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ANALYTICAL STRESS ANALYSIS SOLUTION FOR A
SIMPLIFIED MODEL OF A REACTOR FUEL ELEMENT

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ANALYTICAL STRESS ANALYSIS SOLUTION FOR A SIMPLIFIED
MODEL OF A REACTOR FUEL ELEMENT

by

David Ernest Lamkin

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ABSTRACT

A simplified mathematical model for fuel element stress analysis is proposed. The principal idealizations introduced are:

1. The fuel element is represented geometrically by one fuel region surrounded by a cladding region.
2. Mechanical properties are taken to be space independent within each of the two regions.
3. Creep rates are assumed to be related linearly to the deviatoric stress.
4. A state of plane strain with azimuthal symmetry is assumed.

Using the method of the viscoelastic-elastic analogy, closed-form analytical solutions to this model are obtained giving the stresses in the fuel element. In these solutions, the contributions to the stresses from the several "driving forces," namely fission gas pressure, coolant pressure, thermal expansion, and irradiation induced swelling, are all clearly visible and therefore easily compared. The relative importance of various parameters such as properties and geometry can also be evaluated.

Next, based upon this analytical solution, a numerical evaluation of the cladding stresses for a typical

LMFBR fuel element is undertaken. For the properties and geometry selected, it is found that no fuel-cladding interaction develops. Also, the major stress producing mechanism is identified to be the spatial variation of the stainless steel swelling through the cladding wall. This points out the need for further research to increase the accuracy with which stainless steel swelling can be predicted. It is further found that irradiation creep produces a considerable relaxation of the cladding stresses. Again, this indicates the need for more reliable creep data.

Finally, in order to examine the effect of a fuel-cladding interaction pressure, and also to exercise the full capability of the analytical solution, a second numerical evaluation is carried out with a lower fluence to burnup ratio. It is found that the fuel-cladding interaction results in much higher cladding stresses. Again, this indicates the need for more reliable creep data.

This solution, while containing many idealizing assumptions which may remove it too far from reality for design application, should have useful application nevertheless. This approach does provide exact solutions, with no computational uncertainty, to certain problems which retain, at least qualitatively, descriptions of the important deformation mechanisms in an actual fuel element. Such results should be useful as a standard for comparison

with results from more realistic (but approximate) computer analyses.

CHAPTER 1

INTRODUCTION

In recent years there has been considerable effort within the nuclear community directed toward the development of calculational models for prediction of reactor fuel element mechanical behavior. This interest is motivated largely by the desire to estimate accurately the stresses and strains which develop within the fuel pin cladding during operation. This predictive capability is essential if the fuel element designer is to prevent cladding failure without incorporating an undue degree of conservatism into the design and/or compromising the performance capability of the fuel pin. Adequate computational models would facilitate the interpretation and correlation of experimental data obtained from fuel element burnup experiments, and make extrapolation to other operating conditions possible. Additionally, such models are helpful in identifying important parameters, and supply guidance for further refinements.

Fuel element mechanical behavior is an especially important problem in connection with liquid metal cooled fast breeder reactor development. If such a system is to realize its full potential, very high burnup levels (on the

order of 10^5 Mwd/MT) are necessary, and this may place very severe demands upon the fuel element cladding. Several important deformation mechanisms are potentially operative in producing cladding strain in such a reactor:

1. Volumetric expansion of the fuel because of irradiation induced change in the crystal structure, as well as the accommodation of both gaseous and solid fission products.
2. Fuel geometry changes due to complicated and poorly understood restructuring processes.
3. Volumetric expansion of the stainless steel cladding itself due to irradiation effects.
4. Direct gas pressure resulting from the accumulation of gaseous fission products released from the fuel.
5. Thermal expansion of both fuel and cladding.

Prevailing opinion has varied considerably with time regarding the relative importance of these several "driving forces." It was recognized early that the high power densities and small fuel pin diameters in a large fast reactor could produce considerable fission gas pressure. Accordingly, design practice has been either to provide a large plenum to accommodate the gases or to vent the fuel element. Also, early irradiation tests indicated that fuel swelling may be excessive and could result in considerable pressure upon the cladding [1, 2]. The design solution was

to provide lower density fuel to accommodate the swelling. Finally, the discovery of stainless steel swelling has led to the conclusion, at least in some quarters, that fuel-cladding interaction is not likely to be a limitation, and that the chief problem of fast reactor fuel design has shifted from cladding failure to difficulties outside the cladding, such as behavior of the fuel pin spacer system [1].

Whatever the ultimate resolution of this still ongoing controversy may be, the need remains for analytical models to predict the response of the fuel element to all of these mechanisms. The formulation of such a model can be thought of as a several step procedure. First, relationships which describe the stress or strain produced by each of the driving forces must be derived or postulated. Next, relations between stress and strain, together with the conditions of stress equilibrium and strain compatibility produce a set of equations involving only stress and strains as unknowns. Finally, the solution of these equations, if possible, produces a map of the stresses and strains throughout the fuel element.

In connection with the stress-strain relationships, it must be realized that there may well be significant inelastic response, in addition to the classical elastic behavior described by Hooke's law. In view of the high temperatures involved, creep is especially to be

anticipated. Indeed, because of the high neutron flux, so-called "irradiation-enhanced creep" may well be important.

Several computer programs have been produced to attack this problem, and they can more or less be divided into two broad categories, differing primarily in their description of the driving forces. In the first category, the codes are phenomenological in nature, drawing heavily upon macrocosmic experimental data to describe fuel behavior. For example, empirical correlations are used for the relationship of fuel swelling and gas release to temperature and burnup. The SWELL [3] and OLYMPUS [4] codes are examples of programs in this category.

In the second category, detailed models of microscopic processes are incorporated in an attempt to describe the restructuring, swelling, and gas release behavior of the oxide fuel. Oxide fuel under irradiation undergoes restructuring shortly after startup, dividing the fueled portion of the pin into four more or less distinct regions [5]:

1. A central void.
2. A columnar grain region ($> 1800^{\circ}\text{C}$).
3. An equiaxed grain region ($\sim 1500^{\circ}\text{C}$).
4. An outer zone ($\sim 700^{\circ}\text{C}$ to 1500°C) with structure unchanged from the initial condition.

Swelling and gas release characteristics of the three fuel regions vary considerably. Computer codes such as CYGRO [6],

LIFE [7], and FMODEL [8], contain models to describe the nucleation, growth, and migration of fission gas bubbles, and in turn, to predict the fuel swelling. This information is then used to compute mechanical response.

Although the development of relatively elaborate computer programs such as those mentioned above have received considerable attention, there is little evidence in the literature of attempts to obtain analytical solutions to the stress analysis problem. This is perhaps not surprising, since any fuel element model retaining descriptions of the important deformation driving forces which are sufficiently realistic for design purposes seems hopelessly too complicated for analytical solution. The urgent pressure for the development of an adequate design tool has probably led to the apparent neglect of the analytical approach.

Primary among the analytical difficulties are the facts that the problem is inelastic, and the mechanical properties are temperature dependent (and therefore space dependent). In the face of this, even if creep is neglected, the classical texts on the theory of elasticity [9, 10] and thermal stress analysis [11] offer solutions only for single region cylinders with constant mechanical properties. A cylindrical two region solution is, however, given by Faupel [12]. Chang and Chu [13] report an algebraically formidable elastic stress solution for a metal tube with a

particular kind of temperature dependent properties. Some solutions for single region cylinders have also been reported with creep laws which are linear in stress [11, 12].

A fuel element model demands two regions at the minimum: one fuel region and one cladding region. No two region cylindrical solution has been found which contains even the simplest creep law. It is this kind of solution which is presented here.

It is felt that such a solution is of value in at least two important ways, even though it may contain idealizations which remove it so far from reality as to render it useless for design purposes. First a valuable "benchmark" can be provided for use by the programmer or user of more realistic computer models. The computer code can be made to solve the same simplified problem, and any discrepancy could be helpful in revealing programming errors or shortcomings in numerical approximation. This is a procedure often used in engineering practice. One notable example is in the problem of computation of neutron transport so often encountered by the nuclear engineer. Exact solutions of the neutron transport equation are available for a few simple problems, most notably the Milne problem [14, 15]. Although these simple problems usually bear little resemblance to those encountered in practice, they are nevertheless useful since they provide an exact solution

to problems which can also be solved approximately with one of the neutron transport codes in common use. Comparison of the computer solution with the known exact solution can then be of assistance both in revealing possible inadequacies in the code, and in providing guidance to the engineer regarding improvement of accuracy in the approximate solution.

Although it may have been done, no mention has been found in the literature of such a comparison being made in the case of the fuel element modeling codes. Rather, the emphasis seems to have been on comparison of computer computations with experimental results. This is surely a necessary thing to do, but a comparison with analytical results could be very useful as well. The absence of such activity is doubtless due to the pressure for design methods mentioned above, but may also be due in part to the lack of an appropriate analytical solution for comparison. It is hoped that the solution presented here will be helpful in this respect.

Secondly, from a simplified analytical solution some understanding of the overall problem can be gained, and the relative importance of various parameters can be estimated. Again we can find an analogy in the realm of reactor neutronics, if we consider the use of simple one group diffusion theory. Although the analysis of reactor core behavior via one group diffusion theory rarely is

sufficiently accurate for design purposes, this type of approach nevertheless has great pedagogical value. Qualitatively correct conclusions can be drawn and even first order quantitative estimates can be made. Hopefully the simplified fuel element stress analysis presented here can fulfill a similar role. This could be especially useful to engineers and designers previously inexperienced in fuel element stress analysis.

Accordingly, a simplified fuel element model is suggested here. Geometrically the proposed model consists of two concentric cylindrical regions: an inner fuel region (designated region 1) with the possibility of a central hole, and an outer cladding region (designated region 2). There may be a gap between fuel and cladding regions (see Figure 1).

Several important idealizations are introduced which are merely mentioned here and discussed more fully later. A state of plane strain and azimuthal symmetry is assumed. The azimuthal symmetry assumption is perhaps at variance with the observation of radial cracks in irradiated fuel. It is not certain, however, that these cracks are present under operating conditions. It is further necessary to postulate that the mechanical properties are independent of space (and therefore temperature) within each region, and also that creep rates are linearly related to stress. Thermal conductivities and coefficients of linear expansion

Figure 1. Geometrical Model for Fuel Element

can be retained in temperature dependent form. Also, phenomenological swelling relationships of the type used in the first category of computer codes can be used in this analytical study.

~ In Chapter 2, the governing differential equations are introduced. Chapter 3 discusses the method of solution, and Chapters 4 and 5 present the solution of the equations. Chapter 6 contains a numerical evaluation of the solutions for a choice of geometry and driving forces which are believed to represent a fast reactor fuel pin as accurately as possible within the restrictions of the model. It is found that for this problem, the fuel does not contact the cladding. Therefore, in order to investigate the effect of a fuel-cladding interaction, the geometry and fluence-to-burnup ratio are modified somewhat so as to produce an interaction, and numerical evaluation of the solution in this case is presented in Chapter 7. Throughout Chapters 2 through 7, many of the algebraic and computational details which are tedious but not fundamentally important are relegated to appendices. Chapter 8 presents a summary and conclusions.

CHAPTER 2

GOVERNING EQUATIONS AND IDEALIZATIONS

2.1 Stress-Strain Relations

The strains in the fuel element can be expressed symbolically as the sum of several contributions, which are discussed individually below:

$$\epsilon_r = \epsilon_r^E + \epsilon_r^C + \epsilon_r^T + \epsilon^S \quad (1a)$$

$$\epsilon_\theta = \epsilon_\theta^E + \epsilon_\theta^C + \epsilon_\theta^T + \epsilon^S \quad (1b)$$

$$\epsilon_z = \epsilon_z^E + \epsilon_z^C + \epsilon_z^T + \epsilon^S \quad (1c)$$

where

ϵ_j^E = components of elastic strain

ϵ_j^C = components of creep strain

$j = r, \theta, z$

ϵ^T = thermal expansion strain

ϵ^S = volumetric swelling due to irradiation effects.

Both ϵ^T and ϵ^S have been assumed to be isotropic, and it is also assumed that there are no plastic strains.

The elastic strains are given by Hooke's law:

$$\epsilon_r^E = \frac{1}{E} [\sigma_r - \nu(\sigma_\theta + \sigma_z)] \quad (2a)$$

$$\epsilon_{\theta}^E = \frac{1}{E} [\sigma_{\theta} - \nu(\sigma_r + \sigma_z)] \quad (2b)$$

$$\epsilon_z^E = \frac{1}{E} [\sigma_z - \nu(\sigma_r + \sigma_{\theta})] \quad (2c)$$

where E is Young's modulus, ν is Poisson's ratio, and σ_r , σ_{θ} , and σ_z are the radial, azimuthal, and axial components of the stress.

The thermal expansion term can be expressed as

$$\epsilon^T = \int_{T_0}^T \alpha(T') dT'$$

where $\alpha(T')$ is the linear coefficient of thermal expansion and T_0 is a reference temperature (where ϵ^T is considered to be zero). The thermal strain can be determined independently of the stress analysis, except insofar as the temperature distribution is affected by variation in the gap width, an effect which is neglected in the present approach.

The swelling strain will be discussed more fully later when a specific numerical problem is considered. For the present, it is sufficient to note that the swelling strain is also assumed independent of the stress levels. It can, however, vary with time and/or temperature. Thus, for the purposes of the stress analysis, both ϵ^T and ϵ^S may be considered as known.

In the case of the creep strain, most creep laws are formulated in a manner to reflect the experimentally observed fact that creep does not occur in a hydrostatic stress

state [11, p. 461]; i.e., a state of stress where $\sigma_r = \sigma_\theta = \sigma_z$. To incorporate this observation, the deviatoric stress, S_j , is normally introduced:

$$\begin{aligned} S_r &= \sigma_r - \sigma_m \\ S_\theta &= \sigma_\theta - \sigma_m \\ S_z &= \sigma_z - \sigma_m \\ \sigma_m &= \frac{1}{3} (\sigma_r + \sigma_\theta + \sigma_z). \end{aligned}$$

σ_m is called the mean, or "hydrostatic" stress. The usual postulate then is that the stress dependence of creep rates can contain only the deviatoric stress.

Creep strain in a reactor is usually modeled as the sum of two parts: thermally activated creep and radiation induced creep. Creep rates from both parts can be highly temperature-dependent. Data indicate that thermal creep is linearly related to stress at low stresses, but is highly nonlinear at high stresses. Radiation induced creep, on the other hand, is apparently linear in stress and athermal at lower temperatures [7, 16].

For the present approach, it is necessary to postulate that the creep rate is temperature independent and linearly related to stress, since it is this kind of creep law which admits an analytical solution. The simplest law of this type is

$$\dot{\epsilon}_j^C = \frac{d}{dt} \epsilon_j^C = h S_j \quad (3)$$

sometimes referred to as the Maxwell model [11]. Other more general, but still linear, creep laws are permissible within the analysis technique presented here and are discussed in Appendix A. However, it is very difficult with available data to select meaningful values for the parameters which appear in these more complicated expressions. Furthermore, it is felt that Eq. (3) serves adequately to accomplish the limited objectives of the present work.

2.2 Equilibrium and Compatibility Equations

Under the assumptions of azimuthal symmetry and plane strain, all θ and z dependence disappears, and the stress equilibrium equations reduce to [11, Chapter 8]:

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0. \quad (4)$$

By plane strain, it is meant that ϵ_z is assumed to be space independent, although it may be time dependent. Physically, this implies that plane cross sections of the fuel element remain planar after deformation, an approximation often made for long, thin bodies. For example, the fuel element computer models mentioned previously typically invoke the plane strain assumption [7, 8].

The compatibility equation relevant to this model is [11, Chapter 8]:

$$\frac{d}{dr} (r\epsilon_\theta) = \epsilon_r. \quad (5)$$

2.3 Boundary Conditions

If the fuel-cladding gap is open, the relevant stress boundary conditions are

$$\sigma_{r,1}(a_1) = -p_0 \quad (6a)$$

$$\sigma_{r,1}(b_1) = -p_1 \quad (6b)$$

$$\sigma_{r,2}(a_2) = -p_1 \quad (6b)$$

$$\sigma_{r,2}(b_2) = -p_2 \quad (6d)$$

where by $\sigma_{r,n}$ is meant the radial stress in region n . The usual convention is adopted that a compressive stress is negative.

As long as the fuel and cladding are not in contact, Eqs. (6) provide a complete set of boundary conditions, and p_1 must be independently specified. One reasonable assumption would be that $p_1 = p_0$ since the central void and fuel-cladding gap are likely to communicate via the axial plenum.

If the fuel and cladding are in contact, Eqs. (6) still apply, but p_1 becomes an interaction pressure, p_{int} , which is determined by application of the continuity

condition

$$b_1 + u_1(b_1) = a_2 + u_2(a_2) \quad (7)$$

where $u_n(r)$ is the displacement of point r in region n from its initial position. Note that

$$\epsilon_{\theta,n}(r) = \frac{u_n(r)}{r}.$$

In cases where it is not known a priori whether or not the gap has closed, either condition can be assumed and the result will reveal whether or not the assumption is correct. If Eq. (7) is used and p_{int} is found to be negative, then condition (7) is not applicable and p_1 must be taken to be its independently-specified value. (This assumes that the fuel-clad interface cannot support tension.) On the other hand, if Eqs. (6b) and (6c) are applied directly using the fission gas pressure for p_1 , and it is found that

$$u_1(b_1) - u_2(a_2) > a_2 - b_1$$

then clearly the gap has closed and (7) becomes appropriate.

The value of the constant axial strain is evaluated by the condition

$$\int_{a_n}^{b_n} 2\pi r \sigma_{z,n}(r) dr = F_n$$

where F_n is the applied axial load on region n . This implies that if fuel and cladding are in contact, they can

nevertheless move axially independently with no friction between them. An alternative assumption, which still allows a solution, is the opposite extreme, namely that there is "complete" friction between fuel and cladding and that $\epsilon_{z,1} = \epsilon_{z,2}$. The true condition lies somewhere between these extremes, and there is no way to know which assumption is more realistic. The former condition leads to much reduced algebraic complexity and is therefore adopted here. Also, in the present analysis it is assumed that there are no axial loads other than those imposed by fission gas pressure in the axial plenum. These loads are given by

$$F_1 = p_0 \pi (b_1^2 - a_1^2)$$

$$F_2 = p_0 \pi a_2^2 - p_2 \pi b_2^2 .$$

2.4 Heat Conduction Equation

The thermal expansion strain is clearly temperature dependent. In addition, the data for swelling strain exhibit temperature dependence. Thus, computation of these quantities requires knowledge of the temperature distribution, which is determined from solution of the heat conduction equation

$$\frac{1}{r} \frac{d}{dr} \left(r k \frac{dT}{dr} \right) + Q''' = 0 \quad (8)$$

where k is thermal conductivity and Q''' is the volumetric heat generation rate. Solution of this equation is

facilitated by definition of a pseudo temperature Θ

$$\Theta = \frac{1}{k_r} \int_{T_r}^T k(T') dT'$$

where k_r and T_r are convenient constants. Noting that

$$k \frac{dT}{dr} = k_r \frac{d\Theta}{dr}.$$

Eq. (8) can be written

$$k_r \frac{1}{r} \frac{d}{dr} \left(r \frac{d\Theta}{dr} \right) + Q'' = 0. \quad (9)$$

This equation is readily solved, given appropriate boundary conditions. A large catalog of such solutions to Eq. (9) exists [17]. Since $\Theta(T)$ is easily constructed knowing $k(T)$, the temperature distribution is determined.

In the formulation of Eqs. (8) and (9), axial conduction is neglected, and also a quasi-steady state is assumed. That is, it is assumed that any thermal transients which occur take place with time constants which are short compared to the time scale of interest in the problem. Thus, the temperature is assumed to "follow" the power density, Q'' , with zero time lag.

CHAPTER 3

METHOD OF SOLUTION

With the linear creep law of Eq. (3), a closed form solution for the stresses and strains can be found by use of the "viscoelastic-elastic analogy," a method discussed in classic texts [11]. This approach utilizes the Laplace transform of the solution to a related elastic problem to deduce the Laplace transform of the solution to the problem with creep. A simple inversion of the transform then yields the solution to the creep problem. To illustrate, consider the related thermoelastic problem represented by Eqs. (1) through (9) but with no creep; that is, with $\epsilon_j^C = 0$. Denote the stresses and strains in this related problem with an asterisk (*).

$$\epsilon_r^* = \frac{1}{E} [\sigma_r^* - \nu(\sigma_\theta^* + \sigma_z^*)] + \epsilon^T + \epsilon^S \quad (10.a)$$

$$\epsilon_\theta^* = \frac{1}{E} [\sigma_\theta^* - \nu(\sigma_r^* + \sigma_z^*)] + \epsilon^T + \epsilon^S \quad (10.b)$$

$$\epsilon_z^* = \frac{1}{E} [\sigma_z^* - \nu(\sigma_r^* + \sigma_\theta^*)] + \epsilon^T + \epsilon^S \quad (10.c)$$

$$\frac{d}{dr} (r\sigma_r^*) = \sigma_\theta^* \quad (10.d)$$

$$\frac{d}{dr} (r\epsilon_\theta^*) = \epsilon_r^* \quad (10.e)$$

It will be seen in the next chapter that the problem posed

by these equations together with boundary conditions is easily solved. Therefore, for the moment, the solution to this related problem may be considered known.

This solution can now serve as a basis to obtain the solution to the original problem, including creep. Since parallel remarks apply to all three components, only the radial component will be discussed. Equation (10.a) can be recast into the form

$$\epsilon_r^* = \frac{1+\nu}{E} S_r^* + \frac{1-2\nu}{E} \sigma_m^* + \epsilon^T + \epsilon^S \quad (11)$$

where, as previously,

$$S_r^* = \sigma_r^* - \sigma_m^*$$

$$\sigma_m^* = \frac{1}{3} (\sigma_r^* + \sigma_\theta^* + \sigma_z^*).$$

Now consider the original problem with creep included. From Eqs. (1.a), (2.a), and (3), application of the Laplace transform yields

$$\bar{\epsilon}_r = \left(\frac{1+\nu}{E} + \frac{h}{s} \right) \bar{S}_r - \frac{1-2\nu}{E} \bar{\sigma}_m + \bar{\epsilon}^T + \bar{\epsilon}^S \quad (12)$$

where

$$\bar{\epsilon}_r = \int_0^\infty e^{-st} \epsilon_r(t) dt.$$

Define quantities E' and ν' such that

$$\frac{1+\nu'}{E'} = \frac{1+\nu}{E} + \frac{h}{s} \quad (13)$$

$$\frac{1-2\nu'}{E'} = \frac{1-2\nu}{E}. \quad (14)$$

Equation (12) is then identical in form to the transform of Eq. (11), except that the new "constants" E' and ν' have appeared. The following solution procedure is therefore suggested:

1. Obtain the solution to the related thermoelastic problem.
2. Form the Laplace transform of the result of Step 1.
3. Substitute E' and ν' for E and ν respectively into the result of Step 2.
4. Invert the Laplace transform obtained from Step 3 into the time domain. This is the solution to the original problem with creep.

This approach is quite powerful as long as the linear creep law is acceptable. Solutions to thermoelastic problems normally involve the constants E and ν only in rational functions. Since the "pseudo-constants" E' and ν' are rational functions of the transform variable, it follows that the Laplace transform obtained by the analogy method will contain only rational functions of the transform variable. The inversion of the transform should therefore present no great difficulty, and the final result in general will involve convolution integrals and/or time derivatives of the elastic solution.

The analogy method can be applied to problems which are more difficult than the one considered here. For

example, the method will work with more general linear creep laws (see Appendix A). The quantities E' and ν' are then determined by more complex relations than Eqs. (13) and (14) but the methodology is unchanged. This approach even has application to problems which are too complicated to admit an exact analytical solution for the related elastic problem. Examples of problems in this category would be those of geometrical complexity, or with temperature-dependent mechanical properties. For problems of this type, one of the approximate analytical methods, for example the method of stationary complementary energy, can be used to attack the related elastic problem. The approximate elastic solution thus obtained is then Laplace transformed, the quantities E and ν replaced by E' and ν' , and the resulting transform inverted to obtain an approximate solution to the problem with creep. A serious shortcoming of these approximate methods is that there is no a priori way to estimate the error. This approach is therefore at variance with the objective of providing an exact "benchmark" and consequently is not used here.

The next chapter presents the solution of the related thermoelastic problem for the fuel element model proposed earlier. Chapter 5 then utilizes the analogy method to produce the solution for stresses and strains in the fuel element in the presence of creep.

CHAPTER 4

SOLUTION OF THE RELATED THERMOELASTIC PROBLEM

Algebraic manipulations (presented in Appendix B.1) reduce Eqs. (10.a) through (10.e) to a single differential equation in σ_r^* for each region.

$$\frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} (r^2 \sigma_{r,n}^*) \right] = \frac{E_n}{1-\nu_n} \frac{d}{dr} (\epsilon^T + \epsilon^S) \quad (15)$$

$n = 1$ for fuel region

$n = 2$ for cladding region.

In the development of this equation, the properties E_n and ν_n were taken to be independent of r (and therefore temperature).

This equation is readily integrated to determine $\sigma_{r,n}^*$, and when the boundary conditions (6) are applied to evaluate the integration constants, the result is (see Appendix B.2 for the details):

$$\begin{aligned} \sigma_{r,n}^* = & -\left(\frac{b_n^2}{r^2} - 1\right) \frac{p_{n-1}}{b_n^2/a_n^2 - 1} - \left(1 - \frac{a_n^2}{r^2}\right) \frac{p_n}{1 - a_n^2/b_n^2} \\ & + \frac{1}{2} \frac{E_n}{1-\nu_n} \left(1 - \frac{a_n^2}{r^2}\right) (\hat{\epsilon}_n^T + \hat{\epsilon}_n^S) \\ & - \frac{1}{r^2} \frac{E_n}{1-\nu_n} \int_{a_n}^r r' (\epsilon^T + \epsilon^S) dr'. \end{aligned} \quad (16)$$

$\hat{\epsilon}_n^T$ and $\hat{\epsilon}_n^S$ are area averages of ϵ^T and ϵ^S over region n. For example

$$\hat{\epsilon}_n^T = \frac{2\pi \int_{a_n}^{b_n} r \epsilon^T(r) dr}{\pi(b_n^2 - a_n^2)}.$$

Physically, the first two terms in Eq. (16) represent the compressive radial stress due to the pressures at the boundaries of the region. The last two terms give the stress due to spatial variations in ϵ^T and ϵ^S . This is easily confirmed by considering the case where no such variations exist, or where $\frac{d}{dr} (\epsilon^T + \epsilon^S) \equiv 0$. In this event, $(\epsilon^T + \epsilon^S) \equiv (\hat{\epsilon}_n^T + \hat{\epsilon}_n^S) = \text{a constant}$, and the final two terms exactly cancel.

With the radial stress, $\sigma_{r,n}^*$, known, all other stresses and strains are readily computed. The hoop stress, $\sigma_{\theta,n}^*$, is determined by Eqs. (16) and (10.d).

$$\begin{aligned} \sigma_{\theta,n}^* = & \left(1 + \frac{b_n^2}{a_n^2}\right) \frac{p_{n-1}}{b_n^2/a_n^2 - 1} - \left(1 + \frac{a_n^2}{r^2}\right) \frac{p_n}{1 - a_n^2/b_n^2} \\ & + \frac{1}{2} \frac{E_n}{1-\nu_n} \left(1 + \frac{a_n^2}{r^2}\right) (\hat{\epsilon}_n^T + \hat{\epsilon}_n^S) \\ & + \frac{1}{r^2} \frac{E_n}{1-\nu_n} \int_{a_n}^r r' (\epsilon^T + \epsilon^S) dr' \\ & - \frac{E_n}{1-\nu_n} (\epsilon^T + \epsilon^S). \end{aligned} \quad (17)$$

Here again, the last three terms give the stress contribution due to radial variations in the swelling and thermal expansion.

The axial stress and strain are now found (see Appendix B.3).

$$\epsilon_{z,n}^* = (\hat{\epsilon}_n^T + \hat{\epsilon}_n^S) + \frac{1}{E_n} \frac{F_n}{\pi(b_n^2 - a_n^2)} - \frac{2\nu_n}{E_n} \frac{b_n^2 p_n - a_n^2 p_{n-1}}{b_n^2 - a_n^2} \quad (18)$$

$$\sigma_{z,n}^* = \frac{E_n}{1-\nu_n} [(\hat{\epsilon}_n^T + \hat{\epsilon}_n^S) - (\epsilon_n^T + \epsilon_n^S)] + \frac{F_n}{\pi(b_n^2 - a_n^2)} \quad (19)$$

The last two terms in (18) give the axial strain due to the axial load and the boundary pressures respectively, and the first term represents the average swelling and thermal expansion strain. The axial stress is entirely due to the departure of the local values of swelling and thermal expansion from the average values, plus that due directly to the imposed axial load.

With all stresses known, ϵ_r^* and ϵ_θ^* are directly available from Eqs (10.a) and (10.b). For example, the hoop strain is given by

$$\begin{aligned} \epsilon_{\theta,n}^* = & \frac{1}{E_n} \left\{ \left[(1-\nu_n) + \frac{b_n^2}{r^2} (1+\nu_n) \right] \frac{p_{n-1}}{b_n^2/a_n^2 - 1} \right. \\ & - \left. \left[(1-\nu_n) + \frac{a_n^2}{r^2} (1+\nu_n) \right] \frac{p_n}{1 - a_n^2/b_n^2} \right. \\ & + \left. \frac{1}{2} \frac{E_n}{1-\nu_n} (\hat{\epsilon}_n^T + \hat{\epsilon}_n^S) \left[(1-3\nu_n) + (1+\nu_n) \frac{a_n^2}{r^2} \right] \right\} \end{aligned}$$

$$\begin{aligned}
& + \frac{E_n}{1-\nu_n} \frac{1}{r^2} (1+\nu_n) \int_{a_n}^r r' (\epsilon^T + \epsilon^S) dr' \} \\
& - \frac{\nu_n}{E_n} \frac{F_n}{\pi(b_n^2 - a_n^2)} .
\end{aligned} \tag{20}$$

At the boundaries of the region, this expression simplifies to

$$\begin{aligned}
\epsilon_{\theta,n}^*(r=a_n) &= \frac{p_{n-1}}{E_n} \left[\frac{b_n^2 + a_n^2}{b_n^2 - a_n^2} + \nu_n \right] - \frac{2b_n^2}{b_n^2 - a_n^2} \frac{p_n}{E_n} \\
&+ (\hat{\epsilon}_n^T + \hat{\epsilon}_n^S) - \frac{\nu_n}{E_n} \frac{F_n}{\pi(b_n^2 - a_n^2)}
\end{aligned} \tag{21}$$

$$\begin{aligned}
\epsilon_{\theta,n}^*(r=b_n) &= \frac{p_{n-1}}{E_n} \frac{2a_n^2}{b_n^2 - a_n^2} - \frac{p_n}{E_n} \left[\frac{b_n^2 + a_n^2}{b_n^2 - a_n^2} - \nu_n \right] \\
&+ (\hat{\epsilon}_n^T + \hat{\epsilon}_n^S) - \frac{\nu_n}{E_n} \frac{F_n}{\pi(b_n^2 - a_n^2)} .
\end{aligned} \tag{22}$$

The solution of the related thermoelastic problem is therefore complete, provided that the fuel-cladding gap is open and the pressure p_1 is therefore known. When the fuel and cladding regions are in contact, condition (7) must be applied with the aid of (21) and (22) to determine p_{int}^* .

If this is done, we find

$$p_{int}^* = \frac{b_1 - a_2}{I} + \frac{1}{I} \left[\frac{a_1^2}{b_1^2 - a_1^2} \frac{2b_1 p_0}{E_1} + \frac{b_2^2}{b_2^2 - a_2^2} \frac{2a_2 p_2}{E_2} \right]$$

$$\begin{aligned}
& + \frac{\nu_1 b_1 p_o}{E_1} + \frac{\nu_2 a_2}{E_2} \frac{a_2^2 p_o - b_2^2 p_2}{b_2^2 - a_2^2}] \\
& + \frac{1}{I} [b_1(\hat{\epsilon}_1^T + \hat{\epsilon}_1^S) - a_2(\hat{\epsilon}_2^T + \hat{\epsilon}_2^S)] \quad (23)
\end{aligned}$$

where the "interaction constant," I , is defined as

$$I = \frac{a_2}{E_2} \left(\frac{b_2^2 + a_2^2}{b_2^2 - a_2^2} + \nu_2 \right) + \frac{b_1}{E_1} \left(\frac{b_1^2 + a_1^2}{b_1^2 - a_1^2} - \nu_1 \right). \quad (24)$$

The interaction constant contains the geometries and elastic properties of both regions. The two physical mechanisms which contribute to the interaction pressure are clearly displayed in Eq. (23) by the second and third terms. The second term gives the contribution due to the pressure which exists in the central void (p_o) and the coolant pressure (p_2) exterior to the cladding. The third term is the contribution due to unequal values of the average swelling and thermal expansion in fuel and cladding. When an initial gap is present, $a_2 > b_1$, the first term of (23) reduces p_{int} accordingly.

Even this elastic solution is not given in the classic treatises on the theory of elasticity such as those by Timoshenko and Goodier [9] and by Love [10], consideration of cylinders there being limited to single regions. Faupel [12] presents a solution in essentially the form given here. The differences incorporated here are the

provision for an initial fuel-cladding gap and the inclusion of swelling strain.

CHAPTER 5

SOLUTION WITH CREEP INCLUDED

The procedure outlined in Chapter 3 can now be used with the results from Chapter 4 to obtain the solution to the original problem with the effects of creep included. For example, consider first the axial stress since it is by far the simplest algebraically. From Eq. (19) and the discussion of Chapter 3, the Laplace transform of the axial stress can be written directly as

$$\bar{\sigma}_{z,n} = \frac{E'_n}{1-\nu'_n} [\bar{\epsilon}_n^T + \bar{\epsilon}_n^S - \bar{\epsilon}^T - \bar{\epsilon}^S] + \frac{\bar{F}_n}{\pi(b_n^2 - a_n^2)}. \quad (25)$$

From (13) and (14),

$$\begin{aligned} \frac{1-\nu'_n}{E'_n} &= \frac{1-\nu_n}{E_n} + \frac{1}{3} \frac{h_n}{s} \\ \frac{E'_n}{1-\nu'_n} &= \frac{E_n}{1-\nu_n} \frac{s}{s + \frac{1}{3} \frac{h_n E_n}{1-\nu_n}}. \end{aligned}$$

Define a new constant, β_n ,

$$\beta_n = \frac{1}{3} \frac{h_n E_n}{1-\nu_n}.$$

Then

$$\frac{E'_n}{1-\nu'_n} = \frac{E_n}{1-\nu_n} \frac{s}{s + \beta_n}.$$

The inverse Laplace transform of this latter expression can be directly evaluated.

$$L^{-1}\left\{\frac{E'_n}{1-\nu'_n}\right\} = \frac{E_n}{1-\nu_n} [\delta(t) - \beta_n e^{-\beta_n t}] \quad (26)$$

where $\delta(t)$ is the Dirac delta function [18, p. 27]. The inverse transform of Eq. (25) can now be evaluated with the aid of the convolution theorem.

$$\begin{aligned} \sigma_{z,n}(r,t) = & \frac{E_n}{1-\nu_n} \{[\hat{\epsilon}_n^T(t) + \hat{\epsilon}_n^S(t)] - [\epsilon^T(r,t) + \epsilon^S(r,t)]\} \\ & - \frac{E_n}{1-\nu_n} \beta_n \int_0^t e^{-\beta_n(t-\tau)} \{[\hat{\epsilon}_n^T(\tau) + \hat{\epsilon}_n^S(\tau)] - [\epsilon^T(r,\tau) + \epsilon^S(r,\tau)]\} d\tau \\ & + \frac{F_n(t)}{\pi(b_n^2 - a_n^2)}. \end{aligned}$$

Utilizing (19), this can be written more compactly as

$$\begin{aligned} \sigma_{z,n}(r,t) = & \sigma_{z,n}^*(r,t) - \beta_n \int_0^t e^{-\beta_n(t-\tau)} \\ & [\sigma_{z,n}^* - \frac{F_n}{\pi(b_n^2 - a_n^2)}] d\tau. \end{aligned} \quad (27)$$

Thus, the axial stress in the presence of creep is equal to the axial stress in the related thermoelastic problem reduced by the "corrective" term due to the influence of creep. The latter exhibits properties which are consistent with physical intuition. First, Eq. (27) shows that $\sigma_{z,n}(r,t) \leq \sigma_{z,n}^*(r,t)$, reflecting the fact that creep

produces a stress relaxation with time. Secondly, $\sigma_{z,n}(r,t)$ depends upon the entire previous stress history. Finally, Eq. (27) exhibits a "fading memory;" that is, in the "correction" term, stresses at recent times are weighted more heavily than stresses at remote times through the $e^{-\beta_n(t-\tau)}$ factor. All of these phenomena are characteristic of creep behavior.

In a similar manner, Eq. (26) can be used in conjunction with Eqs. (16) and (17) to obtain the other stresses:

$$\begin{aligned} \sigma_{r,n}(r,t) = & -\left(\frac{b_n^2}{r^2} - 1\right) \frac{a_n^2}{b_n^2 - a_n^2} p_{n-1}(t) - \left(1 - \frac{a_n^2}{r^2}\right) \frac{b_n^2}{b_n^2 - a_n^2} p_n(t) \\ & + \frac{1}{2} \frac{E_n}{1-\nu_n} \left(1 - \frac{a_n^2}{r^2}\right) [\hat{\epsilon}_n^T(t) + \hat{\epsilon}_n^S(t)] - \frac{1}{r^2} \frac{E_n}{1-\nu_n} \int_{a_n}^r r' (\epsilon^T + \epsilon^S) dr' \\ & - \beta_n \frac{E_n}{1-\nu_n} \int_0^t e^{-\beta_n(t-\tau)} \left\{ \frac{1}{2} \left(1 - \frac{a_n^2}{r^2}\right) [\hat{\epsilon}_n^T(\tau) + \hat{\epsilon}_n^S(\tau)] \right. \\ & \left. - \frac{1}{r^2} \int_0^r r' [\epsilon^T(r',\tau) + \epsilon^S(r',\tau)] dr' \right\} d\tau. \end{aligned} \quad (28)$$

$$\begin{aligned} \sigma_{\theta,n}(r,t) = & \left(1 + \frac{b_n^2}{r^2}\right) \frac{a_n^2}{b_n^2 - a_n^2} p_{n-1}(t) - \left(1 + \frac{a_n^2}{r^2}\right) \frac{b_n^2}{b_n^2 - a_n^2} p_n(t) \\ & + \frac{E_n}{1-\nu_n} \left\{ \frac{1}{2} \left(1 + \frac{a_n^2}{r^2}\right) [\hat{\epsilon}_n^T(t) + \hat{\epsilon}_n^S(t)] + \frac{1}{r^2} \int_{a_n}^r r' \right. \\ & \left. [\epsilon^T(r',t) + \epsilon^S(r',t)] dr' \right\} \end{aligned}$$

$$\begin{aligned}
& -[\epsilon^T(r', t) + \epsilon^S(r', t)] \} \\
& - \beta_n \frac{E_n}{1-\nu_n} \int_0^t e^{-\beta_n(t-\tau)} \left\{ \frac{1}{2} \left(1 + \frac{a_n^2}{r^2} \right) [\hat{\epsilon}_n^T(\tau) + \hat{\epsilon}_n^S(\tau)] \right. \\
& \quad + \frac{1}{r^2} \int_{a_n}^r r' [\epsilon^T(r', \tau) + \hat{\epsilon}_n^S(\tau)] dr' \\
& \quad \left. - [\epsilon^T(r', \tau) + \epsilon^S(r', \tau)] \right\} d\tau. \quad (29)
\end{aligned}$$

When the fuel and cladding are in contact, the interface pressure, p_{int} , also must be determined by the analogy method, utilizing Eqs. (23), (24), (13), and (14). This algebraically tedious procedure is outlined in Appendix B.4. The result is

$$\begin{aligned}
p_{int}(t) &= p_{int}^*(t) + \int_{t_{int}}^t e^{-\lambda(t-\tau)} [f(\tau) - \lambda p_{int}^*(\tau)] d\tau \\
f(t) &= \frac{1}{I} \left[\frac{4b_1 a_1^2 h_1}{3(b_1^2 - a_1^2)} p_0(t) + \frac{4a_2 b_2^2 h_2}{3(b_2^2 - a_2^2)} p_2(t) \right. \\
&\quad \left. + \frac{b_1 h_1}{3} p_0(t) + \frac{a_2 h_2}{3} \frac{a_2^2 p_0(t) - b_2^2 p_2(t)}{b_2^2 - a_2^2} \right] \quad (30)
\end{aligned}$$

where λ is defined as

$$\lambda = \frac{1}{3I} \left[a_2 h_2 \left(2 \frac{b_2^2 + a_2^2}{b_2^2 - a_2^2} + 1 \right) + b_1 h_1 \left(2 \frac{b_1^2 + a_1^2}{b_1^2 - a_1^2} - 1 \right) \right]. \quad (31)$$

λ can be thought of physically as an "interaction relaxation constant." Note that it involves the geometries, elastic properties (in I), and creep properties of both regions.

Here t_{int} is the time at which fuel and cladding most recently made contact and boundary condition (7) therefore commenced to apply.

An example of the application of these results follows in the next chapter.

CHAPTER 6

NUMERICAL EXAMPLE

6.1 Introductory Remarks

In this chapter, the stress analysis solution presented earlier is applied to a specific hypothetical fuel element operating at a constant linear heat rate of 15 kw/ft. An attempt has been made to select dimensions and other parameters which are representative of a 1000 MWe LMFBFR. A summary of the values for several of the important parameters is given in Table 1.

The geometry is as shown in Figure 1, with UO_2 fuel and stainless steel cladding. The value for a_1 was arbitrarily selected as one third of the fuel outer radius. The only justification for this assumption is that it appears to be roughly consistent with void sizes observed in UO_2 fuel tests reported in the literature.

The basis for the selection or computation of the remaining parameters is discussed in Sections 6.3 to 6.6 and the values are summarized in Table 1. Several of these quantities require knowledge of the temperature distribution in the fuel element, and this is dealt with in Section 6.2. Finally, results of the stress analysis computation are presented in Section 6.7.

Table 1. Summary of Important Parameters

	Fuel Value (region 1)	Cladding Value (region 2)
Geometry		
Inner diameter, inches	0.07167	0.2200
Outer diameter, inches	0.2150	0.2500
Elastic Mechanical Constants		
Young's modulus, psi	4.00×10^7	2.340×10^7
Poisson's ratio	0.325	0.348
Average Thermal Expansion		
Strain, $\hat{\epsilon}_n^T$	0.01177	0.000494
Average Irradiation		
Swelling Strain, $\hat{\epsilon}_n^S$	Figure 6	$0.02154 \left(\frac{B}{10^5} \right)^{3/2}$
Creep Constants		
h_n , $\text{hr}^{-1} \text{psi}^{-1}$	1.086×10^{-11}	3.4×10^{-27}
β_n^t	515	1.6857
Burnup Rate or Power		
Density, B , kw/kgU		265
Linear Power, Q' kw/ft		15.0
Fluence to Burnup Ratio		
neutron/cm ² -sec/Mw/MT		1.492×10^{18}
Coolant Pressure, psi		75
Fission Gas Pressure, psi		$750 \left(\frac{B}{10^5} \right)$
Maximum Cladding Temperature, °C		600

Note: Some of the "standard" terminology and abbreviations used in this report may not be familiar to all readers. Accordingly, the following clarifications are presented. LMFBR: Liquid Metal Fast Breeder Reactor; Fluence: Time integrated neutron flux, Φt ; Linear Power: Power/unit length of fuel; Mw/MT: Thermal energy generated, expressed in megawatt-days (Mw/MT) per metric ton (1000 kgms) of uranium and/or plutonium metal; Mwe: Electric power, in megawatts.

6.2 Temperature Distribution

The temperature distribution is determined by solution of Eq. (9). Heat generation in the cladding is neglected, and is assumed to be spatially uniform in the fuel.

6.2.1 Cladding Temperature Distribution

$$k_r \frac{1}{r} \frac{d}{dr} \left(r \frac{d\Theta}{dr} \right) = 0$$

$$- 2\pi k_r r \frac{d\Theta}{dr} = Q'$$

where Q' is the linear power (typical units kw/ft) of the fuel element

$$\Theta(r) - \Theta(a_2) = \frac{Q'}{2\pi k_r} \ln \frac{a_2}{r}$$

and

$$\Theta = \frac{1}{k_r} \int_{T_r}^T k(T') dT'. \quad (32)$$

The expression used for the cladding conductivity is [7]

$$k = 0.159 + 1.313 \times 10^{-4} T \quad \text{W/cm-}^\circ\text{C}$$

Q' was taken as 15 kw/ft and $T(a_2)$ as 600°C [19]. Using these values, together with Eq. (32), the outer clad temperature was found to be 588°C . The temperatures at all other points could be similarly determined and the temperature distribution thus established. However, since the conductivity does not vary greatly, an average value $\bar{k}$ was selected to simplify subsequent computations. The

value $\tilde{k} = 0.2372 \text{ W/cm} - ^\circ\text{C}$ when used in the equation

$$T(r) - T(a_2) = \frac{Q'}{2\pi\tilde{k}} \ln \frac{a_2}{r} \quad (33)$$

gives the correct value for $T(b_2)$.

6.2.2 Fuel Temperature Distribution

A similar procedure was followed for the fuel, using the thermal conductivity function [20]

$$k = 0.011 + \frac{1}{(0.485 - 0.4465D)T} \text{ W/cm} - ^\circ\text{C}$$

where D is the fraction of theoretical density for UO_2 . D was arbitrarily selected as 0.92, reducing the above expression to

$$k = 0.011 + 13.473 \frac{1}{T} \text{ W/cm} - ^\circ\text{C}$$

With $T_r = 600^\circ\text{C}$ and $k_r = k(600^\circ\text{C})$, $\Theta(T)$ was calculated.

In the fuel, the temperature distribution obeys Eq.

(9)

$$k_r \frac{1}{r} \frac{d}{dr} \left(r \frac{d\Theta}{dr} \right) + Q''' = 0.$$

For a fast reactor, it is a reasonable approximation to assume the power generation term, Q''' , is independent of r , since spatial variation of the neutron flux will be small. In this case

$$Q''' = \frac{Q'}{\pi(b_1^2 - a_1^2)}$$

$$\frac{d}{dr} \left(r \frac{d\Theta}{dr} \right) = - \frac{Q'}{\pi k_r (b_1^2 - a_1^2)} r$$

Integrating once,

$$\frac{d\Theta}{dr} = - \frac{Q'}{2\pi k_r (b_1^2 - a_1^2)} \left(r - \frac{a_1^2}{r} \right).$$

Here it is assumed that $\frac{d\Theta}{dr} \Big|_{r=a_1} = 0$, or the heat transfer to any gas in the central void is neglected. Integrating again, the result can be written

$$\Theta(r) = \Theta(b_1) + \frac{Q'}{4\pi k_r} \frac{b_1^2}{b_1^2 - a_1^2} \left[\left(1 - \frac{r_2^2}{b_1^2} \right) + \frac{a_1^2}{b_1^2} \ln \frac{r^2}{b_1^2} \right]. \quad (34)$$

Using $T(b_1) = 600^\circ\text{C}$, and therefore $\Theta(b_1) = 0$, together with $a_1/b_1 = \frac{1}{3}$, $Q' = 15 \text{ kw/ft}$, and the conductivity function given above, the fuel temperature distribution of Figure 2 and Table 3, p. 43, was computed.

It is possibly appropriate here to say a word about the neglect of any temperature difference across the fuel-cladding gap. If ultimate interest is focused upon the computation of stresses in the cladding (as it is here), then the temperature distribution in the fuel is of primary interest only when the fuel and cladding are actually in contact. This provides the major rationalization for neglect of any gap temperature drop. Of course, even in the event of contact, there may well be a contact resistance. This effect could be included in the present analysis with no complication provided a value is specified for the resistance. However, it is neglected here.

Note:
 $a_1 = \frac{1}{3} b_1$
 $b_1 = 0.215 \text{ in.}$
 $T(b_1) = 600^\circ\text{C}$
 $Q''' = \text{Constant}$
 $Q' = 15 \text{ kw/ft}$

Figure 2. Temperature Distribution in Fuel

The equations presented in Chapters 4 and 5 contain certain space integrals of the thermal expansion strain. They are evaluated below with the aid of the temperature distribution determined above.

6.2.3 Cladding Thermal Expansion Strain Integrals

An expression which has been suggested for the linear coefficient of expansion of the stainless steel cladding, α_2 , is [21]

$$\alpha_2 = 8.783 \times 10^{-6} + 2.639 \times 10^{-10} T \text{ } (^{\circ}\text{C})^{-1}$$

Over the temperature range of interest in the clad, the variation in α_2 is quite small.

$$\alpha_2(600^{\circ}\text{C}) = 8.94 \times 10^{-6} \text{ } (^{\circ}\text{C})^{-1}$$

$$\alpha_2(558^{\circ}\text{C}) = 8.93 \times 10^{-6} \text{ } (^{\circ}\text{C})^{-1}$$

Therefore, in order to simplify computation, α_2 was taken to be constant and equal to $8.935 \times 10^{-6} \text{ } (^{\circ}\text{C})^{-1}$. With this assumption and Eq. (33) for the temperature distribution, the thermal strain integral can be directly evaluated in closed form. Thus,

$$\epsilon^T = \alpha_2 (T - T_o)$$

or

$$\epsilon^T = \alpha_2 \left[T(a_2) - T_o - \frac{Q'}{2\pi k} \ln \frac{r}{a} \right]$$

$$\frac{1}{r^2} \int_{a_2}^r r' \epsilon^T dr' = \frac{\alpha_2}{2} \left\{ [T(a_2) - T_o + \frac{Q'}{4\pi k}] \left(1 - \frac{a_2^2}{r^2}\right) - \frac{Q'}{4\pi k} \ln \frac{r^2}{a_2^2} \right\}$$

where T_0 is the reference temperature at which the thermal expansion is considered to be zero. Using these results, with $T_0 = 25^\circ\text{C}$, Table 2 was constructed. From the final entry in Table 2, the value of $\hat{\epsilon}_2^T$, the area averaged thermal expansion strain for the cladding, was computed.

Table 2. Thermal Strain Integral for Cladding Region

$2r$ (in)	T $^\circ\text{C}$	ϵ^T	$\frac{1}{r^2} \int_{a_2}^r r' \epsilon^T dr'$
.220	600.0	.005138	0.0
.225	592.6	.005071	1.1217×10^{-4}
.230	585.3	.005007	2.1569×10^{-4}
.235	578.2	.004931	3.1133×10^{-4}
.240	571.3	.004881	3.9975×10^{-4}
.245	564.5	.004820	4.8158×10^{-4}
.250	557.8	.004761	5.5736×10^{-4}

$$\hat{\epsilon}_2^T = \frac{2\pi \int_{a_2}^{b_2} r' \epsilon^T dr'}{\pi(b_2^2 - a_2^2)} = 0.004941$$

6.2.4 Fuel Thermal Expansion Strain Integral

The expression used for the coefficient of thermal expansion for the oxide fuel is [20]

$$\alpha_1(T) = A + BT \quad (^\circ\text{C})^{-1}$$

where

$$A = 8.033 \times 10^{-6}$$

$$B = 2.4336 \times 10^{-9}$$

and T is in $^\circ\text{C}$. The thermal expansion is

$$\epsilon^T = \int_{T_0}^T \alpha_1(T') dT' = \frac{T-T_0}{2} [\alpha(T) + \alpha(T_0)].$$

Again using $T_0 = 25^\circ\text{C}$, Table 3 was constructed. The thermal strain integral of interest for the fuel is tabulated in Column 5. This integral was evaluated numerically using Simpson's rule. From the final entry in Column 5, the value of $\hat{\epsilon}_1^T$ was calculated to be $\hat{\epsilon}_1^T = 0.01177$.

6.3 Elastic Mechanical Constants

6.3.1 Elastic Constants for Fuel

An expression available in the literature giving Young's modulus for oxide fuel is [22]

$$E_1 = 3.59 \times 10^7 (1 - 2.35P)(1 - T/22500) \text{ psi}$$

where T is in $^\circ\text{F}$ and P is the fuel porosity. For $P = 0.08$,

$$E_1(600^\circ\text{C}) = 4.31 \times 10^7 \text{ psi}$$

$$E_1(1823^\circ\text{C}) = 3.87 \times 10^7 \text{ psi.}$$

Table 3. Thermal Strain Integral for Fuel Region

r/b_2	Θ ($^{\circ}\text{C}$)	T ($^{\circ}\text{C}$)	$\epsilon^T(r)$	$\frac{1}{r^2} \int_{a_1}^r r' \epsilon^T dr'$
0.333	849.0	1823	.01849	0.0
0.400	838.1	1803	.01824	.002810
0.467	807.1	1747	.01755	.004442
0.533	758.4	1660	.01649	.005397
0.600	693.3	1547	.01514	.005925
0.667	613.0	1415	.01360	.006164
0.733	517.9	1263	.01189	.006199
0.800	408.8	1100	.01011	.006086
0.867	285.9	931	.00833	.005867
0.933	149.5	762	.00663	.005573
1.000	0.0	600	.00506	.005229

In the analysis to be carried out here, a single, temperature independent value is required for E_1 . On the basis of the above values, a value of 4×10^7 psi was selected.

With regard to the Poisson's ratio, ν , following the suggestion of reference [7], a linear temperature dependence was assumed. Over the fuel temperature range, values found are

$$\nu_1(600^\circ\text{C}) = 0.314.$$

$$\nu_1(1823^\circ\text{C}) = 0.334.$$

The constant value selected for use was $\nu_1 = 0.325$.

6.3.2 Elastic Constants for Cladding

The expressions in the LIFE [7] code for the Young's modulus and Poisson's ratio for stainless steel were examined. Over the range of temperature in the cladding, E_2 varies between 2.314×10^7 psi and 2.369×10^7 psi while ν_2 ranges from 0.3497 to 0.3461. Constant values of $E_2 = 2.34 \times 10^7$ psi and $\nu_2 = 0.348$ were selected.

6.4 Creep Constants

The creep constants required by Eq. (3) are the most difficult parameters to select with a degree of realism. This is partly due to the fact that experimental data are rather sparse and conflicting, and partly because under some circumstances creep rates show strong temperature

dependence and/or a highly non-linear stress dependence. This situation is seriously at variance with the formulation of Eq. (3).

As mentioned earlier, in-reactor creep is usually modeled as composed of two parts: the classical thermally activated creep, plus a "radiation enhanced" component. As will be seen presently when specific creep formulations for fuel and cladding will be discussed separately, the irradiation creep component may be strongly dominant. Furthermore, most investigators have concluded, admittedly upon the basis of limited data, that irradiation induced creep rates show approximately a linear stress dependence.

6.4.1 Fuel Creep Constant

A 1971 review of UO_2 creep data [16] found that in-pile creep rates ranged from five times the out-of-pile creep rates to many orders of magnitude higher. It was further concluded that the in-pile creep rate is approximately linearly related to both stress and fission rate. Assuming this linear relation to hold, data given in the report at 1100°C indicate

$$\dot{\epsilon}(\text{hr}^{-1}) \approx 10^{-10} \dot{B} \sigma \quad (35)$$

where $\dot{B}$ is power density in kw/kg UO_2 . The data show strong temperature dependence, with the strain rate varying about two orders of magnitude in the range 1000°C to 1200°C .

Since the fission rate of interest in the fuel element under

consideration in this paper at 15 kw/ft is over six times the maximum value used in the data cited in reference [16], the conclusion that the irradiation induced creep component is strongly dominant should be even more valid here.

Other UO_2 creep data were also examined [23]. These data gave creep rates corresponding to

$$\dot{\epsilon}(\text{hr}^{-1}) = 1.086 \times 10^{-11} \dot{\sigma} \quad (36)$$

independent of temperature up to 850°C . This does not appear to be grossly inconsistent with that of Eq. (35) above if the temperature difference is taken into account.

For present purposes, expression (36) was adopted. This is primarily because the most important role played by fuel creep is the relaxation of any interaction pressure which may develop between the fuel and the cladding. The lower value of the creep rate is therefore the conservative one. Furthermore, the temperature range of the data upon which (36) is based corresponds to the temperature of that portion of the fuel nearest the cladding.

6.4.2 Cladding Creep Constant

The creep law for 20% cold worked 316 stainless steel cladding which is used in the FMODEL computer code is

$$\begin{aligned} \dot{\epsilon} = & 0.25 \exp\left(-\frac{89000}{RT}\right) \sigma^4 \\ & + 8.174 \times 10^{-27} \sigma^6 \exp(1.405 - 0.0027T) \quad (37) \end{aligned}$$

$$R = 1.986 \text{ cal/g - mole}^{\circ}\text{K}$$

$$T = \text{absolute temperature, }^{\circ}\text{K}$$

$$\sigma = \text{stress, psi}$$

$$\bar{\phi} = \text{neutron flux, cm}^{-2}\text{-sec.}$$

The first term of this expression represents the thermal creep and shows both strong temperature dependence and highly non-linear stress dependence. The second term is the flux enhanced creep contribution, and shows much weaker temperature dependence and a linear stress dependence.

Evaluation of this expression over the temperature range of interest for the cladding reveals the following values:

$$\dot{\epsilon}(T=558^{\circ}\text{C}) = 9.50 \times 10^{-25} \sigma^4 + 3.53 \times 10^{-27} \sigma \bar{\phi}$$

$$\dot{\epsilon}(T=600^{\circ}\text{C}) = 1.27 \times 10^{-23} \sigma^4 + 3.15 \times 10^{-27} \sigma \bar{\phi}.$$

At 600°C , the thermal creep contributes less than 20% to the total strain rate for $\sigma \leq 6571$ psi, while at 550°C , the 20% dividing line falls at $\sigma = 16200$ psi. Thus, according to this expression, the thermal creep contributes less than 20% of the total creep for at least moderately high stresses. With this somewhat weak justification the creep law adopted for this analysis is

$$\dot{\epsilon} = 3.4 \times 10^{-27} \sigma \bar{\phi}.$$

In order to evaluate the validity of this approximation, it will be necessary to examine the magnitudes of the stresses

which develop in the cladding. Provided that these stresses do not fall too far outside the boundaries suggested above, it is probably safe to say that the data are not sufficiently dependable to render this creep law unacceptable.

In any event, it might be mentioned in passing that whether or not this creep law is an adequate representation of reality, the objectives of this analysis as delineated in Chapter 1 can still be achieved.

6.5 Swelling Strains

There are many correlations presented in the literature giving the swelling strains for oxide fuels and stainless steel cladding. There are considerable differences among them, and they exhibit significant historical variation as more experiments are performed and greater understanding is gained regarding these phenomena. This simply reflects the fact that knowledge of these matters is still in a far from satisfactory state. It is not the purpose of this work to enter into a discussion of the merits of these various expressions. Rather, for purposes of computational illustration, specific relations are arbitrarily selected. In passing, it is mentioned that the solution herein presented could provide one means of estimating in a simple manner the importance of these uncertainties with respect to fuel element stress analysis.

6.5.1 Swelling Strain for Stainless Steel Cladding

Experimental data reflect a considerable difference in swelling behavior between cold worked and solution treated stainless steels. LMFBR design attention has recently focused upon cold worked steel because the swelling is much less severe. An expression giving the fluence and temperature dependence of the volumetric swelling for 20% cold worked stainless is [1]

$$\% \frac{\Delta V}{V} = 300 \epsilon^S = 9.0 \times 10^{-35} (\Phi t)^{1.5} .$$

$$[4.028 - 0.03712T + 1.0145 \times 10^{-4} T^2 - 7.879 \times 10^{-8} T^3]$$

where (Φt) is the neutron fluence and T is the temperature in $^{\circ}\text{C}$. Using this expression, Table 4 was computed. The integral in the final column was determined numerically. Any spatial variation of the fast neutron flux was neglected. The average value, $\hat{\epsilon}_2^S$, is also indicated at the end of the table, and is plotted in Figure 3.

6.5.2 Swelling Strain for Oxide Fuel

The fuel swelling expression used in the COMETHE-II code [24] is adaptable to this analysis.

$$3\epsilon^S = \frac{\Delta V}{V} = 0.149 P_o^{-0.746} P_o \exp \left(-100 \frac{\Delta V}{V} \right) - 1.0 \\ + 1.60 \times 10^{-6} (1 - P_o) B$$

where P_o is the initial fuel porosity, taken here to be 0.08, and B is the burnup in Mwd/MT (heavy metal). The

Table 4. Swelling Strain Integral for Cladding Region

$2r$ (in)	T $^{\circ}\text{C}$	$\frac{\epsilon^S}{(\Phi t)^{3/2}}$	$\frac{1}{r^2} \frac{1}{(\Phi t)^{3/2}} \int_{a_2}^r r' \epsilon^S(r') dr'$
.220	600	3.7781×10^{-37}	0.0
.225	593	3.7820×10^{-37}	8.3069×10^{-39}
.230	585	3.7735×10^{-37}	1.6073×10^{-38}
.235	578	3.7439×10^{-37}	2.3319×10^{-38}
.240	571	3.7246×10^{-37}	3.0066×10^{-38}
.245	565	3.6864×10^{-37}	3.6336×10^{-38}
.250	558	3.6401×10^{-37}	4.2150×10^{-38}

$$\hat{\epsilon}_2^S = 3.7367 \times 10^{-37} (\Phi t)^{3/2}$$

Figure 3. Irradiation Swelling for Fuel and Cladding

"fuel" curve in Figure 3 was generated with this expression. For a fast reactor, it is a reasonable approximation to neglect any radial variations of the burnup within the fuel pellet. With this assumption, there is no spatial variation of ϵ^S and therefore $\hat{\epsilon}_1^S = \epsilon^S$ for the fuel region. $\hat{\epsilon}_1^S$ is displayed in Figure 3 and also Table 5.

The curve of Figure 3 exhibits an interesting property typical of fuel swelling curves. At low burnup, the fuel swells at a relatively slow rate as the swelling is "absorbed into the as-fabricated porosity." Eventually, however, the initial porosity is "used up" and swelling proceeds at a higher rate characteristic of 100% dense fuel material.

6.5.3 Fluence to Burnup Ratio

Cladding swelling is controlled by the fast neutron fluence, whereas fuel swelling is governed by burnup. Therefore, an important parameter relevant to fuel-cladding interaction is the fluence to burnup ratio, or, equivalently, the flux to power density ratio. For a given reactor, this quantity is readily determined by examination of the detailed core neutronics. For purposes of the present analysis, however, a value which is representative of a large LMFBR was simply selected, namely 1.4×10^{22} nvt/atom % burnup [1]. This is equivalent to 1.492×10^{18} $\frac{\text{neutron/cm}^2}{\text{kw-day/kg}}$, assuming 3.121×10^{10} fission/watt-sec or 200

Table 5. Average Swelling Strains for Fuel and Cladding

$\frac{B}{Mwd/MTU}$	$\hat{\epsilon}_1^S$	$\hat{\epsilon}_2^S$
0.0	0.0	0.0
15850	0.001	0.001359
37739	0.002	0.004991
51591	0.005	0.007978
56711	0.006	0.009195
61032	0.007	0.01027
64761	0.008	0.01122
68053	0.009	0.01209
71019	0.010	0.01289
73745	0.011	0.01364
76292	0.012	0.01435
78708	0.013	0.01503
81026	0.014	0.01570
83270	0.015	0.01636
85462	0.016	0.01701
87613	0.017	0.01766
89736	0.018	0.01830
91837	0.019	0.01895
102165	0.024	0.02223

Mev/fission. With this value, $\hat{\epsilon}_2^S = 6.81 \times 10^{-10} (B)^{1.5}$, and the "cladding" curve of Figure 3 was determined. It is interesting to note that the cladding swelling exceeds the fuel swelling until very high burnups. This has important consequences in that the probability of fuel-clad interaction is greatly reduced.

Assuming a fuel density of $10.12 \text{ g UO}_2/\text{cm}^3$ (92% of theoretical density), there are $56.61 \text{ g UO}_2/\text{ft}$. A linear power of 15 kw/ft is therefore equivalent to a specific power (or burnup rate) of $\dot{B} = 265 \text{ watts/gU}$ or 234 watts/g UO_2 . Therefore, the time required to achieve a burnup of $100,000 \text{ Mwd/MT}$, t_{100} , is

$$t_{100} = 377.4 \text{ days} = 9057 \text{ hours.}$$

6.6 Coolant Pressure and Internal Gas Pressure

6.6.1 Coolant Pressure

The coolant pressure, p_2 , was chosen as 75 psi , a value representative of the sodium pressure in the core of a LMFBR.

6.6.2 Internal Gas Pressure

During operation of a fuel element, part of the fission product gas generated is retained within the solid fuel, and some is released into the gas plenum which is provided in most fuel element designs. The build-up of gas

is a possible source of stress upon the cladding. Therefore, some estimate is required for the gas pressure.

The release process is governed by complex diffusion mechanisms, and computer programs such as LIFE [7] and FMODEL [8] include detailed models for the prediction of release rates. These considerations are outside the scope of the present analysis, and a much more simplistic approach is adopted.

It is here assumed that the number of moles of gas released is directly proportional to the burnup of the fuel pin.

$$n = C f y V_f B$$

n = number of moles of gas released

f = release fraction, taken to be 0.67 [25]

y = atoms gas produced per fission = 0.27 [7, p. 52]

V_f = fuel volume

B = pin average burnup, Mwd/MT (heavy metal)

C = units conversion constant

The plenum pressure, p_o , is then estimated from the ideal gas law

$$p_o = \frac{nR}{\frac{V_v}{T_v} + \frac{V_p}{T_p}}$$

R = gas constant

V_v = volume of central void

V_p = volume of gas plenum

T_v = average gas temperature in central void

T_p = average gas temperature in plenum.

T_v was taken to be the inside fuel surface temperature, and T_p was taken to be the inside cladding surface temperature. Assuming $V_f/V_v = 9.0$, the plenum pressure after 100,000 Mwd/MT was estimated to be about 750 psi. Therefore, the expression used for p_o is

$$p_p = 750 \chi$$

where $\chi = 10^{-5} B$.

6.7 Stress Analysis Computations

6.7.1 Determination of Fuel-Cladding Interaction Time

The first step in the stress computation is to determine whether or not the fuel and cladding come into contact, and if so, the time at which the interaction occurs. The condition for gap closure can be stated

$$\left(1 - \frac{b_1}{a_2}\right) + \epsilon_{\theta,2}(a_2, t) \leq \frac{b_1}{a_2} \epsilon_{\theta,1}(b_1, t). \quad (38)$$

The two sides of this inequality were computed as described in Appendix C, tabulated in Tables 6 and 7, and are plotted in Figure 4. It is seen that no interaction can take place until burnups far in excess of 100,000 Mwd/MT are achieved. This circumstance is due both to the provision of an adequate initial gap, and also because of the fact that the cladding undergoes greater swelling than the fuel over most

Table 6. Hoop Strain at Inner Surface of Cladding

x	Thermal Expansion Contribution ($\times 10^5$)	Coolant Pressure Contribution ($\times 10^5$)	Fission Gas Pressure Contribution ($\times 10^5$)	Cladding Swelling Contribution ($\times 10^5$)	Total 100 $\epsilon_{\theta_2}(a_2, x)$ (%)
0.0	494.11	-2.8414	0.0	0.0	0.4913
0.1	494.11	-3.4660	2.4838	68.099	0.5612
0.2	494.11	-4.0906	5.4361	192.612	0.6881
0.3	494.11	-4.7152	8.8568	353.851	0.8521
0.4	494.11	-5.3399	12.7460	544.790	1.0463
0.5	494.11	-5.9645	17.1036	761.367	1.2666
0.6	494.11	-6.5891	21.1930	1000.842	1.5103
0.7	494.11	-7.2137	27.2242	1261.121	1.7753
0.8	494.11	-7.8383	32.9872	1540.898	2.0602
0.9	494.11	-8.4629	39.2186	1838.660	2.3635
1.0	494.11	-9.0875	45.9185	2153.470	2.6844

Table 7. Hoop Strain at Outer Surface of Fuel

x	ϵ_1^S	$\epsilon_{\theta,1}(b_2, x)$
0.0	0.0	0.01177
0.15850	0.001	0.01277
0.28120	0.002	0.01376
0.37739	0.003	0.01476
0.45393	0.004	0.01576
0.51591	0.005	0.01676
0.56711	0.006	0.01776
0.61032	0.007	0.01876
0.64761	0.008	0.01976
0.68053	0.009	0.02076
0.71019	0.010	0.02176
0.73745	0.011	0.02276
0.76292	0.012	0.02376
0.78708	0.013	0.02476
0.81026	0.014	0.02576
0.83270	0.015	0.02676
0.85462	0.016	0.02776
0.87613	0.017	0.02876
0.89736	0.018	0.02976
0.91837	0.019	0.03076
0.93920	0.020	0.03176

Figure 4. Determination of Interaction Time

of the pin lifetime. This latter point was mentioned earlier in connection with Figure 3. It is relevant to note that this situation is strongly sensitive to the fluence to burnup ratio, as well as to the particular expressions employed to estimate the swelling for fuel and cladding, as pointed out by Murray [1] among others.

6.7.2 Evaluation of Hoop Stress in the Cladding

The hoop stress in the cladding is given by Eq. (29) with $n = 2$. Since it was found above that no interaction pressure develops in this problem, $p_1(t) = p_0(t) = 750 \text{ X}$. Recalling that the thermal strains contain no time variation, Eq. (29) produces

$$\begin{aligned} \sigma_{\theta,2}(r,X) = & \left(1 + \frac{b_2^2}{r^2}\right) \frac{p_1(X)}{b_2^2/a_2^2 - 1} - \left(1 + \frac{a_2^2}{r^2}\right) \frac{p_2}{1 - a_2^2/b_2^2} \\ & + e^{-\beta_2 t_{100} X} \frac{E_2}{1 - \nu_2} \left\{ \frac{1}{2} \left(1 + \frac{a_2^2}{r^2}\right) \hat{\epsilon}_2^T + \frac{1}{r^2} \int_{a_2}^r r' \epsilon^T dr' - \epsilon^T \right\} \\ & + \frac{E_2}{1 - \nu_2} \left(\frac{\Phi_t}{B} \times 10^5 \right)^{3/2} \left[X^{3/2} - \beta_2 t_{100} \int_0^X e^{-\beta_2 t_{100} (X - X')} \right. \\ & \quad \left. (X')^{3/2} dX' \right] \\ & \left\{ \frac{1}{2} \left(1 + \frac{a_2^2}{r^2}\right) \frac{\hat{\epsilon}_2^S}{(\Phi_t)^{3/2}} - \frac{\epsilon^S(r)}{(\Phi_t)^{3/2}} + \frac{1}{r^2} \int_{a_2}^r r' \frac{\epsilon^S}{(\Phi_t)^{3/2}} dr' \right\}. \end{aligned} \quad (39)$$

Numerical evaluation of this rather formidable expression was carried out using data given earlier. The quantity in

the square brackets is given in Table 8, where

$$J_2(x) = x^{3/2} - \beta_2 t_{100} \int_0^x e^{-\beta_2 t_{100}(x-x')} (x')^{3/2} dx'.$$

The integral was computed numerically using Simpson's rule. The final results of the hoop stress computation are given in Figures 5, 6, and 7.

Table 8. Creep Relaxation Factor for Cladding Swelling Stress

x	$J_2(x)$
0.00	0.0
0.10	0.02958
0.20	0.07845
0.30	0.1354
0.40	0.1961
0.50	0.2584
0.60	0.3208
0.70	0.3824
0.80	0.4427
0.90	0.5014
1.00	0.5582

Note: $\chi = B/10^5$
B = burnup in Mwd/MT

Figure 5. Cladding Hoop Stress as a Function of Position and Burnup

Figure 6. Maximum Cladding Hoop Stress as a Function of Burnup

Figure 7. Components of Hoop Stress at Outer Surface of Cladding

In Figure 7, the contributions to the stress from the several operative mechanisms are indicated. Particularly noteworthy is the large stress produced by spatial variation of the swelling through the cladding wall. This component is given by the final term in Eq. (39) and is labelled "non-uniform swelling" in Figure 7. A previous study [26] also observed that this mechanism can produce large cladding hoop stresses. The prediction of this large stress is directly attributable to the strong temperature dependence displayed by the empirical stainless steel swelling equation. Further research would seem appropriate to verify this dependence.

It is also interesting to note the effect of creep upon the results of Figure 7. The stress due to non-uniform thermal expansion is seen to relax considerably from an initial value of 6500 psi to 1200 psi at 10^5 Mwd/MT. Furthermore, according to Table 8, the stress at 100,000 Mwd/MT due to non-uniform swelling is only about 56% of the value which would result if there were no creep. Thus, if it were not for stress relaxation due to creep, the present model would predict a maximum cladding hoop stress of about 31,000 psi in contrast to the 17,000 psi reflected in Figure 7. This dramatic difference points out the extreme importance of improving confidence in creep data.

6.7.3 Axial and Radial Stresses in Cladding

Using a procedure which exactly parallels that used for the hoop stress, Eqs. (27) and (28) were used to compute the axial and radial stresses. The results are presented in Figures 8 and 9.

As mentioned earlier, now that the stresses are known, it is of interest to examine the validity of neglecting the thermally activated creep. Equation (37) indicates that either high temperature or high stress results in greatest importance for the thermal creep component. Inspection of Figures 7, 8, and 9 reveals that the extreme conditions in this regard are as below:

1. $r = a_2$ at zero burnup, where $T = 600^\circ\text{C}$ and the maximum deviatoric stress is $S_\theta \approx -2800$ psi. Under these conditions, according to Eq. (37), the irradiation creep rate is 98% of the total creep rate.
2. $r = b_2$ at $B = 10^5$ Mwd/MT, where $T = 558^\circ\text{C}$ and the maximum deviatoric stress is $S_\theta \approx 10600$ psi. Under these conditions, the irradiation creep rate is 93.5% of the total creep rate.

Based upon these observations, it is concluded that the creep law used in this analysis is probably quite satisfactory. Indeed, it seems doubtful that confidence in the validity of Eq. (37) could be sufficient to justify any more complex formulation than the simple one used here.

Figure 8. Axial Stress in Cladding as a Function of Position and Burnup

Figure 9. Radial Stress in Cladding as a Function of Position and Burnup

CHAPTER 7

DISCUSSION OF INTERACTION PRESSURE

7.1 Introduction

The problem discussed in Chapter 6 did not result in fuel-cladding interaction. This is interesting in that this problem is a fairly realistic representation of an actual 1000 Mwe LMFBR fuel element, within the restrictions of the simplified analytical model proposed here. The primary reason for the lack of fuel-cladding interaction is the excess in the swelling of the cladding over that of the fuel until very high burnup, as shown in Figure 3. This is consistent with a widely held (but by no means universal) feeling among fuel element designers that the discovery of stainless steel swelling has shifted the major difficulty of LMFBR fuel design from fuel pin failure to problems outside the fuel pin [1, 27].

It is interesting, however, to examine a problem where the fuel does exert pressure upon the cladding. This was accomplished by carrying out computations similar to those of Chapter 6, but with two important modifications. First, the fluence to burnup ratio was reduced by a factor of 0.75 to a value of 1.119×10^{18} neutrons/cm²-sec per Mwd/MT. This serves to reduce the swelling of the cladding

relative to that of the fuel. Secondly, the initial gap size was modified such that the gap is closed when the fuel pin is brought to operating temperature at zero burnup. To accomplish this, it is required that

$$b_1 + b_1 \epsilon_{\theta,1}(b_1, \chi=0) = a_2 + a_2 \epsilon_{\theta,2}(a_2, \chi=0).$$

Utilizing Eqs. (21) and (22), this implies

$$b_1 + b_1 \hat{\epsilon}_1^T = a_2 + a_2 \left[\hat{\epsilon}_2^T - \frac{2-\nu_2}{E_2} \frac{b_2^2}{b_2^2 - a_2^2} p_2 \right].$$

Any small changes in the temperature distribution due to the small perturbation in the dimensions were neglected. Insertion of values for $\hat{\epsilon}_1^T$, $\hat{\epsilon}_2^T$, ν_2 , E_2 , and p_2 from Chapter 6 produced the result

$$\frac{b_1}{a_1} = 0.9932$$

for gap closure at zero burnup. Retaining the cladding dimension as $a_2 = 0.110$ ", the new fuel diameter is 0.2185".

This change in geometry, together with the appropriate modifications made to the cladding swelling rate and creep constant, were the only modifications made to the problem considered in Chapter 6.

7.2 Determination of Interaction Time

Although by design the fuel-cladding gap closes immediately at zero burnup due to thermal expansion, the

cladding swelling rate exceeds the fuel swelling rate early in life, causing the gap to re-open. Eventually, however, the fuel swelling rate increases much more rapidly than that of the cladding, the gap closes, and the fuel exerts appreciable pressure upon the cladding.

As in Section 6.7.1, the condition for gap closure is Eq. (38). Again, evaluation was carried out following the procedure of Appendix C, and interaction was found to occur at $B = 60,000 \text{ Mwd/MT}$ or $X = 0.60$.

7.3 Computation of Interaction Pressure

Using Eqs. (23), (30), and (31), an expression for the interaction pressure can be developed.

$$\begin{aligned}
 \frac{I}{a_2} p_{\text{int}}(t) = & A_2 p_2 [1 - e^{-\lambda t_{100}(X - X_{\text{int}})}] \\
 & + A_0 p_{100} [1 - e^{-\lambda t_{100}(X - X_{\text{int}})}] \\
 & + A_1 p_{100} [\lambda t_{100}(X - X_{\text{int}}) + 1 - e^{-\lambda t_{100}(X - X_{\text{int}})}] \\
 & + G(X) - \lambda t_{100} \int_{X_{\text{int}}}^X e^{-\lambda t_{100}(X - X')} G(X') dX'
 \end{aligned} \tag{40}$$

where the following abbreviations have been used:

$$A_2 = \frac{b_2^2}{b_2^2 - a_2^2} \frac{3(1 - \nu_2)}{E_2} \frac{\beta_2 t_{100}}{\lambda t_{100}}$$

$$A_0 = \frac{1}{\lambda t_{100}} \left[\frac{2a_1^2}{b_1^2 - a_1^2} \frac{b_1}{a_2} \frac{1}{E_1} + \frac{b_1}{a_2} \frac{\nu_1}{E_1} + \frac{a_2^2}{b_2^2 - a_2^2} \frac{\nu_2}{E_2} \right]$$

$$A_1 = \frac{1}{(\lambda t_{100})^2} \left[\frac{a_1^2}{b_1^2 - a_1^2} \frac{b_1}{a_2} \frac{1 - \nu_1}{E_1} 4\beta_1 t_{100} + \frac{b_1}{a_1} \frac{1 - \nu_1}{E_1} \beta_1 t_{100} + \frac{a_2^2}{b_2^2 - a_2^2} \frac{1 - \nu_2}{E_2} \beta_2 t_{100} \right]$$

$$G(X) = \frac{b_1}{a_2} \hat{\epsilon}_1^S - \hat{\epsilon}_2^S .$$

The function $G(X)$ is a measure of the excess in fuel swelling over that of the cladding, and the final two terms of Eq. (40) therefore represent the contribution to the interaction pressure from this mechanism. The first term is the contribution from the coolant pressure, and the second and third terms are due to the fission gas pressure. Table 9 displays numerical values of these several terms for the dimensions and properties under consideration.

Two important facts are immediately apparent from Table 9. First, the interaction pressure produced is quite large, building to a value of over 2300 psi at 100,000 Mw3/MT as compared with the fission gas pressure in the central cavity, which is 750 psi at the same time. Secondly, the overwhelmingly dominant contribution to the

Table 9. Interaction Pressure

X	Differential Swelling Contribution (psi)	Coolant Pressure Contribution (psi)	Fission Gas Pressure Contribution (psi)	Total (psi)
0.60	450.00	0.0	0.0	450.00
0.61032	567.86	0.826	14.75	583.43
0.64761	694.28	2.176	53.54	750.00
0.68053	883.40	2.506	79.63	965.54
0.71019	1079.59	2.600	102.32	1184.50
0.73745	1263.68	2.631	122.20	1388.51
0.76292	1429.17	2.650	140.61	1572.43
0.78708	1571.10	2.650	158.03	1714.40
0.81026	1689.94	2.650	174.70	1867.30
0.83270	1787.37	2.650	190.87	1980.90
0.85462	1862.13	2.650	206.87	2071.40
0.87613	1920.19	2.650	222.11	2144.95
0.89736	1961.25	2.650	237.38	2201.28
0.91837	1990.24	2.650	252.49	2245.38
1.02165	2032.83	2.650	326.78	2362.30

pressure is from the differential swelling between fuel and cladding.

This situation points out an area of considerable uncertainty. The differential swelling component of the interaction pressure is directly proportional to $G(X)$. The function $G(X)$, in turn, is a small difference of two empirically determined quantities, each containing considerable uncertainty. Therefore, these uncertainties can be greatly magnified in $G(X)$. For example, at 81000 Mwd/MT, a 10% uncertainty in the average fuel swelling, $\hat{\epsilon}_1^S$, leads to a 33% uncertainty in the interaction pressure. The function $G(X)$ is also of overriding importance in the determination of the interaction time. Here the difficulty is extreme since of necessity $G(X)$ is very nearly zero at the moment of interaction.

It therefore appears that the prediction of fuel-cladding interaction pressures is a problem of considerable practical difficulty. It inevitably involves the small difference of two empirically determined quantities, each known with a relatively low degree of confidence. This again points out the importance of further research to increase confidence in the expression used to predict irradiation swelling rates.

7.4 Cladding Hoop Stress

The cladding hoop stress was calculated using Eq. (39), with $p_1 = p_{100} \chi$ for $\chi < 0.6$, and $p_1 = p_{int}$ for $\chi > 0.6$. The function $J_2(\chi)$ with the modified cladding creep constant is given in Table 10. Results of the calculation are shown in Figures 10 and 11.

Comparison of Figure 11 with Figure 6 shows the considerable influence of the fuel-cladding interaction upon the maximum cladding hoop stress. At burnups less than 60000 Mwd/MT, Figure 11 shows stresses somewhat lower than Figure 6, due to the lower fluence to burnup ratio. After the fuel and cladding make contact, however, the stress rises rapidly in Figure 11 to a value of nearly 26000 psi at 100,000 Mwd/MT, as compared with about 17000 psi in Figure 6. It is therefore seen that if fuel-cladding interaction is permitted to occur, it can be a very important stress producing mechanism.

Table 10. Creep Relaxation Factor for Cladding Swelling
Stress

χ	$J_2(\chi)$	$J_2(\chi)/\chi^{3/2}$
0.00	0.0	1.00
0.10	0.3270	0.9526
0.20	0.4260	0.9061
0.30	0.4814	0.8638
0.40	0.5135	0.8242
0.50	0.5313	0.7872
0.60	0.5394	0.7527
0.70	0.5409	0.7204
0.80	0.5376	0.6903
0.90	0.5310	0.6620
1.00	0.5221	0.6355

Figure 10. Cladding Hoop Stress as a Function of Position and Burnup with Interaction Pressure

Figure 11. Hoop Stress at Outer Surface of Cladding with Interaction Pressure

CHAPTER 8

SUMMARY AND CONCLUSIONS

A simplified mathematical model for fuel element stress analysis has been proposed. The principal idealizations included in the model are:

1. The fuel element is represented geometrically by one fuel region surrounded by a cladding region.
2. Mechanical properties are taken to be space independent within each of the two regions.
3. Creep rates are assumed to be related linearly to the deviatoric stress.
4. A state of plane strain with azimuthal symmetry is assumed.

Using the method of viscoelastic-elastic analogy, closed-form analytical solutions to this model were obtained giving the stresses in the fuel element. In these solutions, the contributions to the stresses from the several "driving forces," namely fission gas pressure, coolant pressure, thermal expansion, and irradiation induced swelling, are all clearly visible and therefore easily compared. The relative importance of various parameters such as properties and geometry can also be evaluated.

Next, a numerical evaluation of the cladding stresses for a typical LMFBR fuel element was undertaken, based upon this analytical solution. For the properties and geometries selected, it was found that no fuel-cladding interaction developed. Also, the major stress producing mechanism was identified to be the spatial variation of the stainless steel swelling through the cladding wall. This points out the need for further research to increase the accuracy with which stainless steel swelling can be predicted. It was further found that irradiation creep produces a considerable relaxation of the cladding stresses. Again, this indicates the need for more reliable creep data.

Finally, in order to examine the effect of a fuel-cladding interaction pressure and also to exercise the full capability of the analytical solution, a second numerical evaluation was carried out with a lower fluence to burnup ratio. It was found that the fuel-cladding interaction resulted in much higher cladding stresses. It was also observed that the accurate prediction of stresses under such circumstances requires very precise knowledge of fuel and cladding swelling rates.

Several possibilities for the further use of this work can be suggested. One of the objectives of this study was to provide a "benchmark" calculation for comparison with some of the computer codes which have been developed for fuel element stress analysis. Such a comparison was

attempted with the FMODEL code. However, the available computer proved inadequate for the memory requirements of the code. Perhaps the user of such a code may find such a comparison useful.

The solution given here may be useful for survey calculations or rough parameter studies. Since the arithmetic computations are quite tedious, it may be profitable to develop a simple computer program to carry them out. These calculations are probably within the capability of many programmable desk calculators.

The solutions given in Chapter 6 are not necessarily restricted to cases where the temperature is constant in time. Although the calculations would become very laborious, a study of a cyclical temperature variation may be interesting. Probably the most useful periodic function to study would be a sinusoidal variation, since Fourier analysis could extend this to other periodic temperature variation.

An attempt was made to extend the analysis to a problem containing more than one fuel region. This is an attractive possibility, because it would permit some description of the irradiation restructuring of the oxide fuel and would also allow at least approximate description of the temperature dependence of the mechanical properties. The solution of the related elastic problem for an n region cylinder is straightforward, using basically the approach outlined in Chapter 4. Application of the boundary

conditions leads to a system of $n-1$ equations for the $n-1$ unknown interface pressures, p_i , which can be written in the form

$$M_{ij} p_i = C_i. \quad (41)$$

The constants C_i on the right hand side of these equations involve the dimensions as well as the various thermal and mechanical properties, as do the elements of the coefficient matrix M_{ij} . The solution to the related thermoelastic problem is therefore completed by finding the inverse of the matrix M_{ij} , which can be simply accomplished numerically, if not analytically.

However, some difficulty arises if the method of the viscoelastic-elastic analogy is applied to this n -region problem. Since the elements of the coefficient matrix in Eq. (41) involve the constants E_n and ν_n , the insertion of the functions E'_n and ν'_n cause M_{ij} to become a function of the transform variable. Since for $n > 2$ the inversion of M_{ij} becomes very difficult analytically, it appears to be impractical to find the inverse transform and complete the solution in closed form. Perhaps a numerical scheme could be devised to circumvent the difficulty, but since the basic model still includes the assumption of a linear creep law, such a sophisticated approach seems unwarranted.

These last remarks again bring into focus a basic limitation of the present approach which cannot be mitigated.

The viscoelastic-elastic analogy is only useful if the creep law is linear in the stress (and possibly its time derivatives) and therefore amenable to the Laplace transform. If a linear creep law is not acceptable, then the analogy method has no utility. However, if a linear creep law is an adequate representation, the method is quite powerful. In any event, the method does provide a solution with no computational uncertainties to certain problems, and this should be useful as a "benchmark."

APPENDIX A

MORE GENERAL LINEAR CREEP MODELS

The Maxwell solid model proposed in Chapter 2 corresponds to a relationship between the deviatoric components of stress and strain of the form

$$q_1 \dot{e}_j = p_0 s_j + p_1 \dot{s}_j \quad (42)$$

where $q_1 = 1$, $p_0 = h$, and $p_1 = \frac{1+\nu}{E}$. The quantity e is the deviatoric strain, $e_j = \epsilon_j - \epsilon_m$; $\epsilon_m = \frac{1}{3}(\epsilon_r + \epsilon_\theta + \epsilon_j)$. The elastic-viscoelastic analogy approach consists of simply defining new quantities, E' and ν' , such that

$$\bar{e}_j = \frac{1+\nu'}{E'} \bar{s}_j$$

and requiring invariance of the bulk modulus under this transformation, i.e.,

$$\frac{1-2\nu'}{E'} = \frac{1-2\nu}{E}.$$

Thus, application of the Laplace transform to Eq. (42) yields

$$\begin{aligned} \bar{e}_j &= \left(\frac{p_0}{s} + p_1 \right) \bar{s}_j \\ &= \left(\frac{h}{s} + \frac{1+\nu}{E} \right) \bar{s}_j \end{aligned}$$

or, $\frac{1+\nu'}{E'} = \frac{1+\nu}{E}$, in agreement with Eq. (13).

An obvious generalization of Eq. (42), although it has no particular physical justification, is simply to add further time derivatives to one or both sides of the equation.

$$\begin{aligned} & [q_n \frac{d^{(n)}}{dt^{(n)}} + \dots + q_1 \frac{d}{dt} + q_0] e_j \\ & = [p_m \frac{d^{(m)}}{dt^{(m)}} + \dots + p_1 \frac{d}{dt} + p_0] s_j. \end{aligned}$$

Using operator notation for abbreviation, this can be written

$$Qe_j = PS_j \quad (43)$$

where P and Q are the operators indicated. One possible benefit of this type of formulation is that more constants are now available for fitting experimental data.

The analogy method is easily applied to this model as well, in principle. Performing the Laplace transform operation upon (43) gives

$$\bar{e}_j = \frac{P(s)}{Q(s)} \bar{s}_j$$

where $P(s) = p_m s^m + p_{m-1} s^{(m-1)} + \dots + p_1 s + p_0$ and similarly $Q(s)$ is just a polynomial of order n in the transform variable. The quantities E' and v' are then determined from the relations

$$\begin{aligned} \frac{1+v'}{E'} &= \frac{P(s)}{Q(s)} \\ \frac{1-2v'}{E'} &= \frac{1-2v}{E} \end{aligned}$$

Solution of stress analysis problems can now proceed by the same method indicated in Chapter 3, the only difference being that more complicated expressions are encountered for E' and ν' .

One observation can be made immediately regarding such solutions. The quantity $\frac{E'}{1-\nu'}$ appears ubiquitously in the Laplace transforms which must be inverted. In the case of the simple Maxwell model treated previously, $\frac{E'}{1-\nu'} = \frac{E}{1-\nu} \frac{s}{s+\beta}$ (see Chapter 5). Since this quantity has only a single pole, inversion produces only a single time constant for exponential decay. With the formulation of (43) it must be anticipated that there will be more than a single pole, and therefore several time constants will appear. Also, of course, the solution will be greatly more complicated algebraically.

However, a model of the type (43) has the potential of at least qualitatively overcoming some of the shortcomings of the Maxwell model. If a tensile test is performed applying a stress history as in Figure 12a, the Maxwell model predicts the response of Figure 12b. Thus, the specimen undergoes instantaneous elastic deformation, followed by a linear increase in strain due to creep. Upon removal of the load, the elastic portion of the strain is recovered immediately, and the inelastic part is retained indefinitely.

Figure 12. Qualitative Sketches of Creep Behavior

Real creep tests, on the other hand, show behavior qualitatively as shown in Figure 12c. Following the instantaneous elastic deformation is a period of "primary" creep where the strain rate diminishes with time, finally approaching a steady value in the "secondary" creep phase. Upon removal of the load, the elastic part of the strain is instantly recovered, but part of the inelastic strain is also recovered.

Certain higher order differential operator formulations such as in Eq. (43) are more in conformity with the behavior of Figure 12c than the simple Maxwell model. According to Lee [28], the simplest model which exhibits all the features of primary creep, secondary creep, and recovery is

$$(q_2 \frac{d^2}{dt^2} + q_1 \frac{d}{dt})e = (p_0 + p_1 \frac{d}{dt} + p_2 \frac{d^2}{dt^2})S.$$

It is further stated that more complex equations of this type will of course give different quantitative results, but do not introduce qualitatively different behavior.

Still another form of linear creep relationship which is sometimes encountered is the so-called hereditary integral formulation [29]:

$$e_j = \int_0^t J(t-\tau) \frac{d\sigma_j}{d\tau} d\tau$$

where $J(t)$ is a creep compliance kernel. Since this integral is a convolution integral, the analogy works

equally well here. Applying the Laplace transform,

$$\bar{e}_j = s J(s) \bar{\sigma}_j.$$

Therefore E' and ν' are determined from the equation

$$\frac{1+\nu'}{E'} = s J(s)$$

$$\frac{1-2\nu'}{E'} = \frac{1-2\nu}{E}.$$

This formulation includes the previous model of Eq. (43) as a special case if

$$s J(s) = \frac{P(s)}{Q(s)}.$$

It might be interesting to apply some of these more general linear creep laws to a fuel element analysis of the type done in the body of this report. However, the solutions would become even more complex algebraically, perhaps diminishing the value of the work in view of the objectives as stated in the introduction. More importantly, there simply are not sufficient experimental data regarding creep of reactor components to support these more involved creep laws. Therefore, attention was restricted to the Maxwell model.

APPENDIX B

ALGEBRAIC DETAILS

In this appendix, various algebraic developments are outlined which were omitted in the main text of the paper.

B.1 Derivation of Equation (15)

From (10c)

$$\sigma_z^* = E(\epsilon_z^* - \epsilon^T - \epsilon^S) + \nu(\sigma_r^* + \sigma_\theta^*)$$

substituting into (10a) and (10b)

$$\epsilon_r^* = \frac{1+\nu}{E} [(1-\nu)\sigma_r^* - \nu\sigma_\theta^*] - \nu\epsilon_z^* + (1+\nu)(\epsilon^T + \epsilon^S)$$

$$\epsilon_\theta^* = \frac{1+\nu}{E} [(1-\nu)\sigma_\theta^* - \nu\sigma_r^*] - \nu\epsilon_z^* + (1+\nu)(\epsilon^T + \epsilon^S).$$

Now utilize the compatibility Eq. (10e) in the form

$$\epsilon_r^* - \epsilon_\theta^* = r \frac{d\epsilon_\theta^*}{dr}$$

with the above two equations to obtain

$$\sigma_r^* - \sigma_\theta^* = (1-\nu)r \frac{d\sigma_\theta^*}{dr} - \nu r \frac{d\sigma_r^*}{dr} + Er \frac{d}{dr} (\epsilon^T + \epsilon^S).$$

Here the plane strain assumption, $\frac{d\epsilon_z^*}{dr} = 0$, has been incorporated. Also, it is at this point that it is necessary to assume that the mechanical constants E and ν are space independent. From the equilibrium Eq. (10d),

$$\sigma_r^* - \sigma_\theta^* = -r \frac{d\sigma_r^*}{dr}.$$

Substituting,

$$\frac{d}{dr} (\sigma_r^* + \sigma_\theta^*) + \frac{E}{1-\nu} \frac{d}{dr} (\epsilon^T + \epsilon^S) = 0.$$

Use equilibrium again:

$$\frac{d}{dr} (\sigma_r^* - \sigma_\theta^*) + \frac{d}{dr} \left[r \frac{d\sigma_r^*}{dr} \right] = 0.$$

Adding the last two equations gives

$$\frac{d}{dr} (2\sigma_r^*) + \frac{d}{dr} \left[r \frac{d\sigma_r^*}{dr} \right] + \frac{E}{1-\nu} \frac{d}{dr} (\epsilon^T + \epsilon^S) = 0.$$

Finally, this can be written in the form of Eq. (15)

$$\frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} (r^2 \sigma_r^*) \right] = - \frac{E}{1-\nu} \frac{d}{dr} (\epsilon^T + \epsilon^S).$$

B.2 Derivation of Equation (16)

Equation (15) can be directly integrated twice to yield

$$\sigma_{r,n}^* = A_n + \frac{B_n}{r^2} - \frac{1}{r^2} \frac{E_n}{1-\nu_n} \int_{a_n}^r r' (\epsilon^T + \epsilon^S) dr'$$

where A_n and B_n are constants to be determined from the boundary conditions. Application of (8a) or (8c) gives

$$\sigma_{r,n}^*(a_n) = -p_{n-1} = A_n + \frac{B_n}{a_n^2}$$

while (8b) or (8d) gives

$$\sigma_{r,n}^*(b_n) = -p_n = A_n + \frac{B_n}{b_n^2} - \frac{1}{b_n^2} \frac{E_n}{1-\nu_n} \int_{a_n}^{b_n} r' (\epsilon^T + \epsilon^S) dr'$$

$$p_n - p_{n-1} = B_n \left(\frac{1}{2} - \frac{1}{b_n} \right) + \frac{1}{2} \frac{b_n^2 - a_n^2}{b_n^2} \frac{E_n}{1 - v_n} (\hat{\epsilon}_n^T + \hat{\epsilon}_n^S)$$

$$\frac{B_n}{a_n} = \frac{b_n^2}{b_n^2 - a_n^2} (p_n - p_{n-1}) - \frac{1}{2} \frac{E_n}{1 - v_n} (\hat{\epsilon}_n^T + \hat{\epsilon}_n^S)$$

$$A_n = \frac{a_n^2 p_{n-1} - b_n^2 p_n}{b_n^2 - a_n^2} + \frac{1}{2} \frac{E_n}{1 - v_n} (\hat{\epsilon}_n^T + \hat{\epsilon}_n^S).$$

With A_n and B_n thus determined, $\sigma_{r,n}^*$ can now be written

$$\begin{aligned} \sigma_{r,n}^* &= \frac{a_n^2}{b_n^2 - a_n^2} \left(1 - \frac{b_n^2}{r^2} \right) p_{n-1} - \frac{b_n^2}{b_n^2 - a_n^2} \left(1 - \frac{a_n^2}{r^2} \right) p_n \\ &\quad + \frac{1}{2} \frac{E_n}{1 - v_n} \left(1 - \frac{a_n^2}{r^2} \right) (\hat{\epsilon}_n^T + \hat{\epsilon}_n^S) \\ &\quad - \frac{1}{r^2} \frac{E_n}{1 - v_n} \int_{a_n}^r r' (\epsilon^T + \epsilon^S) dr' \end{aligned}$$

which is equivalent to Eq. (16).

B.3 Derivation of Equations (18) and (19)

Insertion of (16) and (17) into (10c) gives

$$\begin{aligned} \epsilon_{z,n}^* &= \frac{1}{E_n} \sigma_{z,n}^* - \frac{v_n}{E_n} \left[\frac{2p_{n-1}}{b_n^2/a_n^2 - 1} - \frac{2p_n}{1 - a_n^2/b_n^2} \right. \\ &\quad \left. + \frac{E_n}{1 - v_n} (\hat{\epsilon}_n^T + \hat{\epsilon}_n^S) - \frac{E_n}{1 - v_n} (\epsilon^T + \epsilon^S) \right] + (\epsilon^T + \epsilon^S). \end{aligned}$$

Now multiply this equation by $2 \, r \, dr$ and integrate from a_n to b_n , recalling that

$$\int_{a_n}^{b_n} 2\pi r \, \sigma_{z,n}^* \, dr = F_n \text{ and } \epsilon_{z,n}^* = \text{constant.}$$

This gives

$$\epsilon_{z,n}^* = - \frac{v_n}{E_n} \frac{2}{b_n^2 - a_n^2} (a_n^2 p_{n-1} - b_n^2 p_n) + (\hat{\epsilon}_n^T + \hat{\epsilon}_n^S) + \frac{1}{E_n} \frac{F_n}{\pi(b_n^2 - a_n^2)}$$

which is Eq. (18). Substitution of this result back into the preceding equation gives $\sigma_{z,n}^*$.

$$\sigma_{z,n}^* = \frac{E_n}{1-v_n} [(\hat{\epsilon}_n^T + \hat{\epsilon}_n^S) - (\epsilon_n^T + \epsilon_n^S)] + \frac{F_n}{\pi(b_n^2 - a_n^2)}$$

which is Eq. (19).

B.4 Derivation of Equation (30)

From Eq. (23) and the discussion of Chapter 3,

$$\begin{aligned} \bar{p}_{int} = & \frac{b_1 - a_2}{I'} + \frac{1}{I'} \left[\frac{a_1^2}{b_1^2 - a_1^2} \frac{2b_1 \bar{p}_0}{E_1'} + \frac{b_2^2}{b_2^2 - a_2^2} \frac{2a_2 \bar{p}_2}{E_2'} \right. \\ & + \frac{v_1' b_1 \bar{p}_0}{E_1'} + \frac{v_2' a_2}{E_2'} \frac{a_2^2 \bar{p}_0 - b_2^2 \bar{p}_2}{b_2^2 - a_2^2} \left. \right] \\ & + \frac{1}{I'} [b_1 (\bar{\epsilon}_1^T + \bar{\epsilon}_1^S) - a_2 (\bar{\epsilon}_2^T + \bar{\epsilon}_2^S)] \end{aligned}$$

where

$$I' = \frac{a_2}{E_2'} \left(\frac{b_2^2 + a_2^2}{b_2^2 - a_2^2} + v_2' \right) + \frac{b_1}{E_1'} \left(\frac{b_1^2 + a_1^2}{b_2^2 - a_2^2} - v_1' \right).$$

Using Eqs. (13) and (14)

$$I' = I + \frac{a_2 h_2}{3s} \left(2 \frac{b_2^2 + a_2^2}{b_2^2 - a_2^2} + 1 \right) + \frac{b_1 h_1}{3s} \left(2 \frac{b_1^2 + a_1^2}{b_1^2 - a_1^2} - 1 \right)$$

or

$$I' = \frac{s + \lambda}{s} I$$

where λ is defined by

$$\lambda = \frac{1}{I} \frac{1}{3} \left[a_2 h_2 \left(2 \frac{b_2^2 + a_2^2}{b_2^2 - a_2^2} + 1 \right) + b_1 h_1 \left(2 \frac{b_1^2 + a_1^2}{b_1^2 - a_1^2} - 1 \right) \right]$$

Then

$$\begin{aligned} \bar{p}_{int} = \frac{s}{s+\lambda} \bar{p}_{int}^* + \frac{1}{I} \left[\frac{4b_1 a_1^2 h_1}{3(b_1^2 - a_1^2)} \bar{p}_o + \frac{4a_2 b_2^2 h_2}{3(b_2^2 - a_2^2)} \bar{p}_2 \right. \\ \left. + \frac{b_1 h_1}{3} \bar{p}_o + \frac{a_2 h_2}{3} \frac{a_2^2 \bar{p}_o - b_2^2 \bar{p}_2}{b_2^2 - a_2^2} \right] \frac{1}{s+\lambda} \end{aligned}$$

Inversion into the time domain gives

$$p_{int}(t) = p_{int}^*(t) + \int_{t_{int}}^t e^{-\lambda(t-\tau)} [f(\tau) - \lambda p_{int}^*(\tau)] d\tau$$

where

$$\begin{aligned} f(\tau) = \frac{1}{I} \left[\frac{4b_1 a_1^2 h_1}{3(b_1^2 - a_1^2)} p_o(t) + \frac{4a_2 b_2^2 h_2}{3(b_2^2 - a_2^2)} p_2(t) \right. \\ \left. + \frac{b_1 h_1}{3} p_o(t) + \frac{a_2 h_2}{3} \frac{a_2^2 p_o(t) - b_2^2 p_2(t)}{b_2^2 - a_2^2} \right] \end{aligned}$$

t_{int} is the time at which fuel-cladding interaction commenced.

APPENDIX C

COMPUTATIONS TO PRODUCE FIGURE 4

From Eq. (21) with $p_2 = \text{constant} = 75 \text{ psi}$, $p_1 = p_0 = 750X$, and $F_2 = \pi(a_2^2 p_0 - b_2^2 p_2)$,

$$\begin{aligned} \epsilon_{\theta,2}^*(a_2, t) = & p_0 \frac{1-2\nu_2}{E_2} \frac{a_2^2}{b_2^2 - a_2^2} + p_0 \frac{1+\nu_2}{E_2} \frac{b_2^2}{b_2^2 - a_2^2} \\ & - \frac{1-2\nu_2}{E_2} p_2 \frac{b_2^2}{b_2^2 - a_2^2} - \frac{1+\nu_2}{E_2} p_2 \frac{b_2^2}{b_2^2 - a_2^2} + \hat{\epsilon}_2^T + \hat{\epsilon}_2^S. \end{aligned}$$

Therefore,

$$\begin{aligned} \bar{\epsilon}_{\theta,2}(a_2, s) = & \frac{1-2\nu_2}{E_2} \left[\frac{a_2^2 \bar{p}_0 - b_2^2 \bar{p}_2}{b_2^2 - a_2^2} \right] \\ & + \frac{1+\nu_2}{E_2} \frac{b_2^2}{b_2^2 - a_2^2} (\bar{p}_0 - \bar{p}_2) + \bar{\epsilon}_2^T + \bar{\epsilon}_2^S. \end{aligned}$$

Insertions of the appropriate expressions for E' , ν' and $\bar{p}_0$, and inversion of the result into the time domain produces the result

$$\begin{aligned} \epsilon_{\theta,2}(a_2, t) = & \frac{1-2\nu_2}{E_2} \left[\frac{a_2^2 p_0(t) - b_2^2 p_2}{b_2^2 - a_2^2} \right] \\ & + \frac{b_2^2}{b_2^2 - a_2^2} \frac{1+\nu_2}{E_2} [p_0(t) - p_2] + \frac{b_2^2 h_2}{b_2^2 - a_2^2} \int_0^t (p_0 - p_2) dt' \\ & + \hat{\epsilon}_2^T + \hat{\epsilon}_2^S. \end{aligned}$$

Numerical values for all of the quantities in the above expression were provided in Chapter 6. Values for $\epsilon_{\theta,2}(a_2,x)$ are given in Table 6.

From Eq. (22) with $p_1 = p_0 = p_{100}x$ and $F_1 = -p_{100}(b_1^2 - a_1^2)$,

$$\epsilon_{\theta,1}^*(b_1,t) = -\frac{1-2\nu_1}{E_1} p_0 + \hat{\epsilon}_1^T + \hat{\epsilon}_1^S.$$

Since $\frac{1-2\nu_1'}{E_1'} = \frac{1-2\nu_1}{E_1}$, it follows directly that

$$\epsilon_{\theta,1}(b_1,t) = \hat{\epsilon}_1^T + \hat{\epsilon}_1^S - \frac{1-2\nu_1}{E_1} p_0.$$

Numerical evaluation of this expression produced Table 7.

APPENDIX D

NOMENCLATURE

a_n	inner radius of region n
b_n	outer radius of region n
$\dot{B}$	burnup rate or specific power, kw/kg
B	burnup Mwd/MT (heavy metal)
E	Young's modulus, psi
F_n	axial load on region n, lb _f
h	creep constant, psi ⁻¹ sec ⁻¹
I	interaction constant [see Eq. (27), p. 30]
J_2	creep relaxation factor (see p. 61)
k	thermal conductivity, Watts/cm-°C
k_r	value of k at some convenient reference temperature
$\bar{k}$	mean value of k
p	pressure, psi
Q'	linear power rating, kw/ft
Q'''	volumetric power density, Watts/cm ³
s	Laplace transform variable
S	deviatoric stress
T	temperature, °C
t	time
t_{100}	time to achieve burnup of 100000 Mwd/MT
t_{int}	interaction time

- u displacement from equilibrium position
 α linear coefficient of thermal expansion
 β creep constant $\beta = \frac{hE}{3(1-\nu)}$
 ϵ_j component of strain in coordinate direction j ,
 $j = r, \theta, z$
 ϵ_j^C component of creep strain in direction j , $j =$
 r, θ, z
 ϵ_j^S component of irradiation swelling strain in
direction j , $j = r, \theta, z$
 Θ Pseudo temperature $\Theta = \frac{1}{k_r} \int_{T_r}^T k(T') dT'$
 λ Creep relaxation time constant
 ν Poisson's ratio
 σ stress, psi
 τ time variable
 Φ neutron flux, $\text{cm}^{-2}\text{sec}^{-1}$
 χ fractional burnup $B/10^5$
 $\dot{}$ time derivative; e.g., $\dot{y} = \frac{dy}{dt}$
 $\bar{}$ Laplace transform of; e.g., $\bar{y} = \int_0^\infty e^{-st} y(t) dt$
 $*$ denotes quantities for related thermoelastic problem
 $\bar{}$ denotes area average of a quantity

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