12.5)
  WRITE(6,44) DELT1,DELT2,TEMPI
44 FORMAT(4X,'DELT1=',E12.5,' DELT2=',E12.5,' TEMPI=',E12.5)

```

C THE FOLLOWING STATEMENTS CONVERT THE DATA TO THE
C MODIFIED SYATEM OF UNITS

C

```

TAU=0.0
QSTR=WDM/(1.-.6*TLQ2)
C2=C2*TL*TL/TK1MBM**3
P=1.+(-.6*TLQ2+6./7.*TLQ*TLQ2-1./3.*TLQ2*TLQ**2)/(1.-1.2*TLQ+3./
C 7.*TLQ**2)
TCQWP=TEMPI*.48/QSTR/P
A=A/TK1MBM

```

CC THIS INITIALIZES THE PARAMETERS

C

```

YM=-1.
Y=-1.
TK1MM=0.
TK1M=0.
TKI=0.
TKO=0.
TKD=0.
N=1
POWERI=PO*TL/QSTR/TK1MBM
POWER=POWERI
POWLN=0.0
ADD=0.0
ALDKM2=ALPHA*TL/TK1MBM/TK1MBM
QD1MQ=TLQ/(1.-TLQ)
SUMK=TKI+TKO+TKD
SUMKM=C.0

```

C THIS PRINTS THE OUTPUT IN COLUMNS

C

```

  WRITE(6,65)
65 FORMAT(9X,'TAU',12X,'Y',11X,'KMAT',11X,'KO',12X,'KD',12X,
  C ',SUM K',12X,'POWER')
  PRINT 50,TAU,Y,TKI,TKO,TKD,SUMK,POWER
50 FORMAT(4X,7E14.5)
  Y=(PO*TL*DELT1/QSTR/TK1MBM)+YM
100 IF(Y.LT.0.) GO TO 110
  TAU=TAU+DELT2
  DELT=DELT2
  GO TO 120
110 TAU=TAU+DELT1
  DELT=DELT1

```

C THIS IS THE RAMP REACTIVITY INPUT

IV. G1 RELEASE 2.0

MAIN

DATE = 74149

11/01/21

C

120 TKO=ALDKM2*TAU
TKD IS THE DOPPLER FEEDBACK

C

C

$$XXX = (Y + 1. + TCDQP) / (TCCQP)$$

$$XXX = ALUG(XXX)$$

$$TKD = A * XXX$$

$$IF(Y.GE.0.) GO TO 300$$

200 SUMK=TKI+TKO+TKD

$$PRINT\ 250, TAU, Y, TKI, TKO, TKD, SUMK, POWER$$

250 FORMAT(4X, 7E14.5)

$$POWLN = POWLN + SUMK * DELT + ADD * DELT * DELT / 2.$$

$$YP = POWER * EXP(POWLN) * DELT + Y$$

$$POWER = (YP - Y) / DELT$$

$$N = N + 1$$

$$IF(N.GE.2000) GO TO 999$$

$$IF(POWER.LE.1.0E-05) GO TO 999$$

$$ADD = (SUMK - SUMKM) / DELT$$

$$SUMKM = SUMK$$

$$YM = Y$$

$$Y = YP$$

$$TKIMM = TKIM$$

$$TKIM = TKI$$

$$GO TO 100$$

C THIS CALCULATES THE DISASSEMBLY REACTIVITY

C

300 XXX=-2.3*CDW/Y

$$IF(XXX.LE.-150.) GO TO 200$$

$$TKI = C2 * EXP(XXX) * DELT * DELT + 2 * TKIM - TKIMM$$

$$GO TO 200$$

999 STOP

END

ITERATION

4247

TKIMBM= 0.10000E-02 TL= 0.50000E-06 PC= 0.17800E+03 ALPHA= 0.23310E+00 A=-0.60000E-02
 TLQ= 0.55000E+00 WDM = 0.53330E+02 CDW= 0.60000E+02 C2=-0.25000E+07 TLQ2= 0.35000E+00
 DELT1= 0.10000E-01 DELT2= 0.10000E-01 TEMPI= 0.15000E+04

ITERATION TO Y=0.= 1176

TAU	Y	KMAT	KO	KD	SUM K	POWER
0.42444E+02	0.16868E+02	-0.22002E+01	0.49469E+01	-0.52250E+01	-0.24783E+01	0.0

ITERATION

3541

TKIMBM= 0.10000E-02 TL= 0.50000E-06 PC= 0.17800E+03 ALPHA= 0.31220E+00 A=-0.60000E-02
 TLQ= 0.55000E+00 WDM = 0.53330E+02 CDW= 0.60000E+02 C2=-0.25000E+07 TLQ2= 0.35000E+00
 DELT1= 0.10000E-01 DELT2= 0.10000E-01 TEMPI= 0.15000E+04

ITERATION TO Y=0.= 1020

TAU	Y	KMAT	KO	KD	SUM K	POWER
0.35388E+02	0.20530E+02	-0.43667E+01	0.55241E+01	-0.59003E+01	-0.47430E+01	0.0

Part Four

The Effect of Neutron Streaming in Voids
on Hypothetical Core Disruptive Accidents (HCDA)

Principal Contributors:

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Prepared by Richard B. Nicholson

I. INTRODUCTION

The classical HCDA is typically initiated by a hypothetical reactivity increase due to sodium voiding, inadvertent rapid control rod movement, or rearrangement of the plutonium fuel following core meltdown. The detailed processes leading up to the HCDA are often exceedingly complex. Nevertheless, in the absence of autocatalytic effects the HCDA itself can be characterized by just a few parameters.

1. The geometrical arrangement of the materials at the instant of the HCDA can usually be approximated by a simple sphere or cylinder or a two or three zone cylinder.
2. The rate of reactivity increase at prompt critical.
3. The Doppler feedback constant.
4. The equation of state of the fuel.
5. The initial conditions of power and temperature.
6. The space distribution of power and reactivity worth of mixed core material.
7. Parameters of small importance such as the prompt neutron lifetime and delayed neutron parameters.

To the extent that these parameters can be correctly specified it has been widely accepted that the energy release in the HCDA can be determined with sufficient accuracy for safety analysis using computer programs such

as VENUS-II (1) and PAD (2) and other closely related programs. However, there is real physical effect associated with neutron streaming through voids (gas spaces) that leads to a positive reactivity feedback effect during the peak of the excursion that has been ignored in the computer programs. It has been ignored because it has been thought to be very small. This effect was first discussed for the Breeder in the literature by Richard Webb (3), who asserted that it could greatly aggravate the consequences of an HCDA; but he did not carry out a detailed evaluation of the problem. A number of LMFBR safety analysts including one of us (RBN) were informed of Webb's ideas in 1972 and in fact earlier but concluded that the effect would be very small and could be ignored.

The streaming Dr. Webb was referring to was streaming through the normal coolant passages after the sodium has voided the core. Because of the close spacing and inter-connection of these passages and because they form a highly ordered array the Behrens theory (4) that Webb applied and which we utilize in Part Five is not strictly applicable. As far as we know, this problem has never been solved. It may require special purpose Monte Carlo techniques. The problem we have undertaken to solve is the effect of streaming through gas bubbles in a molten bubbling pool of fuel. To the extent that the space distribution of the

bubbles can be treated as random, the Behrens theory seems ideally suited as discussed in Part Five. However, since it is still only a transport correction to diffusion theory we recommend further analysis by more accurate methods. Also, while we mistrust the method as applied to the normal coolant channels this does not mean that the effect is small. We find that the effect may be very important for the bubbly core and suspect that the same is true for the normal voided configuration.

It is important that this work be extended to more accurate studies of the bubbly pool and to the normal voided core. We estimate that several man years of effort and perhaps \$100,000 in computer time need be expended to fully evaluate this problem. However, it may also lead to some greater understanding of the more general question of the effect on criticality of heterogeneity in breeder reactors and their mock-up critical assemblies. The sodium has such a long mean free path for neutrons that the streaming should also have a significant effect with the sodium in, especially in critical assemblies with fairly coarse structure.

We define the streaming reactivity to be the difference between the reactivity of a system with voids and the same system with the material and voids locally homogenized, such that there is the same total material content in the two cases. For most purposes this is a small amount of reactivity.

But consider the situation that exists just before peak power is reached in an HCDA when there are voids in the core.

The reactivity in a typical calculation at this point is of the order of ten cents above prompt critical (sometimes only a few cents). If there were a mechanism for suddenly adding even another ten cents at this point in the excursion, that would have a significant effect on the outcome of the accident. The streaming reactivity may be such a mechanism. Just before peak power in a system with voids the dominant features of the HCDA process are a rapid rise in temperature and thermal expansion of the solid and liquid fuel into the internal void spaces, thus decreasing or eliminating the (negative) streaming reactivity and so producing a positive feedback. At the same time the vapor pressure of the fuel is beginning to develop a general outward acceleration, but the inertial restraint causes this to happen on a much longer time scale than the local movements into the voids. The local movement can be treated as instantaneous relative to the global time scale. It thus enters in such a way as to partially compensate or perhaps even overcome the negative Doppler effect. It is well known (5) that the Doppler effect is of great importance in limiting the energy release and so it is self-evident that if the streaming effect is of the same order of magnitude as the Doppler effect, then it is indeed an unpleasant phenomenon.

II. MAGNITUDE OF THE STREAMING EFFECT

The purpose of this Part Four is not to estimate the magnitude of the reactivity effect but rather, assuming a certain magnitude, to then estimate the influence on the HCDA. The magnitude of the reactivity effect is evaluated in the M.S. thesis of G. Goldsmith, which is reproduced as Part Five of this report. From his work we find that when a system with a random distribution of bubbles of one cm radius and void fraction $f = .33$ transforms to a uniform homogeneous system as a result of internal fuel expansion there will be a reactivity increase of the order of $\delta k = .01$ due to elimination of streaming and a random reactivity effect of standard deviation about .001 due to the movement of fuel that accompanies the implosion of the bubbles. The latter can be either positive or negative depending upon the actual voids and can have any magnitude as permitted by the assumption of a random distribution. But one needn't go beyond about three standard deviations. The actual bubble sizes, system size, void fraction and other parameters that should be selected by the conservative safety analyst are open to discussion. The tables of Goldsmith show wide variations in the attainable reactivity. One can obtain reactivity effects as high as .04 without making unduly wild assumptions on bubble size and void fraction. The effect is proportional $k_{\infty}-1$ and so is smaller for the larger 1000 MW plants than for FFTF, for example.

In view of the range of possible values for the reactivity effect and variety of ways of getting a particular value it is appropriate to decouple the analysis at this point and make estimates of the explosion yield as a function of the magnitude of the streaming reactivity. We consider values up to a maximum δk for complete closure of voids of about .02. It will be seen that this is enough to dramatically aggravate an HCDA and that even relatively small values like .002 can be significant.

III. MORE DISCUSSION OF THE PHYSICAL PROCESSES

More discussion of the physical processes is necessary to relate the real world to the models that are employed. The calculations by Goldsmith assume that except for statistical fluctuations from the mean the void distribution is uniform. This is one suitable initial condition for an HCDA calculation. But during the excursion the power distribution will cause the bubbles near the core center to implode before those near the edge. Thus, we require a model of streaming reactivity as a function of position. The effect can be treated as a perturbation in the transport cross section and therefore affects the leakage. It has no effect whatever at the core center and is a maximum toward the core edge. One could calculate the spatial distribution for a particular core by perturbation theory.

The distribution will depend upon the efficiency of any reflector that might surround the molten pool. Instead of carrying out such a calculation we have employed a model that has the qualitative characteristics desired, namely zero at the center of the core and increasing toward the edge.

We took the spatial distribution of the streaming reactivity to be proportional to $1-N(r)$ where $N(r)$ is the power distribution normalized to unity at its maximum position. This is readily available in the VENUS program without introducing new subroutines and has the desired qualitative behavior. It is of course not very accurate. For a perturbed diffusion coefficient the one group diffusion theory for a bare core gives a spatial shape for reactivity that peaks well inside the core edge and is fairly constant over the outer half of the core radius. Our model is a little better for a reflected core. The distribution is normalized such that the integral over the whole core is equal to the reactivity for all the bubbles as calculated by Goldsmith. Our model is non-conservative because it shifts the effect more toward the core edge than it should. During an excursion the power distribution causes more closure of voids near the core center and less toward the edge. Thus, shifting the distribution to the edge results in less positive feedback and a smaller energy release.

This is acceptable for the present purpose, which is to demonstrate that the effect is important. The demonstration is all the more effective in that we obtain considerable aggravation of the accident in spite of our non-conservative model. But it is clearly necessary to match the model more closely to the real world or else use a definitely conservative one when doing an actual safety analysis for a reactor design. One clearly conservative model is a uniform spatial distribution. This is probably much too conservative, at least for those accidents which would be weak in the absence of the streaming effect. For a \$10/sec accident with .002 Doppler constant and with a total streaming reactivity of .021 we find that the streaming effect increases the excursion energy by a factor of 10 with the $1-N(r)$ distribution and by a factor of 27 with the uniform distribution. This serves to show that one must determine the correct distribution to do accurate calculations. The distribution is a little less important in more severe excursions where nearly all the void is ultimately filled by expansion.

In retrospect it might have been preferable to use the distribution $1-N^2(r)$. This happens to be the correct distribution for a perturbation in the diffusion coefficient for a bare slab reactor in the one-energy group diffusion theory approximation.

A little more need be said about the assumption of a random bubble distribution and what it implies about the

physical situation. First of all for the size and number of bubbles we are treating, i.e., sizes of the order of one cm radius and void fractions like 0.3, the statistics are such that the mean void fraction in any region of volume of the order of a liter, for example, is fairly uniform. A liter would contain about 100 bubbles with a standard deviation of 10 or therefore a 10% relative standard deviation on the void fraction. This also means that most of the core material which implodes into the bubbles can come from the immediate vicinity of the bubbles. It is driven by liquid phase pressures acting over short distances so that the void closure can be regarded as instantaneous relative to the time for core disassembly.

But if a liter of volume has a 10% standard deviation in void fraction, then something like 5% of the liquid fuel in each liter must also have to flow in or out before the void closure is total. This movement of material on a macroscopic scale will have a reactivity effect simply due to the variation of material worth with position and independent of the streaming effect. This is the other effect mentioned above that produces a statistical contribution to the void implosion reactivity. Goldsmith has also analyzed this effect in Part Five and found it to be smaller than the streaming effect. Nevertheless it is not negligible if, to be conservative, one elects to use a value equal to three or four standard deviations.

The time scale for the redistribution of fuel to smooth out the statistical variations is not clear. It is definitely longer than the time scale for the bubble implosion but perhaps not as long as the time scale for disassembly. Since the effect is apparently smaller than the streaming effect there is no need to consider it further unless a more accurate analysis of the streaming causes the streaming reactivity estimates to shrink by about a factor of 3 or more. In that case it will be necessary to develop a proper treatment of the statistical fuel relocation. We do not presently see how to do this properly.

The basic assumption of a random bubble distribution is open to question. There are physical effects which will induce order of various kinds. Firstly, the temperature of the bubbly pool will not be uniform at the beginning of the excursion. The nature of the non-uniformity depends upon how the pool accumulated from material in various parts of the original core and it depends upon whether the bubbling is from temperature increase due to fission product heating, or due to fission heating as a result of recriticality. It also depends on upward movement of the bubbles due to gravity. One can imagine that the real bubble distribution may result in either larger or smaller streaming reactivity than is given by our model of a random distribution. Until one can resolve that qualitative uncertainty

there is no point in deviating from the random model, whose principle virtue is the simple analysis that it permits. We believe it will lead to the right order of magnitude. Goldsmith has made a few sample calculations for another model (he calls it Behrens' general equation for intermediate spaced holes) which probably tends to give an underestimate of the streaming for any real distribution. It gives a factor of two to three less streaming reactivity. Thus the random model is in any case not a severely conservative model. On the other hand it is not credible that the streaming reactivity could be very much larger than say .02, which we have taken to be a suitable conservative estimate.

While we are confident that Goldsmith's analysis puts the effect in the right ball park, it will be seen from the VENUS calculations presented below that it is not good enough to just be in the right ball park. It is especially important that the basic premise of Behren's streaming theory be checked by Monte Carlo calculations. The basic premise is that the streaming reactivity is determined by the mean squared distance that a neutron travels between collision in the heterogeneous system relative to the homogenized system. It seems to us that this is a very good assumption, for our system of a bubbly pool, but we require some verification by means of a better transport theory method.

IV. THE VENUS MODIFICATIONS

The VENUS-II modifications involved only the introduction of the additional feedback equation and calculation of the local void fraction as required by the feedback equation. The additional FORTRAN statements and their location in the VENUS program deck are summarized here. To expedite the study we did not attempt to alter the code in a way that would have general applicability. In particular, our changes are only valid for one-region problems. It would not be difficult to provide more general applicability. Before doing this, one would probably want to develop a better description of the space dependence of the streaming reactivity feedback function.

The void reactivity (XKVOID) is calculated in the hydrodynamics subroutine HYDRO.

$$\text{XKVOID} = \text{AVOID} \sum [\text{VFVOD}(\text{I},\text{J}) - \text{VFVOAD}(\text{I},\text{J})] \text{PND}(\text{I},\text{J}) \text{CELVUM}(\text{I},\text{J})$$

where: AVOID - void reactivity weighting factor per
unit volume

PND(I,J) - the mesh weighting factor

CELVUM(I,J) - mesh cell volume at present time step

VFVOAD(I,J) - mesh cell void fraction at time zero

VFVOD(I,J) - mesh cell void fraction at present time
step

$$\text{AVOID} = -\text{AAVOID}/\text{SUMVOL}$$

where AAVOID is the void reactivity weighting factor read in from the data deck in the MAIN program. Input card number 2 has been altered to include the control parameter LMVOID as follows:

Card 2, (12I6)

IMAX = total number of radial mesh intervals

JMAX = total number of axial mesh intervals

LMVOID $\neq$ 1, program calculates void reactivity worth
= 1, ignore option

Card 2a (E12.5), Read AAVOID if and only if LMVOID $\neq$ 1
SUMVOL is the total reactor volume calculated from the cell boundaries in HYDRO.

$$\text{SUMVOL} = \sum_{IJ} \text{CELVUM}(I,J)$$

where CELVUM(I,J) is the cell volume at I,J
(HYDRO statements 1875, 1944, 1946)

The mesh weighting factor $\text{PND}(I,J) = 1 - \frac{H(I,J)}{H(\text{max})}$,

is calculated in the INPUT subroutine at time zero. (INPUT statements between 5330 and 5340)

$H(I,J)$ = relative power density of mesh cell (I,J)

$H(\text{max})$ = maximum relative power density

The void volume fraction at time zero (VFOAD) is calculated and printed out in INPUT by the statements between 5650 and 5660 (also INP 974, 975)

$$VFVOAD(I,J) = 1.0 - VFSS(I,J) - VFNA(I,J) - VFU(I,J)$$

where VFSS = volume fraction of Stainless Steel

VFNA = volume fraction of Sodium

VFU = volume fraction of Uranium Oxide

The present time step void volume fraction (VFVOD) is calculated in HYDRO,

$$VFVOD(I,J) = 1 - VFSSME(I,J) - VFNAME(I,J) - VFUME(I,J)$$

where VFSSME(I,J) = MASSSS(I,J)/RHOSS(I,J)/CELVUM(I,J)

VFNAME(I,J) = MASSNA(I,J)/RHONA(I,J)/CELVUM(I,J)

VFUME(I,J) = MASSU(I,J)/RHOF(I,J)/CELVUM(I,J)

The fuel density (RHOF) is calculated from an empirical relation from the molten fuel coolant interaction model.

$$RHOF = \begin{cases} 11.2892 - 5.1013 \times 10^{-4} (THETA), & \text{for } THETA < T_m \\ 11.2892 - 5.1013 \times 10^{-4} (T_m) - 0.36223 (THETA - T_m), & \text{for } T_m < THETA < T_m + 1 \\ 10.9277 + 1.7169 \times 10^{-4} (T_m) - 6.8182 \times 10^{-4} (THETA), & \text{for } THETA > T_m \end{cases}$$

where THETA = THETA(I,J), the fuel temperature. For temperatures greater than about 20,000°K, the above relation predicts a negative fuel density (RHOF). Two control cards are inserted to prevent the fuel volume fraction from becoming negative or greater than 1.0.


```
IF(VFUME(I,J).GT.1.0) VFUME(I,J) = 1.0
```

```
IF(VFUME(I,J).LE.0.0) VFUME(I,J) = 1.0
```

This second condition is because when the fuel density turns negative VFUME(I,J) switches from large positive values to large negative values.

Similarly for stainless steel and sodium volume fraction becoming greater than 1.0 for very high temperatures, the control statement

```
IF(VFVOD(I,J).LT.0.0) VFVOD(I,J) = 0.0
```

was inserted.

The void volume reactivity worth calculations are located between the statement HYD 2170 and 890. The reactivity worth was added to the Δk and k in statements HYD 890-960. These statements were moved from their original sequence and placed after the void reactivity calculation and before statement HYD 2180.

The void weighting factor (AAVOID) is printed out from statements INP 5331 thru 5333. The void reactivity (XKVOID) is printed out from statements HYD 2351 thru 2353.

The void volume reactivity worth calculation is summarized in Figure 1.

Figure 1

C
C
C

VOID VOLUME REACTIVITY WORTH

```

      IF(LMVOID.EQ.1) GO TO 295
201  AVVOID = - AAVVOID/SUMVOL
      XKVOID = 0.0
      DO 290 N=2,JMAXP1
        J=JMAXP3-N
        DO 290 I=2,IMAXP1
          IF(THETA(I,J).LT.3040.) GO TO 272
          IF(THETA(I,J).GT.3041.) GO TO 280
          GO TO 275
272  RHOF(I,J)= 11.2892-(5.1013D-4)*THETA(I,J)
          GO TO 285
275  RHOF(I,J)=11.2892-(5.1013D-4)*3040.-0.36223*(THETA(I,J)-3040.)
          GO TO 285
280  RHOF(I,J)= 10.9277+(1.7169D-4)*3040.-(6.8182D-4)*THETA(I,J)
285  VFUME(I,J) = MASSU(I,J)/RHOF(I,J)/CELVUM(I,J)
      IF(VFUME(I,J).GT.1.0) VFUME(I,J) = 1.0
      IF(VFUME(I,J).LE.0.0) VFUME(I,J) = 1.0
      VFNAME(I,J) = MASSNA(I,J)/RHONA(I,J)/CELVUM(I,J)
      VFSSME(I,J) = MASSSS(I,J)/RHOSS(I,J)/CELVUM(I,J)
      VFVOD(I,J) = 1.0 - VFSSME(I,J) - VFNAME(I,J) - VFUME(I,J)
      IF(VFVOD(I,J).LT.0.0) VFVOD(I,J) = 0.0
      XKVODD=AVVOID*(VFVOD(I,J)-VFVOD(I,J))*CELVUM(I,J)*0.7024
      XKVOID = XKVOID + XKVODD
290  CONTINUE
      DK = DK + XKVOID
      XK = XK + XKVOID
295  RHOT = DK/XK

```

V. CALCULATIONS AND RESULTS

The reactor model we have chosen for this study is one that had been set up by C. L. Allen and E. L. Fuller (6) to compare VENUS results with the PAD calculations of W. R. Stratton. This was fundamentally a full density liquid UO_2 system that was to represent recriticality in a core meltdown. The agreement was quite good. Allen and Fuller showed also that the addition of voids greatly increased the excursion energy. The voids were added without alteration of the perturbation reactivity worth per unit volume. One might have increased the core volume to reach criticality with the added voids. This would have further increased the excursion energy. Our calculations also contain this same minor inconsistency but that is of secondary importance.

The reactor is a right circular cylinder 26 cm in diameter and 52 cm tall. The material is liquid UO_2 at its melting point of 3041°K . VENUS has the somewhat high liquid density of 9.37 gm/cm^3 . Thus, when there is 10% void the net UO_2 density is 8.433 gm/cm^3 and when there is 30% Void the net UO_2 density is 6.559 gm/cm^3 . There is no reflector. The Doppler constant is $T \frac{dk}{dT} = -.002$, relatively small as it should be in this compact system. The prompt neutron lifetime is $.67 \times 10^{-7}$ sec and the delay fraction is .004. The VENUS ANL equation of state option

was used. The power and reactivity worth distribution are shown in Table 1. The initial power was set at one megawatt thus representing a start-up accident.

Table 1

NORMALIZED POWER DENSITIES

0.400D-05	0.359D-05	0.287D-05	0.182D-05	0.646D-06
0.126D-04	0.113D-04	0.904D-05	0.573D-05	0.204D-05
0.206D-04	0.185D-04	0.148D-04	0.936D-05	0.333D-05
0.274D-04	0.246D-04	0.196D-04	0.124D-04	0.442D-05
0.304D-04	0.274D-04	0.218D-04	0.138D-04	0.492D-05
0.304D-04	0.274D-04	0.218D-04	0.138D-04	0.492D-05
0.274D-04	0.246D-04	0.196D-04	0.124D-04	0.442D-05
0.206D-04	0.185D-04	0.148D-04	0.936D-05	0.333D-05
0.126D-04	0.113D-04	0.904D-05	0.573D-05	0.204D-05
0.400D-05	0.359D-05	0.287D-05	0.182D-05	0.646D-06

INPUT MATERIAL WORTH FOR REGION 1 (DELTA K/CM**3)

0.13250D-05	0.11950D-05	0.79850D-06	0.39860D-06	0.13250D-06	0.66260D-07
0.26560D-05	0.23900D-05	0.15970D-05	0.79850D-06	0.26560D-06	0.13250D-06
0.93410D-05	0.82110D-05	0.58330D-05	0.23900D-05	0.52890D-06	0.26560D-06
0.17220D-04	0.15120D-04	0.10590D-04	0.47790D-05	0.13250D-05	0.66260D-06
0.23840D-04	0.21010D-04	0.14390D-04	0.63990D-05	0.15970D-05	0.79850D-06
0.26560D-04	0.23380D-04	0.16480D-04	0.71920D-05	0.18580D-05	0.93410D-06
0.23840D-04	0.21010D-04	0.14890D-04	0.63990D-05	0.15970D-05	0.79850D-06
0.17220D-04	0.15120D-04	0.10590D-04	0.47790D-05	0.13250D-05	0.66260D-06
0.93410D-05	0.82110D-05	0.58330D-05	0.23900D-05	0.52890D-06	0.26560D-06
0.26560D-05	0.23900D-05	0.15970D-05	0.79850D-06	0.26560D-06	0.13250D-06
0.13250D-05	0.11950D-05	0.79850D-06	0.39860D-06	0.13250D-06	0.66260D-07

One should note that this model involves only 1000 kg of fuel in the no-void situation and 700 kg for a 30% void fraction. Thus, while the energy production calculated is not completely unmanageable it would become very large if one were to assume a large fraction of a Demonstration Plant size core were involved in the accident. One has only to note the maximum temperatures and pressures achieved to realize that some of these cases are violent excursions. Thus, the presence of bubbles may be of even greater importance than anyone had previously assumed.

The results of these calculations are summarized in Table 2. The excursion energy is the total heat energy added in the excursion above the initial energy content. The available work is the isentropic work done by expansion of the core material adiabatically and reversibly from the state in which VENUS-II leaves it to a final pressure of one atmosphere. The information for this calculation comes from Part Two of this report. The temperatures and pressures listed are the values in the center of the core. The temperatures and pressures are substantially lower over most of the core volume. The ANL equation of state is not expected to be very good at such high temperatures. Nevertheless, when a 40,000°K temperature is listed in the table one can assume that this represents a very violent excursion.

Table 2

Results of HCDA Calculations With and
Without Bubble Implosion Reactivity

<u>Reactivity Ramp \$/sec</u>	<u>Void Fraction</u>	<u>AAVoid</u>	<u>Excursion Energy MW-sec</u>	<u>Available Work MW-sec</u>	<u>Max Temp °K</u>	<u>Max Pressure at Core Center Kilobars</u>
10	.10	0	207.	6.5	4747	.026
10	.10	.03	271.	13.	5270	3.8
10	.10	.10	466.	45.	7395	13.5
10	.30	0	159.	4.	4722	.024
10	.30	.03	382.	42.	7056	.92
10	.30	.10	1530.	560.	23,071	127.
100	.10	0	319.	20.	5643	.15
100	.10	.03	389.	31.	6349	20.
100	.10	.10	648.	87.	9836	47.8
100	.30	0	533.	81.	9208	6.5
100	.30	.03	688.	130.	12,407	
100	.30	.10	2292.	1050.	41,983	221.
1000	.10	0	655.	90.	9946	53.9
1000	.10	.03	881.	160.	13,071	
1000	.10	.10	1534.	430.	22,000	227.
1000	.30	0	1082.	300.	19,847	83.6
1000	.30	.03	1504.	540.	27,626	139.
1000	.30	.10	6442.	4000.	118,158	745.

One check case with uniform bubble implosion worth

10	.30	.10	4293	78,000	403.
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Notice the one check case where the space distribution of the reactivity due to bubble implosion was assumed uniform rather than proportional to $1-N(r)$. The energy was nearly tripled. The true distribution will almost surely lie between these extremes. The comparison shows that it is important to pin down the distribution more accurately.

Notice also the strong non-linearity with AAVOID. Beyond a certain point the autocatalysis tends to drive the system to total implosion of all bubbles. There would seem to be a good chance that this occurs beyond the range of physically realizable values of AAVOID, but we cannot conclude this from the work done to date.

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Part Five

Reactivity Due to Neutron Streaming
in the Voids of a Bubbly Pool Core

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SYMBOLS

γ = length of neutron free path through solid fuel.

N = number of collisions made by a neutron.

K = reproductive constant of reactor.

v = volume of fuel material in each cell.

λ = mean free path of a neutron through solid fuel.

ϕ = volume ratio of bubbles to fuel material.

A = surface area of each bubble.

r_H = hydraulic radius of each bubble ($r_H = 2 v \phi / s$).

X = length of straight passage through a bubble.

Ω = unit vector specifying direction.

L_0 = root-mean-square distance travelled by a neutron in N collisions in the absence of bubbles.

$p(\gamma, \Omega, X)$ = probability that a neutron, during the course of a free path of length γ in direction Ω will emerge into a bubble of length X .

$$P(\gamma, \Omega) = \int_0^\infty p(\gamma, \Omega, x) dx$$

$\Psi(\Omega, x) \delta X$ = cross section of passages of length between X and $X + \delta X$ in the direction Ω through a bubble.

Q = ratio of mean square passage length through hole to square of mean passage length.

$P_0(\gamma, \Omega)$ = probability of a neutron moving in direction Ω intersecting zero bubbles in path length γ , where γ may be large.

$P_2(\gamma, X_1)$ = probability of a neutron moving in any direction of path length γ (where γ may be large) intersecting two bubbles to experience lengthening $(X_1 + X_2)$.

- ϵ = probability of entering $n + 1$ bubbles when the neutron can enter only n or $n+1$ bubbles.
- r = radius of each bubble.
- R = radius of spherical mass of bubbly fuel.
- $(\Delta K)_{Dop}$ = reactivity effect due to Doppler broadening.
- $(\Delta K)_B$ = reactivity effect due to implosion of uniform distribution of randomly positioned bubbles.
- $(\Delta K)_{BG}$ = reactivity effect due to implosion of uniform distribution of non random bubbles.
- $(\Delta K)_{fm}$ = reactivity effect due to fuel movement while the fuel is expanding to uniform density.
- l = total number of shells in model of spherical fuel mass.
- $\bar{w}$ = average reactivity worth of each bubble.
- N_i = number of bubbles per shell.
- ϕ = neutron flux.
- f = void fraction (volume of bubbles/volume of bubbly fuel).
- Σ_a = macroscopic absorption cross section.
- Σ_{tr} = macroscopic transport cross section.

I. INTRODUCTION

1.1 Meltdown Accident

The core of a liquid metal fast breeder reactor (LMFBR) will contain large quantities of fissionable material. The core will operate at a temperature just below the melting point of the fuel, and it will operate at very high power densities. To provide a heat transfer medium the core will be interspersed with coolant channels. If, during an accident involving operator error or equipment failure, the power generated in the fuel exceeds the heat transfer rate, then the fuel temperature will increase and melting will occur. Once melting begins, the open core structure would enable the molten fuel to move to lower regions of the core and accumulate. Then more fuel may be melted by fission heating or delayed fission-product heating. Thus it is possible that a molten pool of fuel will form that could exceed a critical mass.

Attention has been drawn to this hypothetical accident by the meltdown of the world's first LMFBR, the EBR-I reactor and by the fact that on October 5, 1966, the United States' first commercially operated LMFBR (ENRICO FERMI I) sustained a partial meltdown of two fuel assemblies as the result of coolant-flow blockage. (1.)(2.)

Subsequent design changes, incorporated in the newer LMFBR concepts have increased the intrinsic safety and reliable operation of safety devices. But because of the potential hazard, detailed studies have been conducted concerning the possible outcome of a super prompt critical power excursion due to fuel meltdown and reassembly.

1.2 Onset of Super Prompt Critical Power Excursion

Initial studies (3.) postulated meltdown and reassembly due to structural collapse. The calculated reactivity insertion rate (40-50 $\$/\text{sec}$) was assumed to continue unchecked till the reactivity was in excess of prompt critical. It was assumed that no negative feedback appeared till the fission heating caused vaporization and subsequent expansion of the fuel. Thus the negative reactivity associated with fuel expansion, during the disassembly of the pool of molten fuel, was responsible for terminating the power excursion. Subsequent studies have established that the shut down mechanism in a large excursion is the combined effect of several phenomena: Fuel expansion, Doppler effect, fuel melting, mechanical shock waves, and coolant vaporization. (4.) (5.) (6.) (7.) (8.). As a rule, the Doppler coefficient of reactivity is the only temperature coefficient that is considered to apply

to the molten and collapsing state of the reactor core.

The Doppler feedback is a monotonic function of the

average fuel temperature: $(\Delta K)_{\text{Dop}} = -A_{\text{Dop}} \int_0^T \frac{1}{T} dT$.

Its importance is that it responds instantaneously to fuel temperatures to reduce the reactivity. On the other hand, the negative reactivity due to overall fuel expansion does not become important till the pressure in the fuel initiates material displacement.

The reactivity effects during the time between the onset of the super prompt critical power excursion and the start of the overall expansion, that will shut down the power transient, will determine the amount of energy that is released. It has been shown that a negative Doppler coefficient of $A_{\text{Dop}} = .004$, or greater, can reduce the explosion work for a large LMFBR (1000 MW_e) by at least a factor of 10.(9.)(10.) However, these calculations were based on the assumption that no positive reactivity feedback (autocatalytic) effects were present during the onset of the power excursion, or at least until overall expansion began. Since the Doppler coefficient is limited to less than 1% ΔK for reasonable temperature increases(8.), any autocatalytic phenomena during the onset of the power excursion having a $\Delta K \geq .1\% \Delta K$ will have a significant effect on the energy release.

The following types of autocatalytic phenomena have been identified and studied:

1) Implosion of core material into voids and gaps in the core may result in a decrease in the neutron leakage and thus be autocatalytic. (8.)(11.)

2) Non-uniform fuel density in the pool of molten fuel has been observed. (12.) This could lead to an autocatalytic effect, if the fuel were forced to the center of the pool by an external pressure. (13.)(14.)

3) Implosion of fuel material into voided coolant channels may also result in autocatalytic effects. (15.)

This thesis, postulates the existence of two additional autocatalytic effects associated with the boiling of nuclear fuel once it has melted and accumulated in a lower region of the reactor.

1-3 Existence of Bubbles in Molten Fuel.

Consider a hypothetical accident in which the core has partially melted. Under the force of gravity the fuel is flowing down and accumulating in a lower region of the reactor. The coolant is assumed to have been expelled from the region and continued heat generation will therefore cause the temperature to further increase. As the temperature increases, more and more of the fission products will become gaseous. Once the temperature

increases to the vaporization temperature, the fuel itself will become gaseous. Thus the pool of molten fuel may be boiling. The actual details of the reassembly will determine if the fuel is vigorously boiling (contains large bubble volume) or is quiescent (contains small bubble volume) when critical mass is achieved.

This thesis is concerned with two autocatalytic effects associated with the implosion of the bubbles in molten fuel as the fuel experiences the onset of a super prompt critical power excursion.

1) The implosion of the bubbles will decrease the neutron leakage by altering the neutron diffusion length. This should introduce a positive reactivity, $(\Delta K)_B$.

2) The distribution of bubbles will be assumed to be random. There will be local fuel movement due to the statistical distribution of the bubbles. The net movement of fuel toward the center of mass will produce a autocatalytic effect. The net movement of fuel away from the center of mass will cause negative feedback. Since the bubble distribution is random, it will be impossible to predict the sign of the reactivity effect. But the variance of the reactivity effect will give an estimate of the order of magnitude of the random reactivity effect associated with this phenomenon.

1-4 Uncertainties of Thesis Calculation

The calculations that are performed in this thesis to determine a value of the reactivity effect, are subject to a number of simplifying assumptions. In order to facilitate hand computation I have assumed the simplest physical model (spherical) for both the mass of molten fuel and for the bubbles. I have idealized the physical properties of the fuel to avoid considering any phenomena but those of interest to this Thesis, in particular I have assumed that the overall bubble distribution is uniform, and that the microscopic location of a bubble is completely random. While volume boiling is generally considered a random process, it has been shown that there is a minimum bubble spacing due to the bubble suppression effect of the local fluid pressure increase resulting from bubble growth (16.) Since the model used does not closely model the real system, I must acknowledge that the calculated values of the reactivity effect will contain considerable uncertainty. I have attempted to vary several parameters to estimate the size of the uncertainty, but the calculated values of reactivity effect are only estimates of the order of magnitude.

II. BUBBLE IMPLOSION DUE TO FUEL EXPANSION

2.1 Calculating Reactivity Effect

The presence of bubbles in molten fuel increases the mean squared distance a neutron will penetrate through the medium before absorption ($\overline{r^2}$). This happens in two ways. First, the low density gases in the bubbles do not significantly scatter or absorb neutrons. Thus the neutron's path length is increased by the width of each hole it intersects. Secondly, certain neutron paths in the bubbly material may intersect a higher bubble density than other paths. Thus, some neutrons may "stream" a relatively long distance. Recall that $L^2 = 1/6 \overline{r^2}$, where (L) is the diffusion length. Thus the appearance or disappearance of bubbles will effect the value of L.

Assuming a one group model, the effective multiplication constant (K) can be shown to be related to (L) by the following equation;

$$K = \frac{K_{\infty}}{1 + L^2 B^2}, \quad (2-1)$$

where (K_{∞}) is the infinite multiplication factor and (B^2) is the geometric buckling. Then the reactivity effect $(\Delta K)_B = K_f - K_i$, where (K_i) is computed for the fuel while it contains bubbles, and (K_f) is computed for the

fuel once it has expanded to fill the bubbles. Since (B^2) is a function of the dimensions of the molten mass under consideration and (K_{∞}) is a function of the composition of the molten mass, they will be constants during the expansion of the fuel to fill the bubbles. Thus

$$(\Delta K)_B = \left[\frac{K_{\infty}}{1 + L_i^2 B^2} \right] \frac{[B^2(L_i^2 - L_f^2)]}{(1 + L_f^2 B^2)}. \quad (2-2)$$

Since fuel expansion would begin once the molten mass was critical, the first factor in equation (2-2) is the effective multiplication constant (K) , which is unity.

Rearranging terms using equation (2-1) we find that

$$(\Delta K)_B = \frac{L_f^2 B^2}{1 + L_f^2 B^2} \left[\frac{L_i^2}{L_f^2} - 1 \right],$$

or
$$(\Delta K)_B = \frac{K_{\infty} - K_f}{K_{\infty}} \left[\frac{L_i^2}{L_f^2} - 1 \right].$$

Since $(\Delta K)_B$ is expected to be small, $K_f \approx 1$. Then

$$(\Delta K)_B \cong \frac{K_{\infty} - 1}{K_{\infty}} \left[\frac{L_i^2}{L_f^2} - 1 \right]. \quad (2-3)$$

The theory that makes it possible to calculate values for (L_f^2) was developed by Behrens (17.) He developed the theory to calculate the diffusion length when there are coolant channels in the core of a reactor.

2.2 Behrens' Theory

This theory deals with the increase in migration length (L) in a reactor core caused by the presence of holes. For all calculations the holes are assumed to be voids. For the case of homogeneous fuel Behrens used the fact that the scattering process is random to show that the probability of a neutron having a mean path length between (z) and ($z + \delta z$) is $\frac{1}{\lambda} \exp[-z/\lambda] \delta z$. Then the mean square distance a neutron will travel thru a homogeneous fuel during (N) collisions is.

$$L^2_0 = \int_0^\infty z^2 \frac{N}{\lambda} \exp[-z/\lambda] dz = 2N\lambda^2, \quad (2-4)$$

where (λ) is the mean free path in the fuel material.

For the case of a fuel material containing holes, he associated a unit cell with each hole. The cell contained a volume (v) of fuel, and the volume ratio of the hole to fuel in the cell was called (ϕ). The volume of a hole (ϕv) can be written as

$$\phi v = \int_x x \psi(x, \Omega) d\Omega, \quad (2-5)$$

where (x) is the length of the hole in a given direction, and $\psi(x, \Omega) \delta x$ is the cross sectional area associated with paths thru the hole of length (x) in direction (Ω). Now let $p(z, \Omega, x) \delta x$ be the probability that a neutron moving in direction (Ω) of path length (z) will enter

a hole and move along paths of length (X to $X + \delta X$). Then

$$p(\gamma, \Omega, X) = \gamma \psi(X, \Omega) / v. \quad (2-6)$$

2-2.1 Widely Spaced Holes

The surface area of a hole (s) times the mean free path in the fuel (λ) represents the volume of fuel in the cell which could produce neutrons that would enter the hole. With $v \gg \lambda$, the probability of a neutron entering two or more holes is small. Thus the mean square distance can be calculated by only allowing for the possibility that a neutron will enter one hole or no hole. That is

$$L^2 = N \int_{\Omega} \int_0^{\infty} \left[\int_0^{\infty} (\gamma + X)^2 p(\gamma, \Omega, X) dX + \gamma^2 \left(1 - \int_0^{\infty} p(\gamma, \Omega, X) dX \right) \right] (1/\lambda) \exp(-\gamma/\lambda) d\gamma \frac{d\Omega}{4\pi}, \quad (2-7)$$

where the first term accounts for the neutron entering one hole such that its mean free path is $(\gamma + X)$. The second term accounts for the neutron entering no hole. By expanding $(\gamma + X)^2$ and simplifying,

$$L^2 = N \int_{\Omega} \int_0^{\infty} \left[\frac{2\gamma^2 v \phi}{v} + \int_0^{\infty} \frac{\gamma X^2}{v} \psi(X, \Omega) dX + \gamma^2 \right] (1/\lambda) \exp(-\gamma/\lambda) d\gamma \frac{d\Omega}{4\pi}. \quad (2-8)$$

Integrating out the γ dependence leaves

$$L^2 = 2N\lambda^2 + 4\phi N\lambda^2 + \frac{N\lambda}{v} \int_{\Omega} \int_0^{\infty} X^2 \psi(X, \Omega) \frac{dX d\Omega}{4\pi}, \quad (2-9)$$

where the integral is a function of the shape of the hole.

Then

$$\frac{L^2}{L_0^2} = 1 + 2\phi + \int_{\Omega} \int_0^{\infty} \frac{X^2}{2\lambda v} \psi(X, \Omega) \frac{dX d\Omega}{4\pi}. \quad (2-10)$$

2-2.2 Closely Spaced Holes

When holes are closely spaced ($s\lambda \gg v$), neutrons will most likely pass thru a number of holes. The $p(z, \Omega, X)$ developed in the last section can still be used as the probability of entering a hole of length X while traveling in direction Ω but it still only applies to values of z of the order of the cell width or smaller. Behrens states that in the limit of closely spaced holes "the probability of entering a hole is independent of any holes which may already have been entered".

Let $P_0(z, \Omega)$ be the probability that a neutron will enter no hole of any size while moving along direction Ω of length z , where z can be large. The assumption that $p(z, \Omega, X)$ is independent of the neutron's history makes it possible to express $P_0(z + \delta z, \Omega)$ as the product of $P_0(z, \Omega)$ and the probability of entering no hole of any size in path length δz ,

$$P_0(z + \delta z, \Omega) = P_0(z, \Omega) \left[1 - \int_0^\infty p(\delta z, \Omega, X) dX \right]. \quad (2-11)$$

To develop an expression for $P_0(z, \Omega)$ we begin with the definition of $\frac{\partial P_0(z, \Omega)}{\partial z}$.

$$\frac{\partial P_0(z, \Omega)}{\partial z} = \lim_{\delta z \rightarrow 0} \left[\frac{P_0(z + \delta z, \Omega) - P_0(z, \Omega)}{\delta z} \right]. \quad (2-12)$$

Using equation (2-11) this becomes

$$\frac{\partial P_0(z, \Omega)}{\partial z} = -P_0(z, \Omega) \lim_{\delta z \rightarrow 0} \left[\frac{1}{\delta z} \int_0^\infty p(\delta z, \Omega, X) dX \right]. \quad (2-13)$$

Substituting equation (2-6) into Equation (2-13) gives

$$\frac{\partial P_0(z, \Omega)}{\partial z} = -P_0(z, \Omega) \lim_{\delta z \rightarrow 0} \left[\int_0^\infty \frac{\delta z \psi(x, \Omega) dx}{v(\delta z)} \right]$$

$$\frac{\partial P_0(z, \Omega)}{\partial z} = -P_0(z, \Omega) \int_0^\infty \frac{\psi(x, \Omega)}{v} dx. \quad (2-14)$$

Using the boundary condition that $P_0(0, \Omega) = 1$, the solution to equation (2-14) is

$$P_0(z, \Omega) = \text{EXP} \left[-z \int_0^\infty \frac{\psi(x, \Omega)}{v} dx \right]. \quad (2-15)$$

Define $P(z, \Omega) \equiv \int_0^\infty \tau(z, \Omega, x) dx,$ (2-16)

then $P_0(z, \Omega) = \text{EXP}[-P(z, \Omega)].$ (2-17)

Now assume that the probability of a neutron interacting with a number of holes (N) of any length in mean free path (z) is independent of its direction Ω . That is we can write $P_N(z) = P_N(z, \Omega)$. If we define (A) to be the integral in equation (2-15), then equation (2-17) becomes $P_0(z) = \text{EXP}(-zA).$ (2-18)

Note that we must maintain the notation $(P_1(\delta z, x_i))$ for the probability of entering one hole in a short mean free path, as the shape of the hole makes $P_1(\delta z, x_i)$ very dependant on x_1 . Then

$$P_1(\delta z, x_i) = \tau(\delta z, x_i), \quad (2-19)$$

and

$$P_0(\delta z) = 1 - P(\delta z) = 1 - A \delta z. \quad (2-20)$$

Calculate $P_1(z, X_i)$

The probability of a neutron, moving in any direction Ω of path length $(z + \delta z)$, intersecting one hole to experience a lengthening (X_i) is the sum of two probabilities. One term is the probability that it will experience one interaction while traveling a length (z) times the probability that it experiences no interaction during the last (δz) of path length. The other term is the probability that the neutron experiences no interaction during path length (z) times the probability of one interaction in the last (δz) of path length. This is written as

$$P_1(z + \delta z, X_i) = P_1(z, X_i) P_0(\delta z) + P_0(z) P_1(\delta z, X_i) \quad (2-21)$$

$$= P_1(z, X_i)(1 - A \delta z) + P_0(z) p(\delta z, X_i). \quad (2-22)$$

Then

$$\begin{aligned} \frac{\partial P_1(z, X_i)}{\partial z} &= \lim_{\delta z \rightarrow 0} \left[\frac{P_1(z + \delta z, X_i) - P_1(z, X_i)}{\delta z} \right] \\ &= \lim_{\delta z \rightarrow 0} \left[\frac{A P_1(z, X_i) \delta z + P_0(z) p(\delta z, X_i)}{\delta z} \right] \\ &= A P_1(z, X_i) + P_0(z) \psi(\Omega, X_i) / v. \quad (2-23) \end{aligned}$$

Solving this equation we find that $P_1(z, x_i)$ will be made up of a homogeneous and particular term,

$$P_1(z, x_i) = \alpha e^{-Az} + \beta z e^{-Az}.$$

Applying the boundary condition $P_1(0, x_i) = 0$ gives that $\alpha = 0$.

Plugging the remainder of the solution into equation (2-23) gives that $\beta = \psi(\Omega, x_i)/v$. Thus

$$P_1(z, x_i) = \frac{\psi(\Omega, x_i) z}{v} e^{-Az} \quad (2-24)$$

Calculate $P_2(z, x_1)$

The probability of a neutron, moving in any direction of path length (z) , intersecting two holes to experience lengthening $(x_1 + x_2)$ will be written as $P_2(z, x_1)$, where in this notation x_1 will be used to express the fact that the probability is a function of the lengthenings the neutron experiences. As before, $P_2(z, x_1)$ is the sum of two terms.

$$P_2(z + \delta z, x_i) = P_1(z, x_i) P_1(\delta z, x_i) + P_2(z, x_i) P_0(\delta z) \quad (2-25)$$

Note that we do not write a term containing $P_2(\delta z, x_i)$, because we have defined $P_1(\delta z, x_i)$ to be so small that $P_2(\delta z, x_i)$ is insignificant.

$$P_2(z + \delta z, x_i) = P_1(z, x_i) p(\delta z, x_i) + P_2(z, x_i)(1 - A\delta z), \quad (2-26)$$

then

$$\begin{aligned} \frac{\partial P_2(z, x_i)}{\partial z} &= \lim_{\delta z \rightarrow 0} \left[\frac{P_2(z + \delta z, x_i) - P_2(z, x_i)}{\delta z} \right] \\ &= \lim_{\delta z \rightarrow 0} \left[\frac{A P_2(z, x_i) \delta z + P_1(z, x_i) p(\delta z, x_i)}{\delta z} \right] \\ &= A P_2(z, x_i) + P_1(z, x_i) \psi(\Omega, x_i)/v. \end{aligned} \quad (2-27)$$

Solving this equation we find that

$$P_2(z, x_i) = \left[\frac{1}{2} \frac{\pi^2}{v} z \psi(z, \Omega, x_i) \right] e^{-Az} \quad (2-28)$$

By a similar procedure we can calculate $P_3(z, x_i)$, it turns out to be

$$P_3(z, x_i) = \left[\frac{1}{(3.2)} \frac{3}{\pi} \frac{z^4(\Omega, x_i)}{V} \right] e^{-Az}. \quad (2-29)$$

By analogy

$$P_n(z, x_i) = \left[\frac{1}{n!} \frac{\pi}{\pi} \frac{z^n(\Omega, x_i)}{V} \right] e^{-Az},$$

which is

$$P_n(z, x_i) = \left[\frac{1}{n!} \frac{\pi}{\pi} p(z, \Omega, x_i) \right] e^{-P(z, \Omega)}. \quad (2-30)$$

To calculate L^2 for the case of closely spaced holes we will replace the expression in equation (2-7) that is inside the square brackets with

$$\left[z^2 P_0(z) + \sum_{n=1}^{\infty} \left\{ \int \dots \int (z + \sum_{i=1}^n x_i)^2 P(z, x_i) dx_1 \dots dx_n \right\} \right]. \quad (2-31)$$

This expression is the summation of an infinite number of terms; thus recognizing that a neutron may intersect an infinite number of holes. Each term is the product of the square of the lengthened path length $(z + \sum_{i=1}^n x_i)$ associated with a neutron that intersects n holes times the probability of that occurrence, $P_n(z, x_i)$. To evaluate (2-31) equation we will need to evaluate each term.

$$\underline{n = 0}$$

For $n = 0$, The corresponding expression is

$$z^2 P_0(z). \quad (2-32)$$

$$\underline{n = 1}$$

When $n = 1$, The first term of the summation becomes

$$P_0(z) \int_0^{\infty} (z + x_1)^2 p(z, \Omega, x_1) dx_1. \quad (2-33)$$

By using equations (2-6) and (2-16) and defining M to be, $M \equiv \int_0^\infty x_i^2 p(z, \Omega, x_i) dx_i$, then equation (2-33) reduces to

$$= P_0(z) [z^2 P(z, \Omega) + 2z^2 \phi + M]. \quad (2-34)$$

$n = 2$

The next term in the summation is

$$= \frac{P_0(z)}{2} \left[\iint (z^2 + x_1 + x_2)^2 p(z, \Omega, x_1) p(z, \Omega, x_2) dx_1 dx_2 \right]. \quad (2-35)$$

Using the same procedures as when n equaled 1, this

becomes

$$= \frac{P_0(z)}{2} \left[z^2 P^2(z, \Omega) + 2MP(z, \Omega) + 4z^2 \phi P(z, \Omega) + 2(z\phi)^2 \right]. \quad (2-36)$$

Continue this procedure for n equal to 3 and 4.

$n = 3$

The third term in the summation of equation (2-31) is

$$= \frac{P_0(z)}{6} \left[z^2 P^3(z, \Omega) + 3P^2(z, \Omega)M + (6z)P^2(z, \Omega)z\phi + 6P(z, \Omega)(z\phi)^2 \right]. \quad (2-37)$$

$n = 4$

The fourth term becomes

$$= \frac{P_0(z)}{4!} \left[z^2 P^4(z, \Omega) + 4P^3(z, \Omega)M + 2(6)P^2(z, \Omega) + 2z(4)P^3(z, \Omega)z\phi \right]. \quad (2-38)$$

Extend n to ∞ and combine terms.

$$z^2 P_0(z) \left\{ \begin{array}{l} (1 + P + P^2/2! + P^3/3! + P^4/4! + \dots) \\ 2\phi(1 + P + P^2/2! + P^3/3! + P^4/4! + \dots) \\ \phi^2(1 + P + P^2/2! + P^3/3! + \dots) \end{array} \right.$$

$$P_0(z) M \{1 + P + P^2/2! + P^3/3! + \dots\}.$$

Since $e^P = 1 + P + P^2/2! + \dots$ and since $P_0(z) = e^{-P(z, \Omega)} \equiv e^{-P}$, equation (2-31) is

$$= z^2(1+\phi)^2 + M. \quad (2-39)$$

So

$$L^2 = N \int_{\Omega} \int_0^{\infty} [z^2(1+\phi)^2 + M] \frac{1}{\lambda} e^{-z/\lambda} \frac{dz d\Omega}{4\pi}, \quad (2-40)$$

which equals

$$= 2N\lambda^2(1+\phi)^2 + \frac{N\lambda}{v} \iint x^2 \psi(x, \Omega) \frac{dx d\Omega}{4\pi}. \quad (2-41)$$

Then

$$\frac{L^2}{L_0^2} = (1+\phi)^2 + \frac{1}{2\lambda v} \iint x^2 \psi(x, \Omega) \frac{dx d\Omega}{4\pi}, \quad (2-42)$$

where the integral is the same as in equation (2-10).

It will be useful to express this integral in terms of $\bar{Q}$ where $\bar{Q}$ is defined as the mean square path length thru a hole divided by the square of the mean path length thru the same hole.

$$Q \equiv (\overline{X^2})/(\bar{X}^2) \quad (2-43)$$

$$\overline{X^2} = \frac{\iint x^2 p(z, \Omega, x) \frac{dx d\Omega}{4\pi}}{\iint p(z, \Omega, x) \frac{dx d\Omega}{4\pi}} \quad (2-44)$$

$$\bar{X} = \frac{\iint x p(z, \Omega, x) \frac{dx d\Omega}{4\pi}}{\iint p(z, \Omega, x) \frac{dx d\Omega}{4\pi}} \quad (2-45)$$

The denominator of the last two equations is evaluated by using equation (2-6)

and the fact that

$$\int_0^{\infty} \Psi(x, \Omega) dx = \frac{1}{2} \iint |\vec{n} \cdot d\vec{s}| ds, \quad (2-46)$$

where the right side is the projected area of the hole perpendicular to $\vec{n}$. Since $|\vec{n} \cdot d\vec{s}|$ is $\cos \theta$, where θ is the angle between $\vec{n}$ and the normal to ds . Then

$$\int_0^{\infty} \Psi(x, \Omega) dx \frac{d\Omega}{4\pi} = \frac{1}{2} \int_s \int_{\phi} \int_{\theta} (\cos \theta \sin \theta) \frac{d\theta d\phi}{4\pi} ds \quad (2-47)$$

$$= \frac{1}{4} \int_s \sin^2 \theta \Big|_0^{\pi/2} ds = \frac{s}{4}. \quad (2-48)$$

Thus equation (2-45) reduces to

$$\bar{x} = 4\phi v/s. \quad (2-49)$$

And then Q is

$$Q = \frac{s}{4(\phi v)^2} \iint x^2 \Psi(x, \Omega) \frac{dx d\Omega}{4\pi}.$$

If we define the hydraulic radius (r_H) to be

$$r_H \equiv 2\phi v/s, \quad (2-50)$$

then equation (2-42) becomes

$$\frac{L^2}{L_0^2} = (1+\phi)^2 + \frac{Q r_H \phi}{\lambda}. \quad (2-51)$$

2-2.3 Intermediate Spaced Holes

When v and $s\lambda$ are of the same order of magnitude, the average number of holes entered ($zs/4v$) during free path z in the solid is unchanged. But now assume that ($zs/4v$) is $(n + \epsilon)$, where n is the integer portion and ϵ is the fraction between the limits of 0 and 1. Behrens' assumes that for free path length z there is a probability ϵ of entering $(n + 1)$ holes and $(1 - \epsilon)$ of entering n holes. Then L^2 can be expressed as the integral over two terms.

$$L^2 = N \int_0^\infty \left[\int_0^{n+1} \int (z + \sum_{i=1}^{n+1} X_i)^2 \epsilon \prod_{i=1}^{n+1} \left(\frac{p(z, \Omega, X_i) dX_i}{P(z, \Omega)} \right) + \right. \\ \left. \int_0^n \int (z + \sum_{i=1}^n X_i)^2 (1 - \epsilon) \prod_{i=1}^n \left(\frac{p(z, \Omega, X_i) dX_i}{P(z, \Omega)} \right) \right] \frac{1}{\lambda} e^{-z/\lambda} \frac{dz d\Omega}{4\pi}. \quad (2-52)$$

In this equation the probability of a neutron experiencing lengthening X_1 is $p(z, \Omega, X_1)/P(z, \Omega)$, where

$P(z, \Omega) \equiv \int_0^\infty p(z, \Omega, X_i) dX_i$. The expression inside the square brackets becomes.

$$\left[\right] = \epsilon \left[z^2 + 2z(n+1)\bar{X} + (n+1)\bar{X}^2 + n(n+1)\bar{X}^2 \right] + \\ (1-\epsilon) \left[z^2 + 2zn\bar{X} + n\bar{X}^2 + n(n-1)\bar{X}^2 \right]. \quad (2-53)$$

Substitute $\bar{X} = 4\phi v/s$ and $\bar{X}^2 = 16Q(\frac{\phi v}{s})^2$ and

$z = (n + \epsilon)\bar{X}/\phi$ into this equation, then

$$\left[\right] = z^2 + 2\phi z^2 + 16Q \left[\frac{35}{64} \right] \left(\frac{\phi v}{s} \right)^2 + 16 \left(\frac{\phi v}{s} \right)^2 (n^2 - n + 2n\epsilon). \quad (2-54)$$

Since the probability of $(z, z + \delta z)$ is $\frac{1}{\lambda} \exp(-z/\lambda) \delta z$,

the probability of $(n + \epsilon, n + \epsilon + \delta \epsilon)$ is $\frac{4v}{s\lambda} \exp\left(-\frac{(n+\epsilon)4v}{s\lambda}\right) d\epsilon$.

So that integration over z can be separated into two parts,

$$L^2 = \int_0^\infty (z^2 + 2\phi z^2 + 4(Qv\phi^2 z/s) \frac{1}{\lambda} \exp(-z/\lambda) dz +$$

$$\int_{\epsilon=0}^1 \sum_n^\infty e^{-4v(n+\epsilon)/s\lambda} \left[\underbrace{\left(\frac{4v}{s\lambda} \right) \frac{16\phi^2 v^2}{s^2}}_{\equiv \beta} (2n\epsilon + n^2 - n) \right] d\epsilon. \quad (2-55)$$

The first integral is

$$= 2\lambda^2 + 2\lambda^2(2\phi) + 4Qv\phi^2\lambda/s. \quad (2-56)$$

The second integral is

$$= \beta \int_0^1 e^{-a\epsilon} \left[\sum_n^\infty e^{-an} (2n\epsilon + n^2 - n) \right] d\epsilon, \quad (2-57)$$

Where $a \equiv 4v/s\lambda$. If we let $e^{-a} = y$ and recall that

$$\sum_{n=0}^\infty y^n = \frac{1}{1-y}, \text{ then}$$

$$\sum_{n=0}^\infty e^{-an} = \frac{1}{1-e^{-a}}. \quad (2-58)$$

Let $F(a) \equiv 1/(1-e^{-a})$, then

$$\sum_{n=0}^\infty n e^{-an} = - \frac{\partial F(a)}{\partial a} = \frac{e^{-a}}{(1-e^{-a})^2} \quad (2-59)$$

And

$$\sum_{n=0}^\infty n^2 e^{-an} = \frac{\partial^2 F(a)}{\partial a^2} = \frac{2e^{-2a}}{(1-e^{-a})^3} + \frac{e^{-a}}{(1-e^{-a})^2}. \quad (2-60)$$

Then

$$\sum_{n=0}^{\infty} (2n\epsilon + n^2 - n) e^{-an} = \left[\frac{\epsilon(1-e^{-a})}{e^{-a}} + 1 \right] \frac{2e^{-2a}}{(1-e^{-a})^3}.$$

Now substituting this into equation (2-57)

$$\begin{aligned} &= \beta \int_0^1 e^{-a\epsilon} \left[\frac{2e^{-2a}}{(1-e^{-a})^3} \right] (1 + \epsilon(e^a - 1)) d\epsilon \\ &= \frac{2\beta e^{-2a}}{(1-e^{-a})^3} a^2 [e^a + e^{-a} - 2] \\ &= (e^a - 1)^{-1} \frac{4v}{s\lambda} 2\phi^2 \lambda^2. \end{aligned} \quad (2-61)$$

So that L^2 is the sum of equations (2-56) and (2-61)

$$L^2 = 2\lambda^2 + 2\lambda^2(2\phi) + \frac{4Qv\phi^2\lambda}{s} + \frac{8v(\phi\lambda)^2}{s\lambda} [e^{4v/s\lambda} - 1]^{-1}. \quad (2-62)$$

Then

$$\frac{L^2}{L_o^2} = 1 + 2\phi + \phi^2 \frac{4v}{s\lambda} [e^{4v/s\lambda} - 1]^{-1} + \frac{2v\phi^2 Q}{s\lambda}. \quad (2-63)$$

And using the hydraulic radius

$$\frac{L^2}{L_o^2} = 1 + 2\phi + \phi^2 \left[\frac{2r_H}{\phi\lambda} \right] [e^{2r_H/\phi\lambda} - 1]^{-1} + \frac{Q\phi r_H}{\lambda}. \quad (2-64)$$

Behrens claimed that equation (2-64) is the general form for calculating (L^2/L_o^2) , as it reduces to the two

other equations, (2-10) and (2-51), at the limiting values of hole spacing. If the bubbles are widely spaced, $v \gg s\lambda$, then the factor in the square brackets in equation (2-63) becomes

$$(e^{\frac{4v}{s\lambda}} - 1)^{-1} \approx e^{-\frac{4v}{s\lambda}}$$

Then the limit of equation (2-63) as $\frac{4v}{s\lambda} \rightarrow \infty$, can be evaluated by using L'Hopital's rule. Thus

$$\frac{L^2}{L_0^2} = 1 + 2\phi + \frac{2v\phi^2 Q}{s\lambda}$$

which is equation (2-10) once it is rewritten using the definition of Q . On the other hand, if the bubbles are closely spaced, $s\lambda \gg v$, then the limit of equation (2-63), as $\frac{4v}{s\lambda} \rightarrow 0$, is.

$$\frac{L^2}{L_0^2} = (1 + \phi)^2 \quad (2-65)$$

which is the same as the limit of equation (2-51) as $\frac{4v}{s\lambda} \rightarrow 0$.

2-3 Theory For Random Distribution of Bubbles

In a pool of molten fuel that is approaching critical mass the temperature will be high. As fission energy is released in the fuel material, it is likely that boiling, and accumulation of fission gases to form bubbles, will occur throughout the molten fuel. I will neglect the fact that heat transfer from the outer areas of the pool will cause a spatially dependant temperature distribution that could produce a spatial distribution of the bubble density. Also, I will ignore the fact that the volume of bubbles per unit volume increases as the bubbles rise due to the accumulation of gases. In short, I will assume the ideal case of a molten fuel mass containing, on a coarse scale, a uniform void fraction. While this distribution is uniform, it can not be represented by a cell model that associates each bubble to a unit of fuel. The random nature of the boiling process means that on the microscopic scale the bubbles will occur randomly. Thus I will redefine ϕ to be the volume ratio of bubbles to fuel averaged over the volume of the molten assembly.

The assumption that the bubbles are randomly distributed throughout the fuel mass means that the probability that a neutron will enter a hole during the next $\delta\chi$ of path length will be independant of its history. This is the assumption that Behrens used to develop his equation

for $(l/L_0)^2$, for the case of closely spaced holes.

$$\frac{L^2}{L_0^2} = (1 + \phi)^2 + Q r_H \phi / \lambda \quad (2-51)$$

The general equation that Behrens developed does not apply to this theory. The basic assumption that he made in developing the general formula was that the average number of holes entered by a neutron of path length z could be expressed as the sum of two parts $(n + \epsilon)$, where n represented the integer portion of the number and ϵ was the fraction. He then assumed that a neutron of path length z could experience only two lengthenings. If it passed thru $(n + 1)$ holes it would experience lengthening $(z + \sum_{i=1}^{n+1} X_i)$. He assigned a probability of (ϵ) to that occurrence. The complimentary probability $(1-\epsilon)$ applied to the neutron entering (n) holes and experiencing lengthening, $(z + \sum_{i=1}^n X_i)$. Restricting the neutron of path length z to entering either (n) or $(n + 1)$ holes corresponds to rigidly spacing the holes so that the distance between any two holes is equal to the average hole spacing. In a fuel material containing a random distribution of bubbles a neutron with mean path length z could intersect any number of holes. Thus Behrens' general theory will not be used. The ratio of L^2 to L_0^2 will be evaluated using his equation for closely spaced holes. The preceeding discussion suggests that this

case is more generally applicable to a random distribution of any mean spacing. The restriction to small spacing only applies if the distribution is not random.

2-4 Computational Model

In order to estimate the reactivity effect of bubble implosion, I will choose a model that will permit simple calculations using equation (2-3).

$$(\Delta K)_B = \frac{K_\infty - 1}{K_\infty} \left[\frac{L_i^2}{L_f^2} - 1 \right] \quad (2-3)$$

Where L_i^2 is the initial value of the diffusion area and L_f^2 is the final value. I have shown that L_i^2 can be represented by L^2 from equation (2-51).

$$\frac{L^2}{L_o^2} = (1 + \phi)^2 + \frac{Q r_H \phi}{\lambda} \quad (2-51)$$

L_o^2 represents the diffusion area in the fuel prior to the formation of bubbles. When the bubbles form they cause the pool of molten fuel to expand by an amount equal to their volume but the density of the fuel among the bubbles is unchanged. With the bubbles in the fuel the diffusion area is L^2 . However, when the fuel expands to implode the bubbles, the density of the fuel itself decreases. In this case the diffusion area is L_f^2 . The value of the diffusion length L_f^2 can be calculated by taking the limit of equation (2-51) as the hydraulic radius goes to zero,

$$\frac{L_f^2}{L_o^2} = \lim_{r_H \rightarrow 0} \left[(1 + \phi)^2 + Q r_H \phi / \lambda \right] = (1 + \phi)^2 \quad (2-66)$$

Then combining equations (2-51) and (2-66),

$$\frac{L_i^2}{L_f^2} = 1 + \frac{Q r_H \phi}{\lambda(1+\phi)^2} \quad (2-67)$$

To evaluate this expression I will consider a spherical pool of molten fuel. Inside this fuel is a random distribution of bubbles that are small enough so that on the macroscopic scale the bubble density is uniform. I also assume that each bubble is a sphere. Since a super prompt critical power excursion would occur in a relatively short period of time compared to bubble movement due to gravity, thermal currents or movement of the fuel, I will assume no time dependence.

The value of Q can be calculated from equation (2-44) and (2-45) by letting $X = 2\sqrt{r^2 - \rho^2}$ where r and ρ are the sides of the triangle shown in Figure 2-1. Thus, equations (2-44) and (2-45) become

$$\overline{X^2} = \frac{4 \int_0^r (r^2 - \rho^2) 2\pi \rho d\rho}{\int_0^r 2\pi \rho d\rho} = 2r^2 \quad (2-68)$$

$$\overline{X} = \frac{2 \int_0^r (r^2 - \rho^2)^{1/2} 2\pi \rho d\rho}{\int_0^r 2\pi \rho d\rho} = \frac{4r}{3} \quad (2-69)$$

$$\text{Then } Q = \frac{2r^2}{\left[\frac{4r}{3}\right]^2} = \frac{18}{16} = 1.125. \quad (2-70)$$

FIGURE 2-1. Spherical Model of Bubble

The (r_H) in equation (2-67) is the hydraulic radius. For a spherical bubble

$$r_H = \frac{2\phi v}{s} = \frac{2(\frac{4}{3}\pi r^3)}{4\pi r^2} = \frac{2r}{3}. \quad (2-71)$$

Thus equation (2-67) becomes

$$\frac{L_i^2}{L_f^2} = 1 + \frac{(0.75) r \phi}{\lambda (1 + \phi)^2}, \quad (2-72)$$

and equation (2-3) reduces to

$$(\Delta K)_B = \frac{K_\infty - 1}{K_\infty} \left[\frac{0.75 r \phi}{\lambda (1 + \phi)^2} \right]. \quad (2-73)$$

- Where
- $(\Delta K)_B$ = Reactivity effect of bubble implosion
 - K_∞ = Infinite multiplication factor
 - r = Radius of Bubble
 - ϕ = Overall volume ratio of bubbles to fuel
 - λ = Neutron mean free path in fuel.

2-5 Order of Magnitude of $(\Delta K)_B$

Equation (2-73) expresses Behrens' reactivity, $(\Delta K)_B = f(K_\infty, \phi, r, \lambda)$. In order to establish the magnitude of $(\Delta K)_B$ and its sensitivity to changes in each of the parameters I will vary the parameters thru the following ranges.

1.1	$\equiv$	K_∞	$\equiv$	2
.1	$\equiv$	ϕ	$\equiv$	1
.25 cm	$\equiv$	r	$\equiv$	2.5 cm
2 cm	$\equiv$	λ	$\equiv$	5 cm

Small LMFBR reactors such as EBR-II and FERMI-I have a K_∞ of about 2(7.). As the size of the core increases or as more blanket material is placed around the core, the neutron leakage will decrease and thus the enrichment of the fuel can be decreased. Thus for a large LMFBR K_∞ can be expected to be between 1 and 2. Most likely it will be close to 1.3. As there is little data concerning the size and density of bubbles that are likely to exist in molten fuel, I have considered a large range of values for the bubble radius (r) and volume ratio of bubbles to fuel (ϕ). The value of λ can be calculated from known values of Σ_{tr} , which are tabulated for various isotopes and ranges of neutron energy (18.). For molten fuel composed of pure U_2O and neutrons with .1 to .4 MEV of kinetic energy, the microscopic transport cross section

is 15.4 b. Then the transport mean free path (λ) is approximately 3.5 cm. If the molten fuel contains impurities that have low transport cross sections, such as iron, nickel and chrome, the λ would be larger. Tables 2-1 thru 2-4. present the results of these calculations. As $(\Delta K)_B$ is a monotone increasing function of r , K_∞ , ϕ , and $1/\lambda$ within the ranges considered, the maximum value of $(\Delta K)_B$ will occur when each parameter is at its maximum: $r = 2.5\text{cm}$, $K_\infty = 2$, $\phi = 1$, and $\lambda = 2\text{ cm}$. From Table 2-4, the maximum $(\Delta K)_B$ is .117188. From Table 2-1, the minimum value of $(\Delta K)_B$ is .000282. A more reasonable value of $(\Delta K)_B$ can be found by carefully selecting the values of the parameters. A value of 1.3 for K_∞ is close to what will exist in a large LMFBR. Also λ can be expected to be

ϕ	r	λ	$(\Delta K)_B$	ϕ	r	λ	$(\Delta K)_B$
1	2.5	2	.021307	.25	2.5	2	.013636
		3	.014204			3	.009091
		4	.010633			4	.006818
		5	.008523			5	.005454
	1.5	2	.012784		1.5	2	.008182
		3	.008523			3	.005454
		4	.006392			4	.004091
		5	.005114			5	.003293
	.5	2	.004261		.5	2	.002727
		3	.002841			3	.001818
		4	.002131			4	.001364
		5	.001705			5	.001091
	.25	2	.002131		.25	2	.001364
		3	.001470			3	.000910
		4	.001065			4	.000682
		5	.000852			5	.000545
.5	2.5	2	.018940	1	2.5	2	.007044
		3	.012626			3	.004696
		4	.009469			4	.003322
		5	.007576			5	.002817
	1.5	2	.011364		1.5	2	.004226
		3	.007576			3	.002817
		4	.005682			4	.002113
		5	.004545			5	.001690
	.5	2	.003788		.5	2	.001409
		3	.002325			3	.000939
		4	.001894			4	.000704
		5	.001515			5	.000563
	.25	2	.001894		.25	2	.000704
		3	.001263			3	.000470
		4	.000947			4	.000352
		5	.000758			5	.000282

TABLE 2-1 $(\Delta K)_B$ For $K_\infty = 1.1$

ϕ	r	λ	$(\Delta K)_B$	ϕ	r	λ	$(\Delta K)_B$
1	2.5 cm	2 cm	.054087	.25	2.5 cm	2 cm	.034615
		3	.036058			3	.023077
		4	.027043			4	.017308
		5	.021635			5	.013846
	1.5	2	.032452		1.5	2	.020769
		3	.021635			3	.013845
		4	.016226			4	.010292
		5	.012981			5	.008308
	.5	2	.010818		.5	2	.006923
		3	.007212			3	.004615
		4	.005409			4	.003462
		5	.004327			5	.002770
	.25	2	.005409		.25	2	.003462
		3	.003606			3	.002308
		4	.002705			4	.001731
		5	.002161			5	.001385
.5	2.5	2	.048077	.1	2.5	2	.017880
		3	.032051			3	.011920
		4	.024038			4	.008940
		5	.019231			5	.007152
	1.5	2	.028754		1.5	2	.010728
		3	.019231			3	.007152
		4	.014423			4	.005364
		5	.011538			5	.004291
	.5	2	.009615		.5	2	.003577
		3	.006410			3	.002382
		4	.004808			4	.001786
		5	.003846			5	.001431
	.25	2	.004808		.25	2	.001786
		3	.003205			3	.001191
		4	.002404			4	.000895
		5	.001923			5	.000715

TABLE 2-2. $(\Delta K)_B$ For $K_\infty = 1.3$

ϕ	r	λ	$(\Delta K)_B$	ϕ	r	λ	$(\Delta K)_B$
1	2.5	2	.078125	.25	2.5	2	.050000
		3	.052083			3	.033333
		4	.039062			4	.025000
		5	.031250			5	.020000
	1.5	2	.046875		1.5	2	.030000
		3	.031250			3	.020000
		4	.023438			4	.015000
		5	.018750			5	.013000
	.5	2	.015625		.5	2	.010000
		3	.010417			3	.006667
		4	.007812			4	.005000
		5	.006250			5	.004000
	.25	2	.007812		.25	2	.005000
		3	.005208			3	.003333
		4	.003906			4	.002500
		5	.003125			5	.002000
.5	2.5	2	.069444	.1	2.5	2	.025826
		3	.046296			3	.017218
		4	.034722			4	.012913
		5	.027777			5	.010331
	1.5	2	.041667		1.5	2	.015500
		3	.027777			3	.010330
		4	.020833			4	.007748
		5	.016666			5	.006198
	.5	2	.013889		.5	2	.005165
		3	.009260			3	.003444
		4	.006944			4	.002583
		5	.005556			5	.002066
	.25	2	.006944		.25	2	.002583
		3	.004630			3	.001722
		4	.003472			4	.001291
		5	.002778			5	.001033

TABLE 2-3. $(\Delta K)_B$ For $K_\infty = 1.5$

ϕ	r	λ	$(\Delta K)_B$	ϕ	r	λ	$(\Delta K)_B$
1	2.5	2	.117188	.25	2.5	2	.075000
		3	.078125			3	.050000
		4	.058594			4	.037500
		5	.046875			5	.030000
	1.5	2	.070312		1.5	2	.045000
		3	.046875			3	.030000
		4	.035156			4	.225000
		5	.028125			5	.018000
	.5	2	.023438		.5	2	.015000
		3	.015625			3	.010000
		4	.011719			4	.007500
		5	.009375			5	.006000
	.25	2	.011719		.25	2	.007500
		3	.007812			3	.005000
		4	.005860			4	.003750
		5	.004688			5	.003000
.5	2.5	2	.104167	.1	2.5	2	.038740
		3	.069444			3	.025826
		4	.052083			4	.019370
		5	.041667			5	.015496
	1.5	2	.062300		1.5	2	.023244
		3	.041667			3	.015496
		4	.031230			4	.011622
		5	.025000			5	.009298
	.5	2	.020833		.5	2	.007748
		3	.013889			3	.005165
		4	.010417			4	.003874
		5	.008333			5	.003099
	.25	2	.010417		.25	2	.003874
		3	.006944			3	.002583
		4	.005208			4	.001937
		5	.004167			5	.001550

TABLE 2-4. $(\Delta K)_B$ For $K_\infty = 2$.

about 4 cm due to fission product accumulation and the inclusion of molten structural material in the mass of molten fuel. While it is difficult to justify selecting a reasonable value for the bubble radius and volume ratio of bubbles to fuel, a conservative safety analyst might choose to consider bubbles with $r = 0.5$ cm and $\phi = 0.50$.

(19.) The value of $(\Delta K)_B$ corresponding to these values of the parameters is 0.004808. To compare this value of $(\Delta K)_B$ to the reactivity effect due to Doppler broadening, $(\Delta K)_{Dop}$, we must calculate $(\Delta K)_{Dop}$ for a reasonable temperature increase. A typical value of the Doppler coefficient (A_D), for a large LMFBR, is 0.004. If the molten fuel temperature is about 3040°K and it increases to a very high temperature during the power transient, 5000°K, then

$$(\Delta K)_{Dop} = -A_D \int_{3040}^{5000} \frac{dT}{T} = -0.004(.497) = -.00199.$$

Thus $(\Delta K)_B$ will be of the same order of magnitude as $(\Delta K)_{Dop}$.

2-6. Calculation of $(\Delta K)_B$ Using Behrens' General Lengthening Formula

To evaluate the effect of using Behrens' lengthening formula for close bubble spacing, equation (2-51), rather than his general formula, equation (2-64), for evaluating the reactive effect of imploding randomly distributed bubbles, I will evaluate the reactivity effect using his

general formula $(\Delta K)_{BG}$ and compare it to $(\Delta K)_B$, evaluated using his close bubble spacing formula. By the same arguments given in Section 2-3, the final migration area (L_f^2) can be found by taking the limit of equation (2-64) as r_H goes to zero.

$$\frac{L_f^2}{L_o^2} = 1 + 2\phi + \phi^2 \left[\frac{2r_H}{\phi\lambda} \right] \left[\exp \left[\frac{2r_H}{\phi\lambda} \right] - 1 \right]^{-1} + \frac{Q\phi r_H}{\lambda} \quad (2-64)$$

$$L_f^2 = L_o^2 (1 + 2\phi + \phi^2) = L_o^2 (1 + \phi)^2 \quad (2-74)$$

Since $L^2 = L_f^2$ in equation (2-64), equation (2-3) becomes

$$\begin{aligned} (\Delta K)_{BG} &= \frac{K_{\infty} - 1}{K_{\infty}} \left[\frac{1 + 2\phi + \phi^2 \left[\frac{2r_H}{\phi\lambda} \right] \left[\exp \left[\frac{2r_H}{\phi\lambda} \right] - 1 \right]^{-1} + \frac{Q\phi r_H}{\lambda}}{(1 + \phi)^2} - 1 \right] \\ &= \frac{K_{\infty} - 1}{K_{\infty}(1 + \phi)^2} \left[\phi^2 \left(\frac{2r_H}{\phi\lambda} \left[\exp \left[\frac{2r_H}{\phi\lambda} \right] - 1 \right]^{-1} - 1 \right) + \frac{Q\phi r_H}{\lambda} \right] \quad (2-75) \end{aligned}$$

Table 2-5 presents values of $(\Delta K)_{BG}$ and $(\Delta K)_B$ for several values of the parameters: K_{∞} , ϕ , r and λ . These values

K_{∞}	ϕ	r	λ	$(\Delta K)_B$	$(\Delta K)_{BG}$
2.0	1	2.5	2	0.117188	0.062401
1.5	.5	1.5	4	0.020833	0.009209
1.5	.5	.5	4	0.006944	0.002400
1.3	.5	1.5	4	0.014423	0.006375
1.3	.5	1.5	5	0.011538	0.004772
1.3	.5	.5	4	0.004808	0.001661
1.3	.5	.5	5	0.003846	0.001291
1.1	.1	.25	5	0.000282	0.000111

TABLE 2-5. Comparison of
 $(\Delta K)_{BG}$ to $(\Delta K)_B$

indicate that over the range of the parameters considered $(\Delta K)_B$ will be two to two and one half times as large as $(\Delta K)_{BG}$. While Beherns' general lengthening equation reduces to his equation for close bubble spacing in the limit of small $\sqrt{s\lambda}$, the values of Table 2-5 indicate that when $\sqrt{s\lambda}$ is not small, the two equations do not give the same result. Thus it is important to use his equation for close bubble spacing when considering random bubble distributions for the range of the parameters under consideration. The values in Table 2-5 also suggest that if the actual bubble distribution departs from a random distribution then the reactivity effect could be significantly different. Also, since the values of $(\Delta K)_{BG}$ are calculated based on the assumption of more or less uniform spacing, they can be taken to give the lower limit of the reactivity effect for any real distribution of bubbles, at least to the extent that other simplifying assumptions are accepted. This is because a uniform spacing of the bubble positions minimizes the variation in the free path length that tends to make the mean square migration length large.

III. FUEL DISPLACEMENT DURING BUBBLE IMPLOSION

During the onset of the power transient associated with the reassembly of a critical mass of molten fuel, the bubbles that are likely to be present will be imploded by expanding fuel. In addition to the reactivity effect associated with the bubble implosion, there will be a reactivity effect associated with fuel movement relative to the center of mass of the molten fuel. For example, if the bubble density has a maximum at the center of the molten fuel and decreases toward the perimeter, fuel expansion will set up an internal pressure that will drive fuel toward the center of mass. The effective increase of fuel density at the center of mass will produce an increase of reactivity. In this thesis I will not be concerned with fuel movement due to deterministically non uniform bubble distributions, as one of my simplifying assumptions is that the bubble density is macroscopically uniform. However, even with a macroscopically uniform bubble density the variance of the bubble density about the uniform value will provide opportunity for fuel movement.

3-1 Poisson Distribution of Bubbles

Consider a mass of molten fuel that is at a constant temperature above the boiling point. The random nature of

the boiling process suggests that the probability of a bubble existing in a small volume (δV) will be a constant ($\xi \delta V$) throughout the molten fuel. Here δV is selected to be so small that the probability of two bubbles existing in δV is insignificant compared to $\xi \delta V$. Let the probability of N bubbles existing in a volume V be $p_N(V)$. Using this notation, $p_0(V + \delta V)$ is the probability that zero bubbles will be in the volume ($V + \delta V$). By assuming that the existence of any bubble is independent of the existence of any other bubble, one can represent $p_0(V + \delta V)$ as the product of the probability of the existence of zero bubbles in volume V and the probability of the existence of zero bubbles in volume δV (20.)

$$\begin{aligned} p_0(V + \delta V) &= p_0(V)p_0(\delta V) \\ &= p_0(V)(1 - \xi \delta V), \end{aligned} \quad (3-1)$$

where I have let $p_0(\delta V)$ equal the complimentary probability of having a bubble in δV . Now consider $p_N(V + \delta V)$ where $N > 0$.

$$\begin{aligned} p_N(V + \delta V) &= p_{N-1}(V) \xi \delta V + p_N(V)p_0(\delta V) \\ &= p_{N-1}(V)\xi \delta V + p_N(V)(1 - \xi \delta V). \end{aligned} \quad (3-2)$$

Rearranging gives

$$p_N \frac{(V + \delta V) - p_N(V)}{\delta V} = -\xi p_N(V) + \xi p_{N-1}(V). \quad (3-3)$$

In the limit of small δV the left side becomes the derivative of $p_N(V)$,

$$\frac{dp_N(V)}{dV} = -\xi p_N(V) + \xi p_{N-1}(V). \quad (3-4)$$

A solution of this equation is

$$p_N(V) = \frac{(\xi V)^N \exp(-\xi V)}{N!}, \quad (3-5)$$

which can be recognized as the poisson distribution.

Now that the number of bubbles per volume V has been identified as a poisson statistic, its variance can be calculated from the following definition.

$$V(N) \equiv E([N - E(N)]^2), \quad (3-6)$$

$$\text{where } E(N) \equiv \sum_{N=0}^{\infty} N p_N(V). \quad (3-7)$$

Now by expanding the squared term

$$V(N) = E(N^2) - [E(N)]^2. \quad (3-8)$$

Add and subtract $E(N)$

$$\begin{aligned} V(N) &= E(N^2) - E(N) - [E(N)]^2 + E(N) \\ &= E(N(N-1)) - E(N)[E(N)-1]. \end{aligned} \quad (3-9)$$

Then

$$E(N(N-1)) = \sum_{N=2}^{\infty} N(N-1) \frac{(\xi V)^N e^{-\xi V}}{N!} \quad (3-10)$$

$$= (\xi V)^2 \sum_{N=2}^{\infty} \frac{(\xi V)^{N-2} e^{-\xi V}}{(N-2)!}. \quad (3-11)$$

Let $M = N-2$.

$$E(N(N-1)) = (\xi V)^2 \sum_{M=0}^{\infty} \frac{(\xi V)^M e^{-\xi V}}{M!} = (\xi V)^2 \quad (3-12)$$

And since

$$\begin{aligned} E(N) &= \sum_{N=0}^{\infty} N p_N(V) \\ &= e^{-\xi V} \left[\frac{0(\xi V)^0}{0!} + \frac{1(\xi V)^1}{1!} + \frac{2(\xi V)^2}{2!} + \dots \right] \end{aligned}$$

$$\begin{aligned}
 &= e^{-\xi V} \left[0 + (\xi V) \left(1 + \frac{(\xi V)}{1!} + \frac{(\xi V)^2}{2!} + \dots \right) \right] \\
 &= e^{-\xi V} (\xi V) e^{\xi V} = \xi V. \quad (3-13)
 \end{aligned}$$

$$\begin{aligned}
 V(N) &= E(N(N-1)) - E(N)[E(N)-1] \\
 &= (\xi V)^2 - (\xi V)(\xi V - 1) \\
 &= \xi V. \quad (3-14)
 \end{aligned}$$

Since the variance of a poisson statistic equals its expectation and since the maximum likelihood estimator of the expectation is the average of the poisson statistic (21.), the variance can be estimated by the average, $V(N) = \bar{N}$. Thus for a molten fuel containing a uniform bubble distribution, the number of bubbles in any volume V will have a standard deviation (σ_N) equal to the square root of the number of bubbles in that volume.

3-2 Calculating Reactivity Effect of Fuel Movement

Once fuel expansion begins the internal pressure will move the fuel around to achieve a more uniform mean fuel density. This will occur prior to complete implosion of the bubbles. By dividing the spherical pool of molten fuel into thin spherical shells one can see that prior to fuel expansion the uniform bubble density may in fact have local variations corresponding to the statistical fluctuations inherent in the bubble density. Figure 3-1 shows a cross section of three of these thin spherical shells. The number of bubbles per shell (N_1) is shown to be variable. After fuel expansion begins the fuel movement will redistribute the fuel so that each shell will contain a void fraction equivalent to exactly $\bar{N}$ bubbles per unit volume.

$$\bar{N} = (\sum_{i=1}^{\ell} N_i) / V_T, \quad (3-15)$$

where ℓ = total number of shells
 V_T = total volume of molten bubbly fuel.

The reactivity effect due to fuel movement from one shell to another in fuel containing a uniform distribution of bubbles will be called δk .

Figure 3-1. Cross-Section of Thin Shells

This δk can be evaluated by expressing it as the reactivity effect of replacing δN_i bubbles of worth w_i with fuel plus the reactivity effect of replacing fuel with those δN_i bubbles at other positions in the sphere such that their worth is $\bar{w}$. $\bar{w}$ is the average worth of a bubble

$$\bar{w} = \frac{\sum_{i=1}^L w_i N_i}{N_T}, \quad (3-16)$$

where all the bubbles are assumed to be of the same size (v) and

$$N_T = \sum_{i=1}^L N_i. \quad (3-17)$$

Thus δk can be expressed as

$$\delta k = \sum_{i=1}^L (w_i - \bar{w}) \delta N_i. \quad (3-18)$$

Then

$$(\delta K)^2 = \left[\sum_i^{\ell} (w_i - \bar{w}) \delta N_i \right] \left[\sum_j^{\ell} (w_j - \bar{w}) \delta N_j \right] \quad (3-19)$$

$$= \sum_{i=j}^{\ell} (w_i^2 - 2w_i\bar{w} + \bar{w}^2) (\delta N_i)^2 + \sum_{i \neq j}^{\ell} (w_i - \bar{w})(w_j - \bar{w}) \delta N_i \delta N_j. \quad (3-20)$$

If $(\delta K)^2$ is averaged over a large number of observations, $\langle (\delta K)^2 \rangle$, then the term where $i \neq j$ will average to zero,

$$\langle (\delta K)^2 \rangle = \sum_i^{\ell} (w_i^2 - 2w_i\bar{w} + \bar{w}^2) \langle (\delta N_i)^2 \rangle. \quad (3-21)$$

Since the average of the square of a statistic is the maximum likelihood estimator of the variance (21),

$$\langle (\delta K)^2 \rangle = V(K), \quad (3-22)$$

and since N_i is a poisson statistic

$$\langle (\delta N_i)^2 \rangle = V(N_i) = \bar{N}_i = N_i.$$

Then equation (3-21) becomes

$$V(K) = (\bar{w}^2 - \bar{w}^2) N_T. \quad (3-23)$$

Where $\bar{w}^2$ is the mean square reactive worth of a bubble

$$\bar{w}^2 = \sum_i^{\ell} w_i^2 N_i / N_T. \quad (3-24)$$

The square root of equation (3-23) is the standard deviation of the reactivity effect due to fuel movement in a molten fuel containing a uniform distribution of bubbles, $(\Delta K)_{fm}$.

$$(\Delta K)_{fm} = \sqrt{(\bar{w}^2 - \bar{w}^2) N_T} \quad (3-25)$$

$(\Delta K)_{fm}$ represents the magnitude of the reactivity effect associated with the statistical variation of the bubble

density from its uniform value. The observed reactivity effect due to fuel movement in any given accident will have a probability of .67 of being in the interval $[-(\Delta K)_{fm}, (\Delta K)_{fm}]$. The variables $\overline{w^2}$ and $\overline{\bar{w}}^2$ are functions of the reactivity worth of a bubble at some position in the bubbly fuel (w_1). In the limit of infinitely thin spherical shells w_1 becomes $w(r)$. Thus to evaluate $(\Delta K)_{fm}$ it will be necessary to develop an expression for $w(r)$.

3.3 Application of Perturbation Theory

The reactivity worth of a bubble as a function of position in the bubbly fuel can be calculated from perturbation theory (22.) The basic relation that will be used is

$$\frac{\Delta K}{K} = \frac{\int_V \left(\delta[(K_\infty - 1)\Sigma_a v] \phi^2 - \delta[Dv] |\nabla \phi|^2 \right) dV}{\int_V K_\infty \Sigma_a v \phi^2 dV}, \quad (3-26)$$

where

- ΔK = change in reactivity worth
- K = effective multiplication constant
- K_∞ = infinite multiplication constant
- Σ_a = macroscopic absorption cross section
- v = velocity
- V = volume
- D = Diffusion coefficient
- ϕ = neutron flux

The fuel motion will be modeled as a small perturbation of the local void fraction (δf), where the void fraction (f) is the ratio of void volume (V_B) to total bubbly fuel volume (V_T). While the local void fraction will be perturbed the overall void fraction will not be affected by fuel movement. I will continue to assume a spherical bubbly fuel mass containing a uniform distribution of spherical bubbles, all of volume v_b .

A perturbation of the void fraction will become a perturbation of the macroscopic absorption cross section (Σ_a) and the diffusion coefficient (D). The macroscopic absorption cross section of the bubbly fuel can be expressed in terms of the macroscopic absorption cross section of the fuel among the bubbles (Σ'_a) and the void fraction, as the bubbles are assumed to be voids.

$$\Sigma_a = \Sigma'_a (1-f) \quad (3-27)$$

Then the perturbation of Σ_a is

$$\begin{aligned} \delta(\Sigma_a) &= \Sigma'_a (-\delta f), \\ &= -\frac{\Sigma_a}{(1-f)} \delta(f). \end{aligned} \quad (3-28)$$

The diffusion coefficient can be written as $D = \frac{1}{3\Sigma_{tr}}$,

where Σ_{tr} , the macroscopic transport cross section, can be related to f by an equation similar to equation (3-27).

Then the perturbation of D is

$$\delta(D) = \delta(f) / (3(1-f)\Sigma_{tr}). \quad (3-29)$$

Assuming that v and K_∞ will not be affected by the perturbation due to fuel movement, equation (3-26) becomes

$$\frac{\Delta K}{K} = \frac{\int_V \left[(K_\infty - 1) v [-\Sigma_a \delta(f) / (1-f)] \Phi^2 - v [\delta(f) / (3(1-f)\Sigma_{tr})] (\nabla \Phi)^2 \right] dV}{v K_\infty \Sigma_a \int_V \Phi^2 dV} \quad (3-30)$$

Now approximate this integral for the volume of a thin spherical shell.

$$\frac{\delta(K)}{K} = \frac{4\pi r^2 \delta(r)}{(1-f)} \left[-(K_\infty - 1) \Sigma_a \delta(f) \Phi^2 - (\delta(f) / 3\Sigma_{tr}) \left(\frac{d\Phi}{dr} \right)^2 \right] \quad (3-31)$$

$$K_\infty \Sigma_a \int_V \Phi^2 dV$$

The integral in the denominator is over the volume of the whole mass of molten fuel ($V = 4/3\pi R^3$). Then substituting the value for the flux in a spherical mass of fuel

($\phi(r) = \frac{\sin Br}{r}$), equation (3-31) becomes.

$$\frac{\delta(K)}{K} = \frac{-2r^2 \delta(r) \delta(f)}{R(1-f) K_{\infty}} \left[(K_{\infty} - 1) \left(\frac{\sin Br}{r} \right)^2 + 3 \Sigma_{tr} \Sigma_a \left(\frac{B \cos Br}{r} - \frac{\sin Br}{r^2} \right)^2 \right] \quad (3-32)$$

Since the diffusion area (L^2) is defined to be

(D/Σ_a), Σ_a can be expressed as a function of Σ_{tr} .

$$\Sigma_a = \frac{B^2}{3(K_{\infty} - 1) \Sigma_{tr}} \quad (3-34)$$

By substituting for Σ_a and letting $K=1$ equation (3-32) becomes

$$\delta(K) = \frac{-2r^2 \delta(r) \delta(f) (K_{\infty} - 1)}{R(1-f) K_{\infty}} \left[\left(\frac{\sin Br}{r} \right)^2 + \frac{1}{B^2} \left(\frac{B \cos Br}{r} - \frac{\sin Br}{r^2} \right)^2 \right] \quad (3-35)$$

Now the reactivity effect per unit bubble volume can be found by dividing the total reactivity affect ($\delta(K)$) by the total bubble volume in the shell ($4\pi r^2 \delta(r) \delta(f)$). Then $w(r)$ is the reactivity effect per unit volume times the volume of a bubble (v_b),

$$w(r) = \frac{-(K_{\infty} - 1) v_b}{K_{\infty} 2\pi R(1-f)} \left[\left(\frac{\sin Br}{r} \right)^2 + \frac{1}{B^2} \left(\frac{B \cos Br}{r} - \frac{\sin Br}{r^2} \right)^2 \right] \quad (3-36)$$

The average reactive worth per unit bubble volume ($\bar{w}$) which was expressed as equation (3-16) can be expressed by the following integral in the limit of thin spherical shells,

$$\bar{w} = \frac{\int_V w(r) N(r) dV}{\int_V N(r) dV} \quad (3-37)$$

Since I am assuming a uniform bubble density, $N(r) =$ constant.

$$\bar{w} = \frac{3}{R^3} \int_0^R w(r) r^2 dr \quad (3-38)$$

Substituting equation (3-36) into equation (3-38) gives

$$\bar{w} = \frac{-3(K_{\infty} - 1) v_0}{2\pi K_{\infty} R^4 (1-f)} \int_0^R r^2 \left[\left(\frac{\sin Br}{r} \right)^2 + \frac{1}{B^2} \left(\frac{B \cos Br}{r} - \frac{\sin Br}{r^2} \right)^2 \right] dr \quad (3-39)$$

Now since B^2 is the geometric buckling, B equals (π/R) for this spherical model,

$$\int_0^R \sin^2(\pi r/R) dr = \frac{R}{2} \quad (3-40)$$

Expanding the square in the second term gives

$$\int \left(\frac{B \cos Br}{r} - \frac{\sin Br}{r^2} \right)^2 r^2 dr = \int \left(B^2 \cos^2 Br + \frac{\sin^2 Br}{r^2} - \frac{2B}{r} \cos Br \sin Br \right) dr \quad (3-41)$$

Then

$$\int_0^R B^2 \cos^2 Br dr = B^2 R/2 \quad (3-42)$$

$$\int_0^R \frac{\sin^2 Br}{r^2} dr = B \int_0^{BR} \frac{\sin^2 x}{x^2} dx, \text{ WHERE } x \equiv Br. \quad (3-43)$$

Since $\sin^2 x = \frac{1}{2} (1 - \cos 2x)$ equation (3-43) becomes

$$= \frac{B}{2} \left[\int_0^{BR} x^{-2} dx - \int_0^{BR} \frac{\cos 2x}{x^2} dx \right], \quad (3-44)$$

$$= -\frac{B}{2x} \Big|_0^{BR} - B \int_0^{2BR} \frac{\cos y}{y^2} dy, \text{ WHERE } y \equiv 2x. \quad (3-45)$$

Then

$$\begin{aligned} \int_0^{2BR} \frac{\cos y}{y^2} dy &= -\frac{\cos y}{y} \Big|_0^{2BR} - \int_0^{2BR} \frac{\sin y}{y} dy, \quad (23.) \\ &= -\frac{\cos 2x}{2x} \Big|_0^{BR} - [Si(2BR) - Si(0)]. \end{aligned}$$

So that equation (3-43) is now

$$= -\frac{B}{2} \left(x^{-1} - \frac{\cos 2x}{x} \right) \Big|_0^{BR} + B [Si(2BR) - Si(0)] \quad (3-46)$$

$$= \frac{\cos 2BR - 1}{2R} - \lim_{r \rightarrow 0} \left(\frac{\cos 2Br - 1}{2r} \right) + B [Si(2BR) - Si(0)]. \quad (3-47)$$

Now using L'Hopital's rule

$$\lim_{r \rightarrow 0} \left(\frac{\cos 2Br - 1}{2r} \right) = \lim_{r \rightarrow 0} \frac{2B(-\sin 2Br)}{2} = 0.$$

Substituting the value of B, equation (3-43) becomes

$$\int_0^R \frac{\sin^2(\pi r/R)}{r^2} dr = B [Si(2\pi) - Si(0)]. \quad (3-48)$$

Now the last term in equation (3-41) can be rewritten

using the identity that $2\cos A \sin C = \sin(A+C) + \sin(A-C)$

$$\begin{aligned} -2B \int_0^R \frac{\cos Br \sin Br}{r} dr &= -B \int_0^R \frac{\sin 2Br}{r} dr, \quad (3-49) \\ &= -B \int_0^{2BR} \frac{\sin y}{y} dy, \text{ where } y = 2Br, \\ &= -B [Si(2BR) - Si(0)], \\ &= -B [Si(2\pi) - Si(0)]. \quad (3-50) \end{aligned}$$

Now combining equations (3-50), (3-48), (3-42) and (3-40); equation (3-39) becomes

$$\bar{w} = \frac{-3(K_{\infty}-1)v_b}{2\pi K_{\infty} R^4(1-f)} [R] = \frac{-3(K_{\infty}-1)v_b}{2\pi K_{\infty} R^3(1-f)}. \quad (3-51)$$

The same result can be achieved directly by writing K as a function of the bubble volume and finding the perturbation of K , (δK), that results from a perturbation of v_b .

An expression for $\bar{w}^2$ can be developed using the definition of $\bar{w}^2$ in the limit of thin spherical shells (See equation (3-24)) and the expression for $w(r)$, equation (3-36),

$$\bar{w}^2 = \frac{3}{R^3} \left[\frac{(1-K_{\infty})3v_b}{2\pi K_{\infty} R(1-f)} \right]^2 \int_0^R \left[\left(\frac{\sin Br}{r} \right)^2 + \frac{1}{B^2} \left(\frac{B \cos Br}{r} - \frac{\sin Br}{r^2} \right)^2 \right] r^2 dr. \quad (3-52)$$

Because of the complexity of this equation I will not attempt to express it in closed form.

3-4 Order of Magnitude of $(\Delta K)_{fm}$

In order to estimate the reactivity effect of fuel movement in a mass of molten fuel containing a uniform distribution of bubbles, I will select a model that will permit simple calculations using equation (3-25).

$$(\Delta K)_{fm} = \sqrt{(\bar{w}^2 - \bar{w}^2) N_T}, \quad (3-25)$$

where $\bar{w}^2$ = mean of squared reactivity worth per bubble

$\bar{w}^2$ = squared mean of reactivity worth per bubble

N_T = total number of bubbles in the mass of molten fuel.

I will use the same assumptions used in the computation

of $(\Delta K)_B$:

- 1) Spherical mass of bubbly molten fuel ($R=50$ cm)
- 2) Spherical bubbles all the same size (r).

The total number of bubbles (N_T) will equal the total volume of the bubbly fuel ($\frac{4}{3}\pi R^3$) times the void fraction (f) divided by the volume of a bubble. Since f is related to the volume ratio of bubbles to fuel (ϕ) by $f = \frac{\phi}{1+\phi}$ and since (ϕ) was used as a variable in the calculation of $(\Delta K)_B$, then N_T becomes

$$N_T = \left[\frac{R}{r} \right]^3 \frac{\phi}{1+\phi} \quad (3-53)$$

Then equation (3-25) becomes

$$(\Delta K)_{fm} = \sqrt{(\bar{w}^2 - \bar{w}^2) \left[\frac{R}{r} \right]^3 \frac{\phi}{1+\phi}} \quad (3-54)$$

Equations (3-51) and (3-52) show that $\bar{w}^2$ and $\bar{w}^2$ will be functions of: K_∞, r, R and ϕ . So that $(\Delta K)_{fm}$ will be a function of three variables since $R = 50$ cm, $(\Delta K)_{fm} = f(K_\infty, r, \phi)$. In order to establish the magnitude of $(\Delta K)_{fm}$ and its sensitivity to changes in each of the parameters, I will vary the parameters thru the following ranges:

$$\begin{array}{rclclcl} 1.1 & \leq & K_\infty & \leq & 2 & \\ .1 & \leq & \phi & \leq & 1 & \\ .25 \text{ cm} & \leq & r & \leq & 2.5 \text{ cm} & \end{array}$$

These are the same ranges used in the calculation of $(\Delta K)_B$ in section 2-5. Using equation (3-51) and the following values: $K_\infty = 1.1$, $\phi = .5$ and $r = .5$ cm,

I was able to compute a value for $\bar{w}$, 2.727×10^{-7} . As it will be necessary to evaluate equation (3-52) by numerical integration, I elected to prepare a program that would use the following trapezoidal integration formula to evaluate $\bar{w}$ and $\bar{w}^2$,

$$\bar{w} = \frac{4\pi}{V_T} \sum_{i=1}^{99} w(R_i) R_i^2 \Delta R + \frac{4\pi}{2V_T} (w(R_1) R_1^2 + w(R_{100}) R_{100}^2) \Delta R, \quad (3-55)$$

$$\bar{w}^2 = \frac{4\pi}{V_T} \sum_{i=1}^{99} w^2(R_i) R_i^2 \Delta R + \frac{4\pi}{2V_T} (w^2(R_1) R_1^2 + w^2(R_{100}) R_{100}^2) \Delta R. \quad (3-56)$$

Where

$V_T = \frac{4}{3}\pi R^3$ = volume of bubbly fuel mass.
 $w(R_1)$ = Reactivity worth of bubble at radius R_1 (from equation (3-36))

R_1 = Distance from center of mass of molten fuel to the bubble

ΔR = Shell thickness $\equiv (R/100)$

The computer calculated value of $\bar{w}$ for $K_\infty = 1.1$, $\phi = .5$, and $r = .5$ checked the hand calculated value, that was determined using equation (3-51). Since the computation of $\bar{w}$ was checked and since the computation of $\bar{w}^2$ only involves squaring the value of $w(R_1)$ prior to summation, one can be reasonably confident that the computed values of $(\Delta K)_{fm}$ are correct.

The results of the calculation of $(\Delta K)_{fm}$ are presented in Tables 3-1 to 3-4. As a further check on the validity of these values, one can substitute equations (3-51) and (3-52) into equation (3-54) to determine the general nature of the functional relationship between

ϕ	r	$(\Delta K)_{fm}^*$
1	2.5	.001896
	1.5	.000881
	.5	.000169
	.25	.000050
.5	2.5	.001161
	1.5	.000540
	.5	.000104
	.25	.000037
.25	2.5	.000749
	1.5	.000348
	.5	.000067
	.25	.000024
.1	2.5	.000444
	1.5	.000207
	.5	.000040
	.25	.000014

* For R = 50 cm .

TABLE 3-1. $(\Delta K)_{fm}$ For $K_{\infty} = 1.1$

ϕ	r	$(\Delta K)_{fm}^*$
1	2.5	.004812
	1.5	.002236
	.5	.000430
	.25	.000152
.5	2.5	.002946
	1.5	.001369
	.5	.000264
	.25	.000093
.25	2.5	.001902
	1.5	.000884
	.5	.000170
	.25	.000060
.10	2.5	.001128
	1.5	.000524
	.5	.000100
	.25	.000036

*For $R = 50$ cm

TABLE 3-2. $(\Delta K)_{fm}$ For $K_{\infty} = 1.3$

ϕ	r	$(\Delta K)_{frn}$ *
1	2.5	.006950
	1.5	.003230
	.5	.000622
	.25	.000220
.5	2.5	.004256
	1.5	.001978
	.5	.000381
	.25	.000135
.25	2.5	.002747
	1.5	.001277
	.5	.000246
	.25	.000087
.1	2.5	.001630
	1.5	.000758
	.5	.000146
	.25	.000052

* For R = 50 cm

TABLE 3-3. $(\Delta K)_{frn}$ for $K_{\infty} = 1.5$

ϕ	r	$(\Delta K)_{f_m}$ *
1	2.5	.010434
	1.5	.004845
	.5	.000932
	.25	.000330
.5	2.5	.006384
	1.5	.002967
	.5	.000571
	.25	.000202
.25	2.5	.004121
	1.5	.001915
	.5	.000369
	.25	.000130
.1	2.5	.002445
	1.5	.001136
	.5	.000219
	.25	.000077

* For R = 50 cm

TABLE 3-4. $(\Delta K)_{f_m}$ For $K_\infty = 2$

$(\Delta K)_{fm}$ and its three parameters,

$$(\Delta K)_{fm} \propto \sqrt{\left(\frac{1-K_{\infty}}{K_{\infty}}\right)^2 \phi (1+\phi) r^3} \quad (3-57)$$

Thus, increasing r from .25 cm to 2.5 cm will yield a factor of 30 increase in $(\Delta K)_{fm}$. Likewise, an increase of K_{∞} from 1.1 to 2.0 will increase $(\Delta K)_{fm}$ by a factor of about 5.5, and increasing ϕ from .1 to 1 will increase $(\Delta K)_{fm}$ by a factor of about 4. All of these variations produced the expected change in the computed values of $(\Delta K)_{fm}$.

Comparing the values of $(\Delta K)_{fm}$ to the values of $(\Delta K)_B$, one finds that both quantities have their relative maximum and minimum at the same combination of values for the parameters. Table 3-5 shows that the relative maximum of $(\Delta K)_B$ is 11 times larger than the relative maximum of $(\Delta K)_{fm}$, while the minimum value of $(\Delta K)_B$ is 18 times larger than

Condition	$(\Delta K)_B$	$(\Delta K)_{fm}$
Maximum * ($K_{\infty}=2, \phi=1,$ $r=2.5, \lambda=5$)	+0.117188	± 0.010434
Minimum * ($K_{\infty}=1.1, \phi=.1$ $r=.25, \lambda=2$)	+0.000252	± 0.000014

* For range of parameters considered.

TABLE 3-5 Range of $(\Delta K)_B$ and $(\Delta K)_{fm}$.

the minimum value of $(\Delta K)_{fm}$. Choosing the same set of values for the parameters (K_{∞}, ϕ, r) , that were used in Section 2-5 to arrive at a "reasonable" value of $(\Delta K)_B$, one finds that the value of $(\Delta K)_B$, $-.004808$, is 18 times larger than the corresponding value of $(\Delta K)_{fm}$, ± 0.000264 . Thus, this value of $(\Delta K)_{fm}$ is only a factor of ten smaller than the value of $(\Delta K)_{Dop}$ calculated in section 2-5.

The relative magnitude of these numbers suggest that the reactivity effect of fuel movement in a molten fuel containing a uniform bubble distribution will be of the order of $\pm 10\%$ of the Doppler feedback. While this reactivity is far from insignificant, it is not clear that it is an effect that will occur at the same time that $(\Delta K)_{Dop}$ and $(\Delta K)_B$ are influencing the power transient. Certainly, inertia effects would lengthen the time until $(\Delta K)_{fm}$ become important in a fuel where a nonuniform bubble distribution would require fuel to move long distances to achieve perfect uniformity. However, in a fuel with uniform bubble distribution the fuel would only need to move small distances. Thus, it is possible that $(\Delta K)_{fm}$ could be effective in the same time frame as $(\Delta K)_B$ and $(\Delta K)_{Dop}$.

IV. EFFECT OF $(\Delta K)_B$ AND $(\Delta K)_{fm}$ ON A POWER EXCURSION

It is almost self evident, that a reactivity increase of as much as 0.01 inserted promptly, due to fuel expansion imploding bubbles, would significantly alter the course of a hypothetical power excursion in which the Doppler Effect is assumed to mitigate the energy release. In this Thesis $(\Delta K)_{fm}$ is shown to be about 5 to 10% of $(\Delta K)_B$ and it is postulated to be effective at the same time as $(\Delta K)_B$. Thus one can absorb $(\Delta K)_{fm}$ into $(\Delta K)_B$ and consider $(\Delta K)_B$ to be the total reactivity effect associated with bubble implosion. However, one must recognize that $(\Delta K)_{fm}$ could be a positive or negative reactivity effect depending on the actual statistical departure of the bubble density from the uniform value. Thus $(\Delta K)_{fm}$ gives the magnitude of the standard deviation associated with $(\Delta K)_B$. The values of $(\Delta K)_B$ calculated in this thesis range from 0.117188 to 0.000252 depending on the choice of the values of these four parameters; K_∞ , ϕ , r , λ . Selection of more or less conservative values of these parameters suggest that $(\Delta K)_B$ will be +.01 to +.02.

4.1 Van Diemen and Nicholson Calculational Model

Paul Van Diemen and R. B. Nicholson have done some calculations to determine the extent of the effect of positive feedback, due to bubble implosion, on the course of a power excursion. (24.) They modified the VENUS-II code to

incorporate a model for calculation of $(\Delta K)_B$. This reactivity feedback is then added to the other VENUS-II feedback terms. The calculations were done assuming a cylindrical mass of molten UO_2 52 cm high and with a diameter of 52 cm. Also, they assumed $f = 10\%$ ($\phi = 11.1\%$) and $f = 30\%$ ($\phi = 42.9\%$). The local reactivity change due to bubble implosion, δk , was assumed to be proportional to the change in local void fraction, δf , which was calculated from the expansion of the liquid as the fuel temperature increased.

$$\delta K = A_{\text{VOID}} \cdot \text{SSF} \cdot \delta f. \quad (4-1)$$

In this equation A_{VOID} is the proportionality constant and SSF is defined to be the space shape function. In their model, bubble implosion is represented by a perturbation of the diffusion coefficient. This means that the δk will be a function of the spatial variables (r, z) that has a minimum at the center of the fuel and a maximum near the edge. In order to give δk the correct spatial dependence they introduced an arbitrary function (SSF) that has the correct shape. SSF has a zero at $(0,0)$ and a maximum at the surface of the cylinder. While they did not attempt to specify what shape function should be used, they did demonstrate that the shape function was an important parameter. The initial fuel temperature was taken to be the melting point of UO_2 and all fuel was assumed to be liquid. The initial power was assumed to be one megawatt. The Doppler Feedback Coefficient was $T\delta k/dT = -0.002$.

Van Diemen and Nicholson did not perform calculations for specific; bubble sizes (r), K_{∞} , and λ . Rather they chose arbitrary values for the reactivity effect for total bubble implosion. For the case of $\phi = 11.1\%$ they selected $(\Delta K)_B$ equal to 0.007 and 0.0021. From Table 2-2 one can see that for $K_{\infty} = 1.3$, $\phi = .1$ and $\lambda = 4$ cm these values approximately correspond to bubbles of radii 1.5 cm and .5 respectively. For the case of $\phi = 42.9\%$ they selected $(\Delta K)_B$ equal to 0.021 and 0.0063. Again these values approximately correspond to bubbles of radii 1.5 cm and .5 cm respectively. Thus their choices of $(\Delta K)_B$ span the range of bubble sizes from what might be considered reasonable sizes to pessimistic sizes.

The results of their calculations are presented in Table 4-1. This table also gives the results of calculations for fuel containing homogenized voids for reference.

TABLE 4-1 Effect of $(\Delta K)_B$ on Power Excursions

$(\Delta K)_B$	$\phi\%$	Excursion Energy Megajoules	Final Central Temperature($^{\circ}\text{K}$)
0	11.1	320	5643
.0021	11.1	390	6349
.007	11.1	650	9836
0	42.9	530	9208
.0063	42.9	690	12,407
.021	42.9	2300	41,983

These results indicate that $(\Delta K)_B$ has offset the $(\Delta K)_{Dop}$ resulting in an increased energy release. The results of this table can not be taken as representative of other accident conditions; i.e., different initial conditions and different insertion rates. Also, they are not necessarily conservative results. In fact, there are two assumptions in the bubble implosion modification to VENUS-II that tend to reduce the influence of the bubble implosion. First, the assumed space shape function is too small in the interior regions where the bubble implosion is greatest. Secondly, the code employs an implicit coefficient of thermal expansion that is probably too small and so gives less bubble implosion than would actually occur. Never the less, the data in Table 4-1 suggests that the reactivity effect due to bubble implosion is an important effect that must be studied in greater detail utilizing more sophisticated models and computational techniques.

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Part Six

Evaluation of Models in the SAS2A
Fast Reactor Analysis Code

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I. INTRODUCTION

A number of accident analysis codes have been developed in recent years to examine the course and consequences of various hypothesized accidents in fast reactors. One of the most sophisticated series of such codes is the SAS series. The first code in the series was SAS1A which was developed in the late 1960's. It modelled a single fuel pin and associated coolant during the initial phases of a transient due to reactivity insertion or flow coastdown.

Because of limitations on the capabilities of computers and on the state of knowledge concerning many of the phenomena relevant to fast reactor accident studies, SAS1A was rather incomplete. Some important phenomena were not treated, and models for others were crude. Work on correcting the defects in SAS1A eventually led to the development of a more sophisticated version, SAS2A. This version was much more complex than the original, and it made much greater demands in terms of computer storage requirements. This report is an evaluation of the adequacy of the models employed in the SAS2A code.

SAS2A is a multi-channel, thermal-hydraulic code for analysis of the initiation and early phases of accidents caused either by a reactivity insertion or by flow coastdown. The code treats up to ten channels; each channel consists of one characteristic fuel pin and its associated coolant and structure. Hence, each channel actually corresponds to one

or more subassemblies in which events are assumed to occur coherently, e.g., voiding in one channel corresponds to voiding across the entire group of subassemblies represented by the channel. The sole exception to this is the fission gas release treatment in which incoherent release in a channel is permitted.

The bulk of this report consists of a comparison of models employed in SAS2A with models that have appeared in the literature since the development of the SAS2A code. An examination of the actual Fortran programming was also carried out, and some of the statements that appear to be incorrect or inconsistent with models employed in the code development are listed. This procedure was necessary because the size and complexity of the code make it impractical to study the code by running it on the Ohio State University computer system.

It should be mentioned, however, that this report is necessarily incomplete because SAS2A itself is not a finished product. The Reactor Analysis and Safety Division of Argonne National Laboratory is currently working on more advanced versions of this code containing greater sophistication in the treatment of many models and also new models not included in earlier versions of the code. Information on much of this developmental work is not available. This is particularly true with regard to new models now being developed.

II. EVALUATION OF SAS2A MODELS

2.1 Material Properties

An important influence on the outcome of a SAS2A calculation is the set of material properties used as data input. These material properties represent the reactor being modelled in terms of geometry and materials. Obviously, those factors concerned with geometry are strictly a matter of user choice. The property treatment, however, must conform to known physical data for the conditions being modelled by the code.

One important group of properties in this respect are fuel properties. For the most part, they are adequately treated in SAS2A. The code allows up to five different fuel types to reflect the effects of burnup, restructuring, blanket composition, etc. Major fuel properties such as thermal conductivity, specific heat, and density are entered as tables of values versus corresponding temperatures for each fuel type. The code then interpolates in these tables. Other fuel properties such as coefficients of linear expansion and Young's modulus are constants in the input.

Cladding properties are also important for the accuracy of the calculation. The SAS2A treatment is less adequate for cladding properties. The clad yield point and Young's modulus are entered as tables, but most of the other clad properties are entered as constants. The constants are to be chosen as average values over the expected transient. It may be reasonable

to expect the user to have a good idea as to what conditions will prevail during the expected transient, but a more accurate treatment should consider the fact that cladding properties can change significantly over the course of a transient because of temperature changes in the cladding. For example, the thermal conductivity of AISI-347 stainless steel increases by 50% over the temperature range from 280°C to 850°C. The specific heat increases by approximately 20% over this temperature range. Type 316 stainless steel is the most commonly mentioned steel for cladding for fast reactors. The data presented above indicate that significant changes in the properties of steels can occur in the temperature range of interest for fast reactor safety.

A simple procedure for improving the accuracy of cladding treatment is to employ a polynomial to represent a cladding property such as thermal conductivity or specific heat. Two or three coefficients could be entered as data input to give a good quadratic representation of cladding properties without any noticeable increase in storage requirements or computational effort.

Sodium properties employed in SAS2A are taken from the work of Golden and Tokar (1). They represent curve fits to empirical data for saturation properties at low and intermediate temperatures. Some dissatisfaction with these particular fits exists, but they are generally accepted as adequate for low and intermediate temperatures, and they are used in a number

of reactor codes. In any event, it would not be difficult to change the values of the coefficients either to obtain new sodium property correlations or to get values for other liquid metals provided the same functional form is retained.

Erdman, Garner, and Reynolds (2) have pointed out, however, that extrapolation of these low temperature properties to high temperature conditions is questionable. For SAS2A, this is not really important because SAS2A does not have a molten fuel-coolant interaction model. Such a model has been included in the most recent version of the code, however, and the issue assumes greater importance since very high sodium temperatures do prevail during a molten fuel-coolant interaction. No information is available concerning any possible alterations in sodium properties for use in the molten fuel-coolant interaction.

The objections to the use of saturation properties stem from the behavior of the thermal expansion coefficient and the isothermal compressibility. At temperatures well below the critical temperature, the use of saturation properties involves only small errors. At the critical point, the thermal expansion coefficient and the isothermal compressibility become infinite. Values for these two properties at high temperatures quickly decrease below saturation values as one proceeds along isotherms into the subcooled region. Erdman, Garner, and Reynolds (2) present results of calculations of pressure and work when different temperature fits for the thermal expansion coefficient were used in a model assuming saturation properties for the

subcooled liquid. They found that work was greater and pressure remained higher when the fit predicting higher values of saturated thermal expansion coefficient was used. These authors remark that use of unrealistically high values for subcooled properties because of the use of saturated liquid values may significantly overestimate the mechanical work done during the single-phase liquid period. Subsequently, another author (3) stressed the importance of using the correct set of sodium properties in the high temperature region and presented a set of properties for this region. Other areas of uncertainty in sodium treatment are in the degree of superheat attainable and the thickness of the film left on the wall during voiding. These matters are discussed in a later section of this report.

Another material of some importance is the fission gas in the fuel pins. In high burnup pins, accumulated fission gas results in high pressures which can contribute to cladding failure. Once the cladding fails, the fission gas can be forced out into the coolant, creating a void and possibly propagating the failure to an adjacent pin. These matters are considered in a later section. Here, however, certain material properties of the fission gas are considered. The fission gas is assumed to obey the perfect gas law in the code, and pressures are determined from that relationship. From considerations of thermodynamics, this procedure is not entirely valid. The perfect gas law is really only valid for low pressures. In the

limit as the pressure goes to zero, the compressibility factor approaches unity, and the perfect gas law becomes a better approximation. At the high pressures prevalent in a fuel pin, the use of the perfect gas law is questionable. A more accurate treatment of fission gas effects would require that another equation of state be used for fission gas at high temperatures and pressures. This might cause problems, however, because of the limits on the current state of knowledge concerning fission gas composition, distribution, and release.

Properties such as the thermal resistance and the product of density and specific heat for fission gas are entered as input constants for each channel. Due to the lack of available data, the adequacy of this procedure cannot be evaluated here, but the matter of the equation of state is probably of more consequence than the variation in other factors.

2.2. Fuel and Cladding Heat Transfer

The transfer of heat in the fuel and cladding involves basic assumptions concerning the direction of heat flow. The code assumes that axial and angular heat conduction are negligible in comparison with radial heat conduction. Neglecting axial heat conduction is a reasonable assumption because the radial temperature gradient is much greater than the axial temperature gradient, and the heat transfer area is much larger in the radial direction. For example, in one calculation, the average fuel temperature varied axially from 316°C at the bottom to 966°C at a position 68 cm up in the core during steady state. The

corresponding radial variation was from 1555°C at $r = 0$ to 523°C at $r = 0.26$ cm. Clearly, the radial temperature gradient will be much more important than the axial temperature gradient.

From a computational standpoint, treating only the radial dependence leads to considerable simplification. If axial and/or angular conduction and temperature distributions were considered in each finite difference equation, core storage problems would be much worse, and the time required to compute fuel and cladding temperatures would increase drastically. The method employed to solve the system of equations would either have to be modified or scrapped because the new system of equations could only be considered tridiagonal in terms of block matrices.

Neglecting the angular temperature distribution has less justification. The computational justification is again the simplification that results. Physically, the validity of this assumption depends on the quantity P/D , the pitch-to-diameter ratio of the fuel pins. As P/D increases, the assumption that angular effects are small becomes better. For very small P/D , i.e., very near unity, angular effects can be large enough to be significant. For P/D above approximately 1.2, angular effects are small enough that they can be neglected without giving rise to any substantial error. This will be discussed further in a later section. A factor that tends to make angular effects less important is the presence of the wire wrap on the fuel pins. The wire wrap increases coolant mixing,

and the increased mixing should decrease the angular temperature variation.

Another effect that is not treated in SAS2A is the possible asymmetry in a fuel pellet. If because of defects in manufacture or some other reason, a fuel pellet should somehow become slightly displaced from the central axis of the fuel rod, the local temperature distribution will change during steady state and during a transient. A local hot spot can occur. This localized effect might be large enough to become significant since the locally higher temperature creates the equivalent of a preferential site for cladding failure. In the present stage of development of SAS2A, this effect is too fine to be treated by the code.

The basic conduction equation for heat transfer in the fuel rod and cladding is the following standard form:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(k(r, T) r \frac{\partial T}{\partial r} \right) + Q(r, t) = C_p(r, T) \frac{\partial T}{\partial t}$$

where $Q(r, t)$ is the heat source. The finite differencing procedure applied to this equation in the code appears to be entirely correct. The Crank-Nicholson scheme employed in the code is one of the more accurate methods for dealing with transient heat conduction problems. For the fuel, the temperature variation of the thermal properties is accounted for, and this makes the equations nonlinear. The code linearizes the equations by obtaining an extrapolated temperature to be used

for obtaining material properties. This approach seems to be simple but accurate, and it has the advantage of not introducing further complications into the program. As mentioned above, the thermal properties for the cladding are treated as constants. The code accounts correctly for the heat source due to fission energy release and for the distribution of this energy among the nodes. Sinks for heat such as the latent heat of melting are also accounted for in the code.

The treatment of the gap between the fuel and the cladding introduces additional approximations. Heat transfer across the gap is allowed either by radiation or by a bond conductance that functions essentially as a convective heat transfer term. SAS2A appears to treat the emissivity in the radiation term as a constant. In fact, emissivity is a temperature dependent quantity. The emissivity for the radiation emitted from the outer fuel surface is not necessarily the same as the absorptivity (or emissivity) for radiation from the inner clad surface, and neither value is necessarily constant as time and temperature change. If the radiation term is much smaller than that due to bond conductance, then this effect is unimportant. If circumstances should arise such that the two heat transfer mechanisms are of approximately equal magnitude, then this effect needs to be treated in some manner.

Very little information is available concerning the bond conductance term. The bond conductance parameter can account

for contact pressure between the fuel and cladding. No other information is supplied in the user's manual and none is to be found in available references.

The treatment of cladding temperatures is quite adequate except for the assumed constancy of cladding thermal properties.

The final matter in this section is the heat transfer between the cladding surface and the coolant. The code uses the standard form of Newton's law of cooling. It has been suggested by Minkowycz and Stein (5) that this is not always accurate because the heat flux, surface temperature, and coolant bulk temperature used to define the heat transfer coefficient h are unknown functions of time and position. A number of alternative formulations have been derived, but none of the alternatives represents a sufficient improvement over the standard form to justify the additional effort and complications involved in their use. This is especially true of the formulation developed by Stein (6) which he maintains is more accurate than Newton's law of cooling. Unfortunately, Stein's form is not only rather complex for use in SAS2A but also runs into convergence problems in certain cases. The form actually employed in the code to obtain h is $hD_h/k_c = A(RePr)^B + C$ where Re is the Reynolds number, Pr is the Prandtl number, k_c is the thermal conductivity of the coolant, D_h is the hydraulic diameter, and A , B , and C are input constants representing a particular empirical correlation for the heat transfer coefficient. It is important that care be used in

choosing a correlation because different correlations sometimes yield values for h that differ significantly.

2.3 Heat Transfer to Structure, Reflectors, and Gas Plenum

The preceeding section examined the detailed heat transfer calculations that are carried out for the core and blanket regions. Although only radial heat conduction is treated, axial effects are included because the coolant couples axial locations.

In contrast, the treatment of heat transfer to the structure, reflectors, and gas plenum is much less detailed and much less complex than heat transfer in the core. Since no axial conduction is considered, convection from the coolant is the only heat transfer mode in the reflectors and gas plenum, and convection and limited direct heating by neutrons and gammas are the only effects treated in the structure. This approximation is valid because direct heating will be very small in the reflectors and gas plenum. The reasons for neglecting axial conduction have been given above. Only a rough calculation is made for heat transfer in the structure, reflectors, and gas plenum. One node is used for each axial location in the structure; one is used in the clad in the gas plenum and reflector regions; and one is used in the reflector. Further, the axial mesh is much less refined in these regions. This results in a calculation that is mainly intended with obtaining a proper heat balance. Detailed information about

temperatures in these regions is unnecessary. This rough treatment does require, however, that the thermal constants chosen to characterize these regions be chosen carefully. This will be discussed later in this section.

The basic equation for heat transfer to the structure is the following:

$$V_s \rho_s \frac{\partial T_s}{\partial t} = h A_s (T_b - T_s) + Q'$$

where the s refers to structure and T_b is the coolant bulk temperature. Q' is the direct heating of the structure. This is a simple heat balance equation to correctly account for the coolant temperature change due to heat flow to the structure. The equation and its finite difference formulation in the code are correct within the limitations imposed by use of a single node and the elimination of axial conduction.

The equations for the reflector are equally simple. They are both conventional forms. For the reflector, the node temperature change is equated to the rate of heat flow from the clad node to the inner reflector node. For the reflector cladding, the change in cladding temperature is the sum of the effects due to heat transfer in from the coolant and heat transfer out to the inner reflector node. The finite differencing appears to be carried out correctly for both equations pertaining to the reflector. SAS2A does allow for detailed treatment of some aspects of the reflector region.

The user can specify the hydraulic diameters in the reflector independently of core and blanket values. The ratio of clad surface area to coolant volume can also be specified independently for each reflector. The thermal characteristics in the reflector can also be specified by the user. As a result, the SAS2A model allows considerable freedom in modelling various reflector designs.

The equations for heat transfer to the gas plenum are exactly the same as the equations for heat transfer to the reflectors in form. The only difference is that thermal constants for the gas plenum are substituted for the thermal constants for the reflector. The treatment of these equations appears to be correct. The finite differencing is correct. The code calculates an average clad temperature and an average gas temperature for the gas plenum region. Both calculations are consistent with standard averaging procedure.

The calculation in the structure, reflector, and plenum regions will not give detailed temperature information in these regions because of the roughness of the treatment, but it does account correctly for the heat that would enter these regions from the coolant. Within the limitations imposed by the roughness of the model, the treatment is correct.

One matter that is important for the accuracy of these calculations is the choice of characteristic thermal constants for these regions. These constants depend principally on the time scale for temperature changes in the coolant. For

very fast changes in coolant temperature, there may not be time for temperature effects to penetrate to the center of the structure or reflector. It is important, then, that the time scale be estimated carefully so that the thermal constants chosen for these regions will be correct. Justification for the forms used to determine the various thermal constants for the structure, reflectors, and gas plenum is presented in Appendix I of the user's manual.

2.4 Point Kinetics

The power-time history of the reactor being modelled is determined using the standard point kinetics equations. These equations are popular in dynamics calculations because they are much easier to solve than the more exact forms, and they are used frequently in reactor codes. SAS2A solves these equations by using the Kaganove method (8). This method is actually a variation on weighted residual methods and is employed in a number of codes where the point kinetics equations must be solved.

Point kinetics equations are much easier to solve than space-dependent formulations, but significant differences sometimes exist between results calculated from point kinetics and results obtained with the more accurate space-energy dependent kinetics. The limited accuracy of the point kinetics formulation arises because point kinetics does not account for spatial effects. Flux level changes due to spectral

hardening, spatial distortion, etc., cannot be treated in point kinetics formulations, and this limits the accuracy of point kinetics calculations.

One calculation that demonstrates the difference between point kinetics and space-dependent kinetics was performed by Bezella and Ott (7). They present results for a synthesized space-time (SST) and consistent point kinetics (CPK) calculation. Both forms were based on adjoint-weighted and volume-integrated kinetics parameters, but neutron shape recalculations were suppressed in the CPK calculation. During the voiding phase, reentry of coolant was not considered, and the calculation was continued until the core was fully voided. Molten fuel-coolant interactions were also neglected. Results of that calculation are presented in Table 1 on the following page.

TABLE 1

COMPARISON OF MODEL RESULTS FOR HYPOTHETICAL RAPID FLOW COASTDOWN
ACCIDENT IN A 1000 MW(e) LMFBR (Ref. 7)

Reactor Conditions	SST	CPK
At Sodium Void Inception		
Void inception time (sec)	1.609	1.619
Flux amplitude	2.219	2.087
Total net reactivity (dollars)	0.440	0.406
Core average fuel temperature ($^{\circ}\text{C}$)	1456	1445
During Sodium Voiding		
Peak flux amplitude	118	109
Number of times at prompt critical	3	1
Fuel melt inception time (sec)	1.650	1.667
Channel void initiation time (sec)		
Channel 2	1.662	1.673
Channel 3	1.685	1.701
Channel 4	1.649	1.658
Channel 5	1.732	1.800
When Core is Entirely Voided		
Time core is voided (sec)	1.785	1.885
Total energy generation (MW-sec)	20,074	14,515
Core fuel melted (%)	55.7	28.5
Flux amplitude	2.00	1.222
Core average fuel temperature ($^{\circ}\text{C}$)	2,491	2,192

Bezella and Ott remark that shape changes were due predominantly to spectral hardening with a minimum of spatial distortion in this case. The channel voiding rate in the SST case was due to self-pressurization effects in voided regions caused by hotter fuel rods. They also suggest that point kinetics may provide acceptable results for this particular accident.

While it is true that conditions for the two cases match rather well during the first two phases listed above, there is significant deviation when the core is entirely voided. The total energy generated, the per cent of fuel melted, the flux amplitude, and the average fuel temperature all show significant differences for the two calculations.

Smith and Ott (9) also performed a calculation comparing point kinetics with space-energy dependent kinetics. Table 2 below shows the results from this calculation.

TABLE 2

ENERGY RELEASE FOR POINT KINETICS AND SPACE KINETICS MODELS

Initial Maximum Temperature ($^{\circ}\text{K}$)

	3900 (.76\$)		3825 (.93\$)		3725 (1.02\$)	
	Point	Space	Point	Space	Point	Space
Q_{tot} (MW-sec)	1443	2712	2509	3107	3395	3687
Absolute difference (MW-sec)	1269		598		292	
Relative error of P.K. (%)	-47		-19		-8	
Relative increase of S.K. (%)	88		24		9	
Q_B (MW-sec)	66	414	309	529	564	687
Absolute difference (MW-sec)	348		220		123	
Relative error of P.K. (%)	-84		-42		-18	
Relative increase of S.K. (%)	527		71		22	

Q_{tot} is the total energy release, and Q_B is the internal energy of the fuel above the vaporization point. Smith and Ott used the two-dimensional safety analysis code QX2-S for this calculation. The computation was performed for a dry core disassembly with a uniform reactivity insertion rate of \$50/sec. The reactivity was inserted into a model of a 750 MW(th) fast reactor core. The initial flux level was ten times the level at normal power. For the point reactor calculations, flux shape recalculations were suppressed. For smaller energy releases, the differences were more severe; for larger energy releases, the error involved in using point kinetics was small enough that point kinetics would yield acceptable results for the accident calculated. The reactivity in Table 2 is the reactivity at the onset of disassembly.

While the calculations cited above are open to question on several grounds, they do illustrate the discrepancies that can appear when the same accident is calculated using both point kinetics and space kinetics. In the case of the SAS2A code, use of point kinetics is almost mandatory because of storage requirements. More elaborate space-time schemes require large amounts of computer storage and large execution times. SAS2A encounters difficulties in both of these areas in its present form, so it would probably not be practical to change the kinetics module to a space-time formulation. However, since this is the case, the possible inaccuracies of the point kinetics calculation must be kept

in mind when considering SAS2A computational results.

Further, it might be of value to conduct a systematic series of comparison calculations to estimate and bound the error involved in using point kinetics, particularly for the more severe accidents for which SAS2A and related codes are used.

SAS2A solves the point kinetics equations using Kaganove's method. The subroutine involved is TSPK. This subroutine follows Reference 8 closely in its formulation and solution of the equations. The reactivity calculations and the summation are conventional in form, and all major sources of reactivity are accounted for in the program except bowing of fuel rods. The theoretical justification for neglecting bowing is not explained in available reference material. It may be that this contribution is small for the types of cores modelled in the program and for the range of accidents considered. From a practical standpoint, neglecting bowing simplifies the calculations because the geometrical distortion involved in bowing would be difficult to treat accurately in SAS2A.

Within the limitations imposed by the use of point kinetics, the method of apportioning calculated power among the nodes is correct.

If the user desires, he can bypass the point kinetics calculations entirely and put in a desired power history for the transient. This can be useful, but it requires that the user know the power-time history in advance. Finally, since all reactivity coefficients employed in the code are user

input, the method used to obtain these coefficients is important, especially when comparisons of calculations are being made.

2.5 Liquid Sodium Flow in Coolant Channels

The flow of liquid sodium is treated in detail in the code, but this treatment is based on several assumptions that simplify the problem. The first of these is that the liquid is assumed to be incompressible, i.e., the mass flow rate in any particular liquid slug is constant. This assumption is justifiable. Under the physical conditions that prevail during normal operation and during those accidents for which SAS2A was intended, any deviation from a constant mass flow rate would be very small. This assumption of incompressibility is common in reactor codes because it is nearly correct and because the incompressible equations are easier to treat than compressible flow equations.

An assumption of greater importance for the code is that turbulent flow of the liquid sodium can be represented by a slug flow regime. This assumption is quite valuable from a programming standpoint, but it must be justified physically. For the program, the assumption of slug flow reduces the storage and the computational effort significantly. If SAS2A attempted to obtain a detailed turbulent velocity distribution in the coolant, a large number of nodes would be required for each axial coolant node location. The solution in the $r-\theta$

directions would probably require between twenty-five and fifty storage locations for necessary coolant nodes at each axial location. For a ten channel problem, this would amount to an array of 7,500 to 15,000 storage locations, and this would be intolerable in a program that has serious storage problems in its present form. Also, the effort required to solve the momentum and continuity equations for the turbulent velocity distribution at each time step would drastically increase execution time for the program.

From a physical standpoint, there is some justification for the slug flow assumption. The most important factor governing the validity of this assumption is P/D , the pitch-to-diameter ratio. For P/D very close to unity, the slug flow assumption becomes rather inadequate. This is discussed in more detail in Reference 12. This reference also presents the results of calculations of heat transfer in tightly packed rod bundles. These results indicate that for P/D near unity, the assumptions of slug flow and uniform circumferential temperature and heat flux are not valid. However, for P/D above about 1.1, the assumption of slug flow is a reasonable representation of the turbulent velocity distribution. Pfann (14) presents results of a study of heat transfer in turbulent longitudinal flow in unbaffled fuel assemblies. His charts and figures are of particular interest. For P/D near unity, they provide a rough estimate of the accuracy of assuming slug flow. They show that slug flow becomes a

better assumption as P/D increases. For example, for P/D of 1.1, Pfann's normalized peak velocity is 1.217 in the center, and the normalized velocity remains above 0.8 over nearly the entire coolant channel. It drops below this value only very near the wall. The contour plots illustrate very well the validity of the slug flow assumption as P/D increases. The results obtained by Pfann and by Marchese indicate that slug flow is a good approximation for P/D above about 1.1, but it breaks down when P/D is very near unity.

A factor that influences the validity of the slug flow assumption is the presence of the wire wrap. This wire wrap increases mixing, and the mixing tends to reduce circumferential effects and improve the slug flow assumption. However, very little has actually been done quantitatively about including the wire wrap in any sort of flow calculations. Novenstern (15) has developed equations for predicting pressure drop in fuel assemblies with wire wrap spacer systems. Also, computer programs have been developed to calculate the mixing motion due to the wire wrap, but in general, little of value has been published concerning other aspects of the presence of the wire wrap in the system. The most useful information needed on the effects of the wrap concerns the possibility that the wire may cause small local hot spots. An accurate estimate of the magnitude of such hot spots would be useful in SAS2A for determining locations of sodium boiling initiation.

Another major assumption of the code is that coolant channels remain essentially intact until a disassembly calculation is initiated. Provided the disassembly calculation is initiated early enough, this assumption is probably adequate. During the early and intermediate stages of the transient, the effects of swelling, thermal expansion, etc. should be small. This assumption also simplifies calculations in SAS2A. The treatment of coolant channels may have been modified recently. The most recent version of the codes in the SAS series contains a fuel-coolant interaction model and a model for treating motion of molten clad. No details of the models are available, so it is not possible to determine what modifications in the treatment of coolant channels have been made.

The equations of continuity and momentum employed in SAS2A are the standard forms given by Bird, Stewart, and Lightfoot (10). These equations have been modified to account for the z-directional slug flow characteristics assumed in the code. Also, the divergence of the shear stress tensor, $\nabla \cdot \underline{\tau}$, has been replaced by an empirical friction pressure gradient term, $\left(\frac{\partial p}{\partial z} \right)_{fr}$. This is common practice because the friction pressure term is much easier to work with than the shear stress tensor.

As mentioned earlier in this report, the temperature dependence of sodium properties is treated adequately. A

matter of some importance is the form chosen for the friction pressure gradient and for $(\Delta p)_{\text{end}}$ which accounts for pressure effects at the top and the bottom of the coolant channel due to flow expansion, contraction, or blockage. The expression for $(\Delta p)_{\text{end}}$ is similar to forms found in standard fluid mechanics texts. Similarly, the expression for the friction pressure gradient is

$$\left(\frac{\partial p}{\partial z}\right)_{\text{fr}} = A Re^B G|G|/2\rho_e D_h$$

and this is similar to standard expressions. A and B are input constants. SAS2A allows up to four sets, one each for liquid slugs, liquid film, vapor, and the primary loop.

Within the limitations imposed by the incompressibility and slug flow assumptions, the treatment of the momentum equation appears to be correct. The steady state treatment is entirely correct. The transient treatment appears to be correct, but this is much more difficult to determine because the transient coolant calculations are done in a group of interconnected subroutines that calculate liquid flow, fission gas release, voiding, and bubble motion. It is almost impossible to go through this section of the code without more information than was available. The dictionary of Fortran variables included in the new user's manual might have made it possible, but that arrived too late to be of any use.

The treatment of interface motion involves one additional assumption. This is that when a liquid slug reenters a voided region, the liquid film thickness is restored to its initial value. The sodium for this restoration must be taken from the liquid slugs. The shrinkage is accounted for in the code by adjusting the interface velocity between the bubble and the slug. For the top and bottom slugs, this shrinkage is neglected. This is probably a good approximation because shrinkage in these slugs would be small in comparison with the amount of sodium in these slugs. These slugs include the sodium above and below the core region. The effect is really only important for intermediate slugs that are small.

Adjusting interface velocities in these intermediate slugs is a simple way of accounting for shrinking, but it carries the assumption that all of the sodium restoring the film thickness comes from the leading edge of the slug. After the interface velocities have been determined, the code computes interface positions. The treatment given here is for SAS2A. The newest version has a more sophisticated treatment of the sodium film, and no information available here gives any idea how film thickness is restored in dried out regions in the model.

The pressure calculation appears to be generally correct. The equation used is consistent with the momentum equation, and the form used for the pressure at the upper end of the upper liquid slug conforms to that given in standard fluid mechanics texts.

SAS2A accounts for sodium in the upper and lower plenums by adding inertial lengths to the upper and lower slugs. These inertial lengths are entered in the code as input constants. Chawla (11) has examined the inertial contribution of the fluid in the plenum to coolant expulsion in single subassembly accidents. By using potential flow theory, he derived an expression for the pressure distribution in the plenum for coolant expulsion. He maintains that a fixed pressure boundary condition may only be valid for slow transients. This matter is relevant to SAS2A because the pressures in the upper plenum will be propagated down through the coolant channel in the sense that they are used to determine the pressure distribution in the coolant channel.

The treatment of liquid sodium flow is generally adequate, but there are uncertainties that deserve further investigation. The pressure distribution in the plenum is one of these since the coolant pressure distribution is rather important. Another of these uncertainties is the influence of the wire wrap. The possible presence of hot spots due to the wire wrap should be investigated because these hot spots can become bubble nucleation sites.

2.6 Liquid Coolant Temperatures

The code uses the following energy equation for liquid slugs:

$$\frac{\partial}{\partial t} (\rho H) + \frac{\partial}{\partial z} (G H) = \gamma \phi(z, t) + Q_c(z, t)$$

where H is enthalpy, G is the mass flow rate, Q_c is the volume heating source due to neutrons and gammas, ϕ is the heat flux from the cladding and structure, and γ is the ratio of clad surface area to coolant volume. Bird, Stewart, and Lightfoot (10) give a slightly different equation for energy conservation in a fluid. Their equation is:

$$\frac{\partial}{\partial t}(\rho H) + \nabla \cdot \rho H \vec{v} = -\nabla \cdot \vec{q} - \underline{\tau} : \nabla \vec{v} + \frac{Dp}{Dt}$$

The left side of the first equation is the one-dimensional equivalent of the second. The first term on the right of each equation corresponds to the heat flux. The remaining terms differ. The Q_c in the SAS equation represents a heat source not considered by Bird, Stewart, and Lightfoot. The remaining terms in the second equation represent energy changes due to viscous dissipation and pressure forces. Both of these terms will be much smaller than the contribution from the heat flux. Viscous dissipation is often neglected in energy balances as being of only minor importance. Sodium has a comparatively low viscosity, so viscous dissipation should be a small effect. Because of the large magnitude of the heat flux and the fact that the liquid is incompressible, the pressure term should also be much smaller than the heat flux term. The equation used in SAS2A appears to be entirely adequate because the terms neglected should be much smaller than the magnitude of the heat flux.

The treatment of the heat flux term is correct in the code. Heat fluxes from the clad and structure are simple standard forms. The proper choice of a correlation for the heat transfer coefficient is particularly important here. An inappropriate correlation may give heat transfer calculations having greater error than if a correct correlation were used with less refined treatment.

The direct heating of the coolant is also treated correctly. The method of apportioning power among coolant nodes is consistent with that employed in the rest of the code. No direct heating is allowed in the gas plenum and reflector regions because neutron and gamma heating in these regions of low power and low density will be much smaller than heating of fuel, cladding, structure, and coolant in the core region.

The steady state coolant temperature calculation is relatively simple and straightforward. All of the power is put into the coolant in steady state, and dT/dt is zero. This allows the energy equation to be simplified. The steady state calculation in SAS2A is entirely correct in finite differencing and solution of the resulting equations for steady state coolant temperatures.

The transient temperature calculation is more complex, and it was not possible to verify the actual programming. Transient calculations can be performed using either an Eulerian scheme or a Lagrangian scheme. The user's manual maintains

that the Eulerian scheme is usually more accurate for high flow rates and large time steps prior to boiling and that the Lagrangian scheme is more accurate for small flow rates. Nothing was found in the literature to support or contradict this statement.

The approximation of derivatives appears to be adequate in both methods. The finite differencing as given by the user's manual is consistent with the rest of the code and appears to be correct. However, the code is too complex to allow verification in the actual programming without additional materials that were not available here.

One point that is worth comment is the treatment of the inlet temperature. The inlet temperature in SAS2A is a constant assigned by the user in the SAS2A input. For many types of transients, this is probably quite adequate because of the influence of the inlet plenum and external coolant loop. There might be circumstances, however, in which this is not true. For example, SAS2A appears to be designed for cases of reactivity insertion and for flow coastdown. Heat transfer capability can be influenced in another way. This is through failure of heat exchangers so that heat removed from the core cannot be transferred to the secondary loop. In this case, the sodium temperature will begin to rise, and this rise will be reflected in the inlet temperature. The severity of this event will depend, of course, on the reactor design and on the degree to which heat removal capability is lost. If the

system design makes this possibility unimportant, then the assumption of constant inlet temperature is adequate. If it is possible for a serious accident to result from a loss of secondary cooling capability, then the assumption of constant inlet temperature might have to be revised. A simple revision would be to make the inlet temperature a function of time that is specified in the input. This would allow SAS2A to be used for accidents of this type. With the exception of this assumption concerning the inlet temperature, the SAS2A treatment of liquid coolant temperatures appears to be correct, at least to the level of sophistication found in the treatment of other phenomena in SAS2A.

2.7 Sodium Boiling

The SAS2A model for sodium boiling is generally supported by recent publications, but it does contain certain assumptions that should be examined more closely.

One of these assumptions is that a maximum of nine bubbles can exist in a channel at any time. There does not appear to be any physical basis for this, so it is probably assumed as a matter of convenience to simplify the procedure of accounting for bubbles. This assumption is probably not restrictive in most cases because the size, shape, and rate of growth for bubbles in the coolant channel makes it difficult to put very many bubbles in a channel. This is because a minimum slug length must exist between bubbles and also because bubbles tend to grow rapidly.

A second assumption of the SAS2A model is that a minimum slug length must exist between bubbles. This concept is supported by references 16 and 19. The minimum slug length arises because the pressure pulse that accompanies nucleation effectively raises the local sodium saturation temperature and suppresses bubble formation in the region around the newly formed bubble. Caldarola (16) discusses the matter in greater detail. The essence of his discussion is that the minimum slug length increases with the degree of superheating and decreases with the rate of change of liquid temperature. For sodium boiling in a reactor coolant channel, the minimum slug length is so large that only a small number of bubbles can be formed and grow. This is the justification for the limitation on the number of bubbles. The code treats this minimum slug length as an input constant, but it actually depends on the degree of superheat and the rate of change of liquid temperature. No information was found concerning how strongly the minimum slug length depends on these two quantities, so it is not possible to estimate how accurate the assumption of a constant minimum slug length is.

The bubbles in SAS2A are assumed to fill the entire coolant channel except for a thin film of liquid left on the cladding and structure. Bubbles start with zero thickness and with an initial temperature equal to the coolant liquid temperature at the time of bubble formation. The vapor bubble pressure is assumed to be always equal to the saturation

pressure corresponding to the bubble temperature. This model is generally supported by the literature, e.g., references 19 and 20. In their experiments, the authors of these papers actually did observe bubbles of the size and shape assumed by the code. The model in the code is based on a paper by Schlechtendahl (18).

Publications generally support the SAS2A model for liquid and vapor slugs, particularly during the later phases of bubble growth and motion. However, there is some question as to whether the model is correct for the initial phase of voiding. Calculations performed using SAS2A are in fair agreement with experiments for later phases of boiling, but there are discrepancies during the initial period. During this initial phase of voiding, SAS2A calculations and experimental results do not agree well. The discrepancies may be due to a deviation from the SAS2A model for the period right after boiling begins. Possibly, there are actually small bubbles formed during the initial phase rather than slugs of vapor as SAS2A assumes.

A matter of considerable uncertainty at the present time is the degree of superheating attainable in sodium under reactor operating conditions. A common opinion is that superheat will be small due to flow characteristics, cavitation, inert gas effects, etc. A range of 0-5°C is often mentioned for the attainable superheat in a reactor. There is a strong opposition view, however. A number of experiments have been

run to measure superheats, but most of them depart from the realistic situation in one way or another. For example, experiments in which electric current is passed directly through sodium in a test section channel have given superheats as high as 400°C . This is an unrealistic value because the experiment was unrealistic. The direct heating of the sodium probably caused nucleation to occur at sites in the bulk liquid rather at sites in wall cavities where it would be most likely to occur due to heat flux in reactor fuel pins. The most realistic experiment performed in this area was carried out by Cramer and Dorr (17). They used electrically heated rods, but their test section has the same size and geometry (169 pins) as the German SNR reactor. They measured superheats from 0°C to approximately 70°C . Further, they noticed that boiling did not always occur at the point of highest temperature first. They suggest that the superheat may not be constant throughout the channel and that boiling will begin at the point where the superheat criterion is first satisfied. Their experiment may not have been completely realistic, but it was sufficiently realistic as to geometry and heating that it may indicate that superheats of appreciable magnitude can occur.

The possibility that superheat may not be constant in a coolant channel has consequences for SAS2A. The SAS2A code uses a single input constant for the superheat. If the superheat varies in the channel, this can change the location of

the initiation of sodium boiling. This affects not only the heat transfer but also the reactivity calculation because of the sodium void worth distribution in the core.

Some investigations of the variation of superheat with various quantities have been made. Dwyer, et al. (21, 22) found that the rate of temperature rise of the heating surface influences the magnitude of the superheat. They also note that the flow rate has a strong influence. The superheat falls off sharply as the flow rate increases. Their experiments were carefully performed but much of their data was for ranges of temperature rise and flow rates that are outside of the range of interest in fast reactor safety. There is a need for good experimental data in this area for physical conditions relevant to fast reactor transients. Impurities may also affect the magnitude of superheat, but no concrete data was found concerning the importance of impurity effects.

2.8 Vapor Bubble Models

Vapor bubbles in SAS2A are assumed to fill the entire cross-section of the coolant channel except for a liquid film on the clad and structure. This assumption is commonly made in the treatment of boiling in liquid metals, and experiments generally support it. As mentioned in the section covering sodium boiling, however, this assumption appears to be questionable during the first phase right after boiling begins. The comparison of SAS2A calculations with experiments

indicates that the initial bubbles formed at boiling inception are small bubbles and that the uniform cross section bubbles form shortly after boiling inception.

The treatment of the vapor slugs is divided into a uniform vapor pressure model for small bubbles and a pressure gradient model applied to large bubbles.

For small bubbles, the initial bubble growth is attributed to the initial pressure jump corresponding to the superheat. This pressure jump drives the liquid slugs apart. The other contribution to bubble growth in small bubbles comes from heat flow through the liquid-vapor interface which produces vapor to fill the bubble and sustain the pressure. The vapor pressure and temperature are assumed uniform in small bubbles. Provided the bubbles are sufficiently small, this approximation will not lead to any substantial errors, especially since the SAS2A mesh itself is limited as to the smallness of mesh intervals axially. The approximations required to treat nonuniformities in small bubbles would probably contribute as much error as the simplifying assumption of uniformity. Problems do arise in choosing a transition point from one model to the other. If the bubble length at the transition is large, then the uniformity assumption becomes worse; if the transition length is small, the uniformity assumption is good, but assumptions in the pressure gradient model cause problems.

The method for evaluationg heat transfer from the cladding

and structure to the vapor in small bubbles appears to be quite adequate. The determination of heat transfer coefficients from the cladding and structure is conventional. The equations used for this heat transfer are also conventional, and the numerical treatment accorded these equations appears to be correct, at least as described in the user's manual.

The treatment of heat flow through the liquid-vapor interfaces is much more complicated for the uniform vapor pressure model. Heat conduction at the interface is equated to the heat added to the vapor. These conduction boundary conditions are correctly formulated in the user's manual. The equation is solved by using Laplace transforms. The Laplace transform technique is a very good approach to many difficult problems, but it leads to very complex expressions. The obvious problem here is that these complex expressions are difficult to program, and, once programmed, they are difficult to check for programming errors. The equations in the user's manual do not appear to have any errors, but the question of possible mistakes in the actual program could not be resolved because of the complexity of the program.

The mechanism for changing the vapor energy and mass appears to be correct. Thermodynamically, no laws are violated, and the method used to obtain changes in energy and volume is consistent with that used in the rest of the code. Changes in the heat of vaporization, vapor density, and pressure are also evaluated correctly.

The pressure gradient model for large bubbles allows for axial variations in pressure due to friction and acceleration of vapor. These effects can be important in large bubbles that extend from the center of the core out into the cooler regions. The continuity and momentum equations for this model are:

$$\frac{dG_v}{dt} + \frac{dP_v}{dt} = \frac{Q}{\lambda}$$

$$\frac{dP}{dz} = -\frac{d}{dz} \left(\frac{G_v^2}{P_v} \right) + \frac{f_v G_v |G_v|}{2 P_v D_h} + F_c - \frac{dG_v}{dt}$$

The subscript v denotes a vapor quantity. Q is the heat added, and λ is the heat of vaporization. F_c accounts for loss of momentum when condensation occurs, and f_v is the vapor friction factor. Both of these equations are conventional expressions, and they include all of the important effects.

One major assumption of the pressure gradient model is that heat flow through the liquid-vapor interface can be neglected. For large bubbles, this is a good approximation because the much larger surface area associated with the clad and structure and the larger temperature differences between these surfaces and the vapor make this contribution much larger than the heat flow through the interfaces. As bubbles become smaller and approach the transition length between

the two vapor models, the contribution from the interfaces becomes more important. As mentioned above, choosing a transition length causes problems because if it is large, then the assumption of uniform vapor pressure for small bubbles becomes poor as the transition length is approached. If the length is small, then neglecting interface heat flow becomes a worse approximation as large bubbles approach the transition length.

The vapor friction factor in the vapor pressure gradient model can be adjusted to account for waves and instabilities in the surface film on the cladding and structure. These waves and instabilities in the film can greatly increase the vapor friction factor, and a multiplicative factor is built into the vapor friction factor to account for this. A problem that might arise here is that this multiplicative factor does not appear to recognize gradations. According to the user's manual, it is either there or not there, depending on whether the factor is set to zero or unity. In fact, waves grow on the film, and unless they grow very rapidly, the vapor friction factor should also grow from a small value to an augmented value gradually rather than in a single step. Otherwise, the treatment of the friction factor is standard and is consistent with the form used in the improved film motion model discussed in the next section.

The treatment of the finite differencing in the user's manual appears to be correct. The discretized equations

correspond to the continuity and momentum equations with only those approximations required in any finite differencing procedure, e.g., choices of where to evaluate certain quantities. The resulting set of equations are solved for vapor pressures and mass flow rates by Gaussian elimination. The corresponding vapor temperatures are found from the saturation condition.

The vapor pressure gradient model does encounter some difficulties. In the case of very small time steps, the numerical scheme becomes unstable. Consequently, for time steps smaller than 0.3 milliseconds, the pressure gradient calculation is bypassed and vapor pressures, temperatures, and mass flow rates are set to their values at the beginning of the time step. When the time step exceeds 0.3 milliseconds, the regular calculation resumes. This technique should not cause any significant error unless small time steps occur with great frequency in a calculation. If very small time steps do appear often in a single calculation, then this approach could lead to errors in the values of vapor pressure, temperature, and flow rate.

A feature that should increase the accuracy of the vapor bubble calculation is the time step control whereby the time step size is reduced and values recalculated if the vapor pressure change exceeds 10% during a time step at any node.

The treatment of nodes that are crossed by the bubble interface during a time step by interpolation and the treatment of bubbles blown out of the top of the subassembly appear to be adequate.

In general, the vapor bubble treatment is adequate for both the uniform pressure and pressure gradient models. As mentioned above, difficulties may arise at the transition between the two models because both employ assumptions that may be questionable for intermediate length bubbles.

One other problem arises when the film decreases to zero thickness and dryout occurs. According to the treatment of the liquid film in the code, if film dryout occurs on the cladding, then no more heat can be transferred to the coolant until rewetting occurs. Condensation onto the dry clad surface is permitted, but heat cannot be added to the vapor from the dry clad surface. In fact, there will be some heat transfer to the vapor, but it will be a much smaller effect than before dryout. If this were taken into account, it would make the treatment of vapor bubbles more accurate, and it would also improve the treatment of cladding heat transfer. Accounting for this heat might delay the onset of clad failure slightly. It would require some sort of correlation for heat transfer coefficients for heat flow from the dry clad to the vapor.

Except for those points noted above, the treatment of vapor bubbles is adequate. The fundamental equations are correct, and the finite differencing appears to be correct. The question of whether any programming error occurred was not resolved.

2.9 Liquid Film Model

Evaluation of the liquid film model is complicated by the fact that a new model for film motion has been added to the most recent code in the SAS series, and few details concerning the model are available. The SAS2A model will be discussed first and then the new model.

The SAS2A model assumes that the film thickness remaining after coolant expulsion can be set initially to a user-supplied initial value. One constant is entered for the clad film and one for the structure film. This assumption can be questioned because experiments have demonstrated that the initial film thickness may not be constant. Takahashi, et al. (27) found that the liquid layer remaining in their experiments ranged from nearly zero thickness to almost one millimeter. Their experiment involved the direct heating of sodium in a channel by passing an electric current through it. They found that the thickness of the film remaining depends on the ejection velocity of the liquid slug. Because the initial film thickness may vary, computed dryout times obtained using a single constant thickness may be inaccurate. A conservative initial film thickness could be used in the code to obtain conservative results, but it would be more accurate to account for the dependence on the ejection velocity in determining the thickness of the film remaining.

An assumption of even greater consequence in SAS2A is that the film is static and that changes in film thickness

occur only because of vaporization and condensation. The validity of this assumption has been discussed by Dunn, et al. (24) and by Fauske (25). Dunn, et al. (24) compared a SAS2A calculation with experimental results. The dryout times in the experiment were much shorter than dryout times calculated by the code, leading these authors to suggest that consideration of other mechanisms in dryout is necessary. Fauske has also commented on the validity of the static film assumption. He suggests that actual dryout times may be smaller than computed dryout times assuming a static film by a factor of ten. This reduction is due to the various types of film motion possible, but the main contribution to the reduction arises from film interaction with streaming vapor. Under certain circumstances, waves may form on the film. If the interaction with the streaming vapor becomes strong enough, the film may be torn apart, leading to dryout much sooner than would be predicted assuming a static film.

An improved film model has been developed and added to the code by Dunn and Hoppner (26). Their model includes the influence of vapor drag, wall friction, gravity, and the axial pressure gradient. The basic equations for this model are:

$$\frac{\partial}{\partial t} (\rho w) + \frac{\partial}{\partial z} (\rho w v) = -\phi / v$$

$$\frac{\partial}{\partial t} (\rho w v) + \frac{\partial}{\partial z} (\rho w v^2) = -\frac{\phi_v}{\lambda} v_v - \rho g w - \frac{dp}{dz} w$$

$$+ \frac{f_v}{8} \rho_v (v_v - v) |v_v - v| - \frac{f}{8} v |v| - \frac{\phi_c}{\lambda} v_v$$

where ρ denotes density, w is film thickness, v is velocity, and ϕ is heat flux. The subscripts v and c denote vapor and condensation respectively. The friction factors are $f = A(Rc)^B$ and $f_v = A_v(Re)^{B_v}(1 + 300w/D_h)$. One calculation performed using this model gave dryout beginning at 0.6 sec. and experimentally observed dryout times agreed within 0.1 second. The finite differencing presented in reference 26 appears to be entirely correct. This improved model should correct most or all of the defects in the existing film treatment except for the matter of initial film thickness and a couple points discussed below.

The treatment of the heat fluxes for vaporization and condensation is correct. Turner, Mills, and Denny (28) have examined the effects of a noncondensable gas on laminar film condensation of liquid metals. They found that the presence of the gas reduces condensation drastically. The situation in SAS2A is slightly different in that condensation is probably turbulent and the gas is fission gas rather than argon as it was in the experiment. However, the presence of the fission gas in the sodium vapor should have a similar effect, reducing condensation to a very low level. The code treats this by reducing the condensation heat transfer coefficient by a

factor of 10^6 when fission gas is present in sodium vapor. This amounts to a complete cutoff of condensation and is the reason why this factor was chosen. There is no physical reason why this particular reduction factor was chosen other than to eliminate condensation. In other respects, the code treatment of vaporization and condensation appears to be correct.

A matter of some importance in the code treatment is the treatment of wave phenomena. The user's manual states that the numerical scheme employed in the code suppresses wave effects to some extent. No further clarification is provided. The point becomes important for the new film motion model. If the waves suppressed are due to numerical instabilities or other mathematical effects, then the matter is not important to the film treatment. If the code suppresses physical waves, the matter becomes more serious. Waves can form in the film, and they can cause the film to be torn apart under certain circumstances. This may be accounted for in the vapor friction factor, but the user's manual does not state that it is. If it is accounted for, then the wave suppression does not matter. If it is not accounted for elsewhere, then it may introduce errors into the film calculation, possibly prolonging the life of the film on the cladding.

The final matter of importance in the film treatment concerns rewetting when a liquid slug reenters a region

where dryout has occurred. The film thickness is restored to its initial value. No comment is made in the user's manual about what happens if the liquid slug should contact a very hot region of cladding where dryout occurred. In this case, one must wonder if wetting can be delayed. The physical effect that comes to mind here is the Leidenfrost phenomenon. An example of this is drops of water dancing around on a hot metal plate. It arises because a layer of vapor is formed between the water and the plate, and this layer of vapor inhibits the heat transfer that would otherwise cause rapid evaporation of the water. If a similar effect can occur in sodium on hot cladding, then rewetting and restoration of the film can be delayed. A similar consideration applies to a liquid film flowing down onto a hot cladding surface. This delay in rewetting and film restoration would influence the time of cladding failure because it would prolong the absence of effective cooling of the cladding surface. No material that bears directly on this problem was found, but it is something that bears consideration.

2.10 Fission Gas Release Model

Fission gas release in SAS2A may be determined using either of two models. The simpler option employs a table lookup procedure to treat voiding due to fission gas release. In the table lookup model, a single gas bubble with uniform pressure is assumed. The user supplies tables of gas pressure

and upper and lower interface positions as a function of time since clad rupture, and these tables determine the voiding dynamics. The obvious point of vulnerability in this approach is the set of values used in the tables. To use the table lookup model requires that the user have an accurate knowledge of the dynamics of the voiding process for the accident being modelled. The values put into the table are going to determine how rapidly voiding due to fission gas release will occur. It is probably easy enough to put in conservative values so that release will correspond to the worst case possible, but specification of realistic tabular values for a given accident is a more difficult task.

A second weakness of this model is the assumption of a single gas bubble with uniform pressure. It is possible for bubbles to be swept away from the rupture point and to subsequently enter a region with a different void reactivity worth. The single bubble assumption introduces a small error into the reactivity calculation. The importance of this error depends on whether the single bubble is in a region of higher or lower void worth than the actual bubble distribution would be.

The clad rupture which initiates gas release is assumed to occur when the pressure of the gas in the plenum exceeds the hoop strength of the clad at any axial node in the core or blanket regions. The user's manual states that the clad

yield strength usually varies more rapidly with temperature than with plenum gas pressure, so clad rupture is usually mainly due to loss of clad strength rather than to an increase in plenum pressure. This may be true for the relative changes in yield strength and plenum pressure, but it neglects possible pressure effects of fuel in contact with the cladding. This will be discussed more fully later in this section.

Table lookup release ceases at a user-specified time after clad rupture; this implies that the user determines the time scale and rapidity of release. When release stops, the gas bubble is converted to a vapor bubble with the same pressure, and it is subsequently treated as a vapor bubble using the uniform pressure model. Because fission gas is noncondensable, condensation is reduced by dividing the condensation heat transfer coefficient by one million for these converted bubbles. According to the user's manual, the pressure gradient model is not used for these converted bubbles because of difficulty in obtaining appropriate conditions to start a pressure gradient calculation.

The uniform pressure model and the pressure gradient model were both discussed in a previous section, and the same remarks apply here. For large fission gas bubbles, the assumption of uniform pressure is likely to be only a rough approximation. Nothing is available in the open literature on possible difficulties in pressure gradient calculations for fission gas bubbles. The

uniform pressure model probably makes an acceptable first approximation, but it may cause distortion of the interface pressures with liquid slugs. This distortion might be important because of its effect on the motion of the liquid slugs.

One serious assumption in the treatment of fission gas release is that fission gas is assumed to obey the ideal gas law. The ideal gas law works well for low pressures, particularly as the pressure decreases to zero. It has the advantage of being simple to use and manipulate. Its defect for use here is that its accuracy is limited in many cases. It is good at low pressure and can be useful under certain circumstances even up to the critical pressure, but it can be very inaccurate in some cases. Andrews (29) discusses this in more detail and also considers other equations of state. A better treatment of fission gas release would involve adoption of a more accurate equation of state, e.g., a parametric form such as the Redlich-Kwong equation. Unfortunately, the current state of uncertainty concerning many of the properties and characteristics of fission gas probably involves so much error and uncertainty that it is the dominant factor in limiting the accuracy of fission gas calculations.

Heat transfer into fission gas bubbles involves several assumptions. Direct heat transfer between the gas and either the cladding or the structure is ignored in SAS2A. Any heat transfer occurs by vaporization of the sodium film

on the walls. No vapor condensation is considered because the fission gas acts as a noncondensable gas. This noncondensable gas inhibits any form of condensation, particularly in the concentration present in a fission gas bubble. Direct heat transfer would certainly be a small effect. It would involve convective heat transfer into the gas from the cladding and structure walls. The convection would be either natural or mixed convection, so the heat transfer coefficient would not be large. Whether it is small enough to be neglected, however, is another question. It might be only a small contribution, but nothing in the available literature provides any estimate of its magnitude.

The treatment of vaporization is more complex. The code treatment evidently assumes that vaporization of the sodium film is a pressure-controlled process. For example, vaporization can only occur if the gas pressure is less than the saturation pressure of the film. Standard heat transfer texts do not discuss the matter at all, but this appears to be a plausible assumption physically. A gas pressure higher than the film pressure should inhibit migration of molecules from the film surface. Granting the validity of this assumption, the code treatment of the process is reasonable. The gas pressure is used to calculate a sodium vapor temperature which is used as the coolant temperature for vaporization.

The explicit gas voiding model is more complex than table lookup, but it is also more realistic and probably

more accurate. It accounts for most of the actual events that occur during fission gas release. At the present time, it does not account for release of gas from the central hole of restructured oxide fuel or for release from fuel heating and melting. According to the user's manual, these effects are to be added in the new fuel-coolant interaction model and fuel slumping model, so they may already have been added to the newest version of SAS.

As mentioned above, clad rupture and initiation of gas release are assumed to occur due to overheating of the cladding. The heating of the cladding causes it to lose strength, and it ruptures when the clad hoop strength becomes insufficient to withstand the gas pressure. Pressure due to swelling of fuel is not considered. Dias, Erdmann and Okrent (30) discuss the matter of fission gas-induced fuel swelling and suggest that it may lead to cladding failure. No quantitative results were found to indicate how much of a contribution fuel swelling is likely to make, but it should be considered in SAS2A if the contribution can become large enough to be significant. The criterion for significance in this case would have to involve the relative values for loss of strength due to clad heating and fuel swelling combined with the rates for clad heating and fuel swelling.

SAS2A generally does not treat incoherency, but the fission gas release is an exception. The user specifies the fraction

of pins in each of three groups and a rupture criterion factor for each group. This allows for incoherent release since the groups can rupture separately, and the plenum gas can be accounted for separately in each group.

The code also allows the user some freedom to specify the location of clad rupture. Since the highest coolant and clad temperatures usually occur in the upper core of the upper blanket region, the initial clad rupture is likely to occur there. SAS2A allows the user to specify the node at which rupture and gas release are to occur. Thus, an initial rupture far from the plenum that would result in little gas release can be ignored in favor of a rupture closer to the plenum which would result in higher release. This option is necessary because the code only allows release from a single rupture location in a pin.

For ruptures in the core and blankets, the user specifies an impedance to gas flow between the gas plenum and the rupture. This impedance is approximated by impedance of a pipe of user-specified length, hydraulic diameter, and flow area. This impedance is particularly important because, as in the case of table lookup, it actually determines the voiding dynamics of the gas release. Fernandez and Thomas (31) discuss a model for gas release from failed rods. They point out that the gas flow network becomes more complex as burnup increases and that studies of release dynamics generally use release rates much larger than experiments indicate to be possible.

One questionable assumption in the explicit release model is that of isothermal gas flow. The plenum gas must flow through the gap between fuel pellets and the cladding or through the cracks in the fuel. If the flow path is short and comparatively simple, there might not be much heat transfer to the gas. If the flow path is long and tortuous, however, as it might be for a rupture far from the plenum and for a complex net-work of cracks, then there might be appreciable heat transfer. The situation would be equivalent to flow through a packed bed. This heat transfer might not carry that much heat away from the fuel to be important there, but it could be sufficient to raise the temperature and/or pressure of the fission gas. This would affect the gas flow rate and the prevailing conditions in the gas bubble in the channel. The gas in the immediate vicinity of the rupture would be affected in a slightly different way. It would be released very soon after rupture; in fact, the code assumes that gas in the neighborhood of the rupture has unrestricted access to the coolant channel. It should be released with a temperature equal to that of the fuel in which it was located. The whole matter of release does raise one large uncertainty. This concerns the distribution of fission gas prior to and during release. The actual distribution of the gas is uncertain, and this introduces errors into any computation that probably make the errors arising from rough approximations in the SAS2A treatment less important.

As in the case of the table lookup model, a uniform pressure is assumed in a gas bubble until it blows out of the top of a subassembly. The problems with this assumption are discussed above. The pressure drop due to friction of gas streaming in the coolant channel is lumped at the end of the subassembly. The pressure variation in the channel is not calculated in detail; the impedance to gas flow in the channel is lumped at the exit. A similar treatment is made at the bottom of the subassembly. For the bubble itself, this treatment is probably adequate, but one must wonder to what extent this procedure distorts the pressures at the interfaces with liquid slugs. If the distortion is appreciable, then there can be a change in the motion of the liquid slugs.

The treatment of bubbles that have blown out of the subassembly involves the restriction that a bubble blown out of a subassembly can only enter the same assembly if it reenters any at all. The validity of this assumption depends mainly on how far the bubble migrates before reentering a subassembly. Since SAS2A only treats single pins in ten channels, each channel actually represents one or more entire subassemblies. If a bubble reenters a subassembly very near its point of origin, then it effectively reenters the same channel, and problems do not arise. If, however, a bubble can migrate far enough for its original point of entry in the plenum, then it might reenter a different channel which could have a different void worth. The restrictiveness

of this requirement that bubbles reenter the same channel depends on their migration characteristics in the plenum. If they cannot move very far in the plenum, then the restriction does not cause problems. If they can move far enough to enter a different channel, then this restriction may introduce an error into the calculation of void reactivity.

The treatment of bubbles that break away from the rupture point and the formation of bubbles at the rupture point appear to be adequate. Gas release stops when the plenum pressure drops below a user-specified minimum, and gas bubbles in the channel are then swept upward by reentering coolant.

When the gas does enter the coolant channel, it is assumed to be adiabatic. The liquid film on the clad can vaporize to cool the clad, but this is assumed to have no effect on the bubble temperature and pressure. The validity of this assumption depends on how much vapor is added to the fission gas bubble. If only a small amount is added in comparison with the mass of the gas in the bubble, then the assumption will not introduce appreciable error. If the sodium vapor added amounts to a significant fraction of the amount of gas in the bubble, then this assumption is weak because the temperature and pressure in the bubble are going to be affected by the presence of the sodium vapor. In its present form, the code accounts for bubbles as either gas bubbles with vapor neglected, vapor bubbles with no gas, or vapor bubbles with enough gas present to inhibit condensation.

Clad rupture has been discussed above. The code allows the user to specify the rupture node. If the user does so, then only this node is checked for rupture at each time step; otherwise, all nodes are checked. The code then assumes that all pin groups rupture at the same node. After the first group ruptures, the remaining groups fail only at that node and add gas to bubbles at that node. This requirement does impose a certain spatial coherence to pin rupture. Incoherence in time of release is allowed by the three groups with different rupture criteria, but spatial incoherence is restricted because all groups in a channel must fail at the same node. This restriction probably is not that serious unless some pins are seriously defected at a different axial plane. If such defects do exist, then the second and third groups could fail at another location. If there are no such defects, then the remaining groups are likely to fail near the same axial plane as the first group. This is because if conditions were severe enough to fail one pin group at a certain location, then they would probably also meet the rupture criteria for the other groups first near the same axial plane.

The method employed to form a volume for a gas bubble being released appears to be adequate. The volume changes due to gas addition from other groups is also correct, and the treatment of bubble volumes that have broken away from the rupture seems correct. The weak point is the calculation

of bubble pressures. These pressures are computed using the perfect gas model, and the defects in this procedure were outlined above.

In keeping with the assumptions of an ideal gas and no heat transfer to the gas bubble in the coolant channel, extensive use is made of the adiabatic relationships for a perfect gas. Ignoring the arguments concerning the perfect gas model discussed above, the gas treatment is generally correct. When two bubbles combine, the method for combining masses, pressures, and temperature is correct. The combined volume is determined by simply adding the volumes of the two bubbles. This is acceptable if the pressures of the bubbles are not too different, but it may not be correct if the pressures differ markedly.

The treatment of gas types appears complete. Each bubble in the channel has a number associated with it to identify its gas type. The ten possible types cover all of the important cases with the possible exception of the case where a bubble consists of approximately equal amounts of sodium vapor and fission gas.

The treatment of time step sizes and convergence is consistent with the approach taken in other sections of the code. Time steps are broken into subintervals for the gas bubble treatment; subinterval sizes are halved and values are recomputed until a convergence criterion is satisfied.

If convergence is not attained within a certain number of subinterval calculations, then extrapolated values are calculated, and extrapolated convergence is tested.

The SAS2A user's manual provides detailed information on the numerical treatment of the various aspects of fission gas release. e.g., plenum temperature, flow rates, etc. The numerical treatment in the code is quite adequate for fission gas release. Any errors that arise due to the numerical treatment are likely to be much less important than the errors caused by the basic models employed in SAS2A for fission gas and the errors caused by the general lack of knowledge concerning fission gas phenomena. Examples of errors from SAS2A assumptions are those arising from the isothermal flow assumption and the adiabatic bubble assumption. Examples of the errors due to lack of knowledge are uncertainties in the distribution of fission gas, uncertainties in release rates, problems in characterizing fission gas, and uncertainties in the equation of state.

III. GENERAL ASPECTS OF SAS2A

The SAS2A code represents a great improvement over the original SAS1A. Nevertheless, SAS2A was incomplete in several respects. It lacked a fuel-coolant interaction model. It failed to treat clad motion or fuel slumping except in terms of reactivity insertion. The SAS2A film model was inadequate. The most recent version of SAS has incorporated models which rectify many of these defects. The new code has models to treat film and clad motion more accurately. Fuel slumping is treated in the new code, and a fuel-coolant interaction model has been added. Unfortunately, no details concerning these new models are available, so they cannot be evaluated. It is certain, however, that they will improve the ability of the SAS code to accurately model fast reactor transients. At the same time, these new models increase the storage requirements and the execution time for the code.

The issue of storage is important for SAS because it already exceeds the storage capabilities of many smaller computers. Even before the changes were made in SAS2A, extensive overlaying was required before it could be executed on many smaller computers. Continued addition and refinement of models may soon make it impossible to run the code on many computers, at least in its most sophisticated form. It may be necessary to make up a stripped down version by eliminating some of the channels or otherwise reducing storage so that it can be run at facilities having smaller machines.

Associated with the problem of storage is that of execution time. SAS is a very expensive code to run, particularly if extensive overlays are required. It may be worthwhile to investigate the possibility of developing a smaller, simplified version that retains the main features of SAS because the sophisticated version is too large and too expensive for use in routine production line-type calculations in many places.

An aspect of more theoretical concern is the assumption that ten channels represent a reactor adequately. The number of channels required is rather an arbitrary matter. What is important theoretically is that these channels correspond to reactor subassemblies; each channel corresponds to one or more subassemblies. Also, only one pin is treated in each channel except in the fission gas model.

This carries with it an implicit requirement that effects be coherent. Each pin in each subassembly represented by a channel is assumed to be represented by the values calculated in the code for the one pin used in each channel. This implies that such effects as voiding are treated as being coherent in the subassembly group represented.

In fact, many of the phenomena involved in the accident calculation are not uniform. There is a temperature distribution over a subassembly that will contribute to an incoherent boiling history rather than the coherent boiling required in the code. This temperature variation should

lead to incoherent boiling of sodium with the initial nucleation in the hotter region toward the center of the subassembly. Other physical phenomena are also likely to be incoherent, but SAS2A has no way of treating this incoherence at the present time. It would be desirable if some effort were devoted toward development of some approximate means of treating incoherence in SAS.

The other fundamental assumption of SAS is that circumferential effects can be neglected. The discussion in the preceeding chapter brought out the fact that the validity of this assumption really depends on the pitch-to-diameter ratio. For P/D near unity, circumferential effects are not necessarily small. This could be important because the fuel pins then have an angular temperature distribution. If this angular temperature distribution is such as to give rise to significantly hotter temperatures where the pins are closest together, then this could give rise to a site for preferential boiling. Dwyer and Berry (33) and Dwyer, Berry, and Hlavac (34) performed calculations to examine heat transfer and temperatures in closely spaced rods for a range of P/D . Their results are worth examining because they give a quantitative answer to the problem of how accurate the assumption that circumferential effects are negligible is. Their results show that for P/D below 1.1, this assumption is not accurate. As P/D increases from 1.1, the assumption becomes better and better.

As mentioned above, there are several effects that are too fine geometrically to be treated by SAS2A, e.g., bowing and the effects of wire wrap. Very little is available on the effects of the wire wrap. No estimates have been made available for the magnitude of possible hot spots arising from the effects of the wire wrap.

Collingham, et al. (32) have conducted experiments on bowing with a seven pin assembly. They found significant but not extreme hot spots. At reactor operating conditions, two pins in contact created a hot spot with less than 100°F increase. The temperature imbalance caused by bowing of the entire bundle amounted to less than 20°F . They found no significant increase in coolant enthalpy two inches downstream from the bowed position.

The preceeding chapter has examined the various models employed in SAS2A and compared them with available material published since development of SAS2A. The defects in SAS2A models have been listed in this evaluation. Some of these can be rectified with ease, e.g., making cladding properties parabolic, while others such as the fission gas model require considerable work and may have to wait for better experimental basis.

This chapter has examined certain general aspects of SAS2A. The areas that seem to most need improvement concern devising a means for roughly treating incoherence and making the simpler alterations possible in the code to make current

models more accurate. Also, the possibility of developing a simplified version of SAS that includes all of the important effects should be considered since SAS is rapidly becoming too large for use in standard calculations.

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Part Seven

The Role of HCDA Analysis
in Fast Breeder Design and Licensing

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I. INTRODUCTION

The question of the role of HCDA analysis in Fast Breeder design and licensing is a difficult one. There is a great diversity of opinion on the subject. We favor a very active and prominent role. Others feel that the HCDA can be so reliably prevented that it can be set aside entirely. The discussion presented here is the text of a paper presented by the author at the 1974 Beverly Hills ANS topical meeting on Fast Reactor Safety.

In the panel discussion following the papers, each of the other panelists expressed some disagreement with our point of view. Two particular points deserve some further discussion. The other panelists seemed uniformly to feel that a great deal of attention given to the HCDA would detract from safety for two reasons. First, they argue that our resources are limited and therefore have to be parceled out in fair shares to the various activities involved in design and construction of a plant. This is of course true. We argue that the enterprise must not be undertaken at all unless those promoting it are willing to dedicate a very large amount of money and effort to the assurance of safety. The very large amounts will probably be required only for the first few plants and will therefore not be an important factor in the total economy of the industry. The funds should be provided through government subsidy since the development of the

Breeder is very much in the public interest. Nearly everyone involved does support large expenditures for safety. It is a matter of degree. We are recommending substantially more attention to safety than are the majority of LMFBR designers.

The second point is a very vulnerable one, and we do not understand how anyone can feel comfortable with it. This one is that if the designers are forced to design for containment of the worst possible HCDA then there is less motivation to pursue more reliable designs and back-up safety features to prevent accidents. This point of view is a terribly pessimistic one that to me is an insult to the engineering profession. The responsible engineers are absolutely obligated to pursue any innovative ideas that could lead to a more safe nuclear power plant, especially at this early developmental stage of the industry.

The text of our Beverly Hills paper follows:

"For years there has been a large world-wide program in fast breeder reactor safety analysis. Two objectives of this work have been to generate the information needed to enable the reactor designers to design a safe reactor and to generate the information needed to establish to the satisfaction of the public and licensing authorities that the public safety requirements are being met. The activities to accomplish these two goals are not always the same, but do overlap considerably. In some cases activities that apparently relate to safety, when critically scrutinized, are found not significantly

to contribute to either of the main objectives except through advancement of our general understanding of nature.

"Over the past few years there has been an increasing body of opinion developing that the studies of hypothetical meltdown and disassembly accidents contribute nothing to the design of safe reactors and detract from the objective of licensing the fast breeder. A principal objective of this paper is to support the opposite thesis that hypothetical accident studies are not only useful but in fact an essential ingredient in both the design process and licensing. In particular, one design criterion must be the containment of all effects resulting from a hypothetical disassembly accident. I believe this statement to be true for any fast breeder that is built within at least the next fifteen years and perhaps longer. There is no way to avoid this design requirement in the foreseeable future. It is well within the capacity of the industry to design for containment, but not without the expenditure of considerable effort. It is essential that all designers accept this challenge and proceed to accommodate the necessary perturbations in the design process.

"The safety analysts investigate a wide range of hypothetical accidents: a wide variety of types and wide range of severity. A very important segment of this work is concerned with insuring that the plant investment will be protected and that it will continue to operate steadily without long periods of shutdown due to malfunctions and equipment failures. This

segment of the work deals with relatively probable kinds of malfunctions and how to prevent them or mitigate the consequences. Having accomplished this goal the designer has gone a long way toward insuring the safety of the public and successful licensing endeavors.

"However, this part of the activity relating to economic assurance does not need to include studies of extremely improbable events involving multiple failures of normally reliable systems. If the probability of total loss of the plant investment is less than about one part in 1000 per year of operation, that is an acceptable economic risk. In contrast, it is necessary to insure to at most one part in 10^6 per year that there will be no major release of activity to the environment. This paper relates to the question of public safety as opposed to the economic assurance. It is more restricted yet in dealing primarily with the hypothetical disassembly accident. It is not that the more probable kinds of accidents are not at least as important. My concentration on the disassembly accident is because this is where the greatest controversy is beginning to surface. If this is not resolved soon it will build into an even greater problem area than emergency core cooling has been for the water reactors.

"The LMFBR has one very important safety advantage over the water reactors. There need be no high pressure components within the reactor containment building. The double-ended failure of a primary sodium pipe is thus more unlikely

relatively to a water reactor pipe failure and secondly such a failure per se does not constitute a direct threat to containment. Because of the low pressure many of us believe the LMFBR may inherently be a safer reactor than a water reactor.

"With no natural source of pressure to threaten containment, the public safety question for the LMFBR is mainly the question of whether there is any combination of malfunctions that can lead to a super-prompt-critical power excursion as a source of pressure. A lesser problem is the potential for sodium-air reactions and another one of considerable importance is ultimate containment of a molten core with or without the prompt-critical excursion.

"There are three main potential initiators for prompt-critical excursions: control rod movement, sodium boiling, and fuel rearrangement. Sodium boiling in at least the central regions of an LMFBR produces positive reactivity and if it is sufficiently fast and coherent over the volume of the core can drive the reactor prompt critical. If a fraction of the oxide fuel of the fast reactor is compacted into a suitable configuration with density of say 5 gm/cc or higher, it will be super-prompt-critical even without any significant neutronic coupling to the rest of the core. When one assumes this to happen he is usually thinking in terms of a meltdown accident. The meltdown could conceivably result from either an over-power transient due to control system malfunction or from a loss of cooling at normal power level. Control rod movement can contribute to prompt criticality only if the

rods are moved out much more rapidly than design rates, for example pressurized ejection due to mechanical failures."

II. DESIGN FOR ACCIDENT PREVENTION

"Now one may ask, why not design fail-safe and redundant control and cooling systems so such accidents can't happen? This is exactly what is done, but most of us realize that the term "can't happen" is too precise if taken literally. It should be replaced with the statement that such accidents have an acceptably low probability of occurrence. The Regulatory Staff of the USAEC in the ATWS report (1) has stated at least for the water reactors that "it is not necessary, nor even possible, to design nuclear power plants, or any other man-made device or system, for all conceivable eventualities that are physically possible." This is approximately equivalent to the statement that accidents of sufficiently low probability need not necessarily be considered in designing protection and containment systems and other engineered safeguards. I agree with this philosophy.

"Some, if not most, of the breeder designers believe that they can design a plant that will have so low a probability for a core disassembly accident that it is unnecessary to consider such an event in design of the containment structures, i.e., that it is a class 9 accident. I do not agree that this accident in all of its possible variations can be shown to be a class 9 accident. Substantial advances in

the technology will be required before such accidents can be ruled out. It is not yet foreseeable when in the future this will be possible. It depends critically upon a vigorous research program with new experimental facilities that have not yet been authorized and are still barely conceived in any degree of detail.

"There is a large body of competent opinion in agreement with my general point of view on this subject. There is not so much agreement on the appropriate measures to accommodate the problem."

III. CONTAINMENT FAILURE PROBABILITIES

"How does one determine whether any particular accident need be considered in design of containment structures? There are two crucial questions.

1. Given that no special containment design measures are taken for the prompt critical accident, what is the probability that there will be such an accident of sufficient strength to fail the containment?

2. What is an acceptable level for this probability? Even more basic, who should decide what is acceptable?

"Let's examine the second question first. Clearly the acceptable probability depends upon the potential for release of plutonium and fission products, which in turn depends upon the strength of the power excursion. The regulatory staff in ATWS concluded that the probability for accidents

beyond the design basis must be less than 10^{-6} per year and that a suitable target probability was 10^{-7} . Some will argue that this is not small enough yet. The ATWS report treats these probabilities as applicable to accidents which 'lead to radiological consequences outside the plant boundary in excess of part 100 guidelines.' It does not address the question of whether these probabilities are still suitable for accidents that are very much worse than the limits of 10 CFR part 100 and the report is not necessarily applicable to the LMFBR.

"Who should decide this question of acceptable probability? It is a sociological question that should be decided by some appropriate representatives of the people, not exclusively by engineers. The decision is certainly related to the risk-cost-benefit balance. There should be significant input from the Environmental Protection Agency on this question. One can expect the ATWS figures to be challenged through the public hearing and judicial procedures. In the meantime the target figure of 10^{-7} per year seems appropriate for the breeder.

"The problem is further compounded by the fact that a 10^{-7} probability is virtually not measurable, i.e., it is not possible to quantitatively monitor compliance with a chosen standard. This is a very different matter from the monitoring of low level releases during normal operation of a plant.

"If such a specification cannot be monitored for compliance, is there then any point in establishing it? Yes, because it tells the designers what to aim for in design of plant protective systems and containment structures and also the nature of the supporting arguments that will be needed to license the plant. It provides a basis for intelligent communication between the safety analyst, the designer, and the licensing agency.

"Having established an acceptable probability for exceeding part 100 limits, the first question remains: What is the probability that there will be an accident severe enough to fail the containment? The designers are beginning to address this question in a semi-quantitative way through fault tree analysis. There are certain kinds of failures and malfunctions that can be assigned meaningful numerical probabilities based upon industrial experience: bearing seizures in a pump, weld failures, electronic component failures in the control system. By considering every conceivable branch of the fault tree that can lead to an excursion, one can in principle develop a network of series and parallel probabilities that can be combined to arrive at an overall probability for a prompt critical excursion of a given severity. However, while some of the elements of the fault tree can be assigned meaningful probabilities, many others at this point in time cannot.

"Let's look briefly at the classical flow failure combined with failure to scram. These may sound like simple

events but both involve multiple failures. There is usually an emergency power supply, a multiplicity of control rods and probably two relatively diverse types of control systems. The designers seem confident that they can provide the desired reliability down to 10^{-7} failure probability or lower if necessary. One can set up the fault tree and analyze the probabilities. There will be some very significant arguments about the numbers chosen for particular events on the fault tree but there is some chance that agreement could be reached on a conservative choice at each step. But there is likely to be objection at the more fundamental level of the drawing of the tree itself. Has the analyst really traced all the paths to the prompt critical accident? Or was a common mode failure missed? If we are really aiming for a 10^{-7} probability there is some chance that an obscure failure mode has been forgotten.

"Given that there is flow failure combined with failure to scram (very unlikely) an extensive core meltdown is assured. The prompt critical excursion is not assured by those two events. Many believe the excursion is unlikely to develop. If a low value can be assigned to this conditional probability, i.e., the probability of getting the excursion given the meltdown, then the requirements on reliability of control and flow are less stringent. For example, if one could show the conditional probability to be of the order 10^{-2} , then the target for prevention of meltdown could be 10^{-5} instead of 10^{-7} . Not only would this be easier for the

designer, but also there would then be much less chance that a relevant mode of failure has been missed.

"I have often heard inferences that the studies of meltdown accidents are meaningless because the designers can and must achieve the low probability goal by designing for prevention. The arguments I have just given are intended to show that this is not necessarily so and that there is potentially a high return on the efforts invested in analysis of reactor meltdown accidents. If we can show that the probability is very high that the reactor melts down safely, the design requirements for the control and coolant flow systems can easily be met.

"To demonstrate safe meltdown is a formidable task and the demonstration is very much dependent on the details of a particular design. Some of the analysts are arguing that their calculations combined with certain experiments show that meltdown will not lead to prompt critical reactivity and a severe power excursion. In my opinion such calculations are not yet reliable enough to accept for licensing purposes. Even if the calculations done now seem to show safe meltdown we must nevertheless still assume otherwise. Gradually we understand the problem better and increase our confidence, but we are still far from our goal. We will not likely reach it until better experimental facilities are provided to verify the analytical modeling of the phenomena. It is important that new facilities be built and that the meltdown

accident studies be pursued vigorously in support of our breeder program."

IV. CONTAINMENT OF BOUNDING ACCIDENTS

"There is another approach to protection against this kind of accident which ought to be vigorously pursued in parallel with the efforts to prevent the accident. This is to determine in some sense the upper bound on severity and design to contain the consequences. There is much discussion about the feasibility of identifying a set of accident conditions that can be agreed upon as a bounding accident. There is no bound in the sense that all the more severe accidents are prevented by the basic laws of nature. There is only in the sense that all the spectrum of more severe accidents has a low integrated probability. I discussed the goal of demonstrating that if a reactor melts down it will melt down safely with no power excursion. Designing for containment is a way of modifying the requirements of that goal so that it can be demonstrated more easily and with greater certainty. The goal is then to show that there is a low probability that meltdown will lead to a power excursion greater than the bounding design basis accident.

"The design basis accident approach does play a role in present discussions between breeder designers and licensing authorities in this country. But the emphasis has been upon efforts to show that specific accidents rather than bounding accidents can be contained within structures that have been

designed for other purposes without significant consideration for core disassembly accidents. Some have argued that to significantly upgrade containment capacity beyond that naturally present would be excessively costly, cause unacceptable interference with the normal functions of the plant, and perhaps decrease overall plant safety while only improving its capacity for an accident that is in any case incredible. I remain hopeful that this is not true.

"What is the nature of the hypothetical accident studies that the designer should do, or commission, to aid himself in making containment design decisions? The popular cases to study in recent years have been an over-power transient at steady flow, and loss of flow combined with failure to scram. These two cases can be regarded as representative of all the class of accidents that produce overheating of the core. The core will not overheat unless the flow is reduced or the power is increased. The nature and severity of the incident of course depends upon the details of the conditions that lead to the over-power transient or loss of flow, as well as the complex physical phenomena of transient boiling, fuel-coolant interactions, fuel element failure, etc.

"These are certainly two of the appropriate accidents to study. After selecting these for study the analyst still has many judgments to make. He must decide whether to do a demonstrably conservative analysis, a best estimate analysis, or some compromise between these limits. Both limits should be done along with a few intermediate cases. The reactor

design is not necessarily determined by the worst case the analyst calculates, but the spectrum of calculations must be available for the responsible engineer to evaluate before he can make an intelligent design decision. Otherwise, the two examples become too specific. One is not attempting to protect only against specific accidents but rather all accidents that have a probability above a certain level. The examples chosen for analysis must in some sense cover the range of possible consequences from not only the specific initiators chosen, but also from all the possible initiators that have escaped the attention of the designer.

"There are many decisions to be made in the course of the analysis, partly because in such complex phenomena apparently identical initiating conditions do not always yield even qualitatively similar results. In addition, our ability to analyze and make predictions about those situations that do follow a definite course to reproducible consequences is not yet sufficiently reliable.

"In the end the safety analyst must be able to provide the designer with a spectrum of accident consequences and an associated probability distribution. The probabilities involved are not all well defined component failure probabilities but also involve more elusive concepts like the probability that one phenomenon model is the correct one relative to other possible models. Perhaps we are not ready to evaluate the probability distribution as a table of numbers but have to rely on less quantitative language.

Whatever terms are used they must provide an adequate communication between the safety analyst and designer and be translated into design criteria. Certainly the trend is to attempt to bring quantitative probability analysis into such decision processes and this is desirable. Numbers assigned to such concepts as the probability of correctness of a model clearly must be involved in the analysis and require a great deal of subjective judgment. Therefore if quantitative numbers are assigned they are at best order of magnitude estimates.

"There are some who object to the notion that one can cover the spectrum of accident consequences, from all possible accidents, including those that have escaped conception, by taking a few representative ones and varying the models and assumptions. For example, in a transient over-power accident one expects the fuel to fail first near the core exit, and this is usually believed to be the most favorable failure location. Does one cover some unknown accident possibility by also doing the analysis with fuel failure at the core center or is this a ridiculous academic exercise? In any particular instance of this type I cannot point to another real accident which gets covered by doing the hypothetical variation. Nevertheless, I firmly believe that if such variations are carried out throughout the analysis then there is a very good chance that the envelope of protection based on this work will be extended over some real unconceived accidents, real in the sense of having non-negligible probability of occurrence.

We are talking here of low probability accidents that are not expected ever actually to occur in the lifetime of a plant. I know of no other way to account for the unpredictable accident in the design of containment structures. One, of course, provides protection through the diverse control systems, but also needs some basis for design of containment structures, which after all are there only because they may someday have a job to do.

V. JUDGMENTS AND PERSONAL RESPONSIBILITY

"The selection of the containment design bases and the verification and certification that the plant meets the licensing safety standards involve a multiplicity of engineering judgments. This is right, fully justifiable, and unavoidable. Major engineering achievements in many industries have involved considerations of public safety that were evaluated with engineering judgment. There is the difference that the potential destruction is much greater in the nuclear reactor industry and this places a greater burden of responsibility on those making the judgments, as well as on the licensing authorities who must evaluate the judgments and in some cases make their own. Because of the potential risks, the public through suitable representatives certainly has the right to demand that these judgments be given visibility.

"Something which in my opinion would advance the cause of safety more than any other single step is a licensing regulation requiring the reactor designer to document for

public view every major design decision relating to containment combined with an identification of the engineer or engineers who accept the responsibility for that decision. There is often, although not always, a balance to be struck between economics and safety. Visibility of the decision process can provide the essential force to balance the economic force and insure that a responsible decision is made.

"Aside from the pressure effect associated with identifying the responsible engineer, this procedure also provides the possibility for the public and licensing agencies to evaluate competence. If public safety is based upon engineering judgment then it is appropriate for the public to inquire into the competence of those individuals who make the decisions. Nothing can hurt the credibility of the nuclear industry more than for the public to find that it is impossible to learn who is responsible for the key decisions on safety."

VI. RECOMMENDED APPROACH

"I have discussed three main options to demonstrate safety relative to hypothetical meltdown and disassembly accidents. One is to design plant protective systems to prevent the accident. Another is to prove that the reactor melts down safely. The third is to design structures to contain relatively severe power excursions. I have explained

that the second two work together to some extent. Most engineers can agree that any single protective system cannot be relied upon to have a failure probability below 10^{-7} per year. To attain this low a probability at least two independent systems are required. They should not merely be independent in the sense of power supplies, sensors and control rods, but in the jargon of reactor safety they must also be diverse, which among other things means that they operate on different principles and have no common failure modes. They must also be capable of regular testing and this is a limitation that eliminates some of the concepts having demonstrable diversity. The proof that there are no common failure modes is really difficult.

"Some of the designers argue that it is much more desirable to attain the 10^{-7} failure probability by employing two diverse control systems rather than one control system and one containment system. I could accept this if I could convince myself that the two control systems do have the required degree of independence and diversity. But it is easier and more reliable to demonstrate independence and diversity between a control system and a strong passive containment system. My view is that one should provide two control systems that are believed to be independent and diverse and also a strong containment system that is believed to have no failure modes in common with either control system. Then we could be really confident that the desired probability has been attained.

"What are the design requirements on this strong passive containment system? The basic requirement is that it must withstand x megawatt-seconds of expansive work by either a sodium-vapor explosion or fuel vapor expansion process with a specified pressure-volume expansion characteristic. Alternatively, a high pressure structure can be provided which restrains the expansion process. By withstand I mean that the design leak-rate of the containment building is not exceeded due to some violent effect and that the core debris does not threaten to melt through the containment. The major objection to this approach has been the difficulty in justifying a particular choice for the number x .

"One can define the conditional probability that given a core meltdown the consequences will be x megawatt-seconds of expansive work or more: this integral probability curve is a decreasing function of x . Most of us would agree that it is a very strongly decreasing function of x although we do not yet have a way of explicitly evaluating it quantitatively. Suppose that a factor of two increase in the design value of x cuts the probability of containment failure by a factor of 10. Then if the natural containment capacity of a particular plant can be increased through design by a factor of even four, one reduces the failure probability by a factor of 100. These numbers mean very little but it should be clear that there is the possibility of vastly reducing the potential hazard to the public.

"It is essential that a breeder design project start with a containment design specification as a target value from the very beginning. I would suggest a number of the order of 1000 Mw-seconds for a demonstration-size plant. Then as the design progresses and detailed accident analysis progresses, this number can be adjusted toward the end goal of an optimized design parameter just as the other design parameters are adjusted. However, the economic value cannot be readily assigned to a certain containment capacity. It is a matter of judgment, which should be guided by the goal of achieving a containment failure probability that is as low as practicable much in the sense of holding environmental discharges to values as low as practicable. One should not stop trying to improve safety once he is satisfied that the failure probability is below the 10^{-6} to 10^{-7} range. I am not here recommending design for class 9 accidents, only providing margin for an accident whose class cannot be clearly demonstrated."

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