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by

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**NUMERICAL SOLUTION OF MELT FLOW AND THERMAL ENERGY TRANSFER
FOR THE LITHOTHERMODYNAMICS OF A ROCK-MELTING PENETRATOR**

by

R. D. McFarland

ABSTRACT

A finite-difference numerical method is described for computing the steady-state thermal and hydrodynamic characteristics of the melt flow around a heated rock-melting penetrator of arbitrary axisymmetric geometry. The major operating and performance results obtained from these calculations are heated-surface power, required thrust load, and the thickness profile of the melt layer. The method described has been incorporated in a computer program, and some typical results are presented.

I. INTRODUCTION

A. Background

The development of rock-melting penetrators¹ has advanced to the point that it is becoming increasingly necessary to be able to predict the relationships between thrust, power, surface temperature, and penetration rate for a wide variety of penetrator shapes and rock or soil media being melted. These predictions are required, for instance, to assess the practical limits of operation, and to aid in determining optimum penetrator shapes for specific applications.²⁻⁴

The analytical and numerical method described herein, which has been incorporated into a computer program, is a preliminary step to satisfy these requirements. The method is based on the assumption that a relatively distinct interface exists between the solid rock and the fluid rock melt at a "melting interface". With the exception of calculations to determine the heat flux at the melting interface, calculations are at present confined to the region between the penetrator surface and this melting interface; this region is termed the "melt layer". The melting-interface temperature and the heat flux out of the interface are required as boundary conditions to the calculation region. In addition,

the penetrator surface temperature must be specified in most models.

B. Method of Analysis

The melt is treated as a homogeneous Newtonian fluid. Although extreme non-Newtonian behavior has been shown for basalt,⁵ this behavior is limited to a narrow temperature range near the lower limit of the melting-temperature range. Because there is little dynamic action in this highly viscous region, the assumption of a Newtonian fluid will be satisfactory for most applications. The effects of rock inhomogeneities and the resulting melting-temperature range, rather than a specific melting temperature, are included by employing temperature-dependent property variations. For instance, the latent heat of fusion may be characterized as an increased specific heat over a certain temperature range rather than as a lumped enthalpy increase at a given temperature. Because the melt formed is usually somewhat transparent, there is a thermal-radiation contribution to the energy transport. The effects of this contribution can, in many cases, be simulated by the use of an effective thermal conductivity which is temperature-dependent.

Two classes of penetrator geometries are considered in these initial computations: (1) A simple

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solid geometry (Fig. 1a) in which the penetrator surface extends to the axis of symmetry; and (2) an annular geometry (Fig. 1b) with central melt extrusion or coring passage. A representative portion of a heated penetrator surface and the associated melt layer are shown in Fig. 2. The frame of reference is fixed on the penetrator, with solid rock approaching at a steady velocity, V_z , the penetration rate. The principal coordinates are:

- Coordinate s , a meridional coordinate tangential to the penetrator surface in a plane through the axis of symmetry. The positive s -direction is directed clockwise as shown in Fig. 2, with the convention that $s = 0$ at the leading edge;
- Coordinate y , a transverse coordinate normal to the penetrator surface, at that surface. The positive y -direction is outward from the penetrator surface with $y = 0$ located at the surface.

In performing the computations the penetrator surface geometry and the boundary conditions are described as functions of s , the meridional position.

Fig. 1. Two types of penetrator geometries considered. (a) Solid geometry; (b) Annular geometry.

Fig. 2. Coordinate conventions for melt layer calculations.

The fluid-rock (melt) properties are provided as functions of temperature, and a penetration rate is imposed. Numerical integrations are then made in the y -direction at a given s -location. These integrations yield the local y -direction distributions of the s and y -components of melt velocity and melt temperature; the local melt-layer thickness, δ ; the local penetrator and melt-surface heat fluxes; and local melt pressure gradients and surface shear stresses in the s -direction. Overall thermal energy, pressure, and shear forces are then found by numerical integration of local gradients, in the s -direction.

Calculations start at the "leading edge" (see Fig. 2). The only special restrictions at the first station are that the s -direction gradients of fluid temperature and melt-layer thickness are assumed to be zero. In an annular geometry with melt flow on both inner and outer surfaces, the outer surface is calculated first, then the inner, each starting at

the leading edge with the melt flow past the leading edge specified. Because, in this case, the location of the stagnation point is not known a priori, the melt flow at the leading edge is not known. The entire calculation must be repeated in an iterative or parametric manner to satisfy known conditions at the "aft" end (such as equal fluid pressures on the inner and outer surfaces, a specified melt flow from one of the surfaces) or to satisfy the consolidation relation at the "aft" end of each surface.

C. General Nomenclature

The following is a list of the various physical and geometric parameters introduced in the report.

Symbol	Description	Units
B	body force per unit volume	N/m ³
C _p	mass heat capacity	J/kg·K
d	thermal diffusivity	m ² /s
ℓ	melt layer thickness*	m
p	pressure	Pa
q	heat flux, conductive	W/m ²
r	perpendicular distance from axis of symmetry*	m
s	meridional distance from leading edge*	m
T	temperature	K
u	s-direction component of melt velocity	m/s
v	ξ-direction component of melt velocity	m/s
V _z	penetration rate	m/s
V' _z	effective penetration rate, V _z ρ _s /ρ _m	m/s
β	ξ-line angle*	($\frac{J}{m^3 \cdot K}$)
γ	volumetric heat capacity	W/m ³ ·K
λ	thermal conductivity	W/m·K
μ	viscosity	Pa·s
ξ	transverse coordinate*	kg/m ³
ρ	density	-
σ	geometric coefficient*	kg/s
ψ	mass flow ÷ 2π	m ⁻¹
ω	curvature*	-

subscripts

a, b	arbitrary points on penetrator surface
o	at penetrator surface
m	at melting interface, or melt
s	solid rock or s direction
ξ	ξ-direction

II. COORDINATE SYSTEM

The coordinate system chosen for this analysis is a curvilinear, axisymmetric system with the orthogonal coordinates, s, a dimensional meridional distance tangential to the penetrator surface; and ξ, a nondimensional transverse coordinate which varies from zero at the penetrator surface to unity at the melt surface. The transverse coordinate was made nondimensional so that it be independent of the surface-wise coordinate. In Fig. 3, $\xi = y/\ell$ where ℓ is the total local melt-layer thickness. Referring to Figs. 3 and 4, a unit differential volume in the melt layer is given by

$$dV = h_1 ds \cdot h_2 d\xi \cdot h_3 d\theta \quad (1)$$

Fig. 3. A portion of the penetrator and melt layer showing the coordinate system and geometric parameters.

*See Figs. 3 and 4.

Fig. 4. Enlarged portion of melt layer showing the differential volume.

where θ is the tangential coordinate about the axis of symmetry.

The geometric coefficients in Eq. (1) are:

$$h_1 = \sigma = (ds^*/ds)_\xi$$

where s^* is the meridional distance along any constant ξ -line;

$h_2 = l$, the total (dimensional) transverse distance or the melt-layer thickness;

$h_3 = r$, the radial distance perpendicular to the axis of symmetry.

Stipulating that β , the angle between the axis of symmetry and a constant ξ -line, varies linearly with ξ at constant s , the radial and axial dimensions are:

$$r = r_0 + \frac{l}{\delta\beta} \left(\sin\beta - \sin\beta_0 \right)$$

$$z = z_0 + \frac{l}{\delta\beta_0} \left(\cos\beta - \cos\beta_0 \right)$$

where $\delta\beta = \beta_m - \beta_0 = \beta(\xi = 1) - \beta(\xi = 0)$; r_0 , z_0 , and β_0 are given penetrator surface geometry

parameters, which vary with s ; and the subscript m denotes parameters at the melting interface. Another geometry parameter required is the penetrator surface curvature:

$$-w_0 = \frac{d\beta_0}{ds}$$

From the r and z relations, expressions for σ and β_m may be found, because

$$\sigma = \sqrt{\left(\frac{\partial r}{\partial s} \right)_\xi^2 + \left(\frac{\partial z}{\partial s} \right)_\xi^2}, \text{ and}$$

$$\tan \beta_m = \left(\frac{\partial r}{\partial s} \right)_m / \left(\frac{\partial z}{\partial s} \right)_m$$

The resulting expressions are, however, very complex algebraically, and do not give explicit relations for the dependent variables. To simplify the coordinate parameters for this analysis, it is assumed that the angle variation ($\delta\beta$) is small so that $\sin \delta\beta \approx \delta\beta$ and $\cos \delta\beta \approx 1$. With this approximation, the following relations are obtained:

$$r \approx r_0 + \xi l \cos \beta_0,$$

$$z \approx z_0 - \xi l \sin \beta_0,$$

$$\sigma = \sqrt{\left(1 + \xi l w_0 \right)^2 + \left(\frac{dl}{ds} \xi \right)^2},$$

$$\tan \delta\beta \approx \frac{dl}{ds} / \left(1 + w_0 l \right).$$

These relations should be adequately accurate for most practical applications because the growth of the melt layer is gradual. The last metric coefficient, l , is either specified or, in most cases, calculated as a function of given thermal boundary conditions at each meridional s -location. In general, two of the metric coefficients (σ and r) are functions of both s and ξ , and the third (l) is a function of s only.

III. PHYSICAL EQUATIONS

The starting points for the differential equations used in the finite-difference analysis are the general steady-state equations in orthogonal curvilinear coordinates⁶ for the conservation of mass, conservation of energy, and the transverse (ξ -direction) and meridional (s -direction) equations of motion. The basic equations are written here in terms of the axisymmetric coordinate system introduced above. Simplifications are then made on the basis that the melt-layer thickness is small compared to the characteristic penetrator dimension. Additional assumptions are that the melt-flow Reynolds number is small compared to unity and that the melt is incompressible.

A. Dimensionless Parameters

The following dimensionless quantities are introduced:

$$\bar{u} = u/V'_z, \quad ,$$

$$\bar{v} = v/V'_z, \quad ,$$

where $V'_z = V_z \rho_s / \rho_m$; ρ_s is the density of the solid rock; and ρ_m is the density of the melt at the melting temperature,

$$\bar{\ell} = \ell/R, \quad ,$$

$$\bar{r} = r/R, \quad ,$$

$$\bar{s} = s/R, \quad ,$$

where R is a characteristic penetrator dimension; and

$$\bar{\mu} = \mu/\mu^*, \quad ,$$

$$\bar{\lambda} = \lambda/\lambda^*, \quad ,$$

$$\bar{\gamma} = \gamma/\gamma^*, \quad ,$$

$$\bar{T} = T/T^*, \quad ,$$

where the starred quantities are quantities averaged over the melt layer;

$$\bar{p} = \frac{pR}{\mu^* V'_z}, \quad ,$$

$$Re_\ell = \frac{\rho V'_z \ell}{\mu^*}, \quad , \text{ a melt Reynolds number,}$$

$$\bar{B} = \frac{B \ell^3}{\mu^* V'_z R}, \quad , \text{ a dimensionless body force,}$$

$$Pr^* = \frac{c_p^* \mu^*}{\lambda^*}, \quad , \text{ a melt Prandtl number,}$$

$$K = \frac{(V'_z/\ell)^2}{c_p^* T^*}, \quad , \text{ a dimensionless kinetic energy parameter,}$$

$$Pe = Re_\ell Pr^* = \frac{V'_z \ell \gamma^*}{\lambda^*}, \quad , \text{ a melt Peclet number, and}$$

$$N = Pr^* K = \frac{\mu^* (V'_z/\ell)^2}{\lambda^* T^*}.$$

For penetration rates up to ~ 1.0 mm/s and values of R up to 0.3 m, and for the expected range of rock properties, it can be shown that the practical range of some of these quantities are:

$$0.001 \leq \bar{\ell} \leq 0.1$$

$$10^{-8} \leq Re_\ell \leq 10^{-4}$$

$$10^3 \leq Pr^* \leq 10^8$$

$$10^{-12} \leq K \leq 10^{-6}$$

$$0.1 \leq Pe \leq 1$$

$$10^{-9} \leq N \leq 10^2$$

$$10^{-8} \leq \bar{B} \leq 10 \quad (\text{for gravity only})$$

It will be shown later that momentum-change terms can be neglected if Re_ℓ is small compared to unity, that viscous energy dissipation can be neglected if N is small compared to unity, that convective energy transport can be neglected if Pe is small compared to unity, and that body forces can be neglected if $\bar{B}$ is small compared to unity.

The basic equations in terms of these dimensionless quantities are written below and simplified.

B. The s -direction Equation of Motion

For steady flow of an incompressible Newtonian fluid:

$$\bar{F}_s - \frac{\bar{\ell}^3}{\sigma} \frac{\partial \bar{p}}{\partial \bar{s}} = \bar{M}_s - \bar{B}_s \quad (2)$$

where

$$\begin{aligned}\bar{F}_s = & \frac{2\bar{\ell}^2}{\sigma\bar{r}} \cdot \frac{\partial}{\partial\bar{s}} \left(\frac{\bar{\ell}\bar{r}\bar{\mu}}{\sigma} \cdot \frac{\partial\bar{u}}{\partial\bar{s}} \right) + \frac{2\bar{\ell}^2}{\sigma\bar{r}} \cdot \frac{\partial}{\partial\bar{s}} \left(\frac{\bar{\ell}\bar{\mu}\bar{v}}{\sigma} \cdot \frac{\partial\sigma}{\partial\bar{s}} \right) \\ & + \frac{\bar{\ell}^3}{\sigma\bar{r}} \cdot \frac{\partial}{\partial\bar{s}} \left[\bar{r}\bar{\mu} \frac{\partial}{\partial\bar{s}} \left(\frac{\bar{v}}{\bar{\ell}} \right) \right] + \frac{\bar{\ell}}{\sigma\bar{r}} \cdot \frac{\partial}{\partial\bar{s}} \left[\sigma^2 \bar{r}\bar{\mu} \frac{\partial}{\partial\bar{s}} \left(\frac{\bar{u}}{\sigma} \right) \right] \\ & + \frac{\bar{\ell}^3\bar{\mu}}{\sigma^2} \cdot \frac{\partial}{\partial\bar{s}} \left(\frac{\bar{v}}{\bar{\ell}} \right) \cdot \frac{\partial\sigma}{\partial\bar{s}} + \bar{r}\bar{\mu} \frac{\partial}{\partial\bar{s}} \left(\frac{\bar{u}}{\sigma} \right) \cdot \frac{\partial\sigma}{\partial\bar{s}} \\ & - \frac{2\bar{\ell}\bar{\mu}\bar{u}}{\sigma^2} \cdot \left(\frac{d\bar{\ell}}{d\bar{s}} \right)^2 - \frac{2\bar{\ell}\bar{\mu}}{\sigma} \cdot \frac{\partial\bar{v}}{\partial\bar{s}} \cdot \frac{d\bar{\ell}}{d\bar{s}} - \frac{2\bar{\ell}^3\bar{\mu}\bar{u}}{\sigma^2\bar{r}} \cdot \left(\frac{\partial\bar{r}}{\partial\bar{s}} \right)^2\end{aligned}$$

and $\bar{M}_s$ is the momentum-change term:

$$\bar{M}_s = \text{Re}_\ell \left[\frac{\bar{\ell}^2\bar{u}}{\sigma} \cdot \frac{\partial\bar{u}}{\partial\bar{s}} + \bar{\ell}\bar{v} \cdot \frac{\partial\bar{u}}{\partial\bar{s}} + \frac{\bar{\ell}}{\sigma} \bar{u}\bar{v} \cdot \frac{\partial\sigma}{\partial\bar{s}} - \frac{\bar{\ell}}{\sigma} \bar{v}^2 \frac{d\bar{\ell}}{d\bar{s}} \right].$$

It can be shown that $\bar{u}$ is of the order of $1/\bar{\ell}$ and that $\bar{v}$ is of the order of unity. Assuming that $\bar{\mu}$, $\bar{\ell}$, $\bar{r}$, $\bar{s}$, and σ are of the order of unity; that $\frac{\partial\sigma}{\partial\bar{s}}$ is of the order $\bar{\ell}$; and that $\bar{\ell}$ is less than unity; then:

- (1) the maximum order of $\bar{M}_s/\text{Re}_\ell$ terms is unity so that $\bar{M}_s$ is of the order of Re_ℓ ;
- (2) all $\bar{F}_s$ terms are of the order $\bar{\ell}^2$ or smaller except for the fourth term (order unity) and sixth term (order $\bar{\ell}$).

Assuming Re_ℓ and $\bar{\ell}^2$ to be small compared to unity, the s-direction equation of motion can be reduced to the following equation of order unity:

$$\begin{aligned}& \frac{\bar{\ell}}{\sigma^2\bar{r}} \left\{ \frac{\partial}{\partial\bar{s}} \left[\sigma^3 \bar{r} \bar{\mu} \frac{\partial}{\partial\bar{s}} \left(\frac{\bar{u}}{\sigma} \right) \right] \right\} \\ & = \frac{\bar{\ell}^3}{\sigma} \left(\frac{\partial\bar{p}}{\partial\bar{s}} \right) - \bar{B}_s, \quad (3)\end{aligned}$$

where the fourth and sixth $\bar{F}_s$ terms have been combined into the term on the left. In dimensional form:

$$\begin{aligned}& \frac{\partial}{\partial\bar{s}} \left[\sigma^3 \bar{r} \bar{\mu} \frac{\partial}{\partial\bar{s}} \left(\frac{\bar{u}}{\sigma} \right) \right] \\ & = \sigma \bar{r} \bar{\ell}^2 \frac{\partial p}{\partial s} - \sigma^2 \bar{r} \bar{\ell}^2 B_s.\end{aligned}$$

C. The ξ -direction Equation of Motion

For steady flow this relation is,

$$\bar{F}_\xi - \bar{\ell}^2 \frac{\partial\bar{p}}{\partial\bar{s}} = \bar{M}_\xi - \bar{B}_\xi, \quad (4)$$

where

$$\begin{aligned}\bar{F}_\xi = & \frac{\bar{\ell}^2}{\sigma\bar{r}} \frac{\partial}{\partial\bar{s}} \left[\frac{\bar{\ell}^2\bar{r}\bar{\mu}}{\sigma} \frac{\partial}{\partial\bar{s}} \left(\frac{\bar{v}}{\bar{\ell}} \right) \right] + \frac{\bar{\ell}^2}{\sigma\bar{r}} \frac{\partial}{\partial\bar{s}} \left[\sigma\bar{r}\bar{\mu} \frac{\partial}{\partial\bar{s}} \left(\frac{\bar{u}}{\sigma} \right) \right] \\ & + \frac{2\bar{\ell}}{\sigma\bar{r}} \frac{\partial}{\partial\bar{s}} \left(\sigma\bar{r}\bar{\mu} \frac{\partial\bar{v}}{\partial\bar{s}} \right) + \frac{2\bar{\ell}}{\sigma\bar{r}} \frac{d\bar{\ell}}{d\bar{s}} \frac{\partial}{\partial\bar{s}} (\bar{r}\bar{\mu}\bar{u}) \\ & + \frac{\bar{\ell}^3\bar{\mu}}{\sigma^2} \frac{d\bar{\ell}}{d\bar{s}} \frac{\partial}{\partial\bar{s}} \left(\frac{\bar{v}}{\bar{\ell}} \right) + \bar{\ell}\bar{\mu} \frac{d\bar{\ell}}{d\bar{s}} \frac{\partial}{\partial\bar{s}} \left(\frac{\bar{u}}{\sigma} \right) - \frac{2\bar{\ell}^2\bar{\mu}\bar{u}}{\sigma\bar{r}^2} \frac{\partial\bar{r}}{\partial\bar{s}} \frac{\partial\bar{r}}{\partial\bar{s}} \\ & - \frac{2\bar{\ell}\bar{\mu}\bar{v}}{\bar{r}^2} \left(\frac{\partial\bar{r}}{\partial\bar{s}} \right)^2 - \frac{2\bar{\ell}^2\bar{\mu}}{\sigma^2} \frac{\partial\bar{u}}{\partial\bar{s}} \frac{\partial\sigma}{\partial\bar{s}} - \frac{2\bar{\ell}\bar{\mu}\bar{v}}{\sigma^2} \left(\frac{\partial\sigma}{\partial\bar{s}} \right)^2,\end{aligned}$$

and

$$\bar{M}_\xi/\text{Re}_\ell = \frac{\bar{\ell}^2\bar{u}}{\sigma} \frac{\partial\bar{v}}{\partial\bar{s}} + \bar{\ell}\bar{v} \frac{\partial\bar{u}}{\partial\bar{s}} + \frac{\bar{\ell}\bar{u}\bar{v}}{\sigma} \frac{d\bar{\ell}}{d\bar{s}} - \frac{\bar{\ell}\bar{u}^2}{\sigma} \frac{\partial\sigma}{\partial\bar{s}}.$$

Again, $\bar{M}_\xi$ is of the order of Re_ℓ and can be neglected. Assuming $\frac{\partial\bar{r}}{\partial\bar{s}}$ is of the order $\bar{\ell}$ and neglecting terms of order $\bar{\ell}^2$ or smaller, there are four $\bar{F}_\xi$ terms (2, 3, 4, and 6) left, all of the order $\bar{\ell}$. Then $\bar{\ell}^2 \frac{\partial\bar{p}}{\partial\bar{s}}$ is of the order $\bar{\ell}$, or $\bar{\ell}^3 \frac{\partial\bar{p}}{\partial\bar{s}}$ is of the order $\bar{\ell}^2$.

But $\bar{\lambda}^3 \frac{\partial \bar{p}}{\partial \xi}$ is of the order of unity, so $\frac{\partial \bar{p}}{\partial \xi}$ can be neglected, and there is no need for the ξ -direction equation of motion. Now:

$$\frac{\partial \bar{p}}{\partial s} = \frac{dp}{ds} \neq f(\xi) \quad .$$

The above conclusions ignore the presence of $\bar{B}$ in the equations. As shown previously, body forces can become significant. This can occur only, however, for very low penetration rates in a medium of relatively low viscosity.

D. The Energy Equation

Energy for enthalpy independent of pressure:

$$\begin{aligned} & \frac{1}{\sigma \bar{r}} \frac{\partial}{\partial \xi} \left(\sigma \bar{r} \bar{\lambda} \frac{\partial \bar{T}}{\partial \xi} \right) + \frac{\bar{\lambda}}{\sigma \bar{r}} \frac{\partial}{\partial \xi} \left(\frac{\bar{\lambda} \bar{r} \bar{\lambda}}{\sigma} \frac{\partial \bar{T}}{\partial \xi} \right) \\ & = Pe \left[\frac{\bar{\lambda} \bar{v} \bar{u}}{\sigma} \frac{\partial \bar{T}}{\partial \xi} + \bar{v} \bar{v} \frac{\partial \bar{T}}{\partial \xi} \right] - \bar{Q} \quad , \end{aligned} \quad (5)$$

where

$$\bar{Q} = \bar{Q}'' + \bar{\Phi} + \frac{D\bar{p}}{Dt} - \bar{Q}_r \quad ,$$

$$\bar{Q}'' = Q'' \left(\frac{\ell^2}{\lambda^* T^*} \right) \text{ where } Q'' \text{ is the volumetric heat-generation rate,}$$

$$\bar{\Phi} = \Phi \left(\frac{\ell^2}{\lambda^* T^*} \right) \text{ where } \Phi \text{ is the "viscous dissipation" term;}$$

$$\bar{\Phi} \approx N \bar{\mu} \left[\sigma \bar{\lambda} \frac{\partial}{\partial \xi} \left(\frac{\bar{u}}{\sigma} \right) \right]^2 \text{ for } \bar{\lambda}^2 \text{ small compared to unity,}$$

$$\frac{D\bar{p}}{Dt} \approx N \left(\frac{\bar{\lambda}^4 \bar{u}}{\sigma} \right) \left(\frac{d\bar{p}}{ds} \right) \quad ,$$

and $\bar{Q}_r = \nabla \cdot \hat{q}_r \left(\frac{\ell^2}{\lambda^* T^*} \right)$ where $\nabla \cdot \hat{q}_r$ is the divergence of the radiation heat-flux vector. Approximately,

$$\bar{Q}_r \approx \left(\frac{\ell}{\sigma \bar{r} \lambda^* T^*} \right) \frac{\partial}{\partial \xi} \left(\sigma \bar{r} \bar{q}_r, \xi \right) \quad .$$

Assuming $\bar{\lambda}$ and $\bar{v}$ to be of the order of unity, the ξ -direction conductive term is also of the order of unity, but the s -direction conductive term is of the order $\bar{\lambda}^2$ and can be neglected. Both convective terms inside the brackets are of the order of unity and cannot be neglected because Pe is not small compared to unity. The quantity $\bar{\Phi}/N$ is of the order of unity as is $\frac{D\bar{p}}{Dt}/N$. The parameter N can become significant only for large penetrators operating at high penetration rates in a medium of very high viscosity. For the present analysis the Φ and $\frac{D\bar{p}}{Dt}$ terms are neglected. If, however, the above conditions are encountered it may be necessary to include them. Heat generation and thermal radiation are completely unknown quantities at this time and are included as a lumped source term $Q = Q'' + \nabla \cdot \hat{q}_r$. In many cases the "optically thick" approximation can be used in these computations. When this is possible, the effects of thermal radiation can be lumped into an effective thermal conductivity. The final energy equation used here is:

$$\begin{aligned} & \frac{1}{\sigma \bar{r}} \left[\frac{\partial}{\partial \xi} \left(\sigma \bar{r} \bar{\lambda} \frac{\partial \bar{T}}{\partial \xi} \right) \right] \\ & = Pe \bar{v} \left(\frac{\bar{\lambda} \bar{u}}{\sigma} \frac{\partial \bar{T}}{\partial \xi} + \bar{v} \frac{\partial \bar{T}}{\partial \xi} \right) - \bar{Q} \end{aligned} \quad (6)$$

where

$$\bar{Q} = Q \left(\frac{\ell^2}{\lambda^* T^*} \right) \quad .$$

In dimensional form,

$$\begin{aligned} & \frac{\partial}{\partial \xi} \left(\sigma r \lambda \frac{\partial T}{\partial \xi} \right) \\ & = \sigma \ell r v \frac{\partial T}{\partial \xi} + \ell^2 r v u \frac{\partial T}{\partial s} - \sigma \ell^2 r Q \quad . \end{aligned}$$

E. The Continuity Equation

Two forms of the continuity equation are used here for steady flow. Neither has been simplified.

(1) Differential continuity,

$$\frac{\partial}{\partial s} (\ell r \rho u) = - \frac{\partial}{\partial \xi} (\sigma r \rho v)$$

(2) Integral continuity,

$$\begin{aligned} \psi &= \left[\ell \int_0^1 r \rho u d\xi \right]_s \\ &= - \int_0^s \left[(\sigma r \rho v)_m - (\sigma r \rho v)_o \right] ds + \psi_o \end{aligned}$$

where $2\pi\psi$ is the mass flow of melt in the s direction.

F. Summary of Simplified Equations

The s -direction equation of motion:

$$\frac{\partial}{\partial \xi} \left[\sigma^3 r \mu \frac{\partial}{\partial \xi} \left(\frac{u}{\sigma} \right) \right] = \sigma r \ell^2 \frac{dp}{ds} - \sigma^2 r \ell^2 B_s, \quad (7)$$

Energy:

$$\frac{\partial}{\partial \xi} \left(\sigma r \lambda \frac{\partial T}{\partial \xi} \right) = \sigma \ell r \gamma v \frac{\partial T}{\partial \xi} + \ell^2 r \gamma u \frac{\partial T}{\partial s} - \sigma \ell^2 r Q, \quad (8)$$

Continuity:

$$\frac{\partial}{\partial s} (\ell r \rho u) = - \frac{\partial}{\partial \xi} (\sigma r \rho v), \quad (9)$$

Integral continuity:

$$\begin{aligned} \psi &= \left[\ell \int_0^1 r \rho u d\xi \right]_s \\ &= - \int_{s_0}^s \left[(\sigma r \rho v)_m - (\sigma r \rho v)_o \right] ds + \psi_o \quad (10) \end{aligned}$$

It can be seen that these equations will not be valid at the stagnation point of a solid-geometry penetrator where r is everywhere zero. From considerations of symmetry, and because $u = 0$ everywhere, it can be shown that r can be replaced by σ

in all differential equations at this singular point. Of course, $\psi = 0$ at the stagnation point.

G. Boundary Conditions

Two classes of boundary conditions are required for the solution of the differential equations. The first, most frequently encountered class, is required when the outer boundary of the melt layer is a free melt surface (Case A) as shown in Fig. 5. The second class is required for penetrators of annular geometry when the entire core is melted and the outer ($\xi = 1$) melt-layer boundary is the axis of symmetry (Case B).

The required boundary conditions are:

(1) The s -direction equation of motion

- (a) $u(\xi = 0) = u_o = 0$ (Case A and Case B)
- (b) $u(\xi = 1) = u_m = V'_z \cos \beta_m$ (Case A only)
- (c) $\frac{\partial u}{\partial \xi}(\xi = 1) = 0$ (Case B only)
- (d) p specified at "aft" end

(2) Continuity

- (a) $v(\xi = 0) = v_o = 0$ for no source or sink (Case A and Case B)
- (b) $u(\xi) = 0$ at $s = s_0$

Fig. 5. A portion of a penetrator and melt layer showing two different boundary condition cases.

The present method of evaluating these integrals is to simply assume that the integrand, $\Xi(\xi)$, varies linearly across the increment, or:

$$\int_{\xi_{i-1}}^{\xi_i} \Xi(\xi) d\xi \approx \frac{\Delta \xi_i}{2} (\Xi_i + \Xi_{i-1}) ,$$

then

$$I_i = I_0 + S_{1,i} G + S_{2,i}$$

where

$$S_{1,i} = \int_0^{\xi_i} g d\xi \approx \sum_{j=1}^i \frac{\Delta \xi_j}{2} (g_j + g_{j-1})$$

$$S_{2,i} = \int_0^{\xi_i} h d\xi \approx \sum_{j=1}^i \frac{\Delta \xi_j}{2} (h_j + h_{j-1}) .$$

Integrating a second time:

$$(u/\sigma)_i = (u/\sigma)_0 + \int_0^{\xi_i} \frac{I}{F} d\xi$$

or for $(\sigma_0 \equiv 1)$:

$$(u/\sigma)_i = u_0 + S_{3,i} I_0 + S_{4,i} G + S_{5,i}$$

where

$$S_{3,i} = \int_0^{\xi_i} \frac{d\xi}{f} \approx \sum_{j=1}^i \frac{\Delta \xi_j}{\frac{1}{2} (f_j + f_{j-1})} .$$

* Note that when r appears in the denominator, the inverse of the average is used rather than the average of the inverse because r may go to zero locally.

$$S_{4,i} = \int_0^{\xi_i} \frac{S_1 d\xi}{f} \approx \sum_{j=1}^i \frac{\frac{\Delta \xi_j}{2} (S_{1,j} + S_{1,j-1})}{\frac{1}{2} (f_j + f_{j-1})}$$

$$S_{5,i} = \int_0^{\xi_i} \frac{S_2 d\xi}{f} \approx \sum_{j=1}^i \frac{\frac{\Delta \xi_j}{2} (S_{2,j} + S_{2,j-1})}{\frac{1}{2} (f_j + f_{j-1})} .$$

A third integration is made to satisfy the integral continuity equation, where ψ is found from s -direction integrations of the melt inflow:

$$\psi/l = \int_0^1 \rho r u d\xi = \int_0^1 w \left(\frac{u}{\sigma} \right) d\xi , \text{ where } w = \sigma \rho$$

or

$$\psi/l = S_{1,n} u_0 + S_{6,n} I_0 + S_{7,n} G + S_{8,n}$$

where

$$S_{j,n} = \int_0^1 w S_{j-3} d\xi$$

$$\approx \sum_{k=1}^n \frac{\Delta \xi}{2} (w_k S_{j-3,k} + w_{k-1} S_{j-3,k-1})$$

where n = total number of increments and $j = 6, 7, 8$. For $i = n$ ($\xi = 1$) there are three linear equations,

$$I_n = I_0 + S_{1,n} G + S_{2,n}$$

$$u_n = \sigma_n (u_0 + S_{3,n} I_0 + S_{4,n} G + S_{5,n})$$

$$\psi/l = S_{1,n} u_0 + S_{6,n} I_0 + S_{7,n} G + S_{8,n} ;$$

there is the known flow, ψ ; and there are two boundary conditions, u_0 and u_n (or $I_n = 0$ for non-stagnation melt at the axis of symmetry). These equations can then be solved for the unknowns G , I_0 , and I_n (or u_n); and the velocity (u) and shear-stress profiles are determined, as well as the s -direction pressure gradient.

The transverse velocity profile, $v(\xi)$, is determined from the differential continuity equation and from the predetermined surface-wise velocity profile, $u(\xi)$. The simplest method of obtaining v would be to use the difference in the function $(\ell r p u)$ between two successive s -direction stations, i.e.,

$$\frac{d}{d\xi}(\ell r p v)_i \approx \frac{(\ell r p u)_{i,j} - (\ell r p u)_{i,j-1}}{\Delta s_j}$$

where i denotes the ξ -step and j the s -step. This method, however, has the disadvantage that the differences in the function $(\ell r p u)$ are much smaller than the values of the functions and errors introduced by the finite-difference mesh, and temperature and melt-layer thickness iteration tolerances are greatly magnified.

The principal contributor to a transverse velocity is the local inflow of melt into the melt layer, and the s -direction gradient of this inflow is relatively independent of computation errors. The present procedure for the determination of the transverse velocity is to differentiate the previously derived equation for u with respect to s , neglecting the s -direction gradients of the functions f , g , and h . Noting that, from integral continuity,

$$\frac{d\psi}{ds} = - \left[(wv)_m - (wv)_o \right], \text{ and}$$

$$\frac{du_m}{ds} \approx - wv_m, \text{ then}$$

$$\begin{aligned} \frac{d}{d\xi} (wv)_i &= w_i \left\{ K_{1,i} \frac{w\ell}{\sigma_m} v_m + K_{2,i} \left[(wv)_m - (wv)_o \right] \right\} \\ &- (r_i p_i u_i - w_i K_{2,i} \frac{\psi}{\ell}) \frac{d\ell}{ds} \end{aligned}$$

where

$$K_1 = \frac{S_{3,i} S_{7,n} - S_{4,i} S_{6,n}}{S_{3,n} S_{7,n} - S_{4,n} S_{6,n}}$$

$$K_2 = \frac{S_{4,i} S_{3,n} - S_{3,i} S_{4,n}}{S_{3,n} S_{7,n} - S_{4,n} S_{6,n}}$$

and all variables have the same definition as in the s -direction equations. Defining

$$J = \frac{d}{d\xi} (wv) ;$$

$$v_i = \frac{1}{w_i} \left(w_o v_o + \int_0^{\xi_i} J d\xi \right)$$

$$\approx \frac{1}{w_i} \left[w_o v_o + \sum_{j=1}^i \frac{\Delta \xi_j}{2} (J_j + J_{j-1}) \right]$$

where v_o is a specified boundary condition. It can be shown that the boundary condition at $\xi = 1$ (v_m) is automatically satisfied through the use of the integral continuity relation.

Although this method is somewhat crude, it does satisfy the boundary conditions (v_o , v_m) and is relatively insensitive to s -direction calculation variations. The transverse velocity profile is used only in the energy equation, and normally does not contribute much to the overall energy transfer.

In deriving a finite-difference form of the energy equation the surfacewise temperature gradient, $\frac{\partial T}{\partial s}$, is approximated by $\frac{\partial T}{\partial s} \approx \frac{|T - T^o|_{\xi}}{\Delta s}$ where T^o is the temperature at the previous s -direction station ($s - \Delta s$) and the same ξ -location. The energy equation can then be written, for $s = \text{constant}$, as

$$\frac{d}{d\xi} (\lambda \phi) = a\phi + bT + c$$

where

$$\phi = g \frac{dT}{d\xi} \quad (\text{again, } g = \sigma r) \text{ and}$$

a , b , and c are known functions of ξ :

$$a = \gamma \ell v$$

$$b = \gamma r \ell^2 u / \Delta s \quad (b = 0 \text{ at the leading edge})$$

$$c = -bT^o - g \ell^2 Q$$

Integrating twice over an increment $\Delta \xi_i$,

$$(\lambda \phi)_i - (\lambda \phi)_{i-1} = \int_{\xi_{i-1}}^{\xi_i} a \phi d\xi + \int_{\xi_{i-1}}^{\xi_i} b T d\xi + \int_{\xi_{i-1}}^{\xi_i} c d\xi$$

and

$$T_i = T_{i-1} + \int_{\xi_{i-1}}^{\xi_i} \frac{\phi}{\delta} d\xi$$

If a linear variation of the integrands is assumed across the increment, the equations become:

$$\phi_i = \alpha_i \phi_{i-1} + \zeta_i T_{i-1} + \delta_i T_i + e_i, \text{ and}$$

$$T_i = T_{i-1} + \Delta \xi_i \left(\frac{\phi_i + \phi_{i-1}}{g_i + g_{i-1}} \right) = T_{i-1} + \eta_i (\phi_i + \phi_{i-1})$$

where

$$\alpha_i = \frac{\lambda_{i-1} + \frac{\Delta \xi_i}{2} a_{i-1}}{\lambda_i - \frac{\Delta \xi_i}{2} a_i},$$

$$\zeta_i = \frac{\frac{\Delta \xi_i}{2} b_{i-1}}{\lambda_i - \frac{\Delta \xi_i}{2} a_i},$$

$$\delta_i = \frac{\frac{\Delta \xi_i}{2} b_i}{\lambda_i - \frac{\Delta \xi_i}{2} a_i},$$

$$e_i = \frac{\frac{\Delta \xi_i}{2} (c_i + c_{i-1})}{\lambda_i - \frac{\Delta \xi_i}{2} a_i}, \text{ and}$$

$$\eta_i = \frac{\Delta \xi_i}{g_i + g_{i-1}}.$$

Inspection of the above equation for ϕ_i reveals that serious errors can be encountered if the quantity $1/2 \Delta \xi_i / \lambda$ is not significantly smaller than unity.

Assuming linear solutions for ϕ_i and T_i of the form (here, the $S_{j,i}$'s are redefined)

$$\phi_i = S_{1,i} \phi_0 + S_{2,i} T_0 + S_{3,i}$$

$$T_i = S_{4,i} \phi_0 + S_{5,i} T_0 + S_{6,i},$$

the following recursion relations are found for the sums, $S_{j,i}$, for $j = 1, 2, 3$, and $k = j + 3$:

$$S_{j,i} = \frac{(\alpha_i + \delta_i \eta_i) S_{j,i-1} + (\zeta_i + \delta_i) S_{k,i-1} + K_{j,i}}{1 - \delta_i \eta_i}$$

$$S_{k,i} = S_{k,i-1} + \eta_i (S_{j,i} + S_{j,i-1})$$

where $K_{j,i} = 0$ for $j = 1, 2$ and $K_{3,i} = e_i$. $S_{1,0} = S_{5,0} = 1$, all other $S_{j,0} = 0$.

For $i = n$ ($\xi = 1$) there are two linear equations:

$$\phi_n = S_{1,n} \phi_0 + S_{2,n} T_0 + S_{3,n}$$

$$T_n = S_{4,n} \phi_0 + S_{5,n} T_0 + S_{6,n}.$$

In the usual case there are three boundary conditions: T_0 ; $T_n = T_m$; and

$\phi_n = - \frac{q_m q_m \ell}{\lambda_m}$. The extra boundary condition is used to determine the melt-layer thickness, ℓ . For the case where the melt reaches the axis of symmetry (e.g., in a central extrusion port), the melt-layer thickness is known and the boundary condition $T_n = T_m$ is no longer valid. In this case the heat flux, q_m , at $\xi = 1$ is zero. In addition, an option is provided wherein the melt-layer thickness can be specified everywhere. When this option is used the boundary condition T_0 becomes a dependent variable.

To determine the melt-layer thickness from the energy equation, the third boundary condition is rearranged;

$$\ell = - \frac{\lambda_m}{g_m} \phi_n(\ell) / q_m(\ell)$$

where

$$\phi_n(\ell) = \frac{S_{i,n}}{S_{4,n}} (T_n - S_{5,n}T_o - S_{6,n}) + S_{2,n}T_o + S_{3,n}$$

and, in general, q_m is a function of $\frac{d\ell}{ds}$. The derivative $\frac{d\ell}{ds}$ is approximated by $\frac{\ell - \ell^o}{\Delta s}$, where ℓ^o is the melt-layer thickness at the previous s-direction station.

Because of strongly divergent behavior of the iteration for ℓ , it was necessary to use a "Newton's Method" iteration scheme. For this purpose the derivatives of the sums, $S_{j,i}$, with respect to the melt-layer thickness $\left(\frac{\partial S_{j,i}}{\partial \ell}\right)$ are computed as well as the sums. Then, for the next iteration the new value, ℓ^* , is given by:

$$\ell^* = \ell - L/L'$$

where

$$L = \ell + \frac{\lambda_m}{q_m} \phi_n/q_m$$

and

$$L' = \frac{\partial L}{\partial \ell} = 1 + \frac{\lambda_m}{q_m} \times \frac{1}{2} \left(q_m \frac{\partial \phi_n}{\partial \ell} - q_n \frac{\partial q_m}{\partial \ell} \right)$$

The term $\frac{\partial \phi_n}{\partial \ell}$ is found as a function of the derivatives $\frac{\partial S_{j,n}}{\partial \ell}$, $j = 1, 2, 3, 4$ by differentiating the above expression for $\phi_n(\ell)$. More discussion of the melting-interface heat flux, q_m , is given below.

B. Solution Procedure

The general procedure used in solving these finite-difference equations at each s-direction station is as follows (for ℓ not specified):

- (1) Make an initial guess of the temperature profile and the melt-layer thickness;
- (2) Determine, at each transverse station, the temperature-dependent fluid properties and the geometric parameters σ , r , etc.;
- (3) Calculate velocity profiles, wall shear stress, and pressure gradient;
- (4) Obtain melting-interface heat flux, penetrator-surface heat flux; melt-layer thickness; and temperature profile by

iteration of the energy equation, until the difference, L , is less than 0.1% of ℓ ;

- (5) Return to Step (2) with the new melt-layer thickness and temperature profile. Repeat until all melt temperatures agree with those from the previous iteration within a specified tolerance.

As a check on the accuracy of the numerical calculations one other quantity is calculated at each s-direction station. This quantity is the total fluid-energy rate given by

$$\begin{aligned} \dot{E} &= \ell \int_0^1 \rho u h d\xi \\ &= \ell \sum_{i=1}^n \frac{\Delta \xi_i}{2} (\rho_i r_i u_i h_i + \rho_{i-1} r_{i-1} u_{i-1} h_{i-1}) \end{aligned}$$

at $s = \text{constant}$ where h is the enthalpy:

$$h = (C_p T)_m + \int_{T_m}^T C_p dT$$

This value of energy can then be compared with an s-direction integration of net energy flow to the melt-layer given in the following section.

C. Incrementation

The increment size $\Delta \xi_i$ is varied across the melt layer by arithmetic progression. The ratio of first-to-last increment sizes can be specified. Usually it is desirable to have relatively small increments near the penetrator surface.

D. Meridional Summations

When the convergence criteria are met for the transverse integrations at a given s-direction (meridional) station, certain s-direction summations are made before proceeding to the next station. These summations all have the form:

$$N_j = N_{j-1} + \frac{\Delta S_j}{2} \left[\left(\frac{dM}{ds} \right)_j + \left(\frac{dM}{ds} \right)_{j-1} \right]$$

where M is a general term for the quantity being summed. The summed quantities are:

(1) Fluid pressure, where $\frac{dp}{ds}$ is defined in the previous section. Initially, the pressure at the first station is set to zero. A pressure increment is added at the end of the problem so that the "outlet" pressure specification is satisfied and the axial load due to pressure is adjusted accordingly.

No other parameters are affected by pressure.

(2) Axial load due to pressure, F_p ;

$$\frac{dF_p}{ds} = 2\pi r_o p \sin \beta_o$$

(3) Axial load due to surface shear stress, F_τ ;

$$\frac{dF_\tau}{ds} = 2\pi r_o \tau_o \cos \beta_o$$

(4) Total penetrator surface power, Q_T ;

$$\frac{dQ_T}{ds} = 2\pi r_c q_o, \text{ where } q_o = -\frac{\lambda_o}{2} \left(\frac{dT}{ds} \right)_o$$

(5) Loss power, Q_L ;

$$\frac{dQ_L}{ds} = 2\pi r_m \sigma_m q_L, \text{ where } q_L = q_m - \rho_s V_z H_f \sin \beta_m$$

q_m is the melting interface heat flux, and H_f is the "total" heat of fusion.

(6) Superheat power, Q_{sh} ;

$$\frac{dQ_{sh}}{ds} = \frac{dQ_t}{ds} - 2\pi r_m \sigma_m q_m + \frac{dQ_G}{ds}$$

where Q_G is heat generation in the melt:

(7) Mass flow, $\dot{m} \times 2\pi$;

$$\frac{d\dot{m}}{ds} = - \left[\left(\sigma r \rho v \right)_m - \left(r \rho v \right)_o \right]$$

(8) Total fluid energy rate, $\dot{E}$;

$$\frac{1}{2\pi} \frac{d\dot{E}}{ds} = - \left[\left(\sigma r v T \right)_m - \left(\sigma r v T \right)_o \right] + \left(q r \right)_o - \left(q \sigma r \right)_m + \frac{dQ_G}{ds} \times \frac{1}{2\pi}$$

The last two items are computed as a check of the solution. The total fluid energy is computed as an s-direction integral of the net energy inflow into the melt layer and as an integral of total energy across the melt layer at each s-direction station. The melt flow can be found simply from

$2\pi\dot{m} = \pi \left(r_m^2 - r_{m,o}^2 \right) \times V'_z$ where $r_{m,o}$ is the melt-radius of the melting interface at the stagnation point. An additional check is made at the end of the problem on the total superheat power. The summation carried along in the s-direction stepping is obtained from the difference between penetrator-surface and melt-surface heat fluxes, plus any generation. The final value is compared to a value computed as the difference between the total power (from heated penetrator surface and the sum of "melt" and "loss" powers). The total and loss powers are sums, but the melt power is obtained from

$$Q_m = \rho_s V_z A_m H_f$$

where A_m is the total cross-sectional area of penetrator plus melt.

E. Melting Interface

The melting-interface heat flux, q_m , is the sum of the heat flux required to supply the latent heat of fusion to the rock entering the melt layer (q_{Lf}) and the flux of heat conducted from the melting interface into the solid rock (q_{mc}). The first component is given by

$$q_{Lf} = \rho_s V_z H_{Lf} \sin \beta_m$$

where H_{Lf} is the latent heat of fusion at the melting temperature.

$$u = V_z \cos \beta_{\text{п}}, \quad v = -V_z \sin \beta_{\text{п}}.$$

(1) Free outer boundary (at $y = Y$),

$T(o)$ = melting interface temperature, T_m .

(2) Outer boundary on axial center line

$$\frac{dT}{dy}(Y) = 0 \quad ,$$

$$\tau(0) = \tau_m.$$

$$\frac{V_z Y}{d_s} = \text{constant.}$$

When the outer boundary reaches the axis of symmetry for annular geometry penetrators, the value of Y is determined from geometric considerations. In the region where the outer boundary first reaches the axis (see Fig. 6) some inaccuracy may be introduced because the lines of constant s are not necessarily perpendicular to the axis. This should not be too serious, however, because the s -direction temperature gradients are generally small.

thickness, ℓ , involves the melting-interface heat flux, the derivative $\frac{\partial q_m}{\partial \ell}$ is also calculated,

V. SOME CALCULATIONAL RESULTS

A. Conical Penetrator - Comparison with Previous Results

Reference 7 gives an approximate closed-form solution to the present problem for a solid penetrator of conical geometry. Some parametric results of Ref. 7 are shown in Figs. 7 and 8 for a cone with a 15-deg half-angle and a radius of 0.10 m at the aft end. The results of calculations using the present computer program are plotted on the figures for comparison. These calculations were made by using the rock properties used in Ref. 7, but in the present case the viscosity was varied with temperature across the melt layer rather than using an effective mean viscosity as was done in Ref. 7. Both thrust ($F_T = F_n + F_t$) and power from

Fig. 7. Conical penetrator thermal power, comparison with analytic results.

the present analysis are seen to be somewhat higher as would be expected. The results of Ref. 8 do not take into account the heat loss to the unmelted rock, and the superheat power was underestimated in Ref. 7. One of the assumptions used in the analysis of Ref. 7 was that the melt layer is of constant thickness over the penetrator surface--this value being the value which would be obtained at the end of a very long penetrator. This melt-layer thickness is larger, then, than the average melt layer indicated by our calculations, resulting in lower average melt velocities relative to the penetrator and, therefore, a lower thrust. In addition, the axial force due to surface shear stress--which is small--was neglected in the results of Ref. 7. Figure 9 shows a plot of the penetrator surface and melt surface resulting from the present analysis.

B. 75-mm Consolidator

An existing consolidating penetrator* with double cone geometry was also analyzed. Figure 10 shows the resulting minimum thrust requirement, surface temperature (assumed uniform), and predicted penetration rate in the consolidation mode for a rock porosity of 48% and the best available

* A consolidating penetrator is used in porous rock for which the melt has a higher density than in situ rock. In the consolidation mode there is no debris removal, only densification, Ref. 8.

Fig. 8. Thrust required for conical penetrator, comparison with analytic results.

properties for local Los Alamos tuff. These results were obtained by assuming various penetration rates for a given surface temperature, and interpolating to find the penetration rate for which the melt-layer thickness at the aft end of the heated section satisfied the consolidation relationship, i.e.,

$$\frac{A_m}{A_z} = \frac{1}{P}$$

where

A_m = total melted rock cross-section area,
 A_z = penetrator cross-section area,
 P = rock porosity (0.48 in this case).

The thrust given is the axial load necessary to overcome the hydrostatic pressure load (due to an increase in pressure from an assumed value of zero at the aft end to a higher value at the

Fig. 9. Surface profiles for a conical penetrator.

leading edge) and the axial component of surface shear stress. An increase in axial load above this value theoretically would only increase hydrostatic pressure levels for the consolidation mode, and would not contribute to higher advance rates unless some melt extrusion resulted (in which case this could not longer be termed a consolidation mode of penetration).

Plotted in Fig. 10 are some data points taken from test results of this penetrator in dried tuff, with approximate corrections to the total power for stem losses. Predicted penetration rates are seen to be in very good agreement with the test data. The agreement or nonagreement of predicted and measured surface temperature is difficult to resolve. The predicted value is uniform over the entire surface, but other analyses have indicated that a mean value of the same magnitude should be similar in effect for a consolidator. The observed values are measured near the "aft" end of the heated surface where the surface temperature should be below average, as indicated by detailed thermal analysis of other penetrators.¹⁰ At this time, however, it is not known how much lower the measured values might be for this case.

Experiments were run on the 75-mm consolidator in an effort to determine the minimum thrust threshold. The results of these experiments indicated a

Fig. 10. Comparison of results with test data for a 75-mm consolidator. Rock porosity = 0.48.

value of roughly 1 to 2 kN over a power range from 3.4 to 7.5 kW. This higher value could be, in part, due to stem drag which is not taken into account in the analysis. Note that the predicted thrust load is extremely sensitive to melt viscosity and thermal-conductivity values assumed, and that the trend of thrust vs. power is very sensitive to the slope of the viscosity-vs.-temperature curve. The properties used in this analysis are shown in Figs. 11 and 12.

C. Other Results

Figures 13 through 15 show some typical velocity and temperature profiles at an s-direction station about halfway along the surface of a consolidating penetrator. Maximum values of the s-component of melt velocity may be very much higher for extruding penetrators, but the profiles are generally similar to those shown.

Figure 16 shows the heated penetrator surface and a typical calculated melting interface profile for the same penetrator. In this case the melting

Fig. 11. Assumed viscosity of local tuff melt.

interface did not reach the axis of symmetry by the end of the heated inner surface.

These and other analyses have indicated that, for a constant surface temperature profile, the thrust requirement varies as the penetration rate to a power which is approximately 3.5, i.e., $F \propto v_z^{3.5}$. This compares to the relation obtained in Refs. 7 and 11 in which a fourth-power dependency was obtained for approximate closed-form solutions.

Fig. 12. Assumed thermal conductivity for local tuff melt.

Fig. 13. Typical profile of s-direction melt velocity component for a penetration rate of 0.3 mm/s.

Results for extruding penetrators take the form of a range of operating conditions with dependence on both power (Q_T) and thrust (F_T), because there is no geometric restriction such as the consolidation relationship. A typical map of such results is shown in Fig. 17 for an annular extruding penetrator operating in basalt. The extremely

Fig. 14. Typical profile of y-direction melt velocity component for a penetration rate of 0.3 mm/s.

Fig. 15. Typical profile of melt temperature for a penetration rate of 0.30 mm/s.

low values of thrust obtained in these analyses are a result of the very low (relative to tuff) viscosity data published for basalt, and are contrary to laboratory experience to date. Again stem drag, and in this case, sticking of the melt to the extruding port cannot be taken into account in the present analysis.

D. Melting Interface Heat Flux Check

The heat flux from a prescribed melting interface was calculated by using AYER,^{12,13} a finite-element code for the solution of the energy equation in two dimensions. These calculations were

Fig. 17. Performance map for 82-mm extruding penetrator in dense rock (basalt).

performed for various penetration rates and solid-rock properties representative of two rock types; local tuff and basalt. Calculations for these same conditions were then made by using the method of this report. A comparison of the results is shown in Fig. 18. The agreement is seen to be very good both for overall heat transfer and for local heat flux. The comparison here is made on the basis of the heat loss, defined as the difference between the total heat-transfer rate from the melting interface, and the heat required to supply sensible and latent heat to the rock entering the melt layer.

Fig. 16. Penetrator surface and melting interface profiles for coring penetrator in porous rock at various penetration rates.

(a) $V_z = 0.10$ mm/s

(b) $V_z = 0.15$ mm/s

(c) $V_z = 0.20$ mm/s

Fig. 18. Heat loss calculation comparison for specified melting interface profile.

(a) Local heat loss, $V_z = 0.30$ mm/s, basalt;

(b) Nondimensional total heat loss; λ = thermal conductivity and d = thermal diffusivity of surrounding rock.

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