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AN EXPERIMENTAL STUDY OF HOT
PLASMA IN CONTACT WITH A COLD WALL

by
Benedict Feinberg
1974

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MASTER

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Submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy, in the Faculty
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ABSTRACT

An experimental study has been made of the energy transfer from a hot ($T \sim 5 \times 10^6$ °K), dense ($n \sim 10^{16} \text{ cm}^{-3}$) deuterium plasma containing a strong magnetic field ($B \sim 10^4$ Gauss) brought into sudden contact with a cold wall. The energy flux from the plasma to the wall was measured using a thin film, fast rise-time bolometer. The construction, calibration, and theory of this bolometer are described. The experimentally measured energy flux was compared with a theoretical prediction employing classical plasma transverse thermal conductivity. The measured energy flux is found to be about 1.5 times that predicted by classical plasma theory. The measured plasma transverse thermal conductivity exhibits the functional dependence on temperature, magnetic field, and density that is expected from classical plasma theory. The extent of damage to the cold wall surface was also measured using a scanning electron microscope to examine various wall samples exposed to the plasma. The observed surface damage to a copper wall sample was in reasonable agreement with the predicted damage due to sputtering. The damage to the surface of a stainless steel wall sample was considerably greater than predicted by sputtering.

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1. INTRODUCTION

When a hot plasma comes into contact with a cold wall, as it must eventually in all plasma devices, energy is transferred from the plasma to the wall. The energy transfer rate determines how fast the plasma cools, and also, to some extent, how hot the surface of the wall becomes. Therefore the problem of heating a plasma to thermonuclear ignition temperatures and the nature of impurities evaporated from the first confinement wall depend critically on the energy transfer from the hot plasma to the wall.

In this thesis, the energy transfer rate to a cold wall from a hot ($T \sim 5 \times 10^6$ °K), dense ($n \sim 10^{16}$ cm⁻³), plasma containing a strong transverse magnetic field ($B \sim 10$ KG) was measured. The plasma was produced in the Columbia University electromagnetically driven coaxial shock tube. A thin film, fast rise-time ($\tau \sim 200$ nsec), bolometer was developed and used to measure the heat flux. The measured energy transfer rate is compared to a relatively simple heat conduction model to determine if classical thermal conduction energy transport across a magnetic field accounts for the actual observed energy transfer. The energy

transport due to photon flux (Bremsstrahlung, $Z = 1$) is about 0.1% of the predicted particle energy flux. Other transport mechanisms are often observed in plasma physics, such as turbulence and plasma instabilities, which substantially increase the energy flux above the classical heat conduction rate. As will be subsequently shown, the measured energy flux in these experiments was found to be in reasonable agreement with classical transport predictions.

A purely heat conduction energy transfer model provides the following qualitative picture. When the hot plasma first comes into contact with a cold wall, a large amount of power is transported from the plasma to the wall. The plasma near the wall (i.e. within a few gyroradii of the wall) rapidly cools, forming a cooler plasma blanket next to the wall¹. This plasma thermal blanket then serves to protect the wall from the remaining hot plasma in the interior, and reduces the energy flux to the wall. Turbulence and instabilities, if present, would disturb this protective cool plasma blanket and greatly increase the heat flux to the wall.

In this thesis the heat flux was measured to determine the physical validity of this heat conduction model. The measured heat flux agrees with the heat

conduction prediction using the classical transverse thermal conductivity. The observed energy transport starts out high, (of the order of all the particles within a gyroradius being deposited in the wall in a gyroperiod, i.e. $\sim 10^6$ watts/cm²). It rapidly decreases as the cooler plasma thermal blanket forms. The functional dependence of the energy flux on ion temperature, magnetic field, and ion density is found to be in rather good agreement with that predicted by assuming simple heat conduction and classical transverse thermal conductivity. The experimentally measured heat flux is about $1.5 \pm .9$ times that predicted from the simple theory when it is modified to include the effects of some convection². The classical heat transport across the magnetic field provides a good estimate of the energy flux to the wall from the hot plasma produced in this experiment.

The damage to wall surface materials, due to exposure to hot plasma and high currents, was also examined. Polished wall samples of various materials were exposed to several hundred shots of the hot plasma and to the wall currents present in the shock tube. These materials were then examined with a scanning electron microscope. The damage to the surface of the copper wall sample was about equal to the damage predicted

due to sputtering. The observed damage to the stainless steel and boron nitride wall samples was greater than that expected due to simple estimates of sputtering and surface heating from the plasma and wall currents. The complex phenomena produced when surfaces are exposed to hot plasma are not well understood, and further research is apparently needed to explain some of the observed surface damage. Explanations of the observed surface damage include possible inaccuracies in the sputtering yields and also possible damage due to the propagating current sheets in these experiments.

This thesis compares the experimentally measured energy flux from a hot, dense plasma containing a strong transverse magnetic field to a cold wall with a classical plasma heat conduction model. The thesis also presents in detail the experimental methods used to measure the energy transfer and the methods used to determine the wall surface damage.

11. HOT PLASMA SOURCE AND ENERGY TRANSFER DIAGNOSTICS

2.1 THE COLUMBIA HIGH ENERGY SHOCK TUBE

This thesis studies the energy transfer rate to a cold wall from a hot plasma produced in the Columbia University high energy shock tube. This shock tube and its characteristics have been described previously^{1,2,3}. Briefly, this device consists of two coaxial conducting cylinders, 3 meters long. The copper inner wall has a 13 cm diameter, and the aluminum outer wall has a 23 cm diameter. The plasma is produced in the 5 cm gap between these coaxial conductors. A "bias" rod runs the axial length of the tube and current through it produces an initial azimuthal magnetic field of the order of several kilogauss.

A 1400 microfarad capacity, 20 KV bias bank stores the energy for the bias azimuthal field. The bias field current pulse has a quarter cycle time of about 100 μ sec. When the bias magnetic field has reached the desired value ($1 \text{ KG} \leq B_0 \leq 4 \text{ KG}$), a fast, high voltage "drive" bank is fired. This 60 microfarad bank

is charged to 85 KV. It has a 3.5 microsecond quarter cycle time, and consists of six modules, each with its own SF_6 filled field distortion switch⁴. The drive bank produces a radial electric field between the inner and outer conductors. This electric field causes a breakdown in the deuterium gas at the boron nitride launching wall, and a radial current sheet forms. This current sheet interacts with its own azimuthal magnetic field to produce an axial $\underline{J} \times \underline{B}$ force.

The $\underline{J} \times \underline{B}$ force propels the current sheet, or "magnetic piston", axially down the shock tube. This magnetic piston produces a shock wave ahead of it, and it is this strong shock wave which heats, ionizes, and compresses the deuterium gas⁵. The shock wave is axially reflected from a boron nitride wall located 180 cm down the tube. The reflected shock passes through the gas a second time, further heating and compressing the plasma. The equations governing this reflected shock are presented in appendix I. A heat transfer gauge whose detecting element is flush with the surface of the outer conductor, is located 15 cm from the reflecting wall.

Figure 2.1 shows an x-t diagram that illustrates the plasma dynamics which occur in the shock tube. Figures 2.2 and 2.3 show 2-dimensional cylindrical computer simulations of the shock wave, plasma dynamics and magnetic piston moving down the tube, heating and compressing the deuterium, and then reflecting off the reflecting wall⁶. These computer simulations illustrate that the magnetic piston travels faster near the inner conductor than near the outer wall. This piston shape has been observed experimentally⁷.

The shock tube, as operated for these experiments, produces a plasma with a typical density of 10^{16} particles per cubic centimeter, an ion temperature of about 5×10^6 °K, and this plasma contains an azimuthal magnetic field of about 10 kilogauss. The ions have a gyroradius of about 4.2 mm and a gyrofrequency of about 5×10^7 radians per second. The ion self-collision time is about 1.5×10^{-6} sec, and the ion-ion mean free path is about 36 cm. The ion gyrofrequency times the ion self-collision time, $\omega_{gi}\tau_{ii}$, is equal to about 60. This is a measure of the anisotropy of the plasma velocity distribution caused by the presence of the azimuthal magnetic field. A value

FIGURE 2.1: $x - t$ (distance - time) diagram of the shock tube. The incident shock wave travels axially down the shock tube, into the deuterium gas, followed by the post shock plasma and the magnetic piston. The shock front reflects off the reflecting wall, and travels back in the $(-x)$ direction. At the intersection of the magnetic piston and the reflected shock front, the twice shocked plasma expands through an expansion fan into the vacuum region. The dashed line represents the "line of sight" of the heat transfer gauge.

FIGURE 2.2: Two-dimensional cylindrical computer simulation of the shock wave, plasma, and magnetic piston traveling down the shock tube; heating and compressing the deuterium. The lines on A1, B1, and C1 represent temperature contours, with the heavy line representing the magnetic piston at the plasma-vacuum interface. The heavier shading on A2, B2, and C2 represents a larger plasma density.

FIGURE 2.3: Two-dimensional cylindrical computer simulation of the shock wave reflecting off the reflecting wall, and further heating and compressing the plasma. See figure 2.2.

of $\omega_{gi}\tau_{ii} \gg 1$ indicates a highly anisotropic velocity distribution. The classical transverse thermal conductivity divided by the thermal conductivity parallel to the magnetic field is about 8×10^{-7} . This ratio varies as $(1/\omega_{gi}\tau_{ii})^2$. The value of the ion plasma frequency for this plasma is 9.3×10^{10} rad/sec, and the electron plasma frequency is 5.6×10^{12} rad/sec.

2.2 THE HEAT TRANSFER GAUGE

The heat transfer sensing element employed in these experiments is made of a sapphire window, vacuum coated* with carbon and aluminum⁸, and is shown in figure 2.4. The surface exposed to the plasma is the thin aluminum film, backed by the carbon and then the sapphire window. The carbon layer is 2000 Å thick. This is the maximum thickness which allows the risetime of the system to be under 200 nanoseconds, as will be discussed below. The aluminum layer is 1500 Å thick. This layer serves to make the gauge opaque to infrared radiation, as will also be discussed below. The thin films on the gauge are heated by the plasma, and the carbon film emits radiation, some of which is collected and focused onto a liquid nitrogen cooled, gold-doped germanium infrared detector.

This heat transfer gauge can be viewed as a semi-

*The vacuum coatings were made by Spectrum Systems Division, Barnes Engineering Company, Waltham, Mass.

HEAT TRANSFER GAUGE

FIGURE 2.4: Pictures of the heat transfer gauge before and after exposure to 3 plasma shots, and a schematic of the heat transfer gauge.

infinite slab of sapphire with a thin but finite layer of carbon in good thermal contact with the sapphire.

It will be shown later that the aluminum layer of the gauge has a negligible effect upon its response time and hence will be ignored for the moment. The solution to the problem of heat transfer in a semi-infinite slab is well known⁹, and is solved in appendix III.

For a composite slab of two materials:

and a heat flux input $q = Q/\sqrt{t}$ at $x = -l$ ($Q = \text{constant}$), the solution for the temperature at the interface surface is,

$$T(x=0) = \frac{2}{1+\sigma} \sum_{n=0}^{\infty} \left(\frac{1-\sigma}{1+\sigma} \right)^n \frac{Q(\pi)^{\frac{1}{2}}}{(k_1 \rho_1 C_1)^{\frac{1}{2}}} \operatorname{erfc} \left(\frac{(2n+1)^2 l^2 \rho_1 C_1}{4k_1 t} \right)^{\frac{1}{2}} \quad (2.1)$$

where $\sigma = (\rho_2 k_2 C_2 / \rho_1 k_1 C_1)^{\frac{1}{2}}$

and ρ is the mass density, k is the thermal conductivity, and C is the specific heat. As $t \rightarrow \infty$, the solution becomes

$$T(x=0, t \rightarrow \infty) \equiv T^* = \frac{Q(\pi)^{\frac{1}{2}}}{(k_2 \rho_2 C_2)^{\frac{1}{2}}} \quad (2.2)$$

For a constant heat flux input, ($q=Q=\text{constant}$), the solution for the temperature at the carbon-sapphire interface is:

$$T(x=0) = \frac{2}{1+\sigma} \sum_{n=0}^{\infty} \left(\frac{1-\sigma}{1+\sigma} \right)^n \frac{Q \sqrt{t}}{(k_1 \pi \rho_1 C_1)^{\frac{1}{2}}} \times$$

$$\left[e^{-(2n+1)^2 \ell^2 \rho_1 C_1 / 4k_1 t} - \left(\frac{\pi (2n+1)^2 \ell^2 \rho_1 C_1}{4k_1 t} \right)^{\frac{1}{2}} \times \right.$$

$$\left. \text{erfc} \left(\frac{(2n+1)^2 \ell^2 \rho_1 C_1}{4k_1 t} \right)^{\frac{1}{2}} \right] \quad (2.3)$$

As $t \rightarrow \infty$, the solution becomes

$$T(x=0, t \rightarrow \infty) \equiv T^* = \frac{Q \sqrt{t}}{(\pi k_2 \rho_2 C_2)^{\frac{1}{2}}} \quad (2.4)$$

These temperatures are plotted as a function of t/τ , where τ is the thermal transit time, in graphs 2.5 and 2.6.

For $\sigma = 3$, the value appropriate for a carbon-sapphire gauge, T is greater than $0.9 T^*$ when $t \geq 50\tau$, where τ equals the thermal transit time of the carbon; i.e.

$$\tau = \ell^2 \rho_1 C_1 / 4k_1 \quad (2.5)$$

For a layer of carbon 2000 Å thick, $\tau = 3 \times 10^{-9}$ sec, giving a gauge risetime of 160 nsec. This calculation

FIGURE 2.5: Temperature of the carbon-sapphire interface versus time for a heat input $q = Q/t^{\frac{1}{2}}$, and $\sigma = 3$.

FIGURE 2.6: Temperature of the carbon-sapphire interface versus time for a heat input $q = Q$, and $\sigma = 3$.

illustrates that the interface thermal risetime depends on the thermal transit time of material 1, and also on σ which in turn is dependent on the properties of both materials. Sapphire was chosen as the substrate because it is transparent to infrared radiation in the wavelength range of interest (2 - 7.5 μm), and a carbon-sapphire system has $\sigma = 3$. The final temperature of the carbon-sapphire interface depends however, only on the properties of material 2, and sapphire yields a higher temperature than other common materials satisfying the above requirements.

The gauge also has an outer aluminum layer, 1500 $\text{\AA}$ thick, vacuum coated upon the carbon layer. This aluminum overcoat is provided to make the gauge opaque to infrared radiation which is emitted from the plasma. In the wavelength range of interest (2 - 7.5 μm), only about 10^{-7} of the incident radiation is transmitted through the thin aluminum outer layer. The thermal transit time of the aluminum layer is about 6×10^{-11} sec, which is only 2% of the thermal transit time of the carbon layer. The aluminum layer has, therefore, a negligible effect on the time response of the gauge when $t \geq 10^{-7}$ seconds. This fact was verified experimentally, as will be described in section 2.5. The thickness of these vacuum deposited layers was determined using a Sloan Monitoring

System¹⁰. This measures the mass deposited on a quartz crystal detector placed in the vacuum deposition chamber. The change in mass introduces a proportional change in the natural oscillation frequency of the quartz crystal, which is monitored. This change in mass is then related to the thickness of the coating.

The temperature of the gauge, $T(x=0, t \rightarrow \infty) = T^*$, depends only on the properties of material 2, i.e. sapphire; the aluminum layer has no effect on T^* . The sole purpose of the aluminum layer is to prevent radiation from the plasma being transmitted to the infrared detector. The total bremsstrahlung energy flux theoretically predicted divided by the particle energy flux hitting the gauge is about 10^{-3} . This indicates that the bremsstrahlung energy flux has a negligible effect upon the temperature rise of carbon-sapphire interface, compared to the effect of the particle energy flux.

2.3 THE INFRARED OPTICAL SYSTEM

The optical system, shown in figure 2.7, was designed to focus a substantial fraction of the infrared radiation emitted from the gauge onto the infrared detector. The 1.27 cm diameter gauge is imaged onto the 2 mm diameter detector element, while allowing the optical system to see a large solid angle.

FIGURE 2.7: The infrared optical system.

The detecting system consists of the heat transfer sensing element, a 7.5 cm diameter first surface mirror with a focal length of 7.5 cm, a 2.5 cm diameter As_2S_3 lens with a focal length of 1.9 cm, a 2.5 cm diameter germanium window, and the gold-doped germanium detector. The mirror serves to collect the light, while the lens, which is 70% transmitting to infrared, focuses the image of the gauge to 2 mm diameter. The Ge window cuts off infrared radiation below about 2 microns. This is desirable because the Ge: Au detector response has a strong, Ge intrinsic peak at 1.5 microns, which would tend to make interpretation of the signal more difficult. This optical system gathers and focuses onto the detector element about 0.5% of the infrared radiation emitted by the heat transfer gauge.

2.4 THE INFRARED DETECTOR

The infrared detector is a gold-doped germanium detector cooled to 77 °K by liquid nitrogen, and it is shown in figure 2.8. The sensing element is mounted in the vacuum section of a metal dewar which contains liquid nitrogen. The Ge: Au element is photoconductive, and its resistance changes in proportion to the infrared radiation flux which impinges on the element. A bias

FIGURE 2.8: Liquid nitrogen cooled, gold-doped germanium infrared detector.

voltage is impressed upon a load resistance connected in series with the detector, as shown in figure 2.9. Any change in the detector resistance produces a change in the voltage drop across the load resistor. It is this voltage change which is amplified and measured. The voltage across the load resistor in the absence of radiation is given by

$$V_{R_L} = V_O \left(\frac{R_L}{R + R_L} \right) \quad (2.6)$$

When radiation hits the detector, the resistance R decreases by ΔR , and the corresponding change in load resistor voltage is,

$$\Delta V_{R_L} = - \frac{V_O R_L \Delta R}{(R + R_L)^2} \quad (2.7)$$

The detectivity, D^* , of the detector is shown in figure 2.10¹¹. D^* is proportional to the detector output divided by the input power for a given frequency. The detectivity, therefore, is a relative measure of the detector response versus the wavelength of the incident radiation. The response time of the detector, as well as its absolute voltage response, is determined by the load resistor R_L .

FIGURE 2.9: Bias circuit for the Infrared detector.

FIGURE 2.10: Detectivity of the infrared detector versus wavelength of the incident radiation.

A larger load resistor will give a larger absolute response, but will lengthen the risetime of the detector. The circuit, shown in figure 2.11, uses a load resistor of $10K\Omega$, and a Motorola video amplifier integrated circuit to give a risetime of less than 100 nsec, and a voltage gain of 260. The amplifier has a flat frequency response from less than 50 KHz to more than 10 MHz.

2.5 CALIBRATION PROCEDURE OF THE ENERGY TRANSFER GAUGE

The heat transfer gauge, the detector and its associated electronics were calibrated directly using a black-body source¹². This source was an oxidized copper cylinder 6.5" long and 1.625" in diameter, with a $\frac{1}{2}$ " aperture in the side of the cylinder. It is shown in figure 2.12. The cylinder was immersed in agitated hot oil, and the temperature monitored with a thermocouple. For a body of this size and shape, the effective emissivity of the cavity is known to be greater than .98 if the emissivity of the material is greater than 0.05. The infrared emissivity of oxidized copper is at least 0.3, and therefore the radiation emitted from the aperture was a blackbody spectrum.

To calibrate the gauge it is necessary to first examine the transmission of infrared radiation through sapphire. The transmission is given by¹⁵

FIGURE 2.11: Amplifier circuit for the infrared detector.

FIGURE 2.12: Black body source used to calibrate the heat transfer system.

$$I = I_0 (1-r)^2 e^{-\alpha x} \quad (2.8)$$

where I_0 is the incident radiation intensity, α is the radiation absorption coefficient, and x is the material thickness. The surface reflectivity, r , is given by¹⁶

$$r = \left(\frac{n-1}{n+1} \right)^2 \quad (2.9)$$

where n is the index of refraction. The factor $(1-r)$ is squared because the radiation passes through 2 surfaces. For sapphire, $.05 \leq r \leq .07$ in the wavelength range $2 \mu\text{m} \leq \lambda \leq 7.5 \mu\text{m}$. When the gauge is used on the shock tube, the test time is short ($\sim 5 \mu\text{sec}$) and therefore the aluminum layer, the carbon layer, and the carbon-sapphire interface heat up, while the bulk of the sapphire (which is .040" thick) remains at room temperature. The output of the detector, V_G , under these conditions, is given by

$$V_G = K \int (I_{bb\lambda}(T) - I_{bb\lambda_0}) R \epsilon_G (1-r) e^{-\alpha x} d\lambda \quad (2.10)$$

where $I_{bb\lambda}(T) - I_{bb\lambda_0}$ is the blackbody spectral intensity at the interface temperature minus the blackbody spectral intensity at room temperature. R is the responsivity of the detector, ϵ_G is the emissivity of the gauge,

and K is a constant which takes into account the losses in the optical system. The blackbody spectral intensity times the emissivity of the carbon layer gives the spectral intensity emitted by the carbon-sapphire interface of the gauge. This is multiplied by $(1-r)$ to take into account the reflection from the surface of the sapphire. The factor $e^{-\alpha x}$ takes into account the losses in the sapphire window. R , the responsivity, which gives the detector voltage output divided by the input power, changes the spectral intensity into a detector voltage intensity. This is then integrated over the wavelength range of the system, in this case $2 \leq \lambda \leq 7.5 \mu\text{m}$. The constant K takes into account the limited solid angle of the infrared optical system as well as the losses inherent in the optical system.

To accurately and realistically calibrate this system, three separate measurements must be made. Heating the carbon-sapphire interface, to obtain the detector response, also involves heating the sapphire window. For this situation

$$V_C = K \int (I_{bb\lambda}(T) - I_{bb\lambda_0}) R (1-r) [\epsilon_G e^{-\alpha x} + (1-e^{-\alpha x})] d\lambda \quad (2.11)$$

where the first term in the brackets represents the opaque carbon layer contribution, and the second term represents the contribution from the heated sapphire.

To obtain the response of the gauge for the shock tube situation we must also measure the detector response to a blackbody source. This is given by

$$V_A = K \int (I_{bb\lambda}(T) - I_{bb\lambda_0}) R d\lambda \quad (2.12)$$

Next a room temperature sapphire window is interposed between the blackbody source and the detector. This gives a response,

$$V_B = K \int (I_{bb\lambda}(T) - I_{bb\lambda_0}) R(1-r)^2 e^{-\alpha x} d\lambda \quad (2.13)$$

These three measurements determine, for each blackbody temperature, the realistic gauge response, V_G . It is,

$$V_G = \frac{V_B}{(1-r)} - (1-r) V_A + V_C \quad (2.14)$$

The actual measurements of V_A , V_B and V_C are shown in figure 2.13, and the detector response V_G as a function of the carbon-sapphire interface temperature is shown in figure 2.14. The optical system used in the calibration procedure was the same as that used in the experiment. The calibration radiation was chopped using a fast, 50 KHz, mechanized chopper.

FIGURE 2.13: Calibration curves of detector output versus temperature. V_A represents the use of the blackbody source as the infrared source. V_B represents the insertion of a room temperature sapphire window between the blackbody source and the optical system. V_C represents the use of an actual heat transfer gauge as the infrared source, where the sapphire window is heated along with the carbon-sapphire interface.

FIGURE 2.14: Calibration curve of detector output versus temperature. V_G represents the detector response to a short pulse ($\sim 5 \mu\text{sec}$) heat input to the heat transfer gauge. The carbon-sapphire interface is heated, but the bulk of the sapphire window remains at room temperature. This is the realistic detector response to the plasma in the shock tube.

The temperature range of this direct calibration as shown in figure 2.14 is not broad enough to enable the use of the calibration curve directly. Curves of the relative signal, S , of the detector as a function of temperature were obtained as follows. The relative detector signal, S , depends upon the power and wavelength of the infrared radiation reaching the detector, as well as the detector response to this radiation. The power is determined by the temperature of the carbon-sapphire interface. The wavelength spectrum of the infrared radiation is determined by the blackbody spectral intensity emitted by the carbon layer, and by the wavelength dependent losses in the sapphire window. The relative signal, S , also depends upon the detectivity of the infrared detector. The blackbody spectral intensity must therefore be multiplied by the detectivity of the infrared detector and by the transmittance of the sapphire, for each wavelength. This product is then integrated over the wavelength interval of interest ($2 - 7.5 \mu\text{m}$) to determine the relative detector signal, S . A curve of $I_{\text{bb}\lambda}(600) \times D^* \times \text{Tr}(\text{sapphire})$ versus wavelength is shown in figure 2.15. $I_{\text{bb}\lambda}(600)$ is the blackbody spectral intensity at 600°K , D^* is the detectivity, and $\text{Tr}(\text{sapphire})$ is the transmittance of the sapphire. This curve is integrated over the wavelength interval

$I_{bb\lambda}(600) \times D^* \times Tr(\text{sapphire})$

FIGURE 2.15: Blackbody spectral intensity at 600 °K multiplied by the detectivity of the infrared detector multiplied by the transmittance of the sapphire window, versus the wavelength of the radiation. The units of the vertical axis are arbitrary.

2 μm - 7.5 μm , giving the relative detector signal, S , for the temperature of 600 $^{\circ}\text{K}$. This was done for different temperatures, and graph 2.16 shows the relative response, S , as a function of T . This curve gives the relative response of the system to a gauge with a carbon-sapphire interface at temperature T . The measured value of V_G for a known temperature now can be extrapolated to any temperature since the relative response, S , of the system is known for all temperatures of practical interest.

The detector signal at temperature T is given by,

$$V_G(T) = S(T) \times \frac{V_G(T_1)}{S(T_1)} \quad (2.15)$$

where $V_G(T_1)$ is obtained from graph 2.14 and $S(T_1)$ is obtained from graph 2.16 for any temperature T_1 . For any temperature T_1 ,

$$V_G(T_1)/S(T_1) \approx 10.9 \quad (2.16)$$

Using this factor, S can be determined from the measured detector voltage, V_G . Using graph 2.16 one can determine the temperature T^* of the carbon-sapphire interface. From a knowledge of the functional dependence of the heat transfer rate q on time t , q can be determined from the measured detector output voltage.

FIGURE 2.16: Relative response of the heat transfer system as a function of the temperature of the carbon-sapphire interface of the heat transfer gauge.

The actual risetime of the heat transfer gauge and detector system was measured directly using a ruby laser. A heat transfer gauge with no aluminum coating (i.e. only carbon on sapphire) was struck by a 30 MW, 50 nsec, laser pulse.¹⁷ Figure 2.17 shows the detector response, and it indicates a measured response risetime of about 200 nsec for the detection system. An aluminum and carbon coated gauge of the type used in the shock tube - plasma experiments was hit by a 100 MW, 30 nsec, ruby laser pulse¹⁸. Figure 2.18 shows the actual response of the detector system and indicates a risetime of about 200 nsec, experimentally demonstrating that the aluminum layer has a negligible effect on the time response of the detection system. The risetime of this system is very close to that which is theoretically expected, since calculations presented in section 2.2 predict a sensing element response time of about 160 nsec.

2.6 PREPARATION OF THE WALL SAMPLES FOR DAMAGE STUDIES

Polished materials were prepared in order to examine the effect of damage caused by plasma bombardment and electric current that impinges on the shock tube outer conducting wall. Three types of materials were used: copper, stainless steel, and boron nitride. Boron nitride is a ceramic dielectric material that is used

FIGURE 2.17: Measured response of the heat transfer system with a carbon coated sapphire heat transfer gauge (i.e. no aluminum) to a 30MW, 50 nsec, ruby laser pulse, versus time.

FIGURE 2.18: Measured response of the heat transfer system with an aluminum and carbon coated sapphire heat transfer gauge to a 100MW, 30nsec, ruby laser pulse, versus time.

for the shock tube launching and reflecting walls. These materials were first polished on polishing wheels using progressively finer grades of emory paper. They were then polished using Linde A and Linde B, which are suspensions in distilled water of fine aluminum oxide particles. A felt polishing wheel coated with the suspension is pressed against the surface. In between the use of the larger Linde A particles and the smaller Linde B particles, the samples were ultrasonically cleaned so that no large particles would be present to scratch the surface. This polishing procedure follows standard metallurgical techniques.

These polished surface samples were placed flush with the inner surface of the shock tube outer conducting wall. The metal samples were in very good electrical contact with the wall. The samples were removed from the shock tube after several hundred plasma shots. Scanning electron microscope pictures were taken of the wall samples before and after insertion. The Cambridge Stereoscan Electron Microscope at Columbia's Lamont-Doherty Geological Observatory was used for this purpose. It was observed that some material was deposited on the wall samples. The atomic nature of these surface deposits was analyzed using an EDAX (Energy Dispersive Analysis of X-Rays) system in conjunction with the scanning electron microscope, and will be discussed in chapter III.

III. THE EXPERIMENTAL DATA

3.1 PLASMA STATE DATA

The heat transfer from a uniform hot plasma containing a strong transverse magnetic field to a cold wall is,

$$q = 3.72 \times 10^{-24} \left(\frac{T_i^{3/2} n_i^3 \ln \Lambda}{B_t^2} \right)^{1/2} \text{ watts/cm}^2 \quad (3.1)$$

where T_i is the ion temperature in $^{\circ}\text{K}$, n_i is the number density in cm^{-3} , and B is the magnetic field in Gauss.

This equation assumes that the only energy transfer is by means of classical plasma thermal conductivity transverse to the magnetic field. This result is derived in the following chapter. In plasma physics, however, many situations have been found where the classical transport properties do not properly describe the actual physics. Instabilities, turbulence, etc. substantially alter the transfer mechanism. To test the validity of this theoretically predicted heat transfer rate, the ion temperature and density, the magnetic field strength, and the actual energy transfer rate must be measured.

The magnetic field strength can be measured experimentally by using small magnetic field probes¹. The plasma density can be inferred from an experimental measurement of the magnetic field compression caused by the transverse shock wave. Frozen flux ($B/\rho = \text{constant}$) has been experimentally

confirmed by Y.G. Chen². The ion temperature can be calculated using the experimentally measured shock speed. Actual ion and electron temperatures have been experimentally measured by S. Robertson³ and D. McNeill⁴. These experimental measurements and the associated calculations are described in this section.

To measure the magnetic field, magnetic field probes¹ consisting of 0.30" diameter wire loops placed in alumina tubing are inserted from the outer conducting wall 1 centimeter into the shock tube. A change in the magnetic flux through the loop provides a probe voltage proportional to dB/dt . This voltage is displayed on an oscilloscope. Using the signal from a magnetic probe placed 150 cm down the tube from the launching wall and the signal from a probe placed at the 175 cm point, the speed of the shock front can be measured. A sample oscilloscope trace from a magnetic field probe is shown in figure 3.1.

This signal, proportional to dB/dt , can be integrated using an RC circuit with an RC time constant much greater than the times of interest in the experiment. The experimental times in these experiments are of the order of microseconds, and the RC time constant used in the integrating circuit is 1 millisecond. Integrating the signal gives a measurement of the magnetic field strength B .

FIGURE 3.1: Response of the magnetic probes at the 150 cm and 175 cm locations, versus time. The first spike on each trace represents the shock front passing the probe. The second peak on each trace represents the magnetic piston passing the probe. The slight rise of the response of the 175 cm probe at 10 μsec is due to the passage of the reflected shock front.

The compression of the magnetic field, from the initial bias field value to the value in the post shock plasma, can therefore be measured. Figure 3.2 shows an oscilloscope trace of the magnetic field compression. This trace also gives the value of the magnetic field in the plasma behind the reflected shot.

The plasma density is compressed along with the magnetic field, in accordance with the physical effects caused by frozen magnetic flux. For times less than τ , the magnetic diffusion time, the field lines can be considered to be frozen into the plasma. The change in magnetic flux is given by:

$$\frac{\partial \Phi}{\partial t} = \frac{\eta}{4\pi} \int \nabla^2 B \cdot ds \quad (3.2)$$

assuming that $\partial j / \partial t$, $\partial v / \partial t$, ∇P , and $\partial E / \partial t$ are zero. Approximating $\nabla^2 B$ by B/L^2 , where L is a characteristic length of the system, the magnetic flux decays exponentially with a time constant τ . τ is the time required for a magnetic field to diffuse through ohmic losses over a distance L .⁵

$$\tau \approx \frac{4\pi L^2}{\eta} \approx \frac{4\pi L^2}{6.53 \times 10^{12}} \frac{T^{3/2}}{\ln \Lambda} \text{ sec} \quad (3.3)$$

For $T \sim 5 \times 10^6$ °K, $\ln \Lambda \sim 12.4$, $L \sim 5$ cm,

$$\tau = 43 \times 10^{-3} \text{ sec}$$

FIGURE 3.2: Integrated signal of the magnetic probe at the 150 cm location, versus time. The signal is proportional to the magnitude of the magnetic field.

Since the time of interest in these experiments is of the order of microseconds, the frozen flux approximation is valid, and the compression of the magnetic field is equal to the compression of the plasma density. Therefore the post shock density may be calculated using the initial filling pressure and the oscilloscope traces of the magnetic field compression.

The use of the frozen flux approximation for the plasma compression behind the reflected shock is valid since the reflected shock is a magnetohydrodynamic shock propagating through a plasma. The use of the frozen flux approximation for the plasma compression behind the incident shock is, however, at first glance not valid. The incident shock is an ionizing shock, ionizing the initially neutral deuterium gas, and the frozen flux approximation is only valid for the case of a highly conducting plasma. Experimental measurements using a Michelson interferometer indicate, however, that even for the initial ionizing shock, the density compression is approximately equal to the magnetic field compression². This comes about because the initial gas is actually a few percent ionized, caused by precursor photoionization and other ionizing effects. Since careful experimental measurements have indicated that the density compression through the incident shock is equal to the magnetic field com-

pression, the post shock density can be calculated from a knowledge of the initial filling pressure and the measured magnetic field compression.

The shock speed has been measured using magnetic field probes at the 150 cm and 175 cm locations. A single fluid computer code was developed which solves the magnetohydrodynamic transverse shock jump conditions and thereby predicts the plasma temperature $T = T_e + T_i$, as a function of the shock speed, the initial magnetic field, and the initial gas density⁶. Appendix I lists the equations solved by the computer code. The temperature has been computed for both the plasma behind the incident shock and the plasma behind the reflected shock. An example of these computational results appears in figure 3.3. The ion temperature is assumed to be equal to $\frac{1}{2}$ of T , ($T = T_e + T_i$). This is a conservative estimate of the ion temperature, since both experimental and theoretical evidence exists that, at least initially, $T_i \gg T_e$ for a shock heated plasma.^{3,4,7,8} The effect of $T_i > T_e$ is analyzed in chapter IV.

These then are the data necessary to compute the theoretically predicted energy transfer from a hot plasma to a cold wall. The magnetic field strength is experimentally measured using the magnetic field probes. The density is calculated from the initial filling pressure and the ex-

FIGURE 3.3

perimentally measured magnetic field probes. The density is calculated from the initial filling pressure and the experimentally measured magnetic field compression. The ion temperature is calculated by solving the single fluid magnetohydrodynamic shock jump conditions using the experimentally measured shock speed.

3.2 HEAT TRANSFER DATA

The oscilloscope trace in figure 3.4 shows the response of the infrared heat transfer detector to the infrared radiation emitted from a heat transfer gauge sensing element which was mounted flush with the inner surface of the outer conducting wall. The gauge was located 165 cm from the launching end and it was in direct contact with the hot plasma. Using the procedure outlined in the calibration section, the temperature of the carbon-sapphire interface was determined. Using equation 2.2, the energy flux to the wall, $q = Q/\sqrt{t}$, can be determined. To determine the heat flux q requires a knowledge of the time that the hot plasma first came into contact with the gauge. This is found by taking the time of the start of the signal in figure 3.4 as $t = 0$. This time, the beginning of the detector response, corresponds quite well to the time that the reflected shock reaches the heat transfer gauge sensing element. This can be seen by comparing the time of arrival

FIGURE 3.4: Response of the infrared detector system as a function of time, to the radiation emitted from an aluminum and carbon coated sapphire heat transfer gauge exposed to the shock tube plasma.

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of the reflected shock at 150 cm, as shown in figure 3.5, with the time of the start of the heat transfer signal. The time difference between these two signals corresponds to the time it takes for the reflected shock to go from 165 cm to 150 cm.

The reflected shock speed is determined by the incident shock speed and the magnetic field oscilloscope trace. The time for the incident shock to go from the magnetic probe to the reflecting wall is calculated by dividing the distance from the magnetic probe to the reflecting wall by the incident shock speed. This time is subtracted from the time interval between the incident shock reaching the probe. This calculation gives the time for the reflected shock to travel from the reflecting wall to the magnetic probe. The distance from the reflecting wall to the probe is divided by this time, giving the reflected shock speed. Typically, the reflected shock speed varies from $1/3$ of the incident shock speed for slow shots to $3/4$ of the incident shock speed for fast shots. For the data given in figures 3.4 and 3.5, the incident shock speed was $14.3 \text{ cm}/\mu\text{sec}$ and the reflected wave speed was $3.8 \text{ cm}/\mu\text{sec}$.

The time for the reflected shock to travel from the heat transfer sensing element at 165 cm to the magnetic probe at 150 cm is easily calculated using the experimentally measured reflected shock speed. For the shot repre-

FIGURE 3.5: Upper trace is the response of the infrared detector system as a function of time to the radiation emitted from a heat transfer gauge at the 165 cm location in the shock tube. Lower trace is the integrated magnetic probe signal at 150 cm versus time. This picture allows comparison of the time of arrival of the reflected shock front with the start of the infrared detector signal, as explained in the text.

sented by figures 3.4 and 3.5 the reflected shock front arrives at the heat transfer sensing element about 4 μ sec before it arrives at the magnetic field probe. The detector signal should start, therefore, about 4 microseconds before the reflected shock front arrives at the 150 cm magnetic field probe. Indeed, this is what is observed, as seen in figure 3.5.

The heat flux q at time t can therefore be determined from the data of figure 3.4 which gives the infrared detector response as a function of time.

3.3 WALL DAMAGE DATA

Polished samples were placed flush with the inner surface of the outer conducting wall of the shock tube. These samples were left in place for periods of the order of several weeks. During these periods various other plasma physics experiments were being performed in the shock tube. The samples were exposed to plasmas with densities ranging from about 10^{15} particles/cm³ to 5×10^{16} particles/cm³, ion temperatures ranging from 10^5 °K to 9×10^6 °K, and shock tube wall currents were of the order of 10^6 amperes. The skin depth in the aluminum outer conductor is about 4×10^{-2} cm for the megampere drive pulse, which has a frequency of about 50 KHz. This pulse gives a current density of the order of 10^5 amp/cm² in the aluminum outer conductor. The effects of these currents,

which travel through the outer conductor and the plasma, are discussed in chapter IV.

Figure 3.6 shows scanning electron micrographs of a stainless steel wall sample, before and after exposure to 625 plasma shots. The picture of the polished surface before exposure to the hot plasma shows very slight scratches and some small pits due to the polishing process. Before exposure the surface looks like a mirror to the naked eye, however. The scanning electron micrograph of the surface after exposure to the plasma and electric currents indicates that melting and pitting of the surface has occurred. It should be emphasized that these pictures are at quite high magnification, and that the surfaces look quite smooth to the naked eye, even after exposure to the plasma.

The distance between the "peaks" and "valleys" was measured using an optical microscope. The microscope was first focused on a valley, and then focused on a peak. The fine focusing knob was calibrated with divisions representing a vertical movement of 2 microns. The stainless steel wall sample showed an average distance of 5 μm between the height of the peaks and the depths of the valleys. The maximum distance measured between the peaks and valleys was 10 μm .

Figures 3.7 and 3.8 show scanning electron micrographs of a copper wall sample, before and after 90

Scanning electron micrograph of a stainless steel surface.

Before exposure;
magnification 2300

After exposure to plasma
and electric currents;
magnification 530
number of shots = 625

53

FIGURE 3.6

Before exposure;
magnification 2150

After exposure to ninety plasma
shots and very high electric currents.
magnification 2250

FIGURE 3.7

Scanning electron micrograph of a copper surface.

After exposure to plasma
and electric currents.

magnification 215

number of shots = 90

After exposure to plasma
and electric currents.

magnification 2250

number of shots = 90

FIGURE 3.8

plasma shots. The polished surface before exposure shows scratches and slight pitting due to polishing, but looks like a mirror to the naked eye. The pictures of the samples after exposure to the plasma and electric currents indicate only very slight melting of the surface. A particularly interesting "crater" formed apparently by a small ($\sim 1 \mu\text{m}$) particle striking the molten surface is visible in figure 3.8. Unipolar arc⁹ tracks were also observed, and an arc track is shown in figure 3.9. The mechanism of this arc will be discussed in chapter IV. Small nodules also appear on the surface of this sample. The height of these nodules was measured using the same technique used for the stainless steel sample. Their height averages $5 \mu\text{m}$, with some nodules ranging up to $15 \mu\text{m}$ above the surface. The atomic nature of the nodules was examined with the EDAX system¹⁰. They were found to be aluminum. The EDAX system also indicated that small amounts of calcium were present on the surface. The outer conductor of the shock tube is made of aluminum, which apparently sputtered and melted thereby depositing small nodules on the copper surface. The boron nitride launching and reflecting walls contain small (about 1.5%) amounts of calcium, which was also probably the source for the small amounts found on the copper sample.

FIGURE 3.9: Photograph of the copper wall sample. The fern-shaped pattern in the center of the photograph, (upper center of the sample), is a unipolar arc track.

Figure 3.10 shows scanning electron micrographs of a boron nitride surface, before and after exposure to 500 plasma shots. Boron nitride is a compressed powder, consisting of, in addition to boron nitride, small amounts of oxygen and calcium. The powdery nature is visible in the photograph of the surface before exposure to the hot plasma. The picture of the surface after exposure to the hot plasma shows evidence of melting. The surface has melted and formed small globules of solid boron nitride. The distance between the peaks and valleys was measured in the same manner as for the other samples. The average distance between the peaks and valleys ranges from 10 μm to 20 μm , with some peaks ranging up to 50 μm higher than the valleys.

The physical properties of these samples, and explanations of the observed surface phenomena are discussed in chapter IV.

Before exposure;
magnification 2000

After exposure to 500 plasma
shots and very high electric currents.
magnification 2150

59

FIGURE 3.10

IV INTERPRETATION AND ANALYSIS OF RESULTS

4.1 ENERGY TRANSFER RESULTS

The section of chapter II on calibration explained the method for obtaining, from the detector signal, the temperature of the carbon-sapphire interface. Figure 4.1 shows the actual interface temperature T^* , as a function of time. This was obtained by dividing the detector signal by the factor given in equation 2.16 and then using the graph of figure 2.16 to obtain T^* . The raw data for figure 4.1 is provided by the oscilloscope trace shown in figure 3.4.

Using equation 2.2, where Q is a constant, the heat flux q can be obtained when T^* and t are known. Figure 4.2 shows a graph of q versus t for the raw data of figure 3.4. The magnitude of the error bars on the graph come from considering the errors which result from the detector noise.

A theoretical analysis of the heat transfer has been developed¹. A simplified analytical model yields the curve shown by the dashed line in figure 4.2. This simple model for the heat transfer assumes that the heat flux is due only to thermal conduction across the magnetic field. The heat conduction² is:

FIGURE 4.1: Temperature of the carbon-sapphire interface of a heat transfer gauge exposed to the shock tube plasma versus time.

FIGURE 4.2: Solid line is the experimentally measured heat transfer from the plasma to the wall, versus time. Dashed line is the plot of the equation:

$$q = 3.72 \times 10^{-24} (T^{3/4} n^{3/2} \ln \Lambda) / B t^{1/2}$$

for the values of T , n , and B shown. This equation is obtained from the simple heat conduction model, using the classical plasma transverse thermal conductivity.

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$$q = K \left. \frac{\partial T}{\partial x} \right|_{x=0} = \frac{KT}{(\pi \kappa t)^{\frac{1}{2}}} \quad (4.1)$$

K is the transverse thermal conductivity, and κ is the thermal diffusivity given by

$$\kappa = K/\rho C \quad (4.2)$$

where ρ is the mass density and C is the heat capacity, given by

$$C = R/(\gamma - 1)$$

R is the gas constant and $2/(\gamma - 1)$ is the number of degrees of freedom. Assuming a perfect gas, with 3 degrees of freedom,

$$C = 3/2 (k/m) \quad (4.3)$$

Therefore,

$$q = T \left(\frac{K \rho k}{m \pi t} \frac{3}{2} \right)^{\frac{1}{2}} = T \left(\frac{3}{2 \pi} \frac{n k K}{t} \right)^{\frac{1}{2}} \quad (4.4)$$

If the magnetic field is very strong, such that the ion cyclotron frequency is much greater than the ion collision time (i.e. $\omega_{ci} \tau_{ii} \gg 1$) then the value of the transverse thermal conductivity³ can be used. For the

plasma produced in these experiments, $\omega_{ci} \tau_{ii} \approx 60$, and therefore

$$K_{\perp} = 14.8 \times 10^{-25} \frac{A_i^{\frac{1}{2}} n_i^2 z^2 \ln \Lambda}{T_i^{\frac{1}{2}} B^2} \frac{\text{joules}}{\text{sec deg cm}} \quad (4.5)$$

where A_i is the atomic weight of the ion, z is the atomic number, and n_i is in cm^{-3} , T_i is in $^{\circ}\text{K}$, and B is in Gauss. For a deuterium plasma, $A_i = 2$, $z = 1$; therefore:

$$q = 3.72 \times 10^{-24} \left(\frac{T_i^{3/2} n_i^3 \ln \Lambda}{B^2 t} \right)^{\frac{1}{2}} \frac{\text{watts}}{\text{cm}^2} \quad (4.6)$$

where T_i , n_i and B have the same units as in equation 4.5, and t is in seconds.

This simple heat conduction equation was used to calculate the lower curve of figure 4.2. The experimental data and the theoretical curve fit each other quite well for the first 0.5 μsec , where the rapid decrease in heat flux is due to the rapid formation of a boundary layer of cool plasma. After about the first microsecond, the experimental curve lies above the prediction of the heat conduction equation, and at about 2 μsec the experimental curve has a value of q about 2 to 3 times the theoretical value. This difference is

discussed at the end of this section. At about $2.5 \mu\text{sec}$, the plasma begins to expand back into the vacuum region and the heat transfer gauge begins to sense the effects of the expansion fan, and the heat flux to the wall decreases.

The dependence of the heat flux q on the magnetic field strength, the ion temperature, and the density was analyzed. Taking the value of q at the maximum gauge temperature for each shot, and normalizing q with respect to all the variables except the one of interest, one can determine the dependence of q on the variable of interest. Figure 4.3 shows a plot of $qt^{1/2}/T^{3/4}n^{3/2}\ln \Lambda$ versus the magnetic field B on log-log paper. If

$$qt^{1/2} \propto T^{3/4} n^{3/2} (\ln \Lambda) B^a$$

then

$$\log \left[\frac{qt^{1/2}}{T^{3/4} n^{3/2} \ln \Lambda} \right] = a \log B + \text{constant}.$$

a is the slope of the line representing the least squares fit of the experimental points on the graph in figure 4.3. Figures 4.4 and 4.5 show log-log plots of q normalized to obtain the dependence of q on ion temperature and density. The slope of these graphs, found by using a least squares fit to the data points, gives the following results.

FIGURE 4.3: Solid line is a least squares fit to a log-log plot of the heat transfer normalized with respect to temperature, density, and time, versus magnetic field. The slope of this plot is -0.9. The dashed line is a plot of the equation (4.6), which has a slope of -1.0.

FIGURE 4.4: Solid line is a least squares fit to a log-log plot of the heat transfer normalized with respect to magnetic field, density, and time, versus temperature. The slope of this plot is 0.72. The dashed line is a plot of the equation (4.6), which has a slope of 0.75.

FIGURE 4.5: Solid line is a least squares fit to a log-log plot of the heat transfer normalized with respect to magnetic field, temperature, and time, versus density. The slope of this plot is 1.0. The dashed line is a plot of equation (4.6), which has a slope of 1.5.

$$\underline{q \propto B^a T^b n^c}$$

EXPERIMENTAL RESULTS

SIMPLE THEORY

$$a = -0.9 \pm 0.35$$

$$a = -1.0$$

$$b = 0.72 \pm 0.24$$

$$b = 0.75$$

$$c = 1.0 \pm 0.4$$

$$c = 1.5$$

The error values in this table are the standard deviations of the slopes.

The measured magnitude of q averages 2.8 times that which the simple theory predicts. The experimental dependence of q on T and B agrees remarkably well with the very simple theory, and the dependence on n is almost within the experimental error.

The experimental data indicates that the actual heat transfer is about 2.8 times that predicted by the simple model. This simple heat transfer equation leaves out several physical effects, some of which are discussed here.

One effect is that of convection of the hot plasma towards the wall. As the plasma near the wall cools, its pressure decreases, and a pressure gradient is set up perpendicular to the wall and the magnetic field. The equations governing this pressure gradient and the resulting convection have been numerically studied by M.S. Chu⁴. Using his numerical results, and interpolating among his results to match the plasma state in the shock tube, the heat transfer rate predicted by the simple conduction model should be multiplied by a factor of about 1.2 to take into account the effect of convection.

Another effect not taken into account by the simple conduction equation is the thermoelectric effect. Spitzer and Harm⁵ have treated the thermoelectric effects in a plasma, and following their arguments, when a magnetic field is present, the heat transfer is,

$$\underline{q}_{\perp} = - \epsilon K_{\perp} \underline{\nabla T} \quad (4.7)$$

$$\text{where } \epsilon = (1 + \frac{9}{4} k^2 n_e^2 T \eta_{\perp} / K_{\perp}) \quad (4.8)$$

where $\eta_{\perp}$, the transverse resistivity⁶, is in e.m.u., k is in joules/^oK, n_e is in cm⁻³, T is in ^oK, and $K_{\perp}$ is given by equation 4.5. For the plasma produced in these experiments, $\epsilon = 1.03$. The thermoelectric effect, therefore, increases the transverse thermal conductivity by

only about 3%, and therefore increases the transverse energy flux to the wall by about 1%, since the energy flux is proportional to the square root of the thermal conductivity, as shown in equation 4.4.

Another effect ignored in the application of the simple heat conduction model is that the shock initially heats the ions more strongly than it heats the electrons. One assumption used in determining the theoretical heat transfer is that $T_i = T/2$, $T = T_e + T_i$, where T is computed from the MHD shock equations. Experiments performed in the shock tube^{7,8} employing an ion energy analyzer and laser Thomson scattering have experimentally shown that the ion temperature is about ten times the electron temperature for the time period of interest. If one therefore assumes that $T_i = .9T$, then the theoretically predicted heat transfer q is larger by a factor of about 55%. Therefore, the experimentally observed energy transfer is only 1.8 times the theoretical value, rather than 2.8 times the value found using $T_e = T_i$.

Taking into account convection, thermoelectric effects and the fact that $T_i \neq T_e$, the actual heat transfer is about 1.5 times the theoretical value.

The dependence of q on temperature, density, and

magnetic field is given by equation (4.6), which is derived from the simple heat conduction model

$$q \propto T^{3/4} n^{3/2} B^{-1}$$

Using this expression, the error in q due to errors in the measurements of T , n , and B can be estimated. Taking the derivative of the above expression:

$$\frac{dq}{q} = \frac{3}{4} \frac{dT}{T} + \frac{3}{2} \frac{dn}{n} - \frac{dB}{B}$$

If the error in the measurement of T , n , and B are represented by ΔT , Δn , and ΔB , then the error in q , Δq , is:

$$\frac{\Delta q}{q} = \frac{3}{4} \frac{\Delta T}{T} + \frac{3}{2} \frac{\Delta n}{n} + \frac{\Delta B}{B}$$

A generous estimate of the errors gives:

$$\Delta T/T = 50\%$$

$$\Delta n/n = 10\%$$

$$\Delta B/B = 10\%$$

Therefore, the error in q is:

$$\Delta q/q = 62\%$$

The error in temperature, ΔT , alone, accounts for a 37% error in q . Therefore, the actual measured heat transfer is $1.5 \pm .9$ times the classical theoretical value.

4.2 WALL SURFACE DAMAGE

The scanning electron micrographs shown in chapter III indicate that the surfaces of all three wall samples show evidence of melting. The pictures of the copper surface indicate also that aluminum from the outer conductor has melted and redeposited on the copper wall sample. The study of the physical effects observed on wall surfaces exposed to a hot plasma and to high electric currents form a very complex subject, which is not well understood. A brief survey of the sputtering expected from a surface exposed to a hot plasma is presented, along with an estimate of the expected surface temperature of the wall samples exposed to the shock tube plasma. The calculated temperature rise and sputtering damage does not account for all the observed damage to the polished samples. A brief explanation of the observed unipolar arc tracks is also presented.

The energy flux from a hot plasma to a cold wall has been measured. For times longer than the first few ion gyroperiods, the simple heat conduction formulation appears valid, at least within a factor of about two. For times less than an ion gyroperiod, the continuum theory of plasma heat conduction is not valid. However, an estimate of the energy flux for this initial time period can be made. Assume that all the ions within one gyroradius of the wall deposit their energy into the wall during one gyroperiod. With an ion gyroradius of

.33 cm, a density of 10^{16} particles/cm³, and an ion temperature of 3×10^6 °K, the ions within one gyroradius of the wall deposit .2 joules of energy into a square centimeter in a time of 1.3×10^{-7} seconds, (i.e. the ion gyroperiod). There is an average energy flux during this early gyroperiod time of 1.5×10^6 watts/cm².

Melting, if it occurs, will happen at the earliest times, namely $t < \tau_g$, because at later times, a boundary layer of cooler plasma forms and helps to protect the wall. The surface temperature of various wall materials at these early times can be estimated using the average heat transfer rate of 1.5×10^6 watts/cm² for the time $0 < t < \tau_g$. The change in surface temperature is:

$$\Delta T_s = \frac{q \sqrt{2t}}{(\pi k \rho C)^{\frac{1}{2}}} \quad (4.9)$$

Equation 4.9 is derived in appendix III. k is the thermal conductivity of the wall material, ρ is the mass density, and C is the thermal capacity. Table 4.6 lists the values of the surface temperature at $t = \tau_g$, using the above assumptions. This table also gives the melting temperature for the various materials. The calculated wall surface temperatures indicate that no melting should take place, a direct contradiction of the experimental evidence shown by the scanning electron micrographs.

These photographs indicate that all three samples, copper,

WALL SAMPLE SURFACE TEMPERATURES

$$q \sim 1.5 \times 10^6 \text{ watts/cm}^2 \text{ for } 0 < t < \tau_g$$

Time	Copper	Stainless Steel	Boron Nitride	Aluminum
$t = \tau_g$	190 °C	456 °C	776 °C	300 °C
Melting Temperature	1080 °C	1500 °C	2800 °C	660 °C

FIGURE 4.6: Table of the early time surface temperature estimates for various wall materials, and the melting temperatures of the materials.

4.6

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stainless steel, and boron nitride, show some evidence of melting, although the copper surface shows only slight evidence of melting. The surface of the aluminum outer conductor also melted, as shown by the aluminum nodules present on the copper wall sample. The experimentally measured heat transfer does not account for the observed wall surface damage.

Another possible explanation for the wall damage is sputtering. Sputtering consists of the ejection of atoms from the wall due to bombarding ion flux. The ion flux can be estimated in the following manner. Assume that the energy transfer q is due solely to ions striking the wall where they give up all their energy to the wall. Thus,

$$q = \frac{3}{2} k T \Phi \quad (4.10)$$

where Φ is the number of ions striking one square centimeter per unit time. Using the previous estimate of q , particle fluxes are obtained for $t = \tau_g = 1.3 \times 10^{-7}$ sec. By the time $t = \tau_g$, about 3×10^{15} ions/cm² have struck the wall. After the first few ion gyroperiods, a cooler plasma boundary layer has formed. Assuming that all the sputtering damage occurs during the first gyroperiod, a total of 3×10^{15} , 300 eV ions strike each square centi-

meter of wall surface per shot.

Sputtering yields have been measured for certain combinations of wall materials and bombarding ions^{9,10,11}. For the cases of interest, 300 eV deuterium ions striking copper and stainless steel, sputtering yields are not available. The existing data indicate, however, that for these materials the sputtering yield should be approximately .03 atoms/ion. This means that for every 100 ions striking the wall material, 3 atoms of wall material will be ejected. Thus for one plasma shot, approximately 10^{14} atoms will be ejected from each square centimeter of wall surface.

The thickness of the layer sputtered from the wall surface can be obtained from the knowledge of the material's density and the number of atoms lost. The density of copper¹² is 8.9 gm/cm^3 and assuming 10^{14} atoms are sputtered per square centimeter of wall surface per shot, a layer 10^{-9} cm thick is lost per shot. For 90 plasma shots, the expected sputtering damage indicates the loss of a layer $10^{-3} \mu\text{m}$ thick. This is consistent with the slight amount of damage observed using the scanning electron microscope.

For stainless steel¹³, with a density of about 7.75 gm/cm^3 , assuming 10^{14} atoms/cm² sputtered per shot, a layer about 1.2×10^{-9} cm thick is lost per shot. The

wall sample was exposed to 625 plasma shots, which would indicate a total loss of $7.5 \times 10^{-2} \mu\text{m}$, which is substantially less than the observed surface damage. The observed damage of the stainless steel wall sample indicates that pits of material about $5 \mu\text{m}$ deep have been lost. The observed peaks and valleys cover the entire surface, and the amount of material lost is about equal to the amount of stainless steel in a uniform layer 2.5 microns thick. This is about 300 times the predicted damage due to sputtering.

These estimates of sputtering damage assume that the flux of ions was $3 \times 10^{15} \text{ ions/cm}^2$ for each shot. Many different experiments were run on the shock tube while the samples were in place. For the majority of shots, the ion flux per shot was substantially less than the above value, since the plasma produced was cooler and less dense. These estimates provide, therefore, the maximum theoretical surface damage that would appear to be from sputtering. Therefore sputtering does not appear to account satisfactorily for the extent of the observed wall surface damage.

A third possible explanation for the observed wall damage is the joule heating associated with the

large electric currents traveling through the walls of the shock tube. This current, of the order of 10^6 amperes, has a frequency of about 50 KHz, and the current propagates partly through the tube wall and partly through the plasma next to the wall. Detailed calculations of the shock tube wall electrical skin heating are shown in appendix IV.

For this frequency, the aluminum outer conductor has a skin depth of about .04 cm. During the first ion gyroperiod, when the initial heat transfer heats the surface to its maximum temperature, the current only heats the aluminum by about .02 $^{\circ}\text{K}$, assuming that all the current of 10^6 amperes travels through the wall. For a longer time period ($t \sim 5 \mu\text{sec}$), the metallic skin is further heated, and a temperature increase of the order of 1 $^{\circ}\text{K}$ is obtained, which is still much too low a value to cause melting.

Similar calculations have been carried out for the copper and the stainless steel wall samples inserted into the shock tube. The calculations were made assuming that the copper and stainless steel wall samples were in good electrical contact with the aluminum outer conductor. For a time of the order of five microseconds, the copper surface temperature rises by about 0.5 $^{\circ}\text{K}$, and the stainless steel surface experiences a temperature

rise of about $.02^{\circ}\text{K}$, due to joule heating.

The effects of this current on the wall temperature were also measured by insulating some heat transfer sensing elements from the shock tube wall, and allowing other gauges to be in good electrical contact with the wall. No significant differences in the surface temperatures of the gauges were observed. The wall electrical current does not appear to account for the observed surface damage.

Unipolar arc tracks were observed on the copper surface, and may be explained as follows¹⁴. The wall potential is:

$$V_W = \frac{kT_e}{2e} \ln \frac{m_i T_e}{m_e T_i} \quad (4.11)$$

If this potential is greater than 10 volts, and an arc spot is initiated, electrons can flow from the spot to the plasma. The potential of the conductor will then become less negative than the wall potential, and electrons will flow to the conductor from the plasma, balancing the emission of electrons from the arc spot. For the plasma produced in this experiment, $V_W \approx -76$ volts. This potential is large enough to maintain a unipolar arc. The copper wall

sample shows damage associated with a unipolar arc, as can be seen in figure 3.9.

The observed damage to wall samples exposed to the hot plasma and large electric currents cannot be explained by the experimentally measured heat flux, the expected sputtering damage associated with the estimated ion flux, or by the wall currents.

Another possible explanation for the observed surface damage has to do with the radial currents traveling through the shock front and the magnetic piston. These currents, traveling from the inner conductor to the outer conductor, bombard the outer conductor with a flux of deuterium ions. This flux is in addition to the flux of ions from the hot plasma to the wall. A realistic quantitative estimate of the ion flux due to this current is very difficult to make, and further experimental research is needed to measure this flux.

The sputtering yields used to estimate the expected surface damage are not obtained by experimental measurements. It is therefore possible that the sputtering yields used were too low, and that sputtering could account for a larger proportion of the observed

damage to the surfaces.*

The observed unipolar arc tracks also may help to explain the surface damage. These arcs evaporate material from the metal surfaces, and if they are present during a substantial number of shots, they may account for some of the observed surface damage. Further research on the effects of exposing surfaces to hot plasmas and electric currents should be performed to explain the observed surface damage.

*R. Behrisch, in Nuclear Fusion, 12, 695 (1972), states that for a surface temperature, T_s , greater than 0.7 times the melting temperature, T_m , the sputtering yield increases exponentially. For $T_s \leq 0.7 T_m$, the yield remains nearly constant. At $T_s = 0.8 T_m$, there is a 20-40% increase in the sputtering yield. For an ion temperature of 5×10^6 °K, the stainless steel surface temperature at $t = \tau_g$ is 950 °C, which is $0.63 T_m$. Since the surface temperature rise scales as $T_i^{3/2}$, a small increase in ion temperature will cause a larger increase in the surface temperature, and will yield an exponential increase in the sputtering yield for stainless steel.

V. CONCLUSION

An investigation was made of the energy transfer between a hot plasma containing a strong transverse magnetic field and a cold wall. The damage to the wall surface due to the sudden contact with the hot plasma and high electric currents was also investigated.

Theoretical predictions of the heat transport were made, based on the assumption that the mechanism for energy transport is classical plasma (ion) thermal conduction across a magnetic field. Many other transport processes are possible in a plasma. Different types of instabilities have been observed in actual plasmas, many of which could greatly increase energy transport. The shock tube plasma, which travels with a typical velocity for these experiments of the order of 20 cm/ μ sec before coming to rest against the reflecting wall, might be turbulent, which would greatly increase the energy transport to a cold wall. The very steep temperature gradients in the plasma in the vicinity of a wall may also create instabilities.

The measured energy transport was compared with a theoretical model employing classical plasma thermal conductivity. The actual energy flux was found to average $1.5 \pm .9$ times this classical energy transfer rate,

taking into account cross-field thermal conduction and convection due to cooling. The dependence of the measured heat flux on temperature and magnetic field agreed remarkably well with the theoretical model, and the dependence on density was quite close to what would be expected.

A comparison of this classical plasma energy conduction flux with the measured energy flux indicates that the measured thermal conductivity transverse to the magnetic field imbedded in the plasma may be about 2.2 times the classical thermal conductivity. The functional dependence of this measured thermal conductivity on temperature, density, and magnetic field was found to agree quite well with classical predictions. These results all indicate that the classical thermal conductivity model's predictions for the heat flux are in fairly good agreement with the actual measured heat flux.

Polished wall samples of copper, stainless steel, and boron nitride were placed in contact with the hot plasma. They were also exposed to very high electric currents. The wall samples were then examined for surface damage using a scanning electron microscope and an optical microscope. All surfaces showed evidence of melting, with the copper surface showing little

evidence of surface melting. The copper surface showed only slight melting, and had some micron size aluminum deposits on the surface. These deposits evidently come from the aluminum outer conductor of the shock tube. The stainless steel surface showed pits of the order of $5\text{ }\mu\text{m}$ deep. The boron nitride surface melted and resolidified into globules of boron nitride, with peaks and valleys of the order of $10\text{ }\mu\text{m}$ depth.

The copper surface, with a much higher thermal conductivity than the stainless steel or the boron nitride, sustained the least damage and showed very little evidence of melting. The material loss suffered by the copper surface was essentially consistent with the predicted sputtering damage. However, the observed damage to the stainless steel and boron nitride surfaces cannot be completely accounted for by simple estimates of sputtering damage or by an estimate of the initial temperature rise due to the hot plasma in contact with the wall during the first gyroperiod. The large electric currents through the wall also cannot satisfactorily account for the observed damage. Other possible explanations for the observed surface damage include effects from the flux of ions striking the surface as current sheets pass over the surface, and the

possibility of much greater sputtering yields (for stainless steel and boron nitride) for 300 eV deuterium ions.

Classical transport properties provided a reasonably accurate prediction for the energy transfer transverse to a strong magnetic field ($B \sim 10^4$ Gauss) from a hot ($T \sim 5 \times 10^6$ °K), dense ($n \sim 10^{16}$ cm⁻³), plasma produced in the Columbia University shock tube. The damage to a copper surface was approximately that expected by sputtering from the deuterium ions. However, the observed wall surface damage to stainless steel cannot be satisfactorily accounted for by sputtering, and further research must be performed to determine whether the observed damage is due to contact with the hot plasma, to the current sheets associated with the shock waves and magnetic pistons, or possibly other effects.

APPENDIX I. THE MHD SHOCK JUMP EQUATIONS

A computer code was developed to solve the following magnetohydrodynamic shock jump equations,¹

$$\rho_1 u_1 = \rho_2 u_2 \equiv m$$

$$mu_1 + p_1 + \frac{B_1^2}{2\mu} = mu_2 + p_2 + \frac{B_2^2}{2\mu}$$

$$m\left(\frac{u_1^2}{2} + \frac{\gamma}{\gamma-1} \frac{p_1}{\rho_1}\right) + \frac{u_1 B_1^2}{\mu} = m\left(\frac{u_2^2}{2} + \frac{\gamma}{\gamma-1} \frac{p_2}{\rho_2}\right) + \frac{u_2 B_2^2}{\mu}$$

$$\frac{B_2}{\rho_2} = \frac{B_1}{\rho_1}$$

These equations correspond to conservation of mass, momentum, and energy through the shock front, as well as the equation for frozen flux. The subscript 1 refers to the upstream plasma, and the subscript 2 refers to the downstream plasma. These equations are written in the shock frame of reference. u_1 is therefore the upstream velocity with respect to the shock front, and u_2 is the downstream velocity with respect to the shock front.

The MHD shock jump equations can be put into the following form. Let:

$$\frac{p_2}{p_1} = x_1, \quad \eta_1 = \frac{B_1^2}{2\mu p_1}, \quad \alpha = \frac{\gamma+1}{\gamma-1}, \quad M_1 = \frac{u_1}{a_1}$$

$$\text{Then: } \frac{p_2}{p_1} = \frac{\eta_1(x_1-1)^3 + (\alpha x_1 - 1)}{\alpha - x_1} \quad (\text{A } 1.1)$$

$$u_1 = a_1 \left[\frac{x_1}{\gamma} \left(\frac{\frac{p_2}{p_1} - 1 + \eta_1(x_1^2 - 1)}{x_1 - 1} \right) \right]^{\frac{1}{2}}$$

$$u_2 = \frac{u_1}{x_1}$$

$$B_2 = x_1 B_1$$

$$T_2 = \frac{T_1 p_2}{x_1 p_1}$$

$$\rho_2 = x_1 \rho_1$$

The reflected shock equations can be written in the same way, letting $\tilde{u}$ be the velocity in the reflected shock frame, replacing subscript 1 with subscript 2, and replacing subscript 2 with subscript 5.

Thus:

$$\frac{p_5}{p_2} = \frac{\eta_2 (x_2 - 1)^3 + \alpha (x_2 - 1)}{\alpha - x_2} \quad (\text{A } 1.2)$$

$$\tilde{u}_2 = a_2 \left[\frac{x_2}{\gamma} \left(\frac{\frac{p_5}{p_2} - 1 + \eta_2 (x_2^2 - 1)}{x_2 - 1} \right) \right]^{\frac{1}{2}}$$

$$\tilde{u}_5 = \frac{\tilde{u}_2}{x_2}$$

$$B_5 = x_2 B_2$$

$$T_5 = \frac{T_2 p_5}{x_2 p_2}$$

$$\rho_5 = x_2 \rho_2$$

$$\eta_2 = \frac{B_2^2}{2\mu p_2} = \frac{x_1^2 \eta_1}{(p_2/p_1)} \quad (\text{A } 1.3)$$

The two sets of equations can be related as follows.

Let U be the velocity in the lab frame. U_1 is the velocity of the pre-shock gas, and U_5 is the velocity of the twice shocked plasma. In the lab frame:

$$u_1 = 0$$

$$u_5 = 0$$

Using these boundary conditions, the speed of the plasma in region 2 (behind the incident shock and before the reflected shock), in the lab frame, is:

$$u_2 = u_2 - u_1$$

and
$$u_2 = \tilde{u}_2 - \tilde{u}_5$$

Therefore
$$u_2 - u_1 = \tilde{u}_2 - \tilde{u}_5$$

and
$$u_1^2 \left(\frac{1}{x_1} - 1 \right)^2 = \tilde{u}_2^2 \left(1 - \frac{1}{x_2} \right)^2 \quad (\text{A } 1.4)$$

Equation (A 1.4) can be rewritten using the expressions for u_1 and $\tilde{u}_2$ given above. Therefore,

$$\frac{a_1^2 x_1}{\gamma} \left(\frac{\frac{p_2}{p_1} - 1 + \eta_1 (x_1^2 - 1)}{x_1 - 1} \right) \left(\frac{1}{x_1} - 1 \right)^2 = \quad (\text{A } 1.5)$$

$$\frac{a_2^2 x_2}{\gamma} \left(\frac{\frac{p_5}{p_2} - 1 + \eta_2 (x_2^2 - 1)}{x_2 - 1} \right) \left(\frac{1}{x_2} - 1 \right)^2$$

Using
$$\left(\frac{a_2}{a_1}\right)^2 = \frac{p_2}{p_1} \frac{\rho_1}{\rho_2} = \frac{p_2}{p_1} \frac{1}{x_1}$$

equation (A 1.5) becomes:

$$(x_1 - 1) \left[\frac{p_2}{p_1} - 1 + \eta_1 (x_1^2 - 1) \right] = \frac{p_2}{p_1} \left(1 - \frac{1}{x_2} \right) \left[\frac{p_5}{p_2} - 1 + \eta_2 (x_2^2 - 1) \right] \quad (\text{A } 1.6)$$

Given x_1 , equation (A 1.1) is used to find p_2/p_1 .

Equations (A 1.2) and (A 1.6) are solved by iteration for x_2 and p_5/p_2 . Therefore all the shock variables are determined once x_1 and η_1 have been given.

APPENDIX II. THE TIME DEPENDENCE OF THE HEAT FLUX

The solution for the temperature of the carbon-sapphire interface is derived in appendix III. Two solutions are presented; one for an input heat flux independent of time ($q = Q \equiv \text{constant}$), and one for a heat flux that is inversely proportional to the square root of time ($q = Q/t^{1/2}$). The respective solutions are:

$$(q = Q) \quad T^* = \frac{2q\sqrt{t}}{\sqrt{\pi k \rho C}} \quad (\text{A } 2.1)$$

$$(q = Q/\sqrt{t}) \quad T^* = \frac{q\sqrt{\pi t}}{\sqrt{k \rho C}} \quad (\text{A } 2.2)$$

The temperature given by equation (A 2.2) is $\pi/2$ times the temperature given by equation (A 2.1). To determine the magnitude of the heat flux from the measured carbon-sapphire interface temperature, it is necessary to know the time dependence of the heat flux q .

Figure A 2.1 shows a plot of the heat flux q versus time t on log-log paper. The plot is for $0 < t < 2 \mu\text{sec}$, after which time the gauge senses the

FIGURE A 2.1: Log - log plot of the heat flux versus time for a representative shot. The slope of the line shows the functional dependence of the heat flux on time.

expansion of the plasma into the vacuum region. Let $q = Qt^a$, where Q and a are constants. Then

$$\log q = \log Q + a \log t$$

The slope of the graph in figure A 2.1 is equal to a . For the experimental data (see figure A 2.1 for a representative shot), the slope is found to be -0.42. This value is in very good agreement with the simple theoretical model for classical heat transfer, which predicts a slope of -0.5.

Therefore equation (A 2.2), which is derived assuming that $q = Q/t^{\frac{1}{2}}$, is a valid expression for the temperature of the carbon-sapphire interface.

APPENDIX III. HEAT TRANSFER IN A SEMI-INFINITE SOLID

The problem of heat transfer in a simple semi-infinite solid is treated in Carslaw and Jaeger¹. The solution to the problem of heat transfer in a composite solid, however, is not readily available in the literature. The composite solid problem is treated by M. Camac and R. Feinberg in Avco Research Report 120². The essence of this treatment is reproduced here. Because this treatment makes use of intermediate steps in the well known simple solid solution, the treatment of this problem is also included.

Consider the problem of a semi-infinite solid, extending from $x = 0$ to $x = \infty$, with a heat flux $q(t)$ applied at $x = 0$, and a zero initial temperature for $x \geq 0$. The heat conduction equation is:

$$\kappa \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \quad (\text{A } 3.1)$$

where κ is the thermal diffusivity

$$\kappa = \frac{K}{\rho C} \quad (\text{A } 3.2)$$

and K is the thermal conductivity, ρ is the mass density, and C is the specific heat of the material.

The boundary conditions are:

$$q(t) = -K \frac{\partial T}{\partial x} \quad \text{at } x = 0$$

$$T = 0 \quad \text{at } x = \infty$$
(A 3.3)

Equation (A 3.1) is simply solved using Laplace transforms. The Laplace transform of equation (A 3.1) is:

$$\kappa \frac{\partial^2 \bar{T}}{\partial x^2} = \bar{t} \bar{T}$$
(A 3.4)

The solution to this equation is:

$$\bar{T}(x) = A e^{-px} + B e^{px}$$
(A 3.5)

and $p = \bar{t}/\kappa$

The transformed boundary conditions (A 3.3) are:

$$L[q(t)] = -K \frac{\partial \bar{T}}{\partial x} \quad \text{at } x = 0$$
(A 3.6)

$$\bar{T} = 0 \quad \text{at } x = \infty$$

Therefore, at $x = \infty$ $B = 0$, and

$$\text{at } x = 0 \quad A = \frac{L[q(t)]}{Kp}$$

Equation (A 3.5) becomes:

$$\bar{T} = \frac{L[q(t)]}{Kp} e^{-px} \quad (\text{A } 3.7)$$

For a heat flux $q = Q \equiv \text{constant}$, the solution to this equation (A 3.7) is:

$$q = Q \quad T = \frac{2Q}{K} \left\{ \left(\frac{\kappa t}{\pi} \right)^{\frac{1}{2}} e^{-x^2/4\kappa t} - \frac{x}{2} \operatorname{erfc} \frac{x}{2(\kappa t)^{\frac{1}{2}}} \right\} \quad (\text{A } 3.8)$$

Let τ , the thermal transit time to a depth x , be:

$$\tau = \frac{x^2}{4\kappa} = \frac{x^2 \rho c}{4K} \quad (\text{A } 3.9)$$

For times much greater than τ ($t \gg \tau$) the solution is:

$$q = Q \quad T = \frac{2Q}{K} \left(\frac{\kappa t}{\pi} \right)^{\frac{1}{2}} \quad (\text{A } 3.10)$$

$t \gg \tau$

For a heat flux $q = Q/\sqrt{t}$ ($Q \equiv \text{constant}$), the solution to the heat conduction equation (A 3.7) is:

$$q = Q/\sqrt{t} \quad T = \frac{Q(\kappa\pi)^{\frac{1}{2}}}{K} \operatorname{erfc}\left(\frac{x}{2(\kappa t)^{\frac{1}{2}}}\right) \quad (\text{A } 3.11)$$

For times much greater than τ ($t \gg \tau$),

$$q = Q/\sqrt{t} \quad T = \frac{Q}{K}(\kappa\pi)^{\frac{1}{2}} \quad (\text{A } 3.12)$$

Equation (A 3.10) and (A 3.12) give the solution to the heat conduction equation for $q = Q$ and $q = Q/\sqrt{t}$ respectively, at $t \gg \tau$. These represent solutions for the surface temperature of a solid exposed to a heat flux $q = Q$ and $q = Q/\sqrt{t}$ at $x = 0$, since $\tau = \frac{x^2}{4\kappa} = 0$ at $x = 0$.

Now consider a composite semi-infinite solid, with material 1 extending from $x = -l$ to $x = 0$, and material 2 extending from $x = 0$ to $x = \infty$; with a heat flux $q(t)$ applied at $x = -l$.

The heat conduction equations are:

$$\frac{\partial^2 T_1}{\partial x^2} = \frac{1}{\kappa_1} \frac{\partial T_1}{\partial t} \quad -l \leq x \leq 0 \quad (\text{A } 3.13)$$

$$\frac{\partial^2 T_2}{\partial x^2} = \frac{1}{\kappa_2} \frac{\partial T_2}{\partial t} \quad x \geq 0 \quad (\text{A } 3.14)$$

If the materials are in good thermal contact with one another, as in the heat transfer gauge, the boundary conditions are:

$$\left. \begin{aligned} T_1 &= T_2 \\ K_1 \frac{\partial T_1}{\partial x} &= K_2 \frac{\partial T_2}{\partial x} \end{aligned} \right\} \quad x = 0 \quad (\text{A } 3.15)$$

$$q(t) = -K_1 \frac{\partial T_1}{\partial x} \quad x = -l \quad (\text{A } 3.16)$$

$$T_2 = 0 \quad x = \infty \quad (\text{A } 3.17)$$

Using Laplace transforms, equations (A 3.13) and (A 3.14) become:

$$\frac{\partial^2 \bar{T}_1}{\partial x^2} = \frac{1}{\kappa_1} \bar{t} \bar{T}_1 \quad (\text{A } 3.18)$$

$$\frac{\partial^2 \bar{T}_2}{\partial x^2} = \frac{1}{\kappa_2} \bar{\tau} \bar{T}_2 \quad (\text{A } 3.19)$$

The transformed boundary conditions are:

$$\left. \begin{aligned} \bar{T}_1 &= \bar{T}_2 \\ K_1 \frac{\partial \bar{T}_1}{\partial x} &= K_2 \frac{\partial \bar{T}_2}{\partial x} \end{aligned} \right\} \quad x = 0 \quad (\text{A } 3.20)$$

$$L[q(t)] = -K_1 \frac{\partial \bar{T}_1}{\partial x} \quad x = -l \quad (\text{A } 3.21)$$

$$\bar{T}_2 = 0 \quad x = \infty \quad (\text{A } 3.22)$$

A solution to equations (A 3.18) and (A 3.19) is:

$$\begin{aligned} \bar{T}_1 &= A \cosh p_1 x + B \sinh p_1 x \quad -l \leq x \leq 0 \\ \bar{T}_2 &= C e^{p_2 x} + D e^{-p_2 x} \quad x \geq 0 \end{aligned} \quad (\text{A } 3.23)$$

$$\text{where } p_1^2 = \bar{\tau}/\kappa_1, \quad p_2^2 = \bar{\tau}/\kappa_2$$

Using the transformed boundary conditions (A 3.20, A 3.21, A 3.22):

$$C = 0 \quad \text{at } x = \infty$$

$$A = D \quad \text{at } x = 0$$

$$B = - \frac{p_2 K_2}{p_1 K_1} D \quad \text{at } x = 0$$

$$\text{Let } \sigma = \frac{K_2 p_2}{K_1 p_1} = \left(\frac{K_2 p_2 C_2}{K_1 p_1 C_1} \right)^{\frac{1}{2}} \quad (\text{A } 3.24)$$

$$\text{Then } \frac{1}{K_1} L[q(t)] = p_1 D (\sinh p_1 \ell + \sigma \cosh p_1 \ell) \quad \text{at } x = -\ell$$

$$\text{Therefore } D = \frac{L[q(t)]}{K_1 p_1 (\sinh p_1 \ell + \sigma \cosh p_1 \ell)}$$

[n.b. in the original ref.(2), there is an error in their Eq. (A-15); which is corrected here.]

Equation (A 3.23) now becomes:

$$\bar{T}_1 = \frac{L[q(t)](\cosh p_1 x - \sigma \sinh p_1 x)}{K_1 p_1 (\sinh p_1 \ell + \sigma \cosh p_1 \ell)} \quad -\ell \leq x \leq 0 \quad (\text{A } 3.25)$$

$$\bar{T}_2 = \frac{L[q(t)] e^{-p_2 x}}{K_1 p_1 (\sinh p_1 \ell + \sigma \cosh p_1 \ell)} \quad x \geq 0 \quad (\text{A } 3.26)$$

Writing the denominator of equation (A 3.26) in terms of $e^{p_1 \ell}$, and expanding in a series:

$$\bar{T}_2 = \frac{2}{1+\sigma} \sum_{n=0}^{\infty} \left(\frac{1-\sigma}{1+\sigma} \right)^n \frac{L[q(t)]}{K_1 p_1} e^{-p_1 \left[(2n+1)\ell + \left(\frac{\kappa_1}{\kappa_2} \right)^{\frac{1}{2}} x \right]} \quad (\text{A } 3.27)$$

Each term in equation (A 3.27) is very similar to the solution for the simple solid problem, Eq.(A 3.7).

The solution to the composite problem can therefore be given in terms of the solution to the simple solid problem:

$$T_2 = \frac{2}{1+\sigma} \sum_{n=0}^{\infty} \left(\frac{1-\sigma}{1+\sigma} \right)^n T_S \left[(2n+1)\ell + \left(\frac{\kappa_1}{\kappa_2} \right)^{\frac{1}{2}} x \right] \quad (\text{A } 3.28)$$

T_S is the simple solid solution as a function of

$\left[(2n+1)\ell + \left(\frac{\kappa_1}{\kappa_2} \right)^{\frac{1}{2}} x \right]$ instead of x , with κ and K replaced by κ_1 and K_1 .

At the interface $x = 0$,

$$T \equiv T_1 = T_2 = \frac{2}{1+\sigma} \sum_{n=0}^{\infty} \left(\frac{1-\sigma}{1+\sigma} \right)^n T_S \left[(2n+1)\ell \right] \quad (\text{A } 3.29)$$

Therefore, using equations (A 3.8) and (A 3.11),

$$q = Q \quad T = \frac{4Q}{K_1(1+\sigma)} \sum_{n=0}^{\infty} \left(\frac{1-\sigma}{1+\sigma}\right)^n \left\{ \left(\frac{\kappa_1 t}{\pi}\right)^{\frac{1}{2}} e^{-(2n+1)^2 \ell^2 / 4\kappa_1 t} - \frac{(2n+1)\ell}{2} \operatorname{erfc} \frac{(2n+1)\ell}{2(\kappa_1 t)^{\frac{1}{2}}} \right\} \quad (\text{A } 3.30)$$

$$q = \frac{Q}{\sqrt{t}} \quad T = \frac{2Q(\kappa_1 \pi)^{\frac{1}{2}}}{K_1(1+\sigma)} \sum_{n=0}^{\infty} \left(\frac{1-\sigma}{1+\sigma}\right)^n \operatorname{erfc} \frac{(2n+1)\ell}{2(\kappa_1 t)^{\frac{1}{2}}} \quad (\text{A } 3.31)$$

In long times, as $t \rightarrow \infty$, the solutions to (A 3.30) and (A 3.31) are:

$$\begin{array}{l} q = Q \\ t \rightarrow \infty \end{array} \quad T \equiv T^* = \frac{2Q}{K_2} \left(\frac{\kappa_2 t}{\pi} \right)^{\frac{1}{2}} \quad (\text{A } 3.32)$$

$$\begin{array}{l} q = Q/\sqrt{t} \\ t \rightarrow \infty \end{array} \quad T \equiv T^* = \frac{Q}{K_2} (\kappa_2 \pi)^{\frac{1}{2}} \quad (\text{A } 3.33)$$

These long time asymptotic solutions, (A 3.32, A 3.33), are the same solutions as those for the surface of a simple solid of material 2 alone.

APPENDIX IV. HEATING OF THE WALL SURFACE BY ELECTRIC CURRENTS

A shock tube electric current is typically about 10^6 amperes with a ringing frequency of about 50 KHz. This current flows through the metallic skin of the shock tube wall and some of the current flows through the boundary plasma. The heating of the wall by this current may be estimated as follows. The skin depth of a conductor is given by¹:

$$\delta = \frac{c}{2\pi\omega\mu\sigma} \quad (\text{A } 4.1)$$

where c is the speed of light, ω is the angular frequency of the current, σ is the conductivity of the material, and μ is the permeability of the material.

The current flowing through the outer conducting wall of the shock tube has a current density j equal to

$$j = I/\pi D\delta \quad (\text{A } 4.2)$$

where D is the diameter of the outer conductor ($D = 23$ cm) and δ is the skin depth. The electric power dissipated in the wall surface is I^2R , where R , the resistance of the surface, is

$$R = \frac{L}{\sigma A} \quad (\text{A } 4.3)$$

L is the axial length of the outer conductor and A is the circumference of the outer conductor (πD), times the skin depth δ . The power per square centimeter of wall surface is

$$P = \frac{I^2 L}{\sigma A} \frac{1}{L \pi D} = \frac{j^2 \delta}{\sigma} \quad (\text{A } 4.4)$$

Assume, as a maximum value, that I is a constant 10^6 amperes for the time period of interest. The power delivered to the wall is therefore also constant. The temperature increase of a volume, V , of material with a constant energy input throughout the volume is:

$$\Delta T = \frac{E}{\rho C V}$$

where ρ is the mass density of the material, and C is the specific heat. For the case of interest, $V = L \pi D \delta$, and the total energy delivered to the volume is,

$$E = P t L \pi D$$

Therefore,

$$\Delta T = \frac{P t}{\rho C \delta}$$

This is the temperature rise of a square centimeter of wall material with a thickness δ for a constant power per square centimeter, P , delivered to the volume.

For the aluminum outer conductor, the skin depth for a 50 KHz current is 0.038 cm. Assuming a current of 10^6 amperes, the current density j is 3.7×10^5 amp/cm², and the power delivered to the wall is 1.4×10^4 watts/cm². This results in a surface temperature rise of 0.02 °K for a time of the order of one ion gyroperiod (1.3×10^{-7} sec), and a temperature rise of 0.8 °K for a time of 5 μ sec.

Similar values are obtained for the copper wall sample. The calculations assume that the polished sample is in very good electrical contact with the aluminum outer conductor. The current density is divided about equally between the copper and the aluminum, since their respective conductivities are very close ($\sigma_{Cu} \sim 5.9 \times 10^5$ ohm⁻¹ cm⁻¹, $\sigma_{Al} \sim 3.6 \times 10^5$ ohm⁻¹ cm⁻¹).² The copper sample should experience a 0.01 °K surface temperature rise due to wall current heating for a time of the order of one ion gyroperiod. For a 5 μ sec time the copper surface temperature should rise by about 0.5 °K.

The effects of the current heating on the stainless steel sample must take into account the difference in

the electrical conductivities between the aluminum outer conductor and the stainless steel wall sample ($\sigma_{SS} \sim 1.7 \times 10^4 \text{ ohm}^{-1} \text{ cm}^{-1}$).³ Using equation (A 4.3), one obtains:

$$\frac{R_{SS}}{R_{Al}} = \frac{(\sigma_{Al})^{\frac{1}{2}}}{(\sigma_{SS})^{\frac{1}{2}}}$$

for an equal area of each material. Assuming that the current near the wall sample travels about the same distance whether it propagates through the stainless steel wall sample or the adjacent aluminum, the division of the current can be treated as a simple circuit problem with two resistors in parallel. The current propagating through the stainless steel wall sample is therefore:

$$I_{SS} = \frac{I_{SS} + I_{Al}}{1 + R_{SS}/R_{Al}} = .217 I_{Al} \quad (\text{A } 4.6)$$

Using this value for the current in equation (A 4.4), the stainless steel surface temperature increases by about $5 \times 10^{-4} \text{ } ^\circ\text{K}$ in one ion gyroperiod. For a time of 5 μsec , the surface temperature rises by about .02 $^\circ\text{K}$.

These calculations indicate that the wall surface temperature rise caused by ohmic heating due to the large current flowing through the walls of the shock tube is not significant and should not be the cause of surface melting.

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Quarterly Progress Report
Period Ending Dec. 30, 1973

"Investigation of Pressure Drops and
Heat Transfer Coefficients for
Loss of Coolant Evaluation"

J. Weisman, Principal Investigator

MASTER

Report COO-2152-10

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Contract AT(11-1)2152
"Investigation of Pressure Drops and
Heat Transfer Coefficients for Loss
of Coolant Evaluation

Quarterly Report for Period
Oct. 1, 1973 to Dec. 30, 1973

The objective of this contract is the study of pressure drop and heat transfer phenomena of importance during a loss of coolant accident. The study is divided into three experiments a) Transient Pressure Drop Investigation; b) Two-Phase Pressure Drop and Flow Pattern Study; c) Transition Boiling and Reflooding Heat Transfer Tests.

I. Transient Pressure Drop Investigation

The transient pressure drop apparatus operates by discharging heated Freon 113 from a large storage accumulator to the test lines. The Freon flashes across a needle valve and proceeds to a 3-way valve. There a portion of the two-phase mixture is directed to a storage accumulator and the remainder is directed through the test section. By appropriately varying the position of the 3-way valve, a sinusoidally varying flow is obtained.

In the previous quarter, test data in the range of 0-30% void was reported for two abrupt area change test sections. These had area ratios of .09 and .35 respectively. These data were compared with predictions from one-dimensional momentum theory coupled with the assumption of homogeneous flow. The comparison indicated that the data across the contraction was reasonably well explained but that the data across the expansion-contraction combination was not well explained.

During this quarter, additional data were obtained across test section #2 (area ratio = .35). The range of the data was extended to void

fractions in excess of 60% for operation at ~ 1.25 cps. These data, and that previously obtained, are shown in Figs. 1 and 2. The same general trends as observed previously still appear. However, the disagreement between the observed and predicted phase shift (shift between pressure and flow curves) is considerably reduced at the higher void fractions.

In all runs made to date, the void fraction was estimated by using the average volumetric flow indicated by the turbine flowmeter and assuming homogeneous flow. The latter assumption is questionable and another void fraction indication is desirable. A capacitance type voidmeter, of the design used in the steady-state Freon loop, is therefore being constructed. It will be installed in the transient apparatus during the next quarter.

II. Two Phase Pressure Drop and Flow Pattern Study

Experimental: These experiments, which are being conducted in a recirculating loop using Freon 113, have measured two-phase pressure drop across abrupt area changes. In this study, void fraction measurements have been made using a capacitance type voidmeter. It has been assumed that the capacitance varied linearly with void fraction. This assumption was indicated to be correct by tests in which the voidmeter was immersed to various levels in a Freon pool. To determine whether this linear behavior held under more typical conditions, a second test was conducted following a procedure by GE. Here the voidmeter was placed in a stream of Freon-Freon vapor which was flowing very slowly. The void fraction was determined by computing density from the pressure drop reading across a section of vertical line. Since the frictional Δp is negligible, the measured Δp allows ready computation of the mixture density. The density so computed and that obtained from the voidmeter appear to be the same within

the error limits of the measurements.

The voidmeter used in the above tests is being installed in the boiling Freon loop. This second voidmeter will allow void fraction measurements to be taken both before and after the test section.

Analysis of Expansion Data

The literature data on 2 phase pressure drops across abrupt expansions have been carefully reviewed. The approach followed has been essentially the same as that taken in the previously reported analysis of the contraction data⁽¹⁾. From a force momentum balance⁽²⁾, one obtains

$$\Delta p = \frac{G_1^2}{\rho_f g_c} \left[\frac{\rho_f}{\rho_g} x^2 \left(\frac{1}{\alpha_1 \sigma} - \frac{1}{\alpha_2} \right) + (1 - x)^2 \left(\frac{1}{\sigma(1 - \alpha_1)} - \frac{1}{\alpha_2} \right) \right]$$

where G_1 = mass velocity in small diameter line, lb m/ft²hr

ρ_f = liquid density, lb m/ft³

ρ_g = vapor density, lb m/ft³

α_1 = void fraction in small diameter line

α_2 = void fraction in large diameter line

σ = area ratio

x = mixture quality

To evaluate α_1 and α_2 three models were considered.

Model E - No slip at any point (homogeneous flow)

Model F - Slip at section 1 and 2 correspond to that for straight pipe flow

Model G - No slip after expansion (due to possible mixing after area change) but slip corresponding to that in straight pipe flow prior to expansion.

Figs. 3, 4 and 5 compare the data of Ferrell and McGee⁽³⁾, Janssen and Kervinen⁽²⁾ and Mendler⁽⁴⁾ to these models. It is clear that model F is the best of the three and it is therefore recommended.

Reconsideration of Abrupt Contraction Data

In the previous quarterly, it was found that the homogeneous model provided the best data fit of all the contraction models which were examined. However, the homogeneous model makes little sense from a phenomenological viewpoint. Examination of high speed photographs clearly show that the flow is generally not homogeneous at reasonable distances upstream and downstream. Observation of the vena contracta region indicate mixing in this region. A model based on slip flow away from the expansion and mixing at the vena contracta would seem most plausible.

It was previously observed that the model based on complete mixing at the vena contracta performed well at low α but predicted too high a pressure drop at high α . This suggests a model in which the degree of mixing at the vena contracta depends on the void fraction. It was therefore assumed that the void fraction at the vena contracta, α_{vc} , was intermediate between that computed for homogeneous flow, α_h , and that for slip flow, α_s . That is

$$\alpha_{vc} = \alpha_s + (\alpha_h - \alpha_s) \cdot A$$

where A is a function of α_s (α_s being evaluated in small line). By appropriate choice of A, a very good correlation of the data is obtained. Fig.6 shows the correlation obtained assuming

$$A = 1.0 \quad \text{for } \alpha_s < 60\%$$

$$A = 1.0 - 1.25 (\alpha_s - .6) \quad \alpha_s > 60\%$$

The correlation coefficient for this model is slightly better than that obtained with the homogeneous model. More important, this model would appear to be more in accord with actual conditions.

Transition Boiling and Reflooding Heat Transfer Tests

Experimental - These experiments are being conducted in a heat transfer loop using hot mercury as the heat source. Boiling of water takes place in a vertical glass annulus around the mercury pipe.

It was reported last quarter that the flow indication system was not functioning properly. A new system has been adopted and the loop was successfully operated. A limited set of data has been obtained in the transition boiling regime. The data was at low ΔT (up to 45 °F) and at a flow of $G = 5.0 \times 10^4$ lbm/hr.ft². Attempts to go to a higher range of ΔT and G were foiled by the failure of an 'O' ring seal on a flange on the mercury pipe. This resulted in a substantial Hg leak and shut down of the loop. The flange has now been seal-welded.

Further delays were encountered because of breakage during installation of the thermocouples measuring mercury temperature. A new set of thermocouples has been obtained and installed.

Data Analysis

Additional transition boiling data has been collected from the literature. Fig. 7 compares the data of Bailey⁽⁵⁾ and Peterson et al⁽⁶⁾ with

the correlation reported in the last quarterly report of this series (COO-2152-9). The agreement appears to be reasonable. However, the data of Peterson et al⁽⁶⁾ show behavior which differs from the FLECHT data when examined on a plot of q'' vs. ΔT_{sat} .

In Fig. 8 some of the FLECHT data⁽⁷⁾ at low pressures are plotted in terms of q'' vs. ΔT_{sat} . It is seen that all of this data shows a peak q'' at a ΔT_{sat} of about 150 °F. The data of Ellion⁽⁸⁾ behave similarly. The data of Peterson et al⁽⁶⁾ show a peak q'' at substantially lower values of ΔT_{sat} . (See Fig.9) The value of ΔT_{sat} at which the peak q'' occurs seems to depend on flow rate here. The data obtained in the present investigation, which was obtained at a substantially lower G than that used by Peterson et al, would seem to be showing similar behavior. The reason for the difference in system behavior requires further examination.

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Fig. 1. Comparison of Predicted and Observed Phase Shift for Combined Expansion-Contraction.

Fig. 2 - Comparison of Predicted and Observed Phase Shift for Contraction

Fig. Comparison of ΔP_E Measured with ΔP_E Predicted by all Homogeneous Model (Model #E).

Fig 4 Comparison of ΔP_E measured with respect to ΔP_E predicted by all slip model (Model F.)

Fig. 5 Comparison of Measured Pressure rise in Expansion with Predicted Pressure rise by Model G. (slip in small pipe but Hom. in big pipe)

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- △ Present data

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Measured $\Delta P_c \rightarrow$
(PSF)

Fig 6 Comparison of Pressure Props Across Abrupt Contraction With Final Model

FIG. 7 COMPARISON OF SOME NEW DATA
WITH PREDICTION.

FIG. 7 LOW PRESSURE DATA FROM FLECHT AND ELLION AS q'' Vs. ΔT_{SAT} .

FIG. 9. COMPARISON OF PETERSON'S DATA WITH U.C. DATA AS q'' VS. T_{WALL} .