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ORNL-4899

UC-38 - Engineering and Equipment

**HEALTH PHYSICS DIVISION
Civil Defense Research Section**

SUMMARY

**EFFECTS OF ELECTROMAGNETIC PULSE (EMP) ON THE
SUPERVISORY CONTROL EQUIPMENT OF A POWER SYSTEM**

Final Report

by

James K. Baird and Nicholas J. Frigo

for

**Defense Civil Preparedness Agency
Washington, D.C. 20301**

**Interagency Agreement No. AEC 40-31-64
and OCD-PS-64-284, Work Unit 2213D**

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James K. Baird and Nicholas J. Frigo

SUMMARY

A previous study of the power system has shown that large currents and high voltages are likely to be induced in power lines by an incident electromagnetic pulse (EMP) wave.* That study prompted this investigation of the supervisory control equipment associated with a power system.

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- (2) Direct induction of currents in exposed outdoor relay circuits.
- (3) Direct induction of currents in relay circuits contained in control houses.

Damage to relays connected to the above circuits is assessed. It is found that if the relays are entirely electromechanical that damage which would prevent their further functioning is extremely unlikely, nor is it very likely that the current induced in the circuits feeding the relays will cause them to operate falsely because of their great mechanical inertia. On the other hand, if the relays are constructed from semiconductor components, damage and false operation are very likely.

The following countermeasures are recommended:

- (1) Avoid routing relay control wiring along paths which form loops.
- (2) With semiconductor relay circuits, include circuit elements of high intrinsic impedance to limit the current drawn by the semiconductor devices.
- (3) In control houses, eliminate openings larger than 1 meter on a side.
- (4) Use surge arresters with high current capacity, preferably vacuum tube type.
- (5) When employing redundant relaying, run the wiring of the relays in different directions.

* James H. Marable, James K. Baird, and David B. Nelson, Effects of Electromagnetic Pulse (EMP) on a Power System, ORNL-4836, December 1972.

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1. INTRODUCTION TO EMP AND SUPERVISORY CONTROL

1.1 Objective

This study concerns itself with the effects of the electromagnetic pulse (EMP) produced by a nuclear explosion on the supervisory control circuitry and equipment associated with electric power distribution systems. The term "supervisory control" is a combination of two others, "supervision" and "control". If a substation of a power distribution system is instrumented for supervision, there is equipment monitoring the substation to constantly inform the system dispatcher of the operating state of the substation. If the substation is also instrumented for control, then the dispatcher is in addition able to make changes in the operating state. Supervisory control instrumentation includes equipment for measuring and adjusting the current, voltage, and power and equipment for protecting the substation against malfunction. The role of this latter protective gear is twofold: (1) the prevention of electrical failure, and (2) the disconnection of faulty equipment with a minimum disturbance to the system with which it is associated.

A previous study in this series has dealt with the effects of the EMP on the power circuits and apparatus of a power distribution system.¹

1.2 Origin of EMP

The explosion of a nuclear weapon produces gamma rays which upon colliding with molecules in the earth's atmosphere produce free electrons. The free electrons are accelerated by the earth's magnetic field and radiate their kinetic energy in the form of a pulse of electromagnetic radiation.

The existence of EMP was apparent even in the first nuclear explosion near White Sands, New Mexico in 1945. However, it wasn't until the advent of missile warfare that its importance as a weapon was appreciated. Nuclear explosions near the earth's surface produce EMP, but they also produce overpressures whose destructive mechanical effects

on structures extend out a distance great enough to obscure any damage caused by EMP. However, at altitudes of 60 miles or so where the encounter between an incoming attacking missile and a defending anti-ballistic missile is likely to occur, the air is rare enough that a nuclear explosion causes no significant overpressure, so that one of the major effects experienced on the ground is the EMP.

The EMP from high altitude explosion is in the form of a plane wave which can induce currents in conductors in its path. The induction process is similar to that produced in a radio receiving antenna by the signal from a transmitting antenna except that the signal is not continuous as in the case of radio. Equations (1.1) give the time dependence of the electric, $E(t)$, and magnetic fields, $H(t)$, associated with a typical EMP from a high altitude nuclear explosion.¹

$$E(t) = E_0(e^{-\alpha t} - e^{-\beta t})$$

$$\bar{H}(t) = E(t)/\eta$$

$$E_0 = 5 \times 10^4 \text{ volts/m} \quad (1.1)$$

$$\alpha = 1.5 \times 10^6 \text{ sec}^{-1}$$

$$\beta = 2.6 \times 10^8 \text{ sec}^{-1}$$

$$\eta = 377 \text{ ohms}$$

The EMP may also be compared and contrasted to lightning. EMP induces currents (i.e., separates charges) in conductors while lightning injects charges directly onto the conductor, a current resulting when this charge density tries to equalize itself. Further, lightning is a localized phenomenon associated with a storm center while the EMP effects may be seen anywhere on the earth's surface within sight of the nuclear burst. If the nuclear explosion occurs at high altitude this area may include much of the continental United States. Although lightning and EMP are different in their mechanisms and extent of action, the effects of the currents both produce in conductors may in some instances be similar.¹

1.3 Brief History of Electric Power Protective Equipment²

The earliest automatic protective device was the fuse, which is still widely used, however, in much improved form. The fuse suffers from the disadvantage of requiring replacement before service can be restored. After fuses, simple instruments for fast tripping on fault currents and time-delayed tripping for overload currents were introduced. Because the voltages and currents in power systems were low, the primary conductors, themselves, could be used for the electrical part of electromechanical circuit breakers. An example is shown in Fig. 1.1. In this instrument a surge or overload fault current in the power line magnetizes the iron core on which it is wrapped, releasing the catch. The spring then supplies the energy to open the switch lever. The antenna-like horns attached to the blocks against which the lever seats, help to dissipate the arc which forms as the lever is unseated. As the arc forms, it heats the surrounding air and arc and air rise. The arc, as it rises, must maintain itself across the ever-increasing gap between the horns until a spacing is reached where the arc extinguishes.

As the voltage and current in distribution systems were increased, it became impossible to have protective equipment operating directly from the power lines for reasons of safety and instrument design. Instead, there were introduced current transformers and instrument potential transformers to reduce the sensed current or voltage to a level that was practical and could be safely metered. A separate switching device, the relay, was attached to the sensing transformer secondary contacts. The contacts of the relay then controlled a mechanism such as a spring, compressed gas, or batteries and a dc motor, which could be activated to open the circuit breaker.

This was the state of power system protection prior to 1920. In the early 1920's there was introduced the inverse time overcurrent relay whose speed of action increased with the magnitude of the overcurrent. Later in the decade the high speed differential relay was introduced which permitted the comparison of power currents flowing

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Fig. 1.1. Early Circuit Breaker.

into and out of a piece of apparatus and operated to disconnect the apparatus if the algebraic sum of the currents was not zero. This prevented the apparatus from being damaged due to an internal short circuit to ground. During this same decade, there appeared the distance or impedance-sensing relay which had the capability of comparing the impedance of a line with its normal value. Since the 60 Hz impedance of a line is proportional to its length, if the impedance of a line decreased below its normal value, the line must be shorted somewhere along its length, and the relay was designed to open. The relay also introduced the capability of discriminating against faults so far away that the impedance of the line was not measurably different from its normal value. Such faults should be cleared by relays in closer proximity to the fault. This coordination allowed relays to have "zones of protection." The zones could be arranged such that an entire system could be protected without the hazard of being shut down by the action of any one relay.

In the following years, power systems became connected together in grids, which meant that because the grid was being supplied by more than one generator, it was possible for current to flow in either direction through a particular branch of the grid. This led to the introduction of direction-sensing relays which would operate should the current change from its normal direction. Also the need for intercomparison of currents in different branches led utility companies to lease cables from telephone companies through which they could transmit a number of channels of relay operation signals.

Prior to 1960, nearly all relays relied upon coils and mechanical switches. The last decade, however, has witnessed the introduction of the "static relay" in which semiconductor diodes and transistors perform both the sensing and the switching.³

2. DESCRIPTION OF SUPERVISORY CONTROL EQUIPMENT AND IDENTIFICATION OF EMP POINTS OF ENTRY

2.1 Introduction

In this chapter, the operation of some typical examples of supervisory control equipment will be described. Included in the discussion will be both the older electromechanical devices and the more recent devices constructed largely from semiconductor diodes and transistors. Taking as an example the Dixie Substation of the Knoxville Utilities Board (Knoxville, Tennessee), we shall show how this equipment is connected to form a supervisory control system monitoring power distribution to a metropolitan area of 250,000 people. Finally, from these descriptive efforts, we shall be able to identify some of the connections made by these devices to the external electromagnetic environment through which EMP might enter.

2.2 Description of Equipment

An induction disk overcurrent relay² is shown schematically in Fig. 2.1 and as the manufactured item in Fig. 2.2. This kind of relay exists in several different varieties and is commonly found in most power systems. The type shown in Fig. 2.1 consists of two electromagnetic cores with windings, a permanent magnet, and a conducting disk. Current passes through the winding (1) on the center leg of the upper E-shaped core creating an eddy current in the disk which can rotate about the axis shown. A second winding (2) on the center leg of the E-shaped core magnetizes the U-shaped core below the conducting disk. This field exerts a torque on the eddy current in the disk causing the disk to turn against a restraining torque provided by a spring. When the disk has turned through a preset angle, it makes a contact completing the circuit which the relay controls. The permanent magnet (3) acts as an eddy current brake to control the speed of the disk. The range of values of overcurrent required to operate the relay can be adjusted, as shown in the figure by shorting out various numbers of

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Fig. 2.1. Schematic of Induction Disk Relay.

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Fig. 2.2. Induction Disk Relay.

turns using the plug bridge. The range of times to close the contacts can be adjusted by varying the angle through which the disk must turn. Once these adjustments have been set, the time to close the contacts varies inversely with the magnitude of the overcurrent. For this reason, the relay is often referred to as an inverse overcurrent relay.

The schematic for a balance beam differential relay² is shown in Fig. 2.3. The beam is given a slight mechanical bias to the contact open position by the spring. If the two coils have equal numbers of turns, the relay closes when $I_a^2 \geq I_b^2 + K$, where I_a and I_b are the currents in coils "a" and "b", respectively, and K is proportional to the restoring torque of the spring.

Protecting transformers, there are often relays which sense the condition of the oil in the transformer housing. One such design² sensitive to the evolution of gas and increased oil pressure is shown in Fig. 2.4. Under ordinary operating conditions the bouyant force of the oil is sufficient to raise the pivoted floats, as shown in the detailed drawing on the right in the figure, to positions where the mercury switches are open. If a fault occurs inside the transformer tank, there will be an evolution of gas from the oil due to the Joule heating by the fault current. The relay will fill with gas and the upper float will turn so that the mercury switch closes. This contact is usually connected to an alarm circuit. The rapid generation of gas causes displaced oil to surge through the relay. This surge of oil strikes the baffle connected to the lower pivoted float closing the other mercury switch. This switch is usually connected to circuitry which isolates the transformer electrically.

An example of a "static relay", basically the same as that described by McConnell and Brandt,⁴ employing semiconducting devices is shown in Fig. 2.5.

We first focus our attention on transistor Q_3 in Fig. 2.5. The emitter, base, and collector are the parts of this transistor connected to the power supply, resistor R_2 , and resistor R_3 , respectively. In addition, the emitter is distinguished by the standard arrow head marking. When the base of Q_3 is negative with respect to the power supply, a current will pass from emitter to collector. This is a property of

Fig. 2.3. Schematic of Balance Beam Relay.

Fig. 2.4. Gas- and Oil-Sensitive Overpressure Relay.

Fig. 2.5. Schematic of Static Relay.

the PNP junction transistor. If the arrangement of semiconducting materials were NPN, then the base would have to be positive with respect to the emitter for current to flow from emitter to collector.

When current passes from emitter to collector through Q_3 due to the voltage at the base of Q_3 , there is a voltage drop across R_3 which may be used to control the tripping of a circuit breaker for example. The voltage at the base of Q_3 is negative with respect to the emitter if current is drawn from the power supply through R_1 . This current is controlled by the remaining semiconducting devices in the circuit. Specifically, both Q_1 and Q_2 should be conducting from collector to emitter for there to be a current through R_1 . For this to happen the bases of both of these transistors must be positive with respect to the reference connection, since both transistors are of the type NPN.

The base of Q_1 will be positive if there are signals passing through at least one of the diodes at inputs 1 and 2, respectively. In terms of logic circuitry, diodes D_1 and D_2 are connected as an OR gate. For transistor Q_2 to be conducting, there must be a positive signal at input 3. Thus, for the relay to operate there must be a signal of the proper sign at input 3 and at either or both of inputs 1 and 2. Input 3 may be said to be connected AND with inputs 1 and 2. We thus see that this relay can make decisions whether to operate a circuit breaker or not, depending upon the combinations of signals which it receives.

Figure 2.6 shows a current transformer typical of those which could be used with any of the relays described above with the exception of the pressure-sensing relay in which the overload current is sensed mechanically by the oil rather than electrically by a current transformer. On the left in Fig. 2.6 is shown the complete current transformer. The wire to be monitored for overcurrent passes through the "hole in the donut" (or window) and the relay is connected between two contacts inside the protective cover on the top, left of the transformer. Shown on the right of the figure is the iron core and windings of the transformer with the cover and the external plastic insulation removed. When a wire carrying a current is run through the window, the magnetic flux which surrounds it links the windings wrapped about the window inducing an

Fig. 2.6. Current Transformer.

electromotive force which may be used to drive a relay connected between the contacts shown at the top left. In analogy with the terminology used with voltage transformers, the winding is called the secondary and the wire passing through the window is called the primary. The sensitivity of the current transformer to an overcurrent can be enhanced by winding the primary through the window several times to increase the flux linkage.

As explained in Section 1.3, a relay is a switch which can be activated by some external signal, for example, an overcurrent or an overpressure, to connect a source of power to drive a high voltage circuit breaker disconnecting some part of a power substation from one or more power lines. Shown in Fig. 2.7 is a bank of storage batteries which provide the power necessary to close a circuit breaker. Figures 2.8 and 2.9 illustrate circuit breakers used to interrupt three-phase 66 kV lines and three-phase 13.8 kV lines, respectively. The switching mechanisms of the 66 kV circuit breaker are separated one phase to a tank. Figure 2.8b shows the interior of the cabinet attached to the side of the circuit breaker tank marked "A" in Fig. 2.8a. This breaker is opened by the release of compressed springs and closed by compressed air. Shown at right center in Fig. 2.8b is a battery driven D.C. motor used to compress the gas after the breaker reclosure.

2.3 Supervisory Control at Dixie Substation

The Electrical Engineering Department of the Knoxville Utilities Board (KUB), Knoxville, Tennessee, has kindly made available a one-line drawing of their Dixie Substation for this study. A part of this diagram, shown in Fig. 2.10, represents by single lines the three phases of the power and supervisory control circuitry. The Dixie Substation was selected for description as being a typical substation in a metropolitan area. This substation is fed at 66 kV from the KUB major subtransmission station at Lonsdale where KUB connects to the Tennessee Valley Authority. Dixie contains three power transformers which step the voltage down to 13.8

Fig. 2.7. Storage Batteries Connected in Series. Used to energize circuit breaker.

Fig. 2.8a. 66 kV Circuit Breaker.

Fig. 2.8b. Interior of Control Cabinet for 66 kV Circuit Breaker.

Fig. 2.9. 13.8 kV Circuit Breaker.

kV for distribution to customers. Figure 2.10 shows in detail the power circuitry and supervisory control connections serving only the third transformer (called T3). We shall focus our attention on this third of the substation circuitry since the other two transformers are protected similarly. The wiring in the diagram is coded as follows: heavy solid lines indicate the high voltage (13.8 kV or 66 kV) lines, thin solid lines represent the connection of the secondaries of current transformers and the sensing contacts of relays, and dotted lines represent the lines of action of the relays. A summary of these symbols and others is included in the figure.

On the part of the one-line diagram shown in Fig. 2.10, power can enter the Dixie Substation in two ways: (1) through circuit breaker DX84 or (2) through high voltage disconnect switch DX831. The preferred connection is through the circuit breaker since, as we shall see presently, it is protected by the substation control circuitry while the disconnect switch is not. In fact, power is allowed to enter the substation through DX831 only when DX84 is for some reason out of service. Hence, DX84 is normally closed and DX831 is normally open. Circuit breaker DX85 connects the 66 kV line from the Lonsdale master substation with the distribution substation at West Hills. The pairs of disconnect switches DX84A, DX84B, and DX85A, DX85B permit circuit breakers DX84 and DX85, respectively, to be disconnected from the 66 kV line should servicing be necessary.

Enclosing circuit breakers DX84 and DX85 are current transformers (CT's) with winding ratios 1600/5. (The turns ratio 1600/5 means that when there is a 1600 A current on the power line, there will appear a 5 A current at the secondary terminals of the CT.) These CT's are not otherwise labeled but are the ones with secondaries wired together through switches TB4 and TB5 respectively. Connected with these two CT's is a third CT with turns in the ratio 400/5 wound around the 66 kV terminal of T3 and linked with these two by the auxiliary CT with turns ratio 5/20. These three CT's connected together through the auxiliary CT form a protective "zone of action" with some relays which we shall soon discuss. The auxiliary CT is necessary to balance the turns ratios

of the CT's associated with DX84 and DX85 with that on the 66 kV terminal of T3. Observing the turns ratios of these transformers, we see that $1600/20 = 400/5$ so that the turns ratio of the auxiliary CT is correct at 5/20. The zone protected by these CT's and their associated relays is the 66 kV bus which runs down the center of the substation through disconnect switches DX814, DX824, and DX831. This bus feeds the primaries (66 kV winding) of power transformer T3.

The CT's defining the bus zone of protection are connected so that under ordinary operating conditions the current through the secondary of each is zero. This is the condition under which the difference between the current entering DX84 from Lonsdale and the current leaving DX85 for West Hills is exactly equal to the current entering transformer T3. If the bus were faulted (either open circuited or short circuited) somewhere inside the protective region of the CT's, then these currents would not be properly balanced.

If the secondary CT currents are unbalanced, this will actuate the contacts of overcurrent relays T351 and/or NB51 and NB51N. Since we are using a single line diagram, we should point out here that, in fact, relays T351 and NB51 each represent a bank of three relays, one associated with each of the three phases of the 66 kV circuit. The relay NB51N is connected to the neutral ground wire of the KUB four-wire system. Figure 2.11 shows the bank of three NB51 relays monitoring phases A, B, and C and relay NB51N (labeled G in the photograph) monitoring the neutral ground wire as the relays appear on the electrical panel of the control house.

The dotted lines in Fig. 2.10 leaving these relays represent their lines of action. Following the directions specified by the arrowheads, we see that T351 connects with tripping relay T386A which has three sets of contacts and three lines of action. First T386A trips DX84 and DX85, disconnecting Dixie from Lonsdale and West Hills. It also operates tripping relays 3194 and 3168 which trip circuit breaker DX31 and block its automatic reclosure, thus disconnecting Dixie from its customers. Thirdly, relay T386A signals relay T394 which will trip DX85 and block its automatic reclosure. T394 can also actuate relays 3194 and 3168

NB51
NORTH SECTION
60 AMP BUS DIFF

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Fig. 2.11. Relays NB51 and NB51N in Dixie Substation.

should for some reason they not be actuated by T386A. The arrows indicate that relays NB51 and NB51N may actuate T394 and also relays 8494, 8468 and 8594, 8568 which trip and block the reclosure of circuit breakers DX84 and DX85, respectively.

Another zone of protection which overlaps the one described above is bounded by the 400/5 CT on the primary of power transformer T3 and the 1200/5 CT on the outgoing side (13.8 kV line) of circuit breaker DX31. These two CT's are connected in a loop with the differential current relay T387. The auxiliary CT with turns in the ratio of 5/8 is used to match the turns ratios of the CT's just mentioned. If a fault occurs within the zone defined by these two CT's, the unbalanced current will be detected by T387. The line of action of T387 proceeds to tripping relay T386A whose lines of action have already been explained in the paragraph directly above.

Finally, associated with T3 are the relay T351N connected in series with a CT wound about the neutral of the Y-connected secondary (13.8 kV winding) of T3, and the relay marked SPR (sudden pressure relay) monitoring the pressure of the insulating oil inside the tank of T3. If there is a fault somewhere among the turns of T3's windings, there will likely be an increase in current to ground through the neutral and an increase in the pressure inside the transformer tanks due to heating of the oil. The lines of action of T351N and SPR join so that either may actuate tripping relay T386B. The action contacts of T386B are wired in parallel with those of T386A. This is made apparent by following the arrowheads away from T386B. Thus T386B will also trip DX84 and DX85, actuate 3194 and 3168, and trip T394.

The 13.8 kV line leaving the Dixie Substation is protected by the Y-connected CT's, turns ratio 800/5, located between the disconnect switch DX310 and the circuit breaker DX31. The two relays connected nearest this set of CT's are, respectively, an inverse overcurrent relay and an instantaneous overcurrent relay. These relays are coordinated with fuses on the loads on this 13.8 kV line, so that an overcurrent in any one load will trip the fuse nearest the load and not circuit breaker DX31 which would disconnect T3 from all its customers. The numbers next

to the two relays indicate the range of settings in amperes to which the relays may be made to be sensitive. The inverse overcurrent relay closes in shorter times for larger currents, the closure time being inversely proportional to the current. Thus, if the overcurrent is small, this relay waits to see if the overcurrent is reduced by the action of a customer's circuit breaker or fuse. The instantaneous relay, however, has no time delay and will close "instantaneously" (that is, no intentional delay is built in) if the overcurrent is exceptionally high. This is why upper current setting on the instantaneous relay is higher than the upper setting on the inverse relay.

Beyond the inverse and instantaneous overcurrent relays in the one-line diagram is a three-phase, instantaneous overcurrent relay. This relay operates only if there are overcurrents in two of the three phases simultaneously. A two-phase fault or phase-to-phase fault is considered to be more serious than a single-phase fault and requires faster action. Thus, the overcurrent settings for this relay are low compared with the single-phase instantaneous overcurrent relay discussed in the paragraph above.

Beyond the three-phase relay in the diagram is an ammeter and switch. This meter may be switched onto the "b" phase of the system and read from the central dispatching station at Lonsdale. The meter above it marked CCA is a thermal ammeter with a slower response time to record the demand on transformer T3 as a function of the time of day for a thirty-day period. This ammeter is read out on a circular strip chart recorder at Dixie.

In this CT circuit there is also a "b" phase transducer (B Φ XDCR) which changes the alternating current read by the ammeter into direct current for frequency coding and transmission to Lonsdale for read out. The transducer consists of an isolation transformer followed by a full wave rectifier constructed from solid state diodes. The output from this transducer, which is a direct current, is then fed to an amplitude-to-frequency converter which sends a frequency modulated 900 Hz signal over a telephone cable connected to the Lonsdale dispatching station.

2.4 EMP Points of Entry

A study of the descriptive material of this chapter and the physical layout of the Dixie substation reveals the following mechanisms of entry of the incident EMP space wave into a supervisory control system:

1. By the induction traveling currents in the power lines which in turn induce electromotive forces in the secondary circuits of the current transformers monitoring the lines.
2. Direct induction of currents in exposed outdoor relay wiring.
3. Direct induction of currents in relay wiring contained in control houses.
4. Induction of traveling voltages on power lines which spark over the insulation of the power transformers establishing a fault. At the point of the fault, the ordinary power voltage creates an arc which heats the transformer insulating oil actuating the sudden pressure relay.
5. Direct induction of currents in the telemetry telephone cable connecting the substation to the master substation.

In the next three chapters, the first three of these mechanisms shall be analyzed. The fourth requires data on the dielectric strength of transformer winding insulation at the voltage rates of rise associated with EMP-induced traveling waves on transmission lines ($> 1000 \text{ kV } \mu\text{s}^{-1}$) to determine with certainty whether a fault will occur. To the authors' knowledge, data at such high rates of rise do not presently exist. The fifth mechanism, albeit an important one, will not be examined here due to the limited scope of this study.

In the last chapter of this report, we shall examine the ability of the currents induced by the three mechanisms analyzed in the preceding chapters to cause damage to the sensitive components of relays or to stimulate their unintended operation.

3. EMP COUPLING THROUGH CURRENT TRANSFORMERS

3.1 On the Idealization of the Problem

Current transformers (CT's) are connected into electric power circuitry in a number of ways. On low voltage systems (≤ 5 kV), the insulated wire carrying the power current passes through a metal weather-proof box in which the transformer is mounted. On high voltage systems, the current transformer protecting a large power transformer often encircles the current-carrying conductor which connects the transformer winding to the power line. The current-carrying conductor has a porcelain jacket surrounding it which insulates the high voltage from the transformer tank which is at ground potential. The entire assembly is called a "bushing." In still another design, a high current capacity bar is placed through the window of the CT, contacts made to this bar and to the secondary winding, and the entire assembly potted in an insulating material with only the contacts remaining exposed. This type of CT might be mounted on a pole.

Nearly all of these designs are sufficiently complicated to require full scale experiments to determine the magnitude of EMP coupled currents and voltages. In cases where nonlinear effects such as the flashover of insulation can be expected from EMP induced transients, the experiments must be performed using EMP simulators (antennas, waveguides, or pulse injection power supplies) producing full threat level electromagnetic fields or currents. Hence, we shall idealize the problem to some extent in order to permit a laboratory scale experimentation.

In this idealization we assume that the high voltage breakdown in the transformer insulation is unimportant. This may not be an unreasonable assumption. Although power system insulation is designed to withstand lightning-produced surge voltages which rise to peak value at a lower rate than those induced by the EMP space wave, the dielectric strength of the insulation does increase with increasing rate of rise of voltage and the transformer insulation may survive without dielectric failure.¹

Further, we scale the diameter of the power line wire and its height above ground down to a laboratory scale, keeping the characteristic impedance of the line-ground combination constant. No attempt is made to scale the CT, itself, to this same spacing.

3.2 Experiment to Determine the Coupling to an Idealized CT Model

Figure 3.1 shows the connection of the current transformer according to the idealization described above. The transformer is an Outdoor/Indoor Instrument Current Transformer, 300/5, FW 10 kV 60 Hz ac for operation at 0.6 kV or less, Catalog No. WE0 601-2, manufactured by Associated Engineering Company of Matthews, N.C.

A wire 50 mils in diameter excluding its thin rubber insulation was strung from wooden supports so that it passed through the center circular window of the current transformer. The wire and the aluminum ground plane over which it was strung at a height of 4 inches formed a transmission line with characteristic impedance of approximately 300 ohms. At either end, the wire was terminated in a circuit (T-pad) containing resistors (see Fig. 3.2) to match it to 50 ohms. Through one of these T-pad's, the wire was driven by a 50 ohm rectangular pulse generator, and through the other it was connected to a 50 ohm coaxial terminator. The pulse generator launched a traveling wave of rectangular shape on the transmission line, which because of its matched termination appeared to be infinite in extent.

Measurements were taken of the open circuit voltage and short circuit current at the secondary terminals of the current transformer using a Tektronix 454 oscilloscope fitted with a Polaroid camera when the transmission line was driven repetitively by a square wave which was "on" for 1 ms and "off" for 1 ms. Figures 3.3 shows that the square wave had a rise time (defined as time to go from 10% to 90% full amplitude) of approximately 10 ns. The measurements of the open circuit voltage and short circuit current are sufficient to completely determine the current transformer as far as the voltage across and the current through a load impedance connected to its secondary terminals is concerned. This is made apparent in the following two paragraphs.

Fig. 3.1. Current Transformer Experiment. Shown are the square wave generator in the foreground, transmission line connected to T-pads, and current transformer. The current transformer and the oscilloscope are connected for a measurement of the current transformer secondary open circuit voltage.

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Fig. 3.2. Circuit for Matching $50\ \Omega$ and $300\ \Omega$ Systems.

Fig. 3.3a. Primary Voltage. Abscissa 5 ns/cm. Ordinate 0.1 V/cm.
Primary Current 2 mA.

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Fig. 3.3b. Primary Voltage. Abscissa $200 \mu\text{s}/\text{cm}$. Ordinate $0.1 \text{ V}/\text{cm}$. Primary Current 2 mA .

When the primary winding of the CT is carrying a current, there will be a voltage across and a current through a load impedance connected between the secondary terminals of the CT. Of the voltage induced in the secondary winding of the CT by the primary, not all will be applied across the load, since there will be some voltage drop across the secondary winding itself. As far as the load is concerned, the current transformer is equivalent to a constant voltage source in series with an internal impedance (see Fig. 3.4a). If we replace the load by a voltmeter of infinite impedance, no current flows from the source and there is no voltage drop across the source impedance, so that we measure the value of the voltage available from the source. If as a second measurement, we replace the voltmeter by an ammeter of zero impedance, the voltage source drives current, but there is no voltage drop across the ammeter. The entire voltage available from the source is dropped across the internal impedance. The current determined in this second measurement multiplied by the internal impedance of the source must just equal the voltage of the source measured previously. Hence, the internal impedance may be calculated by forming the ratio of the voltage measured by the infinite impedance voltmeter to the current measured by the zero impedance ammeter. In usual terminology the voltage measured by the voltmeter is called the open circuit voltage and the current measured by the ammeter, the short circuit current. The above description is referred to as Thevenin's Theorem.

The CT may also be considered to be a constant current source in parallel with an internal impedance (see Fig. 3.4b). The impedance is the same as that defined in Thevenin's Theorem. Again measurements of the open circuit voltage and short circuit current are performed. On open circuit, the constant current source can drive current only through the internal impedance. The open circuit voltage measured is equal to the value of the constant current source times the value of the internal impedance. On short circuit, all of the current available from the source passes through the short circuit. A measurement of the short circuit current determines the value of the

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(a)

(b)

Fig. 3.4. (a) Thevenin Equivalent Circuit. (b) Norton Equivalent Circuit.

current source. The measurements of the open circuit voltage and the short circuit current can be combined to determine the internal impedance as before. The description of the measurements of this paragraph is referred to as Norton's Theorem.

3.3 Results of the Experiment

1. The wave shape of the pulse launched on the transmission line was the same whether the transformer was on the line or not, proving that any reflections from the transformer were below the ability of a single channel of the Tektronix 454 scope to distinguish changes in amplitude (approximately 3% of full vertical scale). In principle, any small reflection could have been determined by using both channels of the oscilloscope to display the difference between the incident and the reflected wave. This proved tedious in practice, however. The transmission line being only seven feet long and the signal speed being approximately $1 \text{ ft} \cdot \text{ns}^{-1}$, the reflections returned to the source in a time just slightly in excess of the rise time of the pulse. The ability of the scope to distinguish the rising edge of the reflected pulse from the rising edge of the incident pulse under such circumstances was unsatisfactory.

2. The wave shape of the pulse traveling on the transmission line was independent of whether the secondary terminals of the current transformer were on short circuit or open circuit. These connections are, of course, the two extremes in load impedance. Interestingly enough, this is also the behavior which is observed at 60 Hz, where the CT's primary current is not determined by the burden on its secondary.⁵

3. Figures 3.5 compare the wave shapes of the transmission line voltage, the secondary short circuit current, and the secondary open circuit voltage all recorded at a sweep speed of 50 ns per major division. A function fitting the short circuit current may be determined by studying Figs. 3.5b and 3.6. The first of these figures shows that, except for a transient oscillation lasting about 10 ns on its

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Fig. 3.5a. Primary Voltage.. Abscissa 50 ns/cm.
Ordinate 0.1 V/cm. Primary Current 2 mA.

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Fig. 3.5b. Secondary Short Circuit Current. Abscissa 50 ns/cm.
Ordinate 0.1 mA/cm. Primary Current 2 mA.

Fig. 3.5c. Secondary Open Circuit Voltage. Abscissa 50 ns/cm.
Ordinate 10 mV/cm. Primary Current 2 mA.

leading edge, the short circuit current had an essentially rectangular front. The second shows that after its initial rise, the short circuit current remained constant for 1 ms and then fell to zero for 1 ms. This is the same timing as the current on the transmission line, the transmission line current being just the transmission line voltage divided by the characteristic impedance of 300 ohms.

From this, we may safely conclude that had the traveling wave on the transmission line been a Heaviside step function, the secondary short circuit current would also have been a Heaviside step function. Consequently, from the numerical data of Fig. 3.6, we have for the secondary short circuit current

$$I_u^{ss}(t) = I_0 u(t)$$

$$I_0 = 2.5 \times 10^{-2} \text{ amp} \cdot \text{amp}_l^{-1} \quad (3.1)$$

where the subscript "u" indicates that the current on the transmission line is a Heaviside step function of unit amplitude, amp_l stands for one ampere of transmission line current, and $u(t)$ is a Heaviside unit step function defined as

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases} \quad (3.2)$$

4. Figure 3.5c shows that the secondary open circuit voltage also has an initial fast transient before assuming a smooth upward-going form. Ignoring this transient, the trace of the secondary open circuit voltage as shown in Fig. 3.7 was fitted to the function

$$V_u^{oc}(t) = V_0 t e^{-ct} \quad (3.3)$$

$$V_0 = 1.24 \times 10^8 \text{ volt} \cdot \text{sec}^{-1} \cdot \text{amp}_l^{-1}$$

$$c = 1.82 \times 10^6 \text{ sec}^{-1}$$

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Fig. 3.6. Secondary Short Circuit Current. Abscissa $200 \mu\text{s}/\text{cm}$.
Ordinate $0.1 \text{ mA}/\text{cm}$. Primary Current 2 mA .

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Fig. 3.7. Secondary Open Circuit Voltage. Abscissa $0.5 \mu\text{s/cm}$.
Ordinate 10 mV/cm . Primary Current 2 mA .

where the subscript "u" has the same meaning as before. Equation (3.3) matches the measured values well around the peak but falls too fast at larger times. Here again we may assume that the transmission line current was a step function. This may be justified as follows: The secondary open circuit voltage decayed to zero in 50 μ s, a time much less than 1 ms, the length of the transmission line pulse. As long as the transmission line pulse amplitude remained constant, the secondary open circuit voltage remained zero and was independent of the length of the pulse as long as that length was greater than the decay time.

3.4 Determining Equivalent Circuits

For a unit step function of current traveling on the transmission lines, Eqs. (3.1) and (3.3) give, respectively, the time dependence of the current source for a Norton equivalent circuit and the time dependence of the voltage source for a Thevenin equivalent circuit. To determine the equivalent source impedance, we need to calculate the Fourier transforms of Eqs. (3.1) and (3.3) and form their ratio. The Fourier transform of a function $f(t)$ is defined by the equation

$$\bar{f}(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt \quad (3.4)$$

The Fourier transforms of Eqs. (3.1) and (3.3), respectively, are

$$\bar{I}_u^{ss}(\omega) = \frac{I_0}{j\omega} \quad (3.5)$$

$$\bar{V}_u^{oc}(\omega) = \frac{V_0}{(j\omega + c)^2} \quad (3.6)$$

Their ratio is the equivalent source impedance

$$Z(\omega) = Z_0 \frac{j\omega}{(j\omega + c)^2} \quad (3.7a)$$

$$Z_0 = \frac{V_0}{I_0} = 0.494 \times 10^{10} \text{ ohms} \cdot \text{sec}^{-1} \quad (3.7b)$$

Plots of the amplitude and phase of Eq. (3.7a) are given in Figs. 3.8a and 3.8b, respectively.

It is of some interest to find a combination of a resistance R , inductance L , and capacitance C which has the impedance given by Eq. (3.7a). Expanding the squared term in the denominator of (3.7a), we find

$$Z(\omega) = Z_0 \frac{j\omega}{-\omega^2 + 2j\omega c + c^2} \quad (3.8)$$

An RLC circuit which forms an impedance with the same dependence on ω as Eq. (3.8) is given by the parallel combination in Fig. 3.9. The impedance of this parallel combination is

$$Z(\omega) = \frac{1}{C} \frac{j\omega}{-\omega^2 + j\omega \frac{1}{RC} + \frac{1}{LC}} \quad (3.9)$$

Identifying similar terms in Eq. (3.8) and Eq. (3.9), we find

$$C = \frac{1}{Z_0} = \frac{I_0}{V_0} = 200 \text{ pf}$$

$$R = \frac{V_0}{2c I_0} = 1360 \text{ ohms} \quad (3.10)$$

$$L = \frac{V_0}{c^2 I_0} = 1.5 \text{ mH}$$

Fig. 3.8a. Amplitude of Thevenin Impedance.

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Fig. 3.8b. Phase of Thevenin Impedance.

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Fig. 3.9. Parallel Combination of R, L, and C.

3.5 Current and Voltage Coupled Through Current Transformer

In this section, we shall calculate the short circuit current and the open circuit voltage to be expected at the secondary terminals of the CT when the transmission line carries a conservatively estimated EMP-induced current. As a conservative estimate, we shall take the current shown in Fig. 3.10, which has been reproduced from Ref. 1.

In Ref. 1, the largest transmission line current calculated had a peak value of 60 kA. This calculation, however, contained no dissipative mechanisms such as wire resistivity, ground resistivity, or corona, so that it should rightfully be considered an overestimate of the largest current induced by EMP in a transmission line. When wire resistivity and ground resistivity are introduced, the worst case current is typically reduced by a factor of ten. If the effects of corona were also included, the reduction in current would be even more. Consequently, Fig. 3.10 may be regarded as a realistic upper bound. The curve in Fig. 3.10 may be fitted to the function

$$I_l(t) = I(e^{-at} - e^{-bt})$$

$$I = 5.13 \times 10^3 \text{ amp,} \quad (3.11)$$

$$a = 1.35 \times 10^6 \text{ sec}^{-1}$$

$$b = 21.5 \times 10^6 \text{ sec}^{-1} .$$

Equations (3.11) were obtained by employing the technique of Ref. 6, whereby the abscissas and ordinates of the peak and the point on the tail where the curve has fallen to half its maximum, respectively, are used to determine the parameters a , b , and I .

For an arbitrary function of time $I_l(t)$, the short circuit current $I^{ss}(t)$ through the secondary contacts of the CT may be calculated by superposition from the formula⁶

Fig. 3.10. Pulse Incident Broadside to the Wire, 60° with Respect to Vertical (Source 30° Above Horizon). Electric Field Horizontally Polarized. Infinitely-Long Perfect Conductor. Conductivity and Dielectric Constant of Ground Independent of Frequency.

$$I^{ss}(t) = \frac{d}{dt} \int_0^t I_u^{ss}(\tau) I_\ell(t - \tau) d\tau, \quad (3.12)$$

where $I_u^{ss}(t)$ is given by Eqs. (3.1). In particular for $I_\ell(t)$ given by Eqs. (3.11), we obtain by performing the derivative indicated in (3.12)

$$\begin{aligned} I^{ss}(t) &= I_u^{ss}(t) I_\ell(0) + \int_0^t I_0 u(\tau) \frac{d}{dt} I_\ell(t - \tau) d\tau \\ &= 0 - I_0 \int_0^t u(\tau) \frac{d}{dt} I_\ell(t - \tau) d\tau \\ &= -I_0 [I_\ell(t - \tau) - I_\ell(t - 0)] \\ &= -I_0 [I_\ell(0) - I_\ell(t)] \\ &= I_0 I_\ell(t) = I_0 I(e^{-at} - e^{-bt}) \end{aligned} \quad (3.13)$$

where we have used twice the fact that $I_\ell(0) = 0$. Thus, we have found that except for the scale factor I_0 , $I^{ss}(t)$ is the same function as $I_\ell(t)$. This is not surprising, since the experiment described in Sections 3.2 and 3.3 showed that when $I(t)$ was a rectangular function, $I_u^{ss}(t)$ was also a rectangular function. The vertical scale on the right in Fig. 3.10 which was obtained from the one on the left by multiplying by $I_0 = 2.5 \times 10^{-2} \text{ amp} \cdot \text{amp}^{-1}$, may be used to read the short circuit current through the secondary terminals of the CT.

For an arbitrary current function of time, $I_\ell(t)$, the open circuit voltage $V^{oc}(t)$ at the secondary terminals of the CT is given by

$$V^{oc}(t) = \frac{d}{dt} \int_0^t V_u^{oc}(\tau) I_\ell(t - \tau) d\tau. \quad (3.14)$$

Substituting Eqs. (3.3) and (3.11) into (3.14), we find

$$\begin{aligned}
 v^{oc}(t) &= \frac{d}{dt} \int_0^t V_o \tau e^{-c\tau} I(e^{-a(t-\tau)} - e^{-b(t-\tau)}) d\tau \\
 &= \frac{d}{dt} V_o I \left[e^{-at} \int_0^t \tau e^{-(c-a)\tau} d\tau - e^{-bt} \int_0^t \tau e^{-(c-b)\tau} d\tau \right] \\
 &= \frac{V_o I}{c-a} \left[\left(at + \frac{a}{c-a} \right) e^{-ct} - \frac{a}{c-a} e^{-at} \right] - \frac{V_o I}{c-b} \left[\left(bt + \frac{b}{c-b} \right) e^{-ct} - \frac{b}{c-b} e^{-bt} \right]
 \end{aligned} \tag{3.15}$$

where the two integrals have been evaluated by parts. Using the numerical values given in Eqs. (3.3) and (3.11), the final formula in Eq. (3.15) has been plotted in Fig. 3.11. From the figure it is apparent that the open circuit voltage has a rise time of the same order of magnitude as the transmission line current driving it shown in Fig. 3.10. Unlike the transmission line current, the open circuit voltage is negative part of the time.

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Fig. 3.11. Current Transformer Open Circuit Voltage When Connected to a Transmission Line Excited by EMP.

4. INDUCTION OF CURRENTS IN CIRCUITS EXPOSED DIRECTLY TO EMP

4.1 On the Idealization of the Problem

In this chapter, we shall concern ourselves with the direct induction by EMP of currents in closed circuits which are unshielded by any conducting materials and which have magnetic induction as their dominant mode of interaction with the EMP. In so doing, we place an upper bound on the EMP coupling to the circuits.

Seldom, however, is supervisory control wiring entirely unshielded. At the Dixie Substation, for example, the wiring is routed through an overhead metal cable tray enclosed on three sides. The fourth side, the bottom, is covered with a grating of large mesh size (~ 1 in.) for water drainage. Clearly, this tray provides some shielding from direct EMP but little from EMP reflected from the ground.

To make the calculation of EMP coupling tractable to simple mathematics we shall:

1. Assume that the circuit consists of a perfectly conducting wire forming a closed circle with radius not larger than 0.6 m.
2. Ignore the reflection of the incident EMP from any nearby objects including the ground.
3. Assume a polarization of the incident magnetic field for which magnetic coupling to the circuit is dominant.

These simplifications will allow us to calculate directly from Faraday's Law, ignoring the time it takes the EMP to travel across the circuit, which if considered would require us to treat the circuit as a loop antenna.

4.2 Application of Faraday's Law to EMP Pick-Up by Loops

Faraday's Law states that the electromotive force $\epsilon(t)$ about a closed path is related to the flux $\phi(t)$ of magnetic induction by the equation

$$\epsilon(t) = \frac{d\phi(t)}{dt} \quad (4.1)$$

For a spatially constant magnetic induction, the flux is given by

$$\phi(t) = B(t)A \quad (4.2)$$

where A is the area enclosed by the path. Consider for example Fig. 4.1a which shows a conductor forming a loop of radius r_1 , and an EMP wave traveling in a direction lying in the plane of the loop with magnetic induction $B(t)$ polarized perpendicular to the plane of the loop. The loop possesses an inductance L and an area πr_1^2 so that the electromotive force may be calculated

$$\frac{L dI(t)}{dt} = \epsilon(t) = \pi r_1^2 \frac{dB(t)}{dt} \quad (4.3)$$

where $I(t)$ is the current in the loop. If we assume that the initial value of the current is zero, then Eq. (4.3) may be integrated to give

$$I(t) = \frac{\pi r_1^2}{L} B(t) \quad (4.4)$$

Since the EMP is a plane wave, the magnetic field $H(t)$ and electric field $E(t)$ are related to the magnetic induction by

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(a)

(b)

Fig. 4.1a. Direction of Propagation and Polarization of an EMP Wave Passing Over a Loop.

Fig. 4.1b. Current in Perfectly Conducting Loop According to Licking and Merewether.

$$B(t) = \mu_0 H(t) = \frac{\mu_0}{\eta} E(t) \quad (4.5)$$

where μ_0 and η are respectively the permeability and the wave impedance of free space. Substituting Eq. (4.5) into Eq. (4.4) we have

$$I(t) = \frac{\mu_0 \pi r_1^2}{r_2} E(t) \quad (4.6)$$

Equation (4.6) gives the current in the loop in terms of the incident electric field. The inductance of the loop needed to evaluate Eq. (4.6) can be calculated from the formula⁷

$$L = \mu_0 r_1 \left(\frac{8r_1}{r_2} \right) - 2 \quad (4.7)$$

where r_2 is the radius of the wire used in forming the loop.

4.3 Relation of Equation (4.7) to the Theory of the Loop Antenna

The EMP pick-up of a perfectly conducting, circular loop antenna has been calculated by Licking and Merewether.⁸ Figure 4.1b reproduces their results for the current divided by the radius in a loop assuming the conditions of incidence for the EMP as shown in Fig. 4.1a. The time dependence of the electric field is a unit step function. The quantity Ω is a shape factor given by the formula

$$\Omega = 2 \ln \frac{2\pi r_1}{r_2} \quad (4.8)$$

As one may show, Ω is related to inductance given in Eq. (4.7) by the expression

$$L = \mu_0 r_1 \left[\frac{\Omega - 4}{2} - \frac{\pi}{4} \right] \quad (4.9)$$

If we substitute Eq. (4.9) into Eq. (4.6), we obtain the expression

$$r_1^{-1} I(t) = \frac{\pi}{\gamma} \frac{E(t)}{\left(\frac{\Omega - 4}{2} - \frac{\pi}{4} \right)} \quad (4.10)$$

which allows us to compare Eq. (4.6) with exact antenna theory given by Fig. 4.1. Taking $E(t)$ to be the unit step function $u(t)$ defined by Eq. (3.2), we find from Eq. (4.10)

$$r_1^{-1} I = \begin{cases} 2.57 u(t) \text{ mA}, & \Omega = 10 \\ 1.96 u(t) \text{ mA}, & \Omega = 12 \\ 1.59 u(t) \text{ mA}, & \Omega = 14 \end{cases} \quad (4.11)$$

Figure 4.1 shows that the exact antenna theory results go through several oscillations before settling down to a constant value specified by Eq. (4.11). Indeed, this agreement is not unexpected as Eq. (4.6) is simply the first term of the Fourier series expansion for the current as calculated in the antenna theory.

Equation (4.2) assumes that the magnetic field associated with the incident EMP is at all times uniform over the entire cross section of the loop. This is not strictly true since the EMP plane wave, which travels at the speed of light, takes a finite time to traverse the diameter of the loop. Equation (4.2) becomes a good approximation if the diameter of the loop is small compared with the distance traveled by the EMP wave in a time during which the magnetic field is appreciably changing. The EMP changes most rapidly on its leading edge, so that the distance of interest is the product of the speed of light by the rise time of the pulse, which is 20 nanoseconds (distance = $3 \times 10^8 \text{ m} \cdot \text{sec}^{-1} \times 2 \times 10^{-8} \text{ sec} = 6 \text{ m}$). In applying the results of Section

4.2, we shall confine our considerations to loops no larger in diameter than 20 percent of this critical distance.

4.4 Equivalent Circuit for Perfectly Conducting, Circular Loop Driven by EMP

It is of interest to determine the voltage and current which a loop driven by EMP can deliver to a load impedance. The equations of Section 4.2 serve as a starting point for this calculation. We shall characterize the loop as a voltage source in series with an internal impedance (Thevenin's Theorem). Combining Eqs. (4.3) and (4.5) we find that the EMP induces an electromotive force about the loop given by

$$\epsilon(t) = \frac{\mu_0 \pi r_1^2}{\eta} \frac{dE(t)}{dt} \quad (4.12)$$

If the loop were broken, this is the voltage which would be measured across a gap in the loop. Using the EMP given by Eq. (1.1) and terming the electromotive force in Eq. (4.12) the open circuit voltage V^{oc} , we find

$$V^{oc}(t) = - \frac{\mu_0 \pi r_1^2 E_0}{\eta} (\alpha e^{-\alpha t} - \beta e^{-\beta t}) \quad (4.13)$$

The maximum value of the open circuit voltage occurs at $t = 0$, and since β is so much larger than α , to a good approximation

$$V_{max}^{oc} = \frac{\mu_0 \pi r_1^2 E_0 \beta}{\eta} \quad (4.14)$$

In evaluating Eq. (4.14) we shall consider two loops of extreme dimensions: a large loop 2 feet (0.6 m) in radius, and a small loop 3 inches (7.62 cm) in radius. Assuming the data of Eq. (1.1) and $\eta = 377$ ohms,

$$\mu_0 = 4\pi \times 10^{-7} \frac{\text{weber}}{\text{amp-m}}.$$

$$V_{\max}^{\text{oc}} = \begin{cases} 49 \text{ kV} & r_1 = 0.6 \text{ m (2 ft)} \\ 0.79 \text{ kV} & r_1 = 7.62 \text{ cm (3 in.)} \end{cases} \quad (4.15)$$

Equation (4.15) expresses the peak voltage that would be measured across a load of infinite impedance. It is apparent that the open circuit voltage, being a function of the square of the loop radius, can vary over a large range of values. For the loop which has a radius of 2 feet, the peak open circuit voltage is large enough to cause dielectric failure of circuit components of low dielectric strength. The effect of the loop on any load may be determined by replacing the loop as in Fig. 4.2 by a voltage source given by Eq. (4.13) in series with the inductance of the loop given by Eq. (4.7).

To obtain a quantitative idea of the maximum current available to the load from the loop, we calculate the current induced in the loop when the loop is intact. This is the same current that would pass through a load of zero impedance connected to the loop. This short circuit current is given by substituting Eq. (1.1) into Eq. (4.6)

$$I^{\text{ss}}(t) = \frac{\mu_0 \pi r_1^2 E_0}{\eta L} (e^{-\alpha t} - e^{-\beta t}) \quad (4.16)$$

From Eq. (4.16) or Eq. (4.6) it is clear that the maximum values of $I(t)$ and $E(t)$ occur at the same time, which is about $t = 20 \text{ nsec}$, at which time the difference between the two exponentials is approximately unity. The maximum short circuit current is thus given by

$$I_{\max}^{\text{ss}} = \frac{\mu_0 \pi r_1^2 E_0}{\eta L} \quad (4.17)$$

Substituting Eq. (4.7) into (4.17) shows that the short circuit current is a more complicated function of the loop geometry than the open circuit voltage, namely:

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Fig. 4.2. Thevenin Equivalent Circuit of a Perfectly Conducting Loop Driven by EMP.

$$I_{\max}^{ss} = \frac{\pi r_1 E_0}{\eta \left(\left(\frac{8r_1}{r_2} \right) - 2 \right)} . \quad (4.18)$$

Within the range of the loop sizes allowed by our approximation that the diameter of the loop be small in comparison with 6 meters, $I_{\max}^{ss}$ could take on extreme values of

$$I_{ss}^{\max} = \begin{cases} 41.6 \text{ A} & r_1 = 0.6 \text{ m (2 ft)} & r_2 = 0.159 \text{ cm (0.0625 in.)} \\ 4.89 \text{ A} & r_1 = 7.62 \text{ cm (3 in.)} & r_2 = 0.125 \text{ mm (0.0005 in.)} \end{cases} . \quad (4.19)$$

4.5 Discussion of Results

In considering the current induced in a circular loop, we have so far assumed that the direction of propagation of the EMP space wave lay in the plane of the loop and the direction of polarization of the magnetic field was perpendicular to this plane. Of course, the direction of propagation depends upon the orientation of the explosion with respect to the loop. The direction of polarization depends upon both the orientation of the explosion and the direction of the earth's magnetic lines of force. When the direction of propagation and the direction of polarization of the magnetic field are different from that specified above, Eq. (4.2) must be replaced by

$$\phi(t) = B(t)A \cos \phi \sin \theta \quad 0 \leq \theta, \phi \leq 180^\circ , \quad (4.20)$$

where θ is the angle made by the direction of propagation of the EMP with the normal to the plane of the loop and ϕ is the angle made by the magnetic field with a plane containing the normal and the direction of propagation (see Fig. 4.3). Consequently, all equations derived above from Eq. (4.2) must be multiplied by the factor $\cos \phi \sin \theta$. In particular, Eqs. (4.14) and (4.17) become, respectively

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Fig. 4.3 Definition of Angles θ and ϕ . AO is normal to the loop. BO is the direction of EMP propagation. DE is the direction of the magnetic field. DG is the component of this field perpendicular to the loop.

$$V_{\max}^{\text{oc}} = \frac{\mu_0 \pi r_1^2 E_0 \beta}{\eta} \cos \phi \sin \theta \quad (4.21)$$

$$I_{\max}^{\text{sc}} = \frac{\mu_0 \pi r_1^2 E_0}{\eta L} \cos \phi \sin \theta \quad . \quad (4.22)$$

It is apparent from these results that for $\phi = 90^\circ$ or $\theta = 0^\circ$, the open circuit voltage and the short circuit current produced by the loop are zero; hence, no energy can be delivered to a load connected in the loop. The factor $\cos \phi \sin \theta$ varies between zero and unity so that the calculations in the preceding sections of this chapter represent a treatment of the worst case as far as direction of incidence and polarization are concerned.

5. INDUCTION OF CURRENTS IN CIRCUITS SHIELDED FROM THE EMP BY METAL BUILDINGS

5.1 On the Idealization of the Problem

Often supervisory control equipment in substations is housed in buildings constructed from sheet metal. Usually the material is iron, commonly with a thickness of 20 mils, covered with a thin layer of zinc to prevent corrosion. For the sake of definiteness in assessing the shielding effects of such enclosures, we shall consider as typical a building used by the Knoxville Utilities Board to house the supervisory control equipment in their Dixie Substation. The approximate dimensions of the building are: length = 31 feet, width = 28 feet, height = 9 feet.

Although the electric and magnetic fields associated with the incident EMP are attenuated differently, the attenuation of both is a strong function of the skin depth in the material, which is the distance which the electromagnetic wave must penetrate into a conductor for its amplitude to be decreased by a factor of e^{-1} . The skin depth δ is expressed by the function⁹

$$\delta = \left(\frac{\rho}{\pi \omega \mu \mu_0} \right)^{1/2} \quad (5.1)$$

where ρ is the resistivity of the material, μ its permeability relative to free space, μ_0 the permeability of free space, and ω is the frequency of the incident electromagnetic field. In addition to the explicit dependence of δ on frequency shown in Eq. (5.1), there is an additional dependence that arises due to the fact that the relative permeability is a function of frequency. A typical variation of μ with ω for iron is shown in Fig. 5.1.

Formulae exist which express the electromagnetic shielding of simple geometric shapes such as cylindrical shells, spherical shells, and parallel planes.¹⁰ In order to apply these formulae to the control house in question, we shall assume that the shape of the house

Fig. 5.1. Typical Variation of Relative Permeability of Iron with Frequency.

is most like that of a sphere and equate the smallest dimension (height = 9 feet) to the diameter of the sphere. This serves to underestimate the shielding effectiveness of the house.¹⁰

The sheet metal of the Dixie Substation control house is penetrated by a number of openings for windows and doors. In order to assess the magnitude of the EMP penetrating such openings, we shall use formulae which apply rigorously only to a circular hole cut in an infinite plane.

Our object in investigating the effectiveness of shielding is to assess the currents induced in circuits contained in control houses. Our model for the circuits will be the loop introduced in Chapter 4. Calculating from these idealizations, we shall find that if the sheet metal is devoid of openings, it will attenuate the incident EMP to a level where currents and voltages induced in circuits are many orders of magnitude below those induced in circuits exposed directly to the EMP. We shall see that both the electric and magnetic fields associated with the incident EMP are attenuated, but the attenuation of the electric field is greater.

5.2 Attenuation of the Magnetic Field

The attenuation $T_H(\omega)$ of the magnetic field associated with an electromagnetic wave by a pair of parallel planes, infinitely long cylindrical shell, or a spherical shell is given by the formula:¹⁰

$$T_H(\omega) = \frac{H_1(\omega)}{H_0(\omega)} = \frac{1}{\cos k_2 \Delta_1 - (K/2) \sin k_2 \Delta_1} \quad (5.2a)$$

$$K = \frac{k_2 \Delta_2}{\mu} \quad (5.2b)$$

$$k_2 = (1 - j) \delta^{-1} \quad (5.2c)$$

$$j = \sqrt{-1} \quad (5.2d)$$

where H_i is the magnitude of the magnetic field inside the enclosure, H_o is the magnitude of the magnetic field outside the enclosure, ν is the frequency of the incident electromagnetic field, and all quantities previously defined in Section 2.1 are here the same. The shielding by all three idealized enclosures is represented by these formulae if specific interpretation is reserved for quantities Δ_1 and Δ_2 to distinguish the three shapes. Depending upon the shape, the quantity Δ_1 is the thickness of one of the identical planes, or the thickness of the cylindrical shell, or the thickness of the spherical shell. Similarly, Δ_2 is the separation between the planes, the radius, or two-thirds of the radius according as the geometry is one of parallel planes, cylinders, or spheres. Equations (5.2) may be applied satisfactorily to a shell in the shape of a parallelepiped, such as a building, if the parallelepiped is approximated as a sphere of radius r equal to half the smallest dimension of the parallelepiped.

Since $T_H(\nu)$ is a strong function of δ , it is helpful to consider limiting cases of this formula for $\delta < \Delta_1$ and $\delta > \Delta_1$, respectively. From Eq. (5.1), we find $\delta = \Delta_1 = 20 \times 10^{-3}$ in. when

$$\mu\nu = \frac{\rho}{\pi \Delta_1^2 \mu_0} = 0.98 \times 10^6 \approx 10^6 \quad (5.3)$$

where we have used for the resistivity of iron $\rho = 10^{-7}$ ohm $\cdot$ m. Since μ is a function of ν we need to solve this equation for ν by successive approximations. Using Fig. 5.1, and taking the lower bound of the range of permeability (i.e., when $\nu = 0$, $\mu = 1000$), we see that a solution is

$$\begin{aligned} \nu &= 100 \text{ Hz} \\ \mu &= 1000 \end{aligned} \quad (5.4)$$

For frequencies less than 100 Hz, $\delta > \Delta_1$ and $T_H(\nu)$ is nearly constant; for frequencies greater than 100 Hz, $\delta < \Delta_1$ and $T_H(\nu)$ falls off rapidly with increasing frequency. As a consequence, it is the frequencies

below 100 Hz which contribute significantly to the magnetic field inside the control house.

If the frequency spectrum of the incident electromagnetic wave is nearly constant at low frequency, then the rapid decrease of $T_H(\omega)$ at large frequencies permits a very useful approximation. This approximation involves the replacement of the incident electromagnetic field by a delta function of appropriate amplitude. A delta function is a function which is zero for all values of its argument except zero at which it is infinitely tall and about which it has infinitesimally small width. The width and the height of the delta function, however, are such that the area under the function is finite. These facts may be summarized by the equation

$$\int_{-\infty}^{+\infty} \delta(t) dt = 1 \quad , \quad (5.5)$$

where $\delta(t)$ is the delta function. If an amplitude A is independent of t , then also

$$\int_{-\infty}^{+\infty} A \delta(t) dt = A \quad . \quad (5.6)$$

The function $\delta(t)$ has the property that its frequency spectrum is a constant independent of frequency. This means that to represent a delta function by a superposition of sinusoidal oscillations, all frequencies are equally important. Thus an incident electromagnetic wave, which has a constant frequency spectrum at low frequencies, has to within a constant multiplicative factor the same frequency spectrum at low frequencies as a delta function. But as we have pointed out previously, only the low frequency components of the incident electromagnetic field significantly penetrate the control house, so that we are permitted to approximate the incident electromagnetic field with a constant low frequency spectrum by a delta function. (For a detailed justification of this technique, see Appendix A.)

If the incident electromagnetic field is given by Eq. (1.1), we may find the amplitude of the delta function which is the equivalent of Eq. (1.1) by equating the area under this function to the amplitude A of a delta function. Integrating, we find

$$A = \int_0^{\infty} H(t) dt = \int_0^{\infty} \frac{E_0}{r} \left(e^{-\alpha t} - e^{-\beta t} \right) dt = \frac{E_0}{r} \left(\frac{1}{\alpha} - \frac{1}{\beta} \right) \approx \frac{E_0}{\alpha r} \quad (5.7)$$

$$= 0.884 \times 10^{-4} \text{ A} \cdot \text{m}^{-1} \text{sec} \quad .$$

In addition to A , it is shown in Ref. 10 that the magnetic field inside the enclosure is a function also of the two quantities B and C calculated from the data; $\sigma = \sigma^{-1} = 10^7 \text{ mho} \cdot \text{m}^{-1}$, $\mu = 1000$, $\mu_0 = 4\pi \times 10^{-7}$, $\Delta_2 = (2/3)r = (2/3) (9 \text{ feet}/2) = 3 \text{ feet}$.

$$B = \Delta_1 \sqrt{\mu \mu_0 \sigma} = 5.70 \times 10^{-2} \text{ sec}^{1/2} \quad (5.8)$$

$$C = 2\mu \frac{\Delta_1}{\Delta_2} = 1.11 \quad .$$

Figure 5.2 shows the time dependence of the interior magnetic field as a function of B and C as given in Ref. 10. The time dependence of the magnetic field penetrating the control house we are considering is thus best represented by the curve for $C = 1.0$. From this curve we find in mks units the peak magnetic field interior to the enclosure by multiplying 0.62, the curve maximum, by A and dividing by B^2 to obtain $H_p = 1.7 \times 10^{-2} \text{ amp} \cdot \text{m}^{-1}$. The time scale in seconds is found by multiplying the abscissa values in Fig. 5.1 by $B^2 \times 10^{-2}$. A curve with the proper scaling for our case is shown in Fig. 5.3. Using again the curve fitting technique of Ref. 6, the curve in Fig. 5.3 may be fitted to the function

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Fig. 5.2. Parametric Curves for Magnetic Fields Inside Shielding Enclosures Due to a Unit Delta Function Incident Field.

Fig. 5.3. Magnetic Field Inside Control House.

$$H(t) = H \left(e^{-a't} - e^{-b't} \right)$$

$$H = 2.39 \times 10^{-2} \text{ amp} \cdot \text{m}^{-1}$$

(5.9)

$$a = 0.238 \times 10^3 \text{ sec}^{-1}$$

$$b = 2.50 \times 10^3 \text{ sec}^{-1} .$$

5.3 Effect of the Interior Magnetic Field on Circuits

In order to assess the magnetic shielding provided by the control house, it is useful to compare the open circuit voltage and the short circuit current available from a loop circuit illuminated by the magnetic field of Eqs. (5.9) with the values for these quantities calculated in Chapter 4 for an unshielded loop.

For the open circuit voltage, we refer to Eq. (4.3) and calculate using Eq. (5.9)

$$\begin{aligned} \epsilon(t) &= \pi r_1^2 \frac{dB(t)}{dt} = \pi r_1^2 \mu_0 H \frac{d}{dt} \left(e^{-a't} - e^{-b't} \right) \\ &= - \pi r_1^2 \mu_0 H \left(a' e^{-a't} - b' e^{-b't} \right) . \end{aligned} \quad (5.10)$$

As in Eq. (4.13), the right hand side of this expression takes on its maximum value at $t = 0$, so that we have for the maximum value of the open circuit voltage

$$V_{\text{max}}^{\text{oc}} = \pi r_1^2 \mu_0 H b' = \begin{cases} 85 \text{ } \mu\text{V}, & r_1 = 0.6 \text{ m} \\ 1.4 \text{ } \mu\text{V}, & r_1 = 7.62 \text{ cm} \end{cases} . \quad (5.11)$$

These are trivial voltages compared with those summarized by Eqs. (4.15).

The great reduction in $V_{\max}^{\text{OC}}$ shown by comparison of Eqs. (4.15) and (5.11) is the result of a reduction in both the amplitude and the rate of rise of the incident EMP by the shielding action of the control house. The quantity $E_0/\eta = 133$ in Eq. (4.14), which is the amplitude of the magnetic field incident on the control house, is reduced by a multiplicative factor of approximately 2×10^{-4} to the value $H = 2.39 \times 10^{-2}$. Also because the walls absorb the high frequency components of the incident magnetic field, the field inside the house is much more slowly rising. Thus in passing from Eq. (4.14) to Eq. (5.11), the factor $a = 2.6 \times 10^8 \text{ sec}^{-1}$ is replaced by the factor $b' = 2.5 \times 10^3 \text{ sec}^{-1}$ resulting in a reduction by another multiplicative factor of 10^{-5} in induced EMP.

To calculate the short circuit current, we combine Eqs. (4.4) and (5.9)

$$I(t) = \frac{\mu_0 \pi r_1^2 H}{L} \left(-a't - e^{-b't} \right) \quad (5.12)$$

From Fig. 5.2, it is apparent that the peak of the magnetic field inside the control house occurs at approximately $t = 1 \text{ ms}$. At this time, the time dependent factor in Eq. (5.12) has the value 0.706. Hence, as the analog of Eq. (4.18), we have

$$I_{\max}^{\text{ss}} = \frac{0.706 \pi r_1 H}{\ln\left(\frac{8r_1}{r_2}\right) - 2} \quad (5.13)$$

from which we calculate the short circuit current to be

$$I_{\max}^{\text{ss}} = \begin{cases} 5 \text{ ma} & r_1 = 0.6 \text{ m} & r_2 = 0.159 \text{ cm} \\ 0.6 \text{ ma} & r_1 = 7.62 \text{ cm} & r_2 = 0.125 \text{ mm} \end{cases} \quad (5.14)$$

This reduction of $I_{\max}^{\text{ss}}$ by a multiplicative factor of 1.3×10^{-4} due to

the shielding is the result of the replacement of the factor $E_0/r = 133$ in Eq. (4.18) by the $0.706 H = 1.68 \times 10^{-2}$. Of course, since the short circuit current follows the time dependence of the electromagnetic field, the shielded short circuit current rises and falls much more slowly than the unshielded current.

5.4 Attenuation of the Electric Field

The attenuation of the electric field by cylinders or spheres is given by¹⁰

$$T_E(\omega) = \frac{E_i(\omega)}{E_o(\omega)} = \frac{2 \left(\frac{2\pi c}{V} \Delta_2 \right)^2}{K \sin k_2 \Delta_1}, \quad (5.15)$$

where $E_o(\omega)$ is the electric field outside the enclosure, $E_i(\omega)$ is the electric field inside, $V = 3 \times 10^8 \text{ m} \cdot \text{sec}^{-1}$ is the speed of light, and the other quantities are as previously defined in this chapter.

In addition to the quantity B previously defined in Section 5.2, the attenuation of the electric field is a function also of the quantity D_1

$$D_1 = \frac{2\lambda_2}{\mu_0 V^2 B^2} = 2.72 \times 10^{-5} \quad (5.16)$$

which has been evaluated using Eq. (5.9) and $\lambda_2 = 3$ feet. Figure 5.4, taken from Ref. 10, shows the time dependence of the electric field inside an enclosure if the field incident on the outside has a delta function time dependence. Since the time dependence of the electric and magnetic fields associated with an incident EMP wave are the same, the arguments advanced in Section 5.2 concerning replacement of the EMP by a delta function again apply to the attenuation of the electric field. Since the electric field of the incident EMP is just $\eta = 377$ times the magnetic field, the amplitude of the equivalent delta function

Fig. 5.4. Electric Field Inside Shielding Enclosure Due to Unit Delta Function Applied Field.

A_1 for calculating the attenuation of the electric field is from Eq. (5.7) just

$$A_1 = \pi A = 0.333 \times 10^{-1} \quad . \quad (5.17)$$

If we multiply the ordinates in Fig. 5.4 by $\pi^4 D_1 A_1$ and the abscissas by B^2 we have the time dependence of the electric field inside the enclosure in mks units shown in Fig. 5.5.

The electric field shown in Fig. 5.5 would seem to be entirely trivial and unlikely to cause any upset or damage to equipment inside the control house.

5.5 Role of Corners and Scattering Surfaces Inside the Enclosure

Reference 10 points out that a rectangular structure, such as a substation control house, can be approximated as a sphere, provided it is recognized that the solution is inaccurate in the neighborhood of corners. In the vicinity of a corner, the time dependence of the interior field is not changed, but the amplitude of the field can increase very significantly as the corner is approached. Figure 5.5 taken from Ref. 10 gives H/H_1 which is the ratio of the magnetic field near the corner to the field near the center of the enclosure as a function of the ratio of Δr , the distance from the corner, to x_0 , the half-width of the enclosure. In the case of the Dixie Substation control house, we may conservatively set $x_0 = 4.5$ ft. which is half the smallest dimension (the height = 9 feet) of the house. Interpreting the horizontal scale of Fig. 5.6 we see that as we approach to within 0.45 feet of one of the corners of the house, the magnetic field increases by a factor of about five. Since $V_{\max}^{oc}$ and $I_{\max}^{ss}$, as calculated in the preceding section, depend linearly upon H , we need simply multiply the values given in Eqs. (5.11) and (5.14) by a factor of five to determine the peak values of the open circuit voltage and short circuit current induced in a loop located 0.45 feet from the corner of the house.

Fig. 5.5. Electric Field Inside Shielding Enclosure Due to Applied Field Given by Equation (1.1).

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Fig. 5.6. Increase of Magnetic Field Strength When Approaching the Corner of a Shielded Space.

The calculations of the previous sections have been based upon the assumption that the fields penetrate an empty enclosure. The presence of conducting structures such as instrument racks inside the enclosure will scatter the penetrating field to a certain extent and can result in constructive reinforcement at some places. However, the wavelengths corresponding to the frequencies which contribute significantly to the interior field $\omega < 100$ Hz, $\lambda > 3 \times 10^6$ m are much larger than the size of any interior structure so that scattering inside the enclosure will be negligible.

5.6 Penetration of EMP Fields Through Holes in Enclosures

The fields penetrating holes in a shielding enclosure have the same time dependence as the fields outside the enclosure but different spatial dependence. By using formulae given in Ref. 10, we shall be able to assess the effect of windows and doors on the performance of the control house as an electromagnetic shield.

Reference 10 considers the penetration of the EMP through a circular hole in an infinite plane shield. As shown in Fig. 5.7, for this calculation the EMP is assumed to be traveling in a direction parallel to the plane with the electric field pointing in a direction normal to the plane and the direction of the magnetic field lying in the plane. The radius of the hole in the plane is r_0 and the point of observation of the EMP penetrating the hole is a distance r from the center of the hole. When r satisfies the inequalities $r_0 < r < \lambda/2\pi$ where λ is the shortest wavelength of importance in the EMP spectrum, then the magnitude of the incident electric and magnetic fields, E_0 and H_0 , respectively, are related to the r , θ , and ϕ components of the respective fields penetrating the enclosure by the formulae

$$E_r = \frac{2}{3\pi} \left(\frac{r_0}{r} \right)^3 E_0 \cos \theta \quad (5.18a)$$

Fig. 5.7. Penetration of Electric and Magnetic Fields Through a Circular Hole.

$$E_{\theta} = \frac{1}{3\pi} \left(\frac{r_0}{r} \right)^3 E_0 \sin \theta \quad (5.18b)$$

$$E_{\phi} = 0 \quad (5.18c)$$

$$H_r = - \frac{4}{3\pi} \left(\frac{r_0}{r} \right)^3 H_0 \sin \phi \sin \theta \quad (5.18d)$$

$$H_{\phi} = \frac{2}{3\pi} \left(\frac{r_0}{r} \right)^3 H_0 \cos \phi \quad (5.18e)$$

$$H_{\theta} = \frac{2}{3\pi} \left(\frac{r_0}{r} \right)^3 H_0 \sin \phi \cos \theta \quad (5.18f)$$

Consider a control house having a double door and a window which we may conservatively represent by circles of radius 1.5 m and 0.5 m, respectively. In applying Eqs. (5.18) we consider the EMP to penetrate the door or window along the most adverse direction, namely the one for which the sines and cosines in the formulae have unit values. The largest constant coefficient in any of Eqs. (5.18) has magnitude $(4/3\pi)$, so in the interest of making a worst case assessment of the penetration, we shall confine our attention to Eq. (5.18d). From Eq. (5.18d), it is clear that the magnitude of the penetrating field decreases proportional to $1/r^3$, so that in locating sensitive electrical equipment inside the enclosure, there is a premium on selecting locations as far away from doors and windows as possible. Since the floor dimensions of the Dixie Substation control house are 28 ft x 31 ft with windows and doors on several sides, we could conceivably crowd all the equipment into the center of the building a distance of perhaps some 3 meters from a door or window. We estimate the field strengths penetrating through the openings a distance of 3 meters using the r dependence of Eq. (5.18d)

$$\left. \begin{matrix} H/H_0 \\ E/E_0 \end{matrix} \right\} \approx \frac{4}{3\pi} \left(\frac{r_0}{r} \right)^3 = \begin{cases} 5.3 \times 10^{-2} & r_0 = 1.5 \text{ m door} \\ 2 \times 10^{-3} & r_0 = 0.5 \text{ m window} \end{cases} \quad (5.19)$$

Because the field penetrating the doors and windows has the same time dependence as the field outside the building, we need only multiply the numerical results summarized by Eqs. (4.15) and (4.19) by the factors given in Eq. (5.19) to obtain the open circuit voltage and the short circuit current expected in a loop illuminated by the penetrating field. Thus for a door

$$V_{\max}^{\text{oc}} = \begin{cases} 2600 \text{ V} & r_1 = 0.6 \text{ m} \\ 42 \text{ V} & r_1 = 7.62 \text{ cm} \end{cases} \quad (5.20a)$$

$$I_{\max}^{\text{ss}} = \begin{cases} 2.2 \text{ A} & r_1 = 0.6 \text{ m} & r_2 = 0.159 \text{ cm} \\ 0.26 \text{ A} & r_1 = 7.62 \text{ cm} & r_2 = 0.125 \text{ mm} \end{cases} \quad (5.20b)$$

and for a window

$$V_{\max}^{\text{oc}} = \begin{cases} 98 \text{ V} & r_1 = 0.6 \text{ m} \\ 1.6 \text{ V} & r_1 = 7.62 \text{ cm} \end{cases} \quad (5.21a)$$

$$I_{\max}^{\text{ss}} = \begin{cases} 83 \text{ mA} & r_1 = 0.6 \text{ m} & r_2 = 0.159 \text{ cm} \\ 9.8 \text{ mA} & r_1 = 7.62 \text{ cm} & r_2 = 0.125 \text{ mm} \end{cases} \quad (5.21b)$$

In the next and final chapter, we shall assess the damage to be expected to semiconducting diodes connected in circuits carrying currents induced by EMP as calculated in Chapters 3 through 5.

6. EFFECTS OF EMP COUPLED CURRENTS ON RELAYS AND FUSES AND PROPOSAL OF COUNTERMEASURES

6.1 On the Idealization of the Problem

We have seen in Chapter 2 some of the variety of supervisory control equipment used in power substations. Assessing the effects of EMP on all of these devices in detail is outside the scope of this effort. One may on good grounds simplify the problem, however, by distinguishing between the electromechanical devices and those employing semiconducting circuit elements. These two groups are greatly different in their speed of operation and ability to dissipate energy. The electromechanical devices will be discussed in the next section. The remainder of the chapter will be devoted to the semiconducting devices which are more likely to suffer malfunction or damage and for which data are available to permit some quantitative analysis.

In Section 2.2 of Chapter 2, we described in ~~some~~ detail the functioning of a "static relay" constructed from transistors and semiconducting diodes. Of the two types of semiconducting devices used in this relay, the transistors are, in general, more vulnerable to damage from transient currents.¹¹ The damage is caused primarily by the heat dissipated by the current as it passes through the junctions between the semiconducting materials.¹² Consequently, we shall confine our attention to the diodes under the assumption that if we find that they will be damaged by EMP-induced currents, we may conclude that the transistors will be also. To draw some quantitative conclusions, we shall assume that a single diode is connected to the perfectly conducting loop whose circuit analysis has been described in the preceding chapters.

6.2 EMP Effects in Electromechanical Relays

Electromechanical relays are very unlikely to be tripped by EMP when connected in any of the circuits analyzed in Chapters 3, 4, and 5, namely,

- (1) Secondary circuits of current transformers (Chapter 3).
- (2) Loop circuits of radius less than 0.6 meters exposed directly to EMP (Chapter 4).
- (3) Loop circuits of radius less than 0.6 meters shielded by control houses like that used at Dixie Substation (Chapter 5).

The reason for this is that these transient signals last too short a time to overcome the mechanical inertia of most electromechanical relays. Typical response times for such relays are of the order of one cycle (at 60 Hz this is 16.7 ms) or more. Of the currents calculated in the previous chapters, only that induced in a loop circuit shielded by a control house lasts as long as one millisecond [see Eqs. (5.9) and (5.12) and Fig. 5.2], and even so, it is doubtful that such a small current could actually trip an electromechanical relay [see Eq. (5.14)].

It is quite possible, on the other hand, that the voltage coupled through current transformers (see Fig. 3.11) and the voltage coupled directly to loop circuits [see Eq. (4.15)] might be high enough to cause dielectric failure of the insulation of the coils used with most electromechanical relays. A detail of one of the coils associated with the relay shown in Fig. 2.2 is pictured in Fig. 6.1. The windings shown in this figure are insulated to withstand the normal operating voltage of 120 volts at 60 Hz. From all appearances a sizeable safety factor in insulation thickness has been added. This might not be enough to prevent failure of the insulation by EMP-induced voltages; however, it is very likely that the wires are so far apart as to continue to withstand the normal operating voltage despite the failure. Even if the failure were effective in shorting out a turn, it is likely that the coil would still produce enough flux to operate the eddy current disk.

We may thus conclude with confidence that electromechanical relays are not likely to operate falsely nor fail when connected in loop circuits or circuits excited by current transformers. In addition to the analysis of the preceding chapters, our conclusion is supported by the experience of power companies who have noticed little trouble with electromechanical relays due to lightning or the switching of live high voltage power lines.¹³

PHOTO-0864 73

Fig. 6.1 Coil. (Part of Induction Disk Relay.)

6.3 Pulse Power Dissipation in Semiconductor Diodes

Diodes are often constructed by sticking a slice of P-type semiconductor to a slice of N-type and making electrical connections to each. The rectifying action of the device is a property of the junction between the two types of semiconducting material. The current through and the voltage across the terminals of such a diode are related as shown in Fig. 6.2.

If a positive voltage is applied across the diode, a large current flows with little resistance, as shown in the figure. The voltage drop is primarily across the junction between the two materials, and the diode is said to be conducting in the forward direction. If the applied voltage is greatly increased, the importance of the junction decreases and the diode acts more like a resistor with resistance characteristic of the bulk materials from which it is constructed. This increase in resistance is not shown in the figure.

If, on the other hand, a negative voltage is applied across the diode, little current initially flows and the apparent resistance of the diode is very high. Here the diode is said to be conducting in the reverse direction. With increasing negative voltage, however, the diode fails as an insulator and avalanche conduction sets in. In this state, the diode has a much lower resistance. The negative voltage, $-V_b$, at which avalanche conduction occurs is called the reverse breakdown voltage. The numerical value for this voltage depends upon the device, and among devices of the same manufacture shows also some statistical variation. For applied voltage $V - V_b$, the current I through the diode follows the curve

$$I = \frac{V - V_b}{Z}, \quad (6.1)$$

where Z is called the dynamic resistance. For most diodes Z is an implicit function of the current also.

For a diode biased in the reverse direction by a voltage which exceeds the reverse breakdown voltage, we may use Eq. (6.1) to calculate

Fig. 6.2. Semiconductor Diode V-I Characteristic Including Avalanche Breakdown.

the power P dissipated by the diode:

$$P = IV = IV_b + I^2 Z \quad . \quad (6.2)$$

As was pointed out above, the resistance to high reverse currents is very low, so to a good approximation

$$P = IV_b \quad . \quad (6.3)$$

6.4 Pulse Power Failure of Semiconductor Diodes

Singletary, Collier, and Myers measured the pulse power necessary to cause a number of diodes to become short circuits.¹¹ They applied a rectangular voltage pulse of 10 μ sec maximum duration and measured the current through the device. This test was repeated with pulses of increasing voltage until the device failed. The product of the current and voltage for such a pulse was the failure power. The threshold failure power with these pulses showed a statistical variation of plus or minus a half order of magnitude or so between devices of the same manufacture. They also measured the avalanche breakdown voltage, which showed some statistical variation. The arithmetic average of their measurements of $-V_b$ for each of the diodes they tested is given in Table 6.1.

Singletary, Collier, and Myers found that the minimum power for breakdown in the reverse direction was a function of the time to breakdown, following roughly the law

$$P = Kt^{-1/2} \quad (6.4)$$

where t is the time and K is a constant characteristic of the device. This relationship has also been justified on theoretical grounds by

Table 6.1. Summary of Semiconductor Diode Data¹¹

Name	Average V_b (volts)	K (watt $\cdot$ sec ^{1/2})	K/V_b (watt $\cdot$ volt ⁻¹ $\cdot$ sec ^{1/2})
1N23RF	6.27	0.94×10^{-3}	0.150×10^{-3}
1N23WE	6.10	0.29×10^{-3}	0.475×10^{-4}
1N25	3.42	0.26×10^{-1}	0.760×10^{-2}
1N64	20.1	0.41×10^{-1}	0.204×10^{-2}
1N277	73.0	0.27×10^{-1}	0.370×10^{-3}
1N457	295.1	0.12	0.407×10^{-3}
1N484A	232.3	0.45	0.194×10^{-2}
1N547	908.1	12.1	0.133×10^{-1}
1N643A	249.3	0.101	0.405×10^{-3}
1N645	754.8	0.56	0.742×10^{-3}
1N646	788.9	2.29	0.290×10^{-2}
1N647	871.7	3.90	0.447×10^{-2}
1N658	155.2	0.92	0.593×10^{-2}
1N661	272.4	0.46	0.169×10^{-2}
1N751A	5.2	6.3	1.21
1N914	104.9	0.96×10^{-1}	0.915×10^{-3}
1N1733A	4132.3	11.3	0.273×10^{-2}

Wunsch and Bell¹² who have calculated the temperature at the junction between the semiconductor materials by assuming that as the heat is being delivered at a rate given by Eq. (6.2), it is also being conducted away through the bulk materials on either side of the junction. Failure of the diode occurs when the temperature reaches the melting point of one of the materials forming the junction. They assume that the heat is conducted away much more slowly than it is being generated, so that the power in Eq. (6.3) may be assumed to be a delta function of time. Experimentally determined values of K are also given in Table 6.1.

In this and the previous section, we have emphasized diode failure due to conduction in the reverse direction. Failure also has been observed due to conduction in the forward direction, but occurs in general at a higher power level than for failure in the reverse direction.¹¹ Since we cannot assume a priori that EMP will drive current only in the forward direction of every diode, we are conservative in our analysis of diode failure if we consider conduction in the reverse direction alone.

6.5 Damage to a Semiconductor Diode Connected to a Current Transformer

We shall now calculate the energy deposited in a semiconductor diode connected between the secondary terminals of the current transformer discussed in Chapter 3 when the primary of the transformer carries the EMP-induced current specified by Eq. (3.11). To perform the calculation, we must first establish two things: (1) That the voltage across the secondary terminals of the current transformer exceeds the avalanche breakdown voltage of the diode, otherwise the current drawn by the diode is not sufficient to burn it out. (2) Estimate the current drawn by the diode. With these two quantities established, we can calculate the power dissipated by the diode from Eq. (6.3).

A plot of the open circuit voltage available from the secondary contacts of the current transformer is shown in Fig. 3.11. Since the initial impedance of the diode in the reverse direction is very high, this is essentially the voltage which will initiall appear across the

diode. By comparison with Table 6.1, it is apparent that the open circuit voltage eventually exceeds the reverse breakdown voltages of all the diodes listed. We may thus safely conclude that all of these diodes will be forced into avalanche conduction.

The maximum current which can pass through the diode is the current transformer secondary short circuit current plotted in Fig. 3.10. This entire current will not, of course, ever actually be conducted by a diode, since the diode has some resistance of its own. However, to establish an estimate of the power dissipated by the diode, we shall use the current of Fig. 3.10. This assumption is consistent with the approximation in Eq. (6.3). In cases where the diode resistance is indeed small compared with the impedance of the current transformer at all frequencies (see Fig. 3.8) our calculation will be exact. This approximation is examined quantitatively in Appendix B.

For the diode to fail, the power dissipated must at some time exceed the failure power. Using Eqs. (6.3), (6.4), and (3.13), this means that

$$Kt^{-1/2} < I(t) V_b = I I_o V_b (e^{-at} - e^{-bt}) \quad (6.5)$$

or confining the time dependence to one side of the inequality

$$\frac{K}{I I_o V_b} < t^{1/2} (e^{-at} - e^{-bt}) \quad (6.6)$$

The maximum of the right side of this inequality may be found by numerical or graphical techniques. The maximum is 0.369×10^{-3} when $t = 0.37 \mu\text{sec}$. Taking $I = 5.13 \times 10^3 \text{ amp}$, and $I_o = 2.5 \times 10^{-2} \text{ amp} \cdot \text{amp}_l^{-1}$ Chapter 3, we require for failure

$$\frac{K}{V_b} < 0.473 \times 10^{-1} \text{ watt} \cdot \text{volt}^{-1} \cdot \text{sec}^{-1/2} \quad (6.7)$$

For comparison, the ratio K/V_b has been tabulated in Table 6.1. It is apparent that all diodes except 1N751A may fail in the reverse direction.

6.6 Damage to a Semiconductor Diode Connected in a Loop Circuit

We may proceed in analogy with our treatment of the current transformer in the preceding section. We consider both the loops for which V_{oc}^{max} is calculated in Eq. (4.15). We first observe from this equation that the open circuit voltage of the larger loop ($r_1 = 0.6$ m) is greater than any of the reverse breakdown voltages of the diodes listed in Table 6.1. The open circuit voltage of the smaller loop ($r_1 = 7.62$ cm) is greater than the reverse breakdown voltages of all diodes except 1N1733A. This diode cannot fail in the reverse direction when connected in the smaller loop. We next assume that the entire short circuit current given by Eq. (4.16) passes through the diode. Again this requires the assumption that the diode resistance is small in comparison with the impedance of the loop. The conditions for this approximation to be true are analyzed in Appendix C.

Employing Eqs. (6.3), (6.4), and (4.16), we require for failure

$$Kt^{-1/2} < I(t)V_b = \frac{\mu_0 \pi r_1^2 E_0}{\eta L} V_b (e^{-\alpha t} - e^{-\beta t}) \quad (6.8)$$

Segregating the time dependence on one side

$$\frac{K \eta L}{\mu_0 \pi r_1^2 E_0 V_b} < t^{1/2} (e^{-\alpha t} - e^{-\beta t}) \quad (6.9)$$

The right hand side of this inequality has a maximum of 0.350×10^{-3} at $t = 0.33$ μ sec. Using the numerical values for the various constants specified in Chapter 4, this inequality becomes

$$\frac{K}{V_b} < \begin{cases} 0.146 \times 10^{-1} \text{ amp} \cdot \text{sec}^{1/2}, & r_1 = 0.6 \text{ m}, r_2 = 0.159 \text{ cm} \\ 0.171 \times 10^{-2} \text{ amp} \cdot \text{sec}^{1/2}, & r_1 = 7.62 \text{ cm}, r_2 = 0.125 \text{ mm} \end{cases} \quad (6.10)$$

By comparing Eq. (6.10) with the tabulated values of K/V_b in Table 6.1, it becomes apparent that of all the diodes listed, only 1N751A will probably survive when connected in the larger of the two loops. Survivability, however, is more frequent when the connection is to the smaller loop. Diodes 1N25, 1N64, 1N484A, 1N547, 1N646, 1N647, 1N658, 1N751A, and 1N1733A should all survive with 1N661 being borderline.

6.7 Damage to a Semiconductor Diode Connected in a Loop Circuit Shielded by a Control House

When not compromised by transparent openings, the control house analyzed in Chapter 5 is a formidable barrier to the penetration of EMP. By comparison of Eq. (5.11) with the tabulated values of V_b in Table 6.1, we see that the maximum EMP voltage available from a loop circuit shielded by the control house is not enough to put any of the semiconductor diodes listed into the avalanche conducting state where failure occurs. Thus, we may conclude that the circuits of Chapter 3 will survive if the control house has no transparent openings.

If, however, the control house is penetrated by doors or windows the relevant voltages to be compared with the values of V_b in Table 6.1 are those summarized in Eqs. (5.20a) and (5.21a), respectively. We see that many of the diodes will be driven into avalanche conduction by the penetrating fields. To determine which of the diodes that are driven into avalanche conduction will fail, we multiply the right hand side of Eq. (6.10) by the transmission factors given by Eq. (5.19). We summarize the results in a 2 x 2 array labeling the columns by the radius of the circuit considered and the rows by "door" and "window" for points of entry of the EMP:

		$(r_1 = 7.62 \text{ cm})$	$(r_1 = 0.6 \text{ m})$
$\frac{K}{V_b} <$	window	0.342×10^{-5}	0.292×10^{-4}
	door	0.905×10^{-4}	0.774×10^{-3}

The units are watt · volt⁻¹ · sec^{1/2} as before.

Comparing the entries in the array with the values of K/V_0 in Table 6.1, we may determine the diodes that are likely to survive. Tabulating our results in a similar array, those which will likely survive are:

	SMALL ($r_1 = 7.62$ cm)	LARGE ($r_1 = 0.6$ m)
window	All	All
door	All but 1N23WE	All but 1N23RF, 1N23WE, 1N277, 1N457, 1N643A, 1N645

We may conclude that windows no bigger than 1 meter on a side are unlikely to allow enough EMP field strength to penetrate to damage electronic equipment containing semiconductor diodes if the equipment is located at least 3 meters away from the window. Double doors, say 3 meters on a side, left open present a greater EMP hazard.

6.8 Effects of EMP on Fuses

The Joule heating of a fuse by an over-current causes the fuse to lose structural integrity and become an open circuit thereby interrupting the current. If the over-current is a fast transient like that induced by EMP or lightning, the fuse wire must not only melt but also vaporize to become an open circuit. It is not sufficient for the fuse wire only to reach the melting state and disintegrate under the action of the force of gravity. Consider for example a transient current lasting 100 μ sec. Assuming that the fuse wire is already in the liquid state before the transient arrives, it will have an acceleration $g = 980 \text{ cm} \cdot \text{sec}^{-2}$ and fall a distance

$$s = \frac{1}{2} g t^2 = \frac{1}{2} (980) (10^{-4})^2 = 4.9 \times 10^{-6} \text{ cm} \quad (6.11)$$

in 100 μ sec. Since fuse wires are unlikely to be made with diameters

much less than 10^{-4} cm, the wire falls a distance barely 5% of its diameter in the time it takes the transient to pass.

As a consequence, the energy necessary to cause a fuse to open a circuit should be considered to be the sum of the following: (1) the energy to bring the wire from ambient temperature to its melting point, (2) the latent heat of fusion of the material of the wire, and (3) the energy necessary to bring the wire from a liquid state at its melting temperature to its boiling point. This sum has been calculated by Rudenberg.¹⁴ In so calculating, he makes the assumption that the transient is fast enough that no heat is lost from the wire by such mechanisms as conduction or radiation. For the sum of quantities (1) through (3), Rudenberg gives the values $11.72 \times 10^8 \text{ amp}^2 \cdot \text{cm}^{-4} \cdot \text{sec}$ for copper and $8.00 \times 10^8 \text{ amp}^2 \cdot \text{cm}^{-4} \cdot \text{sec}$ for silver. Denoting either of these numbers by ξ , he gives the formula

$$\int_0^{\infty} I^2(t) dt = \xi (\text{Area})^2 \quad (6.12)$$

for calculating the time integral of the square of the current necessary to vaporize a fuse of a given cross-sectional area.

Of interest to us is the maximum diameter fuse that will be vaporized by an EMP-induced current. We shall consider in turn the currents given by Eq. (3.11) in the primary circuit of a current transformer, by Eq. (3.13) in the secondary circuit of a current transformer, and by Eq. (4.16) in a loop circuit with $r_1 = 0.6$ m and $r_2 = 0.159$ cm. The time integrals of the squares of these currents are, respectively

$$\text{C.T. Primary: } \int_0^{\infty} i^2(t) dt = \frac{(b-a)^2 I^2}{2ab(b+a)} = 8.08 \text{ amp}^2 \cdot \text{sec} \quad (6.13a)$$

$$\text{C.T. Secondary: } \int_0^{\infty} I^2(t) dt = \frac{(b-a)^2 I^2 I_0^2}{2ab(b+a)} = 0.505 \times 10^{-2} \text{ amp}^2 \cdot \text{sec} \quad (6.13b)$$

$$\begin{aligned} \text{Loop Circuit: } \int_0^{\infty} I^2(t) dt &= \frac{(\beta - \alpha)^2}{2\alpha\beta(\beta + \alpha)} \left(\frac{\mu_0 \pi r_1^2 E_0}{\eta L} \right)^2 \\ &= 0.571 \times 10^{-3} \text{ amp}^2 \cdot \text{sec} . \end{aligned} \quad (6.13c)$$

Expressing the cross-sectional area of the fuse wire in terms of its diameter (Area = $\pi d^2/4$), we solve Eq. (6.12) for the maximum fuse diameter

$$d = \left(\frac{4}{\pi} \right)^{1/2} \left(\frac{1}{\xi} \int_0^{\infty} I^2(t) dt \right)^{1/4} . \quad (6.14)$$

Substituting in turn Eqs. (6.13a), (6.13b), and (6.13c) into this expression, we get the results summarized in Table 6.2.

Table 6.2. Maximum Fuse Diameters to Interrupt EMP Currents

Circuit	Silver $d_{\max}$ (mil)	Copper $d_{\max}$ (mil)
Current Transformer Primary Eq. (3.11)	4.5	4.1
Current Transformer Secondary Eq. (3.13)	0.70	0.64
Loop $r_1 = 0.6$ m, $r_2 = 0.159$ cm Eq. (4.16)	0.41	0.37

It is unlikely that copper and silver fuse wires are ever constructed with diameters this small. We may thus safely conclude that copper and silver fuse wires will not interrupt the EMP-induced currents we have considered in this report.

6.9 Recommendation and Discussion of Countermeasures

Based on the analysis presented in this and preceding chapters, we recommend the following countermeasures:

- (i) Avoid routing wiring along paths which form loops,
- (ii) Use circuits with high intrinsic resistance when using semiconducting circuit elements,
- (iii) Eliminate openings in control houses larger than one meter on a side,
- (iv) Use heavy duty surge protectors,
- (v) When employing redundant relaying, run the wiring of the redundant relays in different directions, and
- (vi) Use logic circuitry to prevent false operation.

In what follows, each of these will be discussed in turn.

(i) In Chapter 4, Eqs. (4.13) and (4.16), we analyzed the dependence of the voltage and current induced in a loop circuit by an EMP incident along the most adverse direction. It is clear from this analysis that the voltage and current increase with increasing diameter of the circuit. As a countermeasure, we recommend that wiring be routed through continuous ferromagnetic conduit grounded at both ends. A technique similar to this has already proved itself in reducing the transient surge voltages induced in supervisory control circuits by the fields associated with the switching of high voltage power lines.¹⁵

(ii) Equation (4.3) expresses the relationship between the current in a loop circuit and the incident electromagnetic field. If a resistor R is included in the loop, this equation becomes

$$L \frac{dI}{dt} + RI = \pi r_1^2 \frac{dB}{dt} \quad . \quad (6.15)$$

If the resistor is large enough, the voltage drop RI across it exceeds

in value the term $L \, dI/dt$, so that Eq. (6.5) may be solved for the current

$$I(t) = \frac{\pi r_1^2}{R} \frac{dB}{dt} = - \frac{\mu_0 \pi r_1^2 E_0}{rR} (\alpha e^{-\alpha t} - \beta e^{-\beta t}) \quad (6.16)$$

This current takes on a maximum of

$$I_{\max} = \frac{\mu_0 \pi r_1^2 E_0 \beta}{rR} \quad (6.17)$$

at $t = 0$. Forming the ratio of this maximum current to that expressed by Eq. (4.17), we find

$$I_{\max} = \frac{\beta L}{R} I_{\max}^{ss} = \frac{1180 \text{ ohms}}{R} I_{\max}^{ss}, \quad (6.18)$$

where we have assumed L is that associated with the larger of the two loops considered in Chapter 4, namely $r_1 = 0.6 \text{ m}$, $r_2 = 0.159 \text{ cm}$. Hence, the resistance inserted in the circuit should be large compared with 1180 ohms if it is to limit the current to any appreciable extent. Instead of being dissipated across the junction in the diode, the pulse energy is dissipated in the resistor. This is preferable, since good composition resistors can withstand pulse powers of better than 10,000 times their power rating for microsecond pulses.¹²

(iii) Earlier in this chapter, we discussed the consequences of the fields penetrating windows and doors in control houses for circuits containing semiconductor diodes. It was apparent, that insignificant EMP fields penetrate windows if the ratio of the height of the window to the distance to the circuit in question was no larger than 1:3. By contrast, the EMP leaked by doors was enough to cause burnout of the more sensitive diodes. Hence, we recommend that doors contain no non-ferromagnetic sections larger than 1 meter on a side (equivalent to a window) and be kept shut as much as possible.

(iv) Having shown the vulnerability of semiconducting diodes, we recommend the use of non-semiconducting surge protectors. One such is the Westinghouse WL-642, gas-filled protector tube. This tube contains in a glass envelope three electrodes which can be connected, respectively, to either side of a circuit and to ground. The design is thus effective in reducing both common mode and differential mode transients. The gas is seeded with a small amount of radium which makes ionization continuously available to start a discharge when a voltage appears across any pair of the electrodes.

(v) We have seen in our discussion of the relaying of the Dixie Substation in Chapter 2 that considerable redundancy is typical of the supervisory control of a power substation. Also in Chapter 4, we considered the effect of the EMP polarization on the voltage and current induced in a circuit by the EMP. If the wiring for relays which perform the same function is separated and routed through separate cable trays or conduits in different directions, damage to all of the relays is less likely because of the geometric factors expressed in Eqs. (4.21) and (4.22) will be different for each relay.

(vi) Of the five countermeasures discussed so far in this section, all are effective in reducing the voltages and currents induced by EMP in control circuits. None guarantee, however, that the signals induced will be so low as not to be confused with the ordinary signals on which the circuit is designed to operate. Countermeasure (iv), for instance, shunts the EMP currents induced in a circuit to ground. Despite the existence of the radium, the WL-642 tube requires a finite time to turn on. During this time, part of the EMP-induced signal will pass the tube and continue on toward the load. This part of the EMP-induced signal might be confused with an ordinary operating signal by a "static" relay of the type considered in Section 2.2 of Chapter 2, for example.

This relay, however, may be protected from such an eventuality by the addition of transistors Q_4 and Q_5 wired as shown in Fig. 6.3. As might be imagined, EMP could be expected to induce signals at all three inputs 1-3. In the absence of Q_4 and Q_5 , as discussed in Section 2.2,

Fig. 6.3. Schematic of "Static Relay" Protection Against False Signals.

this could put both Q_1 and Q_2 into conducting states so that a current is drawn through Q_3 producing an output voltage across R_3 . With Q_4 and Q_5 in the circuit, the EMP, of course, could be expected to produce a signal at Input 4 also, but this would turn on either Q_4 or Q_5 depending upon whether the signal were positive or negative with respect to the Reference. In either case, this has the effect of shorting the signal at the base of Q_2 to the Reference, thereby shutting off Q_2 . If Q_2 is not conducting, then there is no signal at the base of Q_3 and no output voltage across R_3 . The "static" relay fails to respond to the EMP.

The question now arises as to what the EMP sensing element connected to Input 4 should be. One possibility is an omnidirectional antenna, which would sense EMP traveling from any direction. This, however, might also be sensitive to electromagnetic noise so much so that the relay would be driven into a nonoperating state most of the time. Assuming that the job of the "static" relay is to monitor power lines for legitimate faults, a more discriminating connection would be to a current transformer whose primary winding was a section of transmission line carrying no power current. There could never be a fault on this dead line. Such a line then is sensitive only to noise and EMP. Since the electromagnetic noise or EMP pick up would be the same on the dead line as on lines carrying power, the relay would be equally sensitive at all inputs to these signals. In this way, the relay could be shut down, as it should be, both by noise signals at all inputs or EMP signals at all inputs.

Even better EMP protection for supervisory control equipment is obtained by having protective devices as described in (iv) on all incoming control cabling and having the equipment wired to discriminate against incoming EMP signals as described in (vi).

6.10 Relative Importance of EMP Protection for the Supervisory Control Circuitry vs the Power Circuitry

It is important to recognize that the object of EMP protection is to maintain the flow of power. As we pointed out in our historical

survey in Chapter 1, supervisory control was for many years greatly inferior in its capabilities compared with its state today. During this earlier period, power was distributed nonetheless satisfactorily for the needs of the day. If the power circuitry of today survived a nuclear attack and the supervisory control equipment did not, the situation of this earlier period might again apply. Power would continue to be distributed as before the attack, but the luxury of being able to monitor the system and change its operating state remotely might be removed. Subject to the restrictions imposed by fallout, even this inconvenience could be removed by operation of substations by men communicating by radio, telephone, or personal contact.

It would seem then that the real hazard associated with the supervisory control circuitry is that it could confuse the incoming EMP with legitimate operating signals and change the state of the system such as to make it unstable by disturbing the balance between generation and load, for example. Countermeasure (vi) of the previous section is designed specifically to prevent this situation. As computer control of power systems becomes more widespread, this countermeasure could easily be superseded by a central computer programmed to distinguish the electromagnetic environment of a nuclear attack from the most complex of the ordinary day-to-day operating conditions. For such a system to be reliable, however, the computer itself would have to be well protected against EMP.

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APPENDIX A

SOME MATHEMATICAL DETAILS CONCERNING SHIELDING

A.1 Justification of the Equivalent Delta Function Method

In order to calculate the attenuation of the EMP by the walls of a typical control house, we assumed in Chapter 5 that the time dependence of the EMP was that of a delta function and equated the area under the incident EMP to the amplitude of the delta function. In this section, we shall justify this approximation.

The Fourier transform (frequency spectrum) $\bar{F}(\omega)$ of a function $f(t)$ is defined by the integral

$$\bar{F}(\omega) = \int_{-\infty}^{+\infty} e^{-j\omega t} f(t) dt \quad . \quad (A.1)$$

The quantity $\omega = 2\pi\nu$ is the frequency in radians and ν the frequency in Hz. Although a physically measurable value of ω must be real, due to the complex number j in the definition, $F(\omega)$ can assume complex values. As an example, we may compute the frequency spectrum of the EMP given by Eq. (1.1)

$$\begin{aligned} \bar{E}(\omega) &= E_0 \int_0^{\infty} (e^{-\alpha t} - e^{-\beta t}) e^{-j\omega t} dt = E_0 \left[\left(\frac{1}{j\omega + \alpha} \right) - \left(\frac{1}{j\omega + \beta} \right) \right] \\ &= E_0 \left[\left(\frac{e^{-j \tan^{-1} \frac{\omega}{\alpha}}}{\sqrt{\omega^2 + \alpha^2}} \right) - \left(\frac{e^{-j \tan^{-1} \frac{\omega}{\beta}}}{\sqrt{\omega^2 + \beta^2}} \right) \right] \quad . \quad (A.2) \end{aligned}$$

The integral here is evaluated only between 0 and ∞ because the EMP is zero for $t < 0$.

Let $g(t)$ be an arbitrary function, then the delta function has the property that

$$\int_{-\infty}^{\infty} \delta(t) g(t) dt = g(0) \quad . \quad (A.3)$$

From this property and the definition of the Fourier transform [Eq. (A.1)], the frequency spectrum of the delta function is given by

$$\bar{\delta}(\omega) = \int_{-\infty}^{+\infty} e^{-j\omega t} \delta(t) dt = 1 \quad . \quad (A.4)$$

The delta function frequency spectrum is a constant independent of frequency with zero imaginary part. This means that all frequencies from zero to infinity are equally represented in the delta function with equal amplitude and phase.

The frequency spectrum of the EMP, Eq. (A.2), becomes nearly independent of frequency for frequencies less than $\omega = 1.5 \times 10^6 \text{ sec}^{-1}$. In this frequency range the spectrum of the EMP like that of a delta function is constant. As was mentioned in Chapter 5 and will be proved below, the walls of the control house strongly attenuate frequencies above 100 Hz; thus, for the frequency components less than 100 Hz which significantly penetrate the control house, the frequency spectrum of the EMP has nearly constant amplitude with zero phase, just as the frequency spectrum of a delta function. The frequency spectra can be made exactly equal at $\omega = 0$ if we multiply the delta function by $\bar{E}(0) = E_0 \left(\frac{1}{\alpha} - \frac{1}{\beta} \right)$. Thus if only the low frequency part of the EMP spectrum is important, we may approximate

$$\bar{E}(\omega) = E_0 \left(\frac{1}{\alpha} - \frac{1}{\beta} \right) \bar{\delta}(\omega) \quad (A.5a)$$

so that

$$\bar{E}(t) = \bar{E}(0) \delta(t) = E_0 \left(\frac{1}{\alpha} - \frac{1}{\beta} \right) \delta(t) \simeq \frac{E_0}{\alpha} \delta(t) \quad . \quad (A.5b)$$

A.2 Approximate Forms of $T_H(\omega)$

The transfer function $T_H(\omega)$ was defined in Section 5.2 of Chapter 5. We shall study the behavior of this function for the cases $\Delta_1/\delta < 1$ and $\Delta_1/\delta > 1$.

First, the case $\Delta_1/\delta > 1$. As shown in Section 5.2, this corresponds to frequencies $\omega \geq 100$ Hz. The sine and the cosine in Eq. (5.2a) may be approximated as

$$\sin k_2 \Delta_1 = \sin \frac{\Delta_1}{\delta} (1 - j) \approx \frac{1}{2j} e^{j \frac{\Delta_1}{\delta} (1-j)} \quad (\text{A.6a})$$

$$\cos k_2 \Delta_1 = \cos \frac{\Delta_1}{\delta} (1 - j) \approx \frac{1}{2} e^{j \frac{\Delta_1}{\delta} (1-j)} \quad (\text{A.6b})$$

so that $T_H(\omega)$ becomes

$$T_H(\omega) \approx \frac{e^{-j \frac{\Delta_1}{\delta} (1-j)}}{\frac{1}{2} - \frac{K}{2} \cdot \frac{1}{2j}} = \frac{4e^{-j \frac{\Delta_1}{\delta} (1-j)}}{2 + jK} \quad (\text{A.7})$$

Substituting Eq. (5.2c) into Eq. (5.2b), we find for K

$$K = \frac{k_2 \Delta_2}{\mu} = \frac{\Delta_2}{\mu \delta} (1 - j) = x(1 - j) \quad , \quad (\text{A.8a})$$

where

$$x \equiv \frac{\Delta_2}{\mu \delta} \quad . \quad (\text{A.8b})$$

Now using Eq. (5.1), we find that the quantity x is greater than unity for any frequency

$$\nu \geq \frac{\mu \rho}{\pi \Delta_2^2 \mu_0} = \frac{\mu}{33} \quad , \quad (\text{A.9})$$

which for the largest value of μ shown in Fig. 5.1 is 30 Hz. At the frequencies $\nu \geq 100$ Hz at which $\Delta_1/\delta > 1$, we are justified in taking $x \gg 1$, so that Eq. (A.7) becomes

$$T_H(\nu) \simeq \frac{2\sqrt{2}}{x} \exp\left(-\frac{\Delta_1}{\delta}\right) \cdot \exp j\left(-\frac{\Delta_1}{\delta} - \tan^{-1} \frac{x}{x+2}\right) \quad . \quad (\text{A.10})$$

The amplitude of $T_H(\nu)$, which is the absolute value of Eq. (A.10),

$$\left| T_H(\nu) \right| \simeq \frac{2\sqrt{2}}{x} e^{-\Delta_1/\delta} \quad (\text{A.11})$$

is a steadily decreasing function as ν increases. This justifies our contention that high frequencies play a negligible part in the EMP signal which penetrates the control house.

For the case $\Delta_1/\delta < 1$, we begin again with Eq. (5.2a), but this time approximate the sine and the cosine, respectively, by

$$\sin k_2 \Delta_1 = \sin \frac{\Delta_1}{\delta} (1 - j) \simeq \frac{\Delta_1}{\delta} (1 - j) \quad (\text{A.12a})$$

$$\cos k_2 \Delta_1 = \cos \frac{\Delta_1}{\delta} (1 - j) \simeq 1 \quad . \quad (\text{A.12b})$$

We substitute these expressions plus Eqs. (A.8a) for K and (5.1) for δ to obtain

$$\begin{aligned} T_H(\nu) &\simeq \frac{1}{1 - \frac{K}{2} \frac{\Delta_1}{\delta} (1 - j)} = \frac{1}{1 - \frac{1}{2} \frac{\Delta_2}{\mu \delta} (1 - j) \Delta_1 (1 - j)} \\ &= \frac{1}{\sqrt{1 + \left(\frac{\pi \mu_0 \Delta_1 \Delta_2}{\rho} \right)^2}} e^{-j \frac{\pi \mu_0 \Delta_1 \Delta_2}{\rho} \nu} \quad , \end{aligned} \quad (\text{A.13})$$

The amplitude of $T_H(\omega)$ is

$$|T_H(\omega)| \approx \frac{1}{\sqrt{1 + \left(\frac{\pi\mu_0\Lambda_1\Lambda_2}{\rho}\right)^2 \omega^2}} \quad , \quad (\text{A.14})$$

which is a slowly varying function of ω for

$$\omega < \frac{\rho}{\pi\mu_0\Lambda_1\Lambda_2} = 55 \text{ Hz} \quad . \quad (\text{A.15})$$

APPENDIX B

ANALYSIS OF THE APPROXIMATIONS OF SECTION 6.5

We shall consider the current transformer to be a current source and use its Norton Equivalent Circuit in Fig. 3.4b. The impedance $Z(\omega)$ is just that given by Eqs. (3.7). The current source $i(t)$ is that given by Eq. (3.13). Summarizing,

$$Z(\omega) = Z_c \frac{j\omega}{(j\omega + c)^2} \quad (\text{B.1})$$

$$I(t) = I I_0 (e^{-at} - e^{-bt}) \quad (\text{B.2})$$

Given a function $f(t)$ we may define its Laplace transform $\bar{f}$

$$\bar{f}(s) = \int_0^{\infty} f(t) e^{st} ds \quad (\text{B.3})$$

from which $f(t)$ may be regained by evaluating

$$f(t) = \frac{1}{2\pi j} \int_{\xi - j\infty}^{\xi + j\infty} \bar{f}(s) e^{st} ds \quad (\text{B.4})$$

where the path of integration in the complex s -plane is to the right of any singularities of $\bar{f}(s)$ (i.e., ξ is real and greater than the real parts of any of the singularities).

Applying Eq. (B.3) to Eq. (B.2) we find

$$\bar{I}(s) = I_0 I \left(\frac{1}{s+a} - \frac{1}{s+b} \right) \quad (\text{B.5})$$

For sources such as Eq. (B.2) which are defined to be identically zero when $t < 0$, it is possible to make the identification $s = j\omega$ in passing from the Fourier to the Laplace transform so that we may validly rewrite Eq. (B.1) as

$$Z(s) = Z_0 \frac{s}{(s + c)^2} \quad . \quad (B.6)$$

Considering for the moment a particular value of the variable s , the current $\bar{I}(s)$ from the source in the Norton's Equivalent Circuit divides, part going through the impedance $Z(s)$ and part through the load impedance Z , which we take to be independent of s (see Fig. B.1). The division is such that the current $\bar{I}_L(s)$ through the load may be calculated from simple circuit considerations to be

$$\bar{I}_L(s) = \frac{Z(s)}{Z(s) + Z} \bar{I}(s) \quad . \quad (B.7)$$

Applying the inversion theorem of Eq. (B.4) to (B.7), we get an integral specifying the time dependence of the load current

$$I_L(t) = \frac{1}{2\pi j} \int_{\xi - j\infty}^{\xi + j\infty} \frac{Z(s)}{Z(s) + Z} \bar{I}(s) e^{st} ds \quad . \quad (B.8)$$

Our approximations involve this integral.

On substituting Eqs. (B.5) and (B.6) into (B.8), we find

$$I_L(t) = \frac{1}{2\pi j} \int_{\eta - j\infty}^{\xi - j\infty} \frac{(Z_0/Z)s}{(s + c)^2 + (Z_0/Z)s} \left(\frac{1}{s + a} - \frac{1}{s + b} \right) e^{-st} ds \quad . \quad (B.9)$$

The poles (zeros of the denominator) of the integrand of Eq. (B.9) determine the decay constants of the current through the load impedance.

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Fig. B.1. Norton Equivalent Circuit with Load Impedance Z .

The poles due to the parallel combination of the source impedance and the load impedance are found by finding the roots of the quadratic equation

$$(s + c)^2 + \frac{Z_0}{Z} s = 0 \quad . \quad (B.10)$$

These are

$$s = \frac{1}{2} \left[-\left(2c + \frac{Z_0}{Z}\right) \pm \sqrt{\left(2c + \frac{Z_0}{Z}\right)^2 - 4c^2} \right] \quad . \quad (B.11)$$

Expanding the square root in the above equation in a binomial series in the variable Zc/Z_0 , we are led to useful approximations to the roots, namely

$$s_1 = -3c \left(\frac{Zc}{Z_0} \right) \quad (B.12)$$

$$s_2 = -c \left(\frac{Z_0}{Zc} \right) \left(1 + 2 \frac{Zc}{Z_0} \right) \quad . \quad (B.13)$$

These approximations are valid if

$$Z < Z_0 c^{-1} \quad . \quad (B.14)$$

The poles contributed by $\bar{I}(s)$ are located at

$$s_3 = -a \quad (B.15)$$

$$s_4 = -b \quad . \quad (B.16)$$

Evaluating the residues of the integral in (B.8) and simplifying the result

slightly algebraically, we have

$$I_L(t) = I I_0 \frac{Z_0}{Z} \left[\frac{s_1(b-a)}{(a-s_1)(b-s_1)} \frac{e^{-s_1 t}}{(s_2-s_1)} - \frac{s_2(b-a)}{(s_2-a)(s_2-b)} \frac{e^{-s_2 t}}{(s_2-s_1)} + \frac{ae^{-at}}{(a-s_1)(s_2-a)} - \frac{be^{-bt}}{(b-s_1)(s_2-b)} \right] . \quad (B.17)$$

For the current transformer and transmission line current in question, we have the numerical values

$$\begin{aligned} a &= 1.35 \times 10^6 \text{ sec}^{-1} \\ b &= 2.15 \times 10^7 \text{ sec}^{-1} \\ c &= 1.81 \times 10^6 \text{ sec}^{-1} \\ Z_0 &= 0.5 \times 10^{10} \text{ ohm} \cdot \text{sec}^{-1} . \end{aligned} \quad (B.18)$$

There is a value of the ratio Z_c/Z_0 such that (B.14) is true and

$$|s_1| = 3c \left(\frac{Z_c}{Z_0} \right) < a \sim c < b < |s_2| . \quad (B.19)$$

For (B.14) and (B.19) to hold it is sufficient that

$$\frac{Z_c}{Z_0} < \frac{a}{3c} < 1 \quad (B.20a)$$

or

$$Z < \frac{a}{3c^2} Z_0 = 686 \text{ ohms} . \quad (B.20b)$$

In this case, we may expand (B.17) in powers of s_1/a , s_1/b , s_1/s_2 , a/s_2 , b/s_2 , and a/b keeping only terms of the order of unity and obtain our desired result, namely

$$I_L(t) = I I_0 \left(e^{-at} - e^{-bt} \right) . \quad (B.21)$$

APPENDIX C

ANALYSIS OF THE APPROXIMATION OF SECTION 6.6

From the analysis of Section 4.2, we may write the equation for the current in a loop containing a diode as

$$L \frac{dI}{dt} + ZI = - \frac{\mu_0 \pi r_1^2 E_0}{\eta} \left(\alpha e^{-\alpha t} - \beta e^{-\beta t} \right) , \quad (C.1)$$

where Z is the dynamic impedance of the diode. In what follows we shall consider Z to be independent of I .

Letting $\lambda = Z/L$, the solution to Eq. (C.1) is given by

$$I(t) = - \left(\frac{\mu_0 \pi r_1^2 E_0}{\eta L} \right) e^{-\lambda t} \int_0^t \left(\alpha e^{-\alpha \tau} - \beta e^{-\beta \tau} \right) e^{\lambda \tau} d\tau . \quad (C.2)$$

Evaluation of the integral in (C.2) gives

$$I(t) = - \frac{\mu_0 \pi b^2 E_0}{\eta L} \left[- \frac{\alpha e^{-\alpha t}}{\alpha - \lambda} + \frac{\beta e^{-\beta t}}{\beta - \lambda} + \frac{\alpha e^{-\lambda t}}{\alpha - \lambda} - \frac{\beta e^{-\lambda t}}{\beta - \lambda} \right] . \quad (C.3)$$

Assuming that

$$\lambda < \alpha, \beta \quad (C.4)$$

we may expand the denominators in (C.3) in a binomial series in λ/α and λ/β and obtain to order λ

$$I(t) \approx \frac{\mu_0 \pi b^2 E_0}{\eta L} \left(e^{-\alpha t} - e^{-\beta t} \right) + \frac{\mu_0 \pi b^2 E_0}{\eta L} \lambda \left(\frac{e^{-\alpha t}}{\alpha} + \frac{e^{-\beta t}}{\beta} + \left(\frac{\beta - \alpha}{\beta \alpha} \right) e^{-\lambda t} \right) . \quad (C.5)$$

The leading term of this expansion is just Eq. (4.16).

We may use Eq. (4.7) to calculate the inductances of the loops described by the parameters in Eq. (4.19). These are

$$L = \begin{cases} 4.53 \mu\text{H} & r_1 = 0.6 \text{ m} & r_2 = 2.159 \text{ cm} \\ 0.622 \mu\text{H} & r_1 = 7.62 \text{ cm} & r_2 = 0.125 \text{ mm} \end{cases} \quad (\text{C.6})$$

Eq. (C.4) then specifies that

$$Z < \alpha L = \begin{cases} 6.79 \text{ ohms} & r_1 = 0.6 \text{ m} & r_2 = 0.159 \text{ cm} \\ 0.93 \text{ ohms} & r_1 = 7.62 \text{ cm} & r_2 = 0.125 \text{ mm} \end{cases} \quad (\text{C.7})$$