THERMAL RATCHETING OF A BEAM

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ABSTRACT

Thermal ratcheting has been analytically investigated for a beam subjected to a linear temperature variation across its solid rectangular cross section. A linear strain-hardening material response exhibiting an idealized Bauschinger effect was assumed. Formulas are given for the associated strain accumulation which are valid over a large range of strain hardening, temperature variation, temperature cycles, and axial load. Specific results are tabulated for the materials and operating conditions typically associated with liquid metal breeder reactors.

INTRODUCTION

Much of the current motivation for understanding and estimating thermal ratcheting is due to the design requirements of nuclear reactors. Presently, the codes require designs which exclude ratcheting based on the conservative criteria developed for an elastic-perfectly plastic one-dimensional continuum [1,2]. However, the extreme operating conditions envisioned for liquid metal breeder reactors may require designs which account for some ratcheting. Since the materials proposed for these reactors exhibit definite Bauschinger effects [3], then information on kinematic hardening ratcheting is required to optimize designs.

Comprehensive studies of the thermal ratcheting of an elastic-perfectly plastic one-dimensional continuum have been made and reviewed by Burgreen [4]. In the discussions which follow the term ratcheting shall always imply thermal ratcheting as defined by Burgreen. For strain-hardening materials some idealized problems have been considered. Idealizations are required because of the variety and the complexity of strain-hardening metal response which exist even for one-dimensional stress states.

Thermal ratcheting of a one-dimensional continuum exhibiting an isotropic hardening material response has received some attention [2], wherein a very small constant hardening modulus was assumed. The strain accumulation per cycle was found to decrease with cycling resulting in a finite strain accumulation for an infinite number of cycles. Thus, the constant strain accumulation per cycle of a perfectly plastic material greatly overestimates isotropic strain-hardening ratcheting. The range of mechanical and thermal loads for which ratcheting occurred was found to be very nearly the same for both types of hardening.

A study [5] which compared the basic ratcheting mechanism associated with kinematic and isotropic hardening has been recently completed. The two-element assembly used by Burgreen [6] to illustrate the basic perfectly-plastic ratcheting mechanism was employed. No assumption was made with regard to the size of the constant hardening modulus. Both the amount of strain accumulation and range of mechanical and thermal loads for which ratcheting occurred was found to be greater for isotropic hardening materials than for kinematic hardening materials, especially at higher temperatures and smaller hardening moduli. Although such results are useful for assessing the need for kinematic hardening ratcheting calculations, its usefulness in the design of components is limited to those which can be idealized as a two-element assembly. Many nuclear reactor components are better represented as beams, plates, or shells having solid cross sections.

PROBLEM DESCRIPTION

The load response relations for a solid rectangular beam having a cross-sectional area of $2A$ are presented here. Plane cross sections are assumed to remain in plane and perpendicular to the axis of the beam, in keeping with the analogy to a cylindrical or spherical shell proposed by Bree [2]. The one-dimensional piecewise linear elastic-kinematic hardening material response shown in Fig. 1 is assumed. No restriction is made regarding the size of the constant hardening modulus. Initially, a constant centroidal axial load of P is applied resulting in a uniform stress which is less than the elastic limit σ_y .

A linear temperature distribution across the depth of the beam is assumed. The difference between the temperature at the end of the heating half cycle and the ambient temperature is

$$\Delta T = -\frac{T}{2} \left(1 + \frac{y}{H} \right) \quad (1)$$

where H is the half depth of the beam and T is the temperature at the top of the beam $y = H$. Distance measured perpendicular to the centroidal axis is represented by y . At the end of the cooling half cycle the ambient temperature is again attained. Without loss of generality, the ambient temperature is assumed to be zero and the maximum temperature variation in (1) is between $-T$ and 0 . Strains associated with other temperature limits, say $(-T + \text{extra temperature})$ to $(\text{extra temperature})$, can be obtained by superposition of the uniform temperature strain associated with $(\text{extra temperature})$ upon the results developed herein.

According to Fig. 1 the stress strain relation during the initial heating half cycle

$$\epsilon_1 = \frac{\sigma_1}{E} + \gamma \frac{k}{E} (\sigma_1 - \sigma_y) + \alpha \Delta T \quad (2)$$

where γ is 1 or 0 if the elastic limit is or is not exceeded, respectively.

For increasing (decreasing) σ_1 the minus (plus) sign is used in (2).

The coefficient of thermal expansion is α , Young's elastic modulus is E ,

and k is related to the linear hardening modulus βE by

$$k = \frac{1-\beta}{\beta} \quad (3)$$

If the fiber of the beam is plastically deformed during the $n-1$ half cycle, then for the n th subsequent half cycle of heating or cooling the material response is

$$\Delta \epsilon_n = \frac{\Delta \sigma_n}{E} + \gamma \frac{k}{E} (\Delta \sigma_n - \gamma_1 2\sigma_y) + \alpha \Delta(\Delta T) \quad (4a)$$

where $\Delta \epsilon_n$, $\Delta \sigma_n$, $\Delta(\Delta T)$ are the changes which occur in the strain, stress, and temperature, respectively, during the half cycle. γ_1 is +1, -1, or zero for $\Delta \sigma_n$ positive and $\Delta \sigma_{n-1}$ negative, $\Delta \sigma_n$ negative and $\Delta \sigma_{n-1}$ positive, and $\Delta \sigma_n$ and $\Delta \sigma_{n-1}$ the same sign, respectively. If the fiber of the beam is elastically deformed in the $(n-1)$ th half cycle, plastically deformed in the n th half cycle, elastically deformed in the $(n+1)$ th half cycle, etc., then during the n th, $(n+2)$ th, etc., half cycle the material response is given by

$$\Delta \epsilon_n = \frac{\Delta \sigma_n}{E} + k(\Delta \sigma_n + \Delta \sigma_{n-1}) + \alpha \Delta(\Delta T) \quad (4b)$$

Elastic cycling response is included (4a). Note that the subscripts on parameters, associated with deformation, indicate the number of the current half cycle.

ELASTIC CYCLING, SHAKEDOWN, AND PLASTIC CYCLING

For a given beam, several modes of deformation can occur depending upon the magnitude of the axial load P and the temperature range T . The loading conditions which produce elastic cycling in every half cycle are the same for any type of initially isotropic hardening response [1].

$$\frac{\alpha TE}{2} < \left(\sigma_y - \frac{P}{2A} \right) \quad (5)$$

Relation (5) defines Region E in Fig. 2.

Shakedown is defined as cycling wherein plastic deformation occurs during the first heating half cycle, but only elastic deformation occurs in subsequent cycles. Two different first half modes of plastic deformation can precede shakedown. Plastic deformation can occur either in the top beam fibers alone, Fig. 3a, with $l_1 \geq H$, or in both top and bottom fibers, Fig. 3a, when $l_1 \leq H$.

For the stress distribution of Fig. 3a with $l_1 \geq H$, the constant values of ϵ_1 and h_1 at the end of the first half cycle are determined by equilibrium condition

$$P = \int \int_A \sigma_1 dA \quad (6)$$

and the yield condition $\sigma_1 = \sigma_y$ at $y = h_1$. Substituting the appropriate forms of the stress-strain relations (2) into these conditions gives two expressions relating ϵ_1 to h_1 , respectively

$$\epsilon_1 E = \sigma_y - \frac{\alpha TE}{2} \left(1 + \frac{h_1}{H} \right) \quad (7a)$$

$$\frac{k\alpha TE}{8} \left(1 + \frac{h_1}{H} \right)^2 + \frac{k\epsilon_1 E}{2} \left(1 + \frac{h_1}{H} \right) + \frac{k\sigma_y}{2} \left(1 - \frac{h_1}{H} \right) = \frac{P}{2A} (1+k) - \epsilon_1 E - \frac{\alpha TE}{2} \quad (7b)$$

Combining (7a) and (7b) gives

$$\frac{h_1}{H} = \left[\frac{4}{k} + \frac{4}{k^2} + \left(\frac{k+1}{k} \right) \frac{8}{\alpha TE} \left(\sigma_y - \frac{P}{2A} \right)^2 \right]^{1/2} - \left(1 + \frac{2}{k} \right) \quad (7c)$$

The mode of deformation represented by Fig. 3a with $\ell_1 \geq H$ remains valid as long as

$$\sigma_1 \geq -\sigma_y \quad \text{at} \quad y = -H \quad (8a)$$

$$h_1 \geq 0 \quad (8b)$$

With (7), both of (8) can be expressed in terms of P and T,

$$\frac{\alpha TE}{2} \frac{P}{2A} (1+k) \leq \left(\frac{\alpha TE}{2} - \sigma_y \right) \left(\frac{\alpha TE}{2} + k\sigma_y \right) \quad (9a)$$

$$\frac{k\alpha TE}{8} \leq (1+k) \left(\sigma_y - \frac{P}{2A} \right) \quad (9b)$$

respectively. One can show that if P and T satisfy (9) and violate (5), Region S_1 in Fig. 2, then only elastic response occurs in subsequent half cycles. Curves C_1 and C_2 in Fig. 2 are graphical representations of (9a) and (9b), respectively, when the equality sign is employed.

If (8a) is violated and (8b) is satisfied, then the other shakedown mode represented by Fig. 3a with $\ell_1 \leq H$ may occur. The values of ϵ_1 , h_1 , and ℓ_1 are determined by the equilibrium condition (6), $\sigma_1 = \sigma_y$ at $y = h_1$, and $\sigma_1 = -\sigma_y$ at $y = -\ell_1$. Combining these conditions with the appropriate form of the stress-strain relation (2) gives

$$\frac{h_1}{H} = \frac{P}{2A} (1+k) \left[\frac{\alpha TE}{2} + k\sigma_y \right]^{-1} - \frac{2\sigma_y}{\alpha TE} \quad (10a)$$

$$\frac{\ell_1}{H} = \frac{4\sigma_y}{\alpha TE} - \frac{h_1}{H} \quad (10b)$$

$$E\epsilon_1 = \sigma_y - \frac{\alpha TE}{2} \left(1 - \frac{h_1}{H} \right) \quad (10c)$$

In the second half cycle, a probable stress distribution including plastic deformation is given by Fig. 3b. Assuming $\ell_2 > h_1$, $h_2 > \ell_1$, the conditions of equilibrium for any half cycle

$$0 = \int \int_A \Delta \sigma_n dA, \quad (11)$$

$\Delta \sigma_2 = -2\sigma_y$ at $y = l_2$, and $\Delta \sigma_2 = 2\sigma_y$ at $y = -h_2$ can be combined with the appropriate form of the stress-strain relations (4) to determine $\Delta \epsilon_2$, h_2 , and l_2 to be

$$\Delta \epsilon_2 = \frac{\alpha TE}{2} \quad (12a)$$

$$\frac{h_2}{H} = \frac{l_2}{H} = \frac{4\sigma_y}{\alpha TE} \quad (12b)$$

Comparing (12) with (10), one sees that $l_2 > l_1$ and $h_2 > h_1$ for positive h_1 , if

$$\frac{\alpha TE}{2} \left[\frac{P}{2A} (1+k) - \sigma_y \right] \leq k \sigma_y^2 \quad (13)$$

and

$$\alpha TE > 4\sigma_y \quad (14)$$

For αTE which violate (14) but not (13), Region S_2 of Fig. 2, shakedown occurs.

When both conditions (13) and (14) are satisfied, Region P in Fig. 2, one can show that repeated plastic strain cycling occurs with a total change of strain

$$\Delta \epsilon_n E = (-1)^n \frac{\alpha TE}{2} \quad (15)$$

in each half cycle. The values of h_n and l_n are given by (12b). The curve C_3 in Fig. 2 is represented by (13) with the equality sign. Note that curve C_3 is asymptotic to the line

$$\frac{P}{2A} = \frac{1}{(1+k)} \sigma_y \quad (16)$$

Further, as $k \rightarrow \infty$ ($\beta \rightarrow 0$), the intersection of curves C_1 , C_2 , and C_3 approach the coordinate $\left(\frac{1}{2} \sigma_y, 4\sigma_y\right)$ and the line given by (16) coincides with the line $P = 0$.

RATCHETING

Obviously, ratcheting cannot occur for the deformation modes described in Figs. 3a and 3b since the central beam fibers are never permanently deformed. The deformation mode illustrated in Fig. 3c can be expected to produce ratcheting.

Case 1

As might be anticipated, the first heating half cycle analysis and results, equations (10a-c), are the same for the deformation modes of Fig. 3a, with $l_1 \leq H$, and Fig. 3c when the h_n are defined as shown. For the n th subsequent heating half cycle, the analysis is based on the simultaneous solution of the equilibrium condition (11), the condition $\Delta\sigma_n = 0$ at $y = -h_n$, and $\Delta\sigma_n = -2\sigma_y$ at $y = -l_n$. For the $(n+1)$ cooling half cycle the conditions are the same, but the latter two are evaluated at $y = +h_{n+1}$ and $y = +l_{n+1}$, respectively. That is, l_n and h_n always are defined so that they are positive. Combining the appropriate form of the stress-strain relations (4) with the three conditions listed above gives the explicit results

$$\frac{h_n}{H} = \left[\frac{\alpha TE}{2k\sigma_y} + 1 \right]^{(1-n)} \frac{h_1}{H} \quad (17a)$$

$$\frac{l_n}{H} = \frac{4\sigma_y}{\alpha TE} + \frac{h_n}{H} \quad (17b)$$

$$\Delta\epsilon_n E = \frac{\alpha TE}{2} \left[\frac{h_n}{H} + (-1)^n \right] \quad (17c)$$

where h_1 is given by (10a).

From (17a) and (17b) one can deduce that $h_n < l_n < l_1$. Therefore the condition $l_1 \geq 0$ is the condition which determines the values of T and P for which the deformation mode of Fig. 3c is valid. Since the results for the first cycle are the same as determined for the deformation

mode of Fig. 3a with $h_1 \leq H$, the bounding curve C_1 is applicable.

Region (FR₁) in Fig. 2 is the region for which (17) is valid.

The strain at the end of any half cycle where $n \geq 2$ can be determined by summing (17c) and (10c)

$$\epsilon_n E = \epsilon_1 E + \frac{\alpha TE}{2} \left[\frac{h_2}{H} + \dots + \frac{h_n}{H} \right] + \frac{\alpha TE}{4} [1 + (-1)^n] \quad (18)$$

where $\epsilon_1 E$ is given by (10c). Combining (18) with (17a) results in a geometric series which has the sum

$$\epsilon_n E = \epsilon_1 E + \frac{\alpha TE}{4} [1 + (-1)^n] + k\sigma_y \left(\frac{h_1}{H} \right) \left[1 - \left(\frac{\alpha TE}{2k\sigma_y} + 1 \right)^{1-n} \right] \quad (19)$$

Note that the sum is finite for $n \rightarrow \infty$.

Case 2

The results of Case 1 for the P and αTE defining curve C_1 indicate that another mode of deformation occurs for which strain accumulates. Figure 3d illustrates the deformation mode, where again the h_n values always are defined so as to be positive. During the first half cycle, the two conditions to be satisfied are the equilibrium condition (6) and $\sigma_1 = \sigma_y$ at $y = -h_1$. Combining these conditions with the appropriate form of the stress-strain relations, one finds

$$\frac{h_1}{H} = \left(1 + \frac{2}{k} \right) - \left[\frac{4}{k} + \frac{4}{k^2} + \left(\frac{k+1}{k} \right) \frac{8}{\alpha TE} \left(\sigma_y - \frac{P}{2A} \right) \right]^{1/2} \quad (20)$$

which is the negative of (7c). The value of ϵ_1 is found by combining (10c) and (20).

For subsequent half cycles, the equilibrium conditions (11) and the condition $\Delta\sigma_n = 0$ at $y = (-1)^n h_n$ must be satisfied. Combining these conditions yields, for this case, a recursion relation for h_n

$$\frac{h_n}{H} = \left(1 + \frac{2}{k} \right) - \left[\frac{4}{k^2} + \frac{4}{k} + \left(1 - \frac{h_{n-1}}{H} \right)^2 \right]^{1/2} \quad (21)$$

Knowing $\frac{h_n}{H}$ from (20), $\Delta\epsilon_n$ can be calculated via (17c). As can be seen by inspection of (21), the deformation in the n th half cycle depends upon the deformation in the $(n-1)$ th half cycle. Both h_n and ϵ_n decrease as cycling continues. Ultimately, as $n \rightarrow \infty$, $h_n \rightarrow 0$ and $\Delta\epsilon_n \rightarrow 0$. The dependence of the current deformation on past deformation is unique to this mode of deformation.

To determine h_n for a finite number of cycles, the successive numerical evaluation of (21) is required. Little chance exists that an explicit expression for h_n can be determined for such a complicated recursion relation. The change in h_n with n is easily calculated on a digital computer for the range of values of k of interest. The results are plotted in Fig. 4 and can be used for computations. The starting value of $\left(1 - \frac{h_1}{H}\right)$, for $N = 1$, corresponds to the value of (20) for $\frac{P}{2A} = \sigma_y$; the largest value of $\frac{h_1}{H}$ for a given k . To determine $\left(1 - \frac{h_n}{H}\right)$ one simply enters the graph on the appropriate $k = \text{constant}$ curve at the N corresponding to $\frac{h_1}{H}$, say $N = N_1$, and then reads the value of the curve at $N = N_1 + n - 1$. For instance, suppose $k = 999$ ($\beta = 0.001$), $\frac{P}{2A} = \sigma_y$, and $\left(1 - \frac{h_{400}}{H}\right)$ is desired. According to (20), $\left(1 - \frac{h_1}{H}\right) = 0.061$. For the $k = 999$ curve in Fig. 4, 0.061 corresponds to $N = N_1 = 1$. Therefore at $N = 400$ one reads $\left(1 - \frac{h_{400}}{H}\right) = 0.80$ for the answer. As another example, suppose $\frac{P}{2A} = 0.95 \sigma_y$, $\alpha TE = 2\sigma_y$, $k = 999$, and $\left(1 - \frac{h_{400}}{H}\right)$ is desired. According to (20), $\left(1 - \frac{h_1}{H}\right) = 0.450$. On the graph of Fig. 4, 0.450 corresponds to $N = N_1 = 72$. The value of $\left(1 - \frac{h_{400}}{H}\right)$ is read at $N = 471$ to be 0.831.

To determine $\Delta\epsilon_n$ for a finite n , $\frac{h_n}{H}$ is obtained as described above and substituted into (17c). The value of ϵ_n is obtained from (18) which in its present form requires calculation of $\sum_{m=2}^n \frac{h_m}{H}$. A more convenient

form can be obtained by use of (21). Noting that (21) can be rewritten as

$$\left(1 - \frac{h_n}{H} + \frac{2}{k}\right)^2 = \left(1 - \frac{h_{n-1}}{H} + \frac{2}{k}\right)^2 + \frac{4}{k} \frac{h_{n-1}}{H}, \quad (21')$$

continued resubstitution of (21') leads to the relation

$$\frac{k}{4} \left[\left(1 - \frac{h_n}{H}\right)^2 - \left(1 - \frac{h_1}{H}\right)^2 \right] = \left(\frac{h_n}{H} + \dots + \frac{h_2}{H}\right). \quad (22)$$

Combining (22) with (18) gives

$$\epsilon_n E = \epsilon_1 E + \frac{k\alpha TE}{8} \left[\left(1 - \frac{h_n}{H}\right)^2 - \left(1 - \frac{h_1}{H}\right)^2 \right] + \frac{\alpha TE}{4} [1 + (-1)^n] \quad (23)$$

where ϵ_1 is determined by (10c) and (20). With (23), evaluation of ϵ_n only requires that $\frac{h_n}{H}$ be determined as outlined previously. For $n \rightarrow \infty$

$$\epsilon_\infty E = \epsilon_1 E + \frac{k\alpha TE}{8} \left[1 - \left(1 - \frac{h_1}{H}\right)^2 \right] + \frac{\alpha TE}{4} [1 + (-1)^n] \quad (24)$$

The sum is explicit and finite.

As an example, suppose $\frac{P}{2A} = 0.95 \sigma_y$, $\alpha TE = 2\sigma_y$, $k = 999$, and the strain at the end of the 400th temperature half cycle is desired. The value of $\left(1 - \frac{h_1}{H}\right) = 0.450$ and $\left(1 - \frac{h_{400}}{H}\right) = 0.831$ is determined from (20) and the graph of Fig. 4, respectively, as previously described. The value of $\epsilon_1 E$ is determined from (10c) to be $0.55 \sigma_y$. Entering these values into (23) yields $\epsilon_{400} E = 123.43 \sigma_y$. For $n \rightarrow \infty$, $\epsilon_\infty E = 200.73 \sigma_y$ as determined from (24).

The deformation mode of Case 2 is valid for all cycles where $h_n \geq 0$ and $l_n \geq H$. The first inequality is seen from (21) to be satisfied for all n . However, the second inequality, which is equivalent to the condition $|\Delta\sigma_n| \leq 2\sigma_y$, requires that

$$\frac{\alpha TE}{2} \left(1 - \frac{h_n}{H}\right) \leq 2\sigma_y \quad (25)$$

Thus, Case 2 cycling will occur for all cycles ($n \rightarrow \infty$) only if $\alpha TE \leq 4\sigma_y$. Region FR in Fig. 2.

Case 3

For the remaining values of T and P , Region (FR-FR₁) of Fig. 2, the deformation will be that of Case 2 for $(R-1)$ half cycles and that of Case 1 for subsequent half cycles, $n \geq R + 1$. The R th cycle, during which the transition from Case 1 to Case 2 type cycling occurs, is the one for which the inequality of (25) is first violated.

For $n \leq R$, Fig. 3d represents a typical stress distribution and h_n , $\Delta\epsilon_n$, ϵ_n , etc., are calculated as discussed in Case 2 above. During the R th half cycle the stress distribution changes from that of Fig. 3d to that of Fig. 3c. Following procedures similar to those outlined previously, the following expression for h_R can be determined:

$$\frac{h_R}{H} = \left[k\sigma_y \left(1 - \frac{2\sigma_y}{\alpha TE} \right) - k \frac{\alpha TE}{8} \left(1 - \frac{h_{R-1}}{H} \right)^2 \right] \left(\frac{\alpha TE}{2} + k\sigma_y \right)^{-1} \quad (26)$$

Expressions (17b) and (17c) remain valid for h_R and $\Delta\epsilon_R$, respectively.

For $n > R$, the solution is the same as for Case 1 except that Case 1 deformation mode is beginning in the $(R+1)$ th half cycle. In direct analogy to (17a)

$$\frac{h_n}{H} = \left[\frac{\alpha TE}{2k\sigma_y} + 1 \right]^{(R-n)} \frac{h_R}{H} \quad n > R \quad (27)$$

and (17b) and (17c) remain valid. In direct analogy to (19)

$$\epsilon_n E = \epsilon_R E + \frac{\alpha TE}{4} [(-1)^n - (-1)^R] + k\sigma_y \frac{h_R}{H} \left[1 - \left(\frac{\alpha TE}{2k\sigma_y} + 1 \right)^{R-n} \right] \quad n > R \quad (28)$$

where $\frac{h_R}{H}$ is given by (26) and ϵ_R is determined as discussed in Case 2.

For $n \rightarrow \infty$, a closed form upper bound to $\epsilon_\infty E$ can be obtained via (28).

Note that the part of the last term involving R goes to zero for $n \rightarrow \infty$, thus only values of $\epsilon_R E$ and $\frac{h_R}{H}$ are required. An upper bound on $\epsilon_R E$ can be obtained from (23), by choosing $\frac{h_n}{H}$ to be the smallest value of $\frac{h_n}{H}$ allowed by (25). An upper bound on $\frac{h_R}{H}$ can be obtained from (21) and

(25). By choosing $\frac{h_n}{H}$ in (21) to be the smallest value of $\frac{h_n}{H}$ allowed by (25), the calculable value of $\frac{h_{n-1}}{H}$ is always larger than $\frac{h_R}{H}$. The closeness of the upper bound to $c_{\infty}E$ depends upon the rate of change of $\frac{h_n}{H}$ per cycle, which depends upon P , T , and k . The smaller the rate of change, the closer the approximation will be. Fortunately, the bounds are poorest for the larger rates of change where the cycle by cycle numerical evaluation is the easiest.

As an example, $c_{400}E$ will be determined for $\frac{P}{2A} = 0.95 \sigma_y$, $\alpha TE = 6\sigma_y$, and $k = 999$. From (20), $\left(1 - \frac{h_1}{H}\right) = 0.264$ and Case 2 is valid as long as $\left(1 - \frac{h_n}{H}\right) < 0.667$, according to (25). From Fig. 4, 0.264 corresponds to $N_1 = 21.5$ and 0.667 corresponds to $N = 213$. Therefore, Case 2 remains valid for ~ 191 half cycles, and $R = 192$ and $\left(1 - \frac{h_{191}}{H}\right) \approx 0.660$. With the above values (23), (26), and (17c) give $c_{191}E = 274.37 \sigma_y$, $\frac{h_{192}}{H} = 0.339$, and $\Delta c_{192}E = 4.02 \sigma_y$ respectively. Using these results, $c_{400}E = 435.5 \sigma_y$ is determined from (28).

DISCUSSION

The values of (P,T) for which ratcheting occurs, call them $(P,T)_R$, are dependent upon the hardening coefficient k . The $(P,T)_R$ and associated strain accumulation will be determined for widely different k to show extremes in behavior. Figure 1 shows the stress-strain curve for a material with large strain hardening $k = 3$, and one with none $k = \infty$: a perfectly plastic response. Also shown is the idealized response of a typical cyclically hardened 304 stainless steel at 1200°F [7], for which $k = 44.5$. The $(P,T)_R$ corresponding to $k = 3$ and $k = 46.5$ are indicated in the Regions FR, FR-FR1, and FR1 shown in Figures 2 and 5. The ratcheting region bounding curve for $k = \infty$ also is provided for reference in both figures. The range of $\alpha TE/\sigma_y$ included in the figures should be sufficient for application to liquid metal breeder reactor design; since $\alpha TE/\sigma_y < 17$ for typical cyclically hardened and annealed 304 stainless steels [7] undergoing a 1200°F temperature change.

Comparison of Figures 2 and 5 show that the $(P,T)_R$ for widely different k have most values in common: for $k = 46.2$ the $(P,T)_R$ are almost identical to those for $k = \infty$; and even for the extremely different $k = 3$ and $k = \infty$, the ratcheting regions only differ in size by about 25%. However, the amount of strain which accumulates during ratcheting is a sensitive function of k . To assess strain accumulation the previously developed relations were assembled in a computer program which calculates strain accumulation for any number of half cycles N , temperature $\alpha TE/\sigma_y$, or axial load $P/2A\sigma_y$. Consider the strain which accumulates after the first half cycle for an infinite number of cycles

$$\delta \epsilon_{\infty}^P = \epsilon_{\infty} - \epsilon_1 - \frac{\alpha TE}{4} [1 + (-1)^N] \quad (29)$$

Figures 2 and 5 show $\delta \epsilon_{\infty}^P$ for $k = 3.0$ and 46.2 , respectively. Obviously,

for a given P and T , considerably more strain accumulates for larger k , P , and T . However, the accumulation is most sensitive to the value of k . This fact is perhaps better illustrated by Fig. 6, which shows for $P = 0.70$ the variation in $\delta\epsilon_{\infty}^P$ with $\alpha TE/\sigma_y$ and several values of k .

Figures 2 and 5 also illustrate that $\delta\epsilon_{\infty}^P$ is relatively insensitive to changes in $\alpha TE/\sigma_y$ at large $\alpha TE/\sigma_y$, as compared to the sensitivity of $\delta\epsilon_{\infty}^P$ at small values of $\alpha TE/\sigma_y$. A similar but more extreme trend was found for a two-element sandwich beam [5]: no change occurred in $\delta\epsilon_{\infty}^P$ for variations in $\alpha TE/\sigma_y$ at large $\alpha TE/\sigma_y$. Another extreme trend of the two-element beam was that repeated plastic cycling would occur for $\alpha TE/\sigma_y$ sufficiently large. For the solid beam, a much less pronounced trend toward smaller $\delta\epsilon_{\infty}^P$ at larger $\alpha TE/\sigma_y$ can be seen in Fig. 2 for $P/2A\sigma_y > 0.9$. In general, more strain will accumulate for a sandwich beam than for a solid beam of the same cross sectional area, except at high temperature variations. The fact that repeated plastic cycling occurs for a sandwich beam at high temperature variations, whereas ratcheting occurs for a solid beam, means that for some conditions an optimal cross section may be determined which minimizes ratcheting and cost while maintaining static strength.

The strain which accumulates for a finite number of half cycles after the first half cycle

$$\delta\epsilon_N^P = \epsilon_N - \epsilon_1 - \frac{\alpha TE}{4} [1 + (-1)^N] \quad (30)$$

shows variations similar to those of $\delta\epsilon_{\infty}^P$: for a given N , more strain accumulates for larger P , T , and k . For different N and a given P , T , and k ; the rate of strain accumulation per half cycle is a decreasing function of the number of half cycles. Additional trends are shown in Figures 7 and 8, which are indicative of the general variation of $\delta\epsilon_N^P$ with N . In Figure 7, the range of the number of half cycles necessary to accumulate 90% of $\delta\epsilon_{\infty}^P$, $N_{90\%}$, is plotted versus k . For a given k , the maximum values of $N_{90\%}$ correspond to the smallest values of $\alpha TE/\sigma_y$, for

which ratcheting occurs, and the largest values of $P/2\Delta\sigma_y$. The minimum values of $N_{90\%}$ corresponds to the largest value of $\alpha TE/\sigma_y = 20$ and are nearly the same for any $P/2\Delta\sigma_y$. In general, the greatest cyclic strain accumulation rate always occurs for the largest values of T , and it is a relatively insensitive function of k and P . The minimum value of the cyclic strain accumulation rate is a sensitive function of k , and it always occurs for the smallest values of T and the larger values of P . The larger amounts of strain accumulation and the larger cyclic rates of strain accumulation associated with larger values of T are vividly illustrated in Figure 8. The desirability of avoiding large temperature variations is clear.

The calculation of strain accumulation is most tedious when $\delta\epsilon_N^P$ is required for Cases 2 and 3, because consecutive cycle by cycle calculations must be made instead of the evaluation of a closed form solution. A method of obtaining a closed form upper bound solution to $\delta\epsilon_m^P$ for Case 3 has already been outlined. Of course, the closed form solutions and bounds for $\delta\epsilon_m^P$ are upper bounds on $\delta\epsilon_N^P$. The closeness of these bounds can be estimated from results like those presented in Fig. 7. For instance, for typical reactor engineering materials and thermal fluctuations, if more than 25 full thermal cycles are expected in a design application; then $\delta\epsilon_m^P$ will be within approximately 10% of $\delta\epsilon_{25}^P$. For other design applications, more pertinent diagrams similar to Fig. 7 can be constructed in order to simplify calculations.

Up to now the strain accumulation discussed has been that which occurs after the first half cycle. The strain which accumulates during the first half cycle can be determined from (19) and (23) to be

$$\epsilon_1^P = \epsilon_1 + \frac{\alpha TE}{2} \quad (31)$$

This accumulation will be the same for any ambient temperature base.

For Case 1 (Region FR), the specific form of (31) is

$$\epsilon_1^P = \left(\frac{P}{2A} \right) \frac{1+k}{1+k \left(\frac{2\sigma_y}{\alpha T E} \right)} \quad (32)$$

For Cases 2 and 3 (Region FR1 and FR1-FR), the specific form of (31) is

$$\epsilon_1^P = 1 + \frac{\alpha T E}{2} \left[\left(1 + \frac{2}{k} \right) - \left[\left(1 + \frac{2}{k} \right)^2 + \left(\frac{k+1}{k} \right) \frac{8\sigma_y}{\alpha T E} \left(1 - \frac{P}{2A\sigma_y} \right) - 1 \right]^{1/2} \right] \quad (33)$$

Inspection of (32) and (33) shows that ϵ_1^P depends directly upon T and P , but is only a sensitive to variations in k for small k . Figure 9 demonstrates the variation with k and T for $P = 0.7$. Comparison of Figures 8 and 9 show that ϵ_1^P can constitute a significant part of the total strain accumulation, especially for small n .

CONCLUSIONS

Thermal ratcheting has been investigated for a solid rectangular beam made of a material exhibiting an idealized Bauschinger effect, as modeled by kinematic hardening. Constant strain-hardening, or equivalently, a constant tangent modulus, and a linear temperature variation across the beam was assumed. Results were obtained for a wide range of parameters: a tangent modulus of zero to a tangent modulus equal to the elastic modulus, an axial load of zero to a yield point axial load, a temperature variation from zero to infinity, and temperature cycles from one to infinity.

Closed form exact expressions or upper bounds are given for determination of the thermal ratcheting strain accumulation. For cases where bounds are available but a more accurate evaluation of the strain accumulation is desired, cycle by cycle numerical evaluation procedures are outlined and examples presented. Also, a simplified graphical method to save calculation time is included. For material response and loads typical of nuclear reactor operating conditions, the strain accumulation for an infinite number of temperature cycles has been calculated and presented in graphical format. The strain accumulation reported for an infinite number of temperature cycles is within 10%, at least, of that which occurs after twenty-five temperature cycles.

Although no simple functional relation exists between the strain accumulation and the parameters, over their entire range, certain general trends were evident. The axial loads and temperature variations for which strain accumulation occurred were found not to be a sensitive function of the tangent modulus; except when the tangent modulus was nearly equal to the elastic modulus. For material response typical of reactor operating conditions, the ratcheting loads and temperature variations were nearly the same as those for a perfectly plastic material. However, the amount

of strain accumulation was found to be a most sensitive function, nearly inversely proportional, to the tangent modulus. The functional relation between the strain accumulation and the temperature variation was less sensitive. It ranged from being nearly proportional, for small temperature variations, to being very insensitive, or a constant, for large temperature variations. The strain accumulation was nearly proportional to the axial load.

The strain accumulation response for a solid beam with a rectangular cross section was found to be smaller than that of a sandwich beam of the same cross sectional area, except for large temperature variations, where the sandwich beam does not ratchet. For the latter case, sandwich beam construction may be more desirable.

References

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Fig. 1 Uniaxial stress-strain curve for a linear, kinematic strain-hardening material (drawn for $\sigma_y = 33,500 \text{ lb/in}^2$, $E = 19 \times 10^6 \text{ lb/in}^2$, and $k = 44.5$)

Fig. 2 Typical regions of elastic (E), shakedown (S₁, S₂), repeated plastic (P), and ratcheting cycling (FR, FR₁-FR, FR₁); and curves (---) of constant strain accumulation. $\delta\epsilon_\omega^P$. (drawn for $k = 3.0$).

Fig. 3 Typical stress distributions for shakedown (a), repeated plastic cycling and shakedown (b), Case 1 thermal ratcheting strain accumulation (c), and Case 2 thermal ratcheting strain accumulation (d).

Fig. 4 Graphical solution of Case 2 recursion relation (21). Curves for $\beta \times 10^3 = 1, 1.5, 2, 3, 4, 5, 6, 8, 10$, etc.

yes shown

N

100

2

3

1000

Fig. 5 Regions of elastic (E), shakedown (S₁, S₂), repeated plastic (P), and ratcheting cycling (FR, FR₁-FR, FR₁); and curves (---) of constant strain accumulation, $\delta \epsilon_{\omega}^P$, for $k = 46.2$.

Fig. 6 Variation of $\delta \epsilon_{\infty}^P$ with $\alpha TE / \sigma_y$ for $P = 0.70$. Each curve represents a different value of k .

Fig. 7 The range of the number of half cycles, $N_{90\%}$, necessary to accumulate 90% of $\delta\epsilon_p^p$ for a given k .

Fig. 8 Variation in $\delta \epsilon_N^P$ with N and selected $\alpha TE / \sigma_y$, for $P / 2A \sigma_y = 0.7$ and $k = 46.2$.

Fig. 9 Variation of ϵ_1^P with T for selected values of k and $P/2A\sigma_y = 0.7$.