IEEE FOR 1972 JACC

A DIRECT ITERATIVE DECOUPLING PROCEDURE FOR
MULTIVARIABLE CONTROL SYSTEM DESIGN

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ABSTRACT

A design technique has been developed, for a limited class of systems, which provides a state variable control design resulting in a decoupled, conditioned system. A controller was designed for both linear and non-linear fourth order models of a nuclear rocket engine. A direct non-linear control synthesis was performed. The non-linear control exhibited constant decoupled performance in control of the highly coupled non-linear engine model and exhibited a stable "type two" control response.

MASTERINTRODUCTION

A design technique has been developed to synthesize a multiple input-output state variable controller with decoupled characteristics of selected outputs. The basis of the technique is to identify command paths and then decouple these paths by selective feedback, input equivalence transformation and pole placement. The process successively reduces the equivalent order of the system by conditioning decoupled input states to where they may be considered algebraic.

For a system described by a set of n differential equations of the form:

$$\dot{\underline{X}} = \underline{A} \underline{X} + \underline{B} \underline{U}$$

with

$\underline{A}$ is an $n \times n$ matrix and

$\underline{B}$ is an $n \times r$ matrix

where

r is the number of inputs

The objective of the technique is to obtain an equivalent sectionally diagonalized system through feedback. For example, for a two input (and two prime outputs) system:

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$$\dot{\underline{X}} = \begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{54} & a_{55} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{64} & a_{65} & a_{66} & 0 & 0 \\ a_{71} & a_{72} & a_{73} & a_{74} & a_{75} & a_{76} & a_{77} & a_{78} \\ a_{81} & a_{82} & a_{83} & a_{84} & a_{85} & a_{86} & a_{87} & a_{88} \end{bmatrix} \underline{X} + \begin{bmatrix} b_{11}^- & 0 \\ b_{21}^- & 0 \\ b_{31}^- & 0 \\ 0 & b_{42}^- \\ 0 & b_{52}^- \\ 0 & b_{62}^- \\ b_{71}^- & b_{72}^- \\ b_{81}^- & b_{82}^- \end{bmatrix} \underline{U}$$

where X_3 and X_6 are the prime variables to be controlled. "Equivalent" inputs b_{11}^- , b_{21}^- and b_{31}^- may individually have values or be zero but b_{12}^- , b_{22}^- and b_{32}^- must be zero. The same holds for the b_{42}^- , b_{52}^- , b_{62}^- and b_{41}^- , b_{51}^- , b_{61}^- relation.

It can be seen by observation that u_1 will effect X_3 and not X_6 and conversely u_2 will effect X_6 and not X_3 .

DISCUSSION

The technique basically consists of:

1. Choose prime parameters (X_p) to be controlled (equal to the number of input variables). For simplicity, state variables are assumed available as outputs.
2. Identify the input variable (U_x) having greatest effect on each prime output.
3. Identify basic structure, i.e.:
 - a. Start from each output X_p and build chain back to U_x by the most direct route of highest gain.*
 - b. Order the state variables in each section, from the ones directly affected by the inputs to the ones directly affecting the output.

*The criteria for this organization has been partially worked out but requires further effort to cover all cases.

4. If one input directly affects the first state of more than one section the procedure is as follows:

a. Calculate the gains for an equivalent input which will separate feedbacks to the individual sections; i.e., for the preceeding example, if:

$$\underline{B} = \begin{bmatrix} b_{11} & b_{12} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ b_{41} & b_{42} \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$$

$$\text{let: } b_{12}^- = -\frac{b_{12}}{b_{11}} \quad b_{11} + b_{12}$$

$$\text{and: } b_{11}^- = -\frac{b_{41}}{b_{42}} \quad b_{12}^- + b_{11}$$

The gain for feedback to the first section (U_1') then is one to U_1 and $\frac{-b_{41}}{b_{42}}$ to U_j . The gains for the second section are one to U_1 and $\frac{-b_{12}}{b_{11}}$ to U_2 . The gains to calculate feedback are defined by the b_{ij} .

b. Feedback for the first section, all a_{1j} (except a_{11}) times $\frac{-1}{b_{11}^-}$ and for the second section all a_{4j} (except a_{44}) times $\frac{-1}{b_{42}^-}$.

5. The first state in each chain should then be conditioned to at least five times the desired dominant root location times the square of the number of states remaining in the chain.

6. The system can then be considered as one which has the first state in each chain as a gain from its input to the states affected by it, which is added to or subtracted from direct inputs to the various states. This is a new system which is reduced in order by one state for each chain.

7. The steps of 4, 5, and 6 above are then repeated as required in each chain until the state identified as the prime variable is reached. The prime variables are then conditioned to the desired root location.

An evaluation of the application of this design technique was made by designing a controller for a linearized fourth order Smith-Stenning model.*

The scaled equations for this model are:

1. $\dot{n} = 65 [\delta n - n + c]$
2. $\dot{c} = 0.1 [n - c]$
3. $\dot{T} = n - P \sqrt{T}$
4. $\dot{P} = 0.4 [V P \sqrt{T} - \frac{P^2}{\sqrt{T}}]$
5. $\delta = \delta_D - 0.001 \sqrt{T} + 0.3 \frac{P}{T}$

Where

- n = Nuclear Power
- c = Delayed Neutrons
- T = Chamber Temperature
- P = Chamber Pressure
- δ = Reactivity
- V = Turbine Control Valve (Input)
- δ_D = Control Drum Reactivity (Input)

The model was linearized at an operating point which gives the following linear equations:

$$\begin{aligned}\dot{n} &= -65n + 65c - 19.5T + 19.5P + 65\delta_D \\ \dot{c} &= .1n - .1c \\ \dot{T} &= n - .5T - P \\ \dot{P} &= .4T - .4P + .4V\end{aligned}$$

Even this linearized version of the model is highly coupled where motion of one of the inputs (V or δ_D) causes significant change in both of the prime variables (T & P) of the system, i.e., (Figure 1) a change in V from V_1 to V_2 causes a 40% change in P and a 40% change in T along a constant δ_D , and a change from δ_{D1} to δ_{D2} causes a 60% change in T and a 30% change in P along a constant V .

*Open Loop Stability and Response of Nuclear Rocket Engines, Harold P. Smith, Jr. and Alan H. Stenning.

FIGURE 1

SMITH-STIRLING MODEL INPUT/OUTPUT
RELATION LINEARIZED AT 100% OF SCALE

When organized as required, the equations give the following matrix description where T and P are chosen as the prime outputs:

$$\begin{bmatrix} \dot{n} \\ \dot{T} \\ \dot{c} \\ \hline \dot{P} \end{bmatrix} = \begin{bmatrix} -65 & -19.5 & 65 & 19.5 \\ 1 & -.5 & 0 & -1 \\ .1 & 0 & -.1 & 0 \\ \hline 0 & .4 & 0 & -.4 \end{bmatrix} \begin{bmatrix} N \\ T \\ c \\ \hline P \end{bmatrix} + \begin{bmatrix} 65 & 0 \\ 0 & 0 \\ 0 & 0 \\ \hline 0 & .4 \end{bmatrix} \begin{bmatrix} \delta_D \\ \hline V \end{bmatrix}$$

(The dashed line shows the two sections)

Since the inputs δ_D and V directly affect only single states $\dot{n}$ and $\dot{P}$ respectively, the other variables are removed from $\dot{n}$ and $\dot{P}$ by:

1. Feedback to δ_D of $\frac{19.5}{65} T$, $-\frac{65}{65} c$, $-19.5/65P$
2. Feedback to V of $-\frac{0.4}{0.4} T$

The new matrix description is:

$$\begin{bmatrix} \dot{n} \\ \dot{T} \\ \dot{c} \\ \hline \dot{P} \end{bmatrix} = \begin{bmatrix} -65 & 0 & 0 & 0 \\ 1 & -.5 & 0 & -1 \\ .1 & 0 & -.1 & 0 \\ \hline 0 & 0 & 0 & -.4 \end{bmatrix} \begin{bmatrix} n \\ T \\ c \\ \hline P \end{bmatrix} + \begin{bmatrix} 65 & 0 \\ 0 & 0 \\ 0 & 0 \\ \hline 0 & .4 \end{bmatrix} \begin{bmatrix} \delta_D \\ \hline V \end{bmatrix}$$

Since the second section is in the final form we work on the first section:

1. Condition pole of n; (want dominant roots at 6.28 radians), therefore the root at 65 radians is already greater than 10X desired root and no conditioning is required.

2. The new equivalent system is:

$$\begin{bmatrix} \dot{T} \\ \dot{c} \\ \hline \dot{P} \end{bmatrix} = \begin{bmatrix} -.5 & 0 & -1 \\ 0 & -.1 & 0 \\ \hline 0 & 0 & -.4 \end{bmatrix} \begin{bmatrix} T \\ c \\ \hline P \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ .1 & 0 \\ \hline 0 & .4 \end{bmatrix} \begin{bmatrix} \delta_D \\ \hline V \end{bmatrix}$$

Now again inputs from δ_D only affect the δ_D section and inputs from V only affect the V section and we can decouple:

1. Remove input to $\dot{T}$ from P by feedback of $1 \times P$ to δ_D .
2. No other feedbacks are necessary to achieve the final conditioning form since the prime parameters $\dot{P}$ and $\dot{T}$ are a function only of themselves and their respective inputs. The state c now has an input from P but since c does not influence P or T (due to the first feedback step) this is not of concern.

The equivalent system at this point is:

$$\begin{bmatrix} \dot{T} \\ \dot{c} \\ \dot{P} \end{bmatrix} = \begin{bmatrix} -.5 & 0 & 0 \\ 0 & -.1 & .1 \\ 0 & 0 & -.4 \end{bmatrix} \begin{bmatrix} T \\ c \\ P \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ .1 & 0 \\ 0 & .4 \end{bmatrix} \begin{bmatrix} \delta_D \\ V \end{bmatrix}$$

Now we are in position to condition the prime parameters:

1. Feedback of $-(6.28 - .5)T$ to δ_D places pole of T at -6.28 .
2. Feedback of $-\frac{(6.28 - .4)}{.4}P$ to V places pole of P at -6.28 .

The gains to each input, therefore, are:

1. $\delta_D = \left(\frac{19.5}{65} - 5.78\right) T - c + \left(\frac{-19.5}{65} + 1\right) P$
 $= \underline{\underline{-5.48T - c + 0.7 P}}$
2. $V = \underline{\underline{-1 T - 14.7 P}}$

Substituting these gains into the original linearized equations, we have:

$$\dot{n} = -65n + 65c - 19.5T + 19.5P + 65(-c - 5.48T + 0.7P) + 65\delta_{DD}$$

$$\dot{c} = .1n - .1c$$

$$\dot{T} = n - 15T - P$$

$$\dot{P} = .4T - .4P + .4(-T - 14.7P) + V_D$$

where δ_{DD} and V_D are demand inputs.

In matrix form this is

$$\begin{bmatrix} \dot{n} \\ \dot{T} \\ \dot{c} \\ \dot{P} \end{bmatrix} = \begin{bmatrix} -65 & -375.7 & 0 & +65 \\ 1 & -0.5 & 0 & -1 \\ .1 & 0 & -0.1 & 0 \\ 0 & 0 & 0 & -6.28 \end{bmatrix} \begin{bmatrix} n \\ T \\ c \\ P \end{bmatrix} + \begin{bmatrix} 65 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & .4 \end{bmatrix} \begin{bmatrix} \delta_{DD} \\ V_D \end{bmatrix}$$

By observation V_D only affects $\dot{P}$ and the root of P is 6.28 . Also by observation c does not affect any of the other states and it has its root at 0.1 .

Now solving for the roots of the system we find that the roots are:

$$S = -0.1, -6.28, -6.96, -58.5$$

Since the state c does not affect other states and it has its root at 0.1 this root will not appear in any other state transfer function. The fact that P does not affect T is less obvious, however, if we again make the assumption that the root of n at 65 is sufficiently high to be considered just a gain the system is:

$$\begin{bmatrix} \dot{T} \\ \dot{c} \\ \dot{P} \end{bmatrix} = \begin{bmatrix} -6.28 & 0 & 0 \\ -.578 & -0.1 & 0.1 \\ 0 & 0 & -6.28 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ .1 & 0 \\ 0 & .4 \end{bmatrix} \begin{bmatrix} \delta_{DD} \\ V_D \end{bmatrix}$$

While c is affected by both T and P as well as δ_{DD} it still does not affect T or P and the roots of T and P are at -6.28 . The actual location of the roots at -6.96 and -58.5 instead of -6.28 and -65 are the result of the approximation of considering the root at -65 to be at minus infinity.

These controller gains were implemented along with the linearized model on an analog computer and evaluated. The results of step demand signals to the two inputs are shown in Figure 2. As can be seen in Figure 2, the response to step inputs, equivalent to approximately 6% on each output, does result in a decoupled action with an approximate one cycle (pole at 6.28 radians) response.

Since some concern existed as to the response of this controller to internal disturbances in the plant, disturbances were input throughout the model for evaluation. To facilitate the evaluation an external integral control was added which generated a δ_D demand proportional to the integral of the error between a temperature demand setpoint and the measured T . An example of the results of this disturbance-input control is shown in Figure 3. In this case an external 10% step in power was added and the controller removed the effect of this disturbance as shown in Figure 3. The peak disturbance of T of 1.5% with essentially full recovery in one second shows very good response to disturbance inputs.

It is felt that this design technique has certain advantages, primarily due to the fact that the real parameters of the system are being manipulated and that at all times it is obvious what is being accomplished by each feedback. Zeroes of the system are cancelled by the poles which create them leading to a degree of insensitivity to pole shifting (although gains between states may shift, changing the cancellation). Limitations of the system are easily identified and this may lend itself to evaluation of tradeoffs where system limits are reached.

Perhaps most important is the fact that the initial evaluation shows that the technique may be used for direct non-linear control synthesis. The non-

FIGURE 2

STEPS IN P AND T DEMAND

LINEARIZED MODEL AND CONTROL

FIGURE 3

10% EXTERNAL STEP IN M

LINEARIZED MODEL AND CONTROL

linear equations for the Smith-Stenning engine model were manipulated to obtain a control design as follows:

$$\begin{aligned}\dot{n} &= 65n (\delta_D - 10^{-3} \sqrt{T} + 0.3 \frac{P}{T} - 1) + 65c \\ \dot{T} &= n - P \sqrt{T} \\ \dot{c} &= 0.1 n - 0.1 c \\ \dot{P} &= -0.4 \frac{P^2}{\sqrt{T}} + 0.4 VP \sqrt{T}\end{aligned}$$

Separate inputs

$$\begin{aligned}\dot{n} &= 65n (-10^{-3} \sqrt{T} + 0.3 \frac{P}{T} - 1) + 65 c & + 65 n \delta_D \\ \dot{T} &= n - P \sqrt{T} \\ \dot{c} &= 0.1 n - 0.1 c \\ \dot{P} &= -0.4 \frac{P^2}{\sqrt{T}} & + 0.4 VP \sqrt{T}\end{aligned}$$

First feedback: $-\frac{c}{n} - \frac{.3P}{T} + 10^{-3} \sqrt{T}$ to δ_D

$$\begin{aligned}\text{Now: } \dot{n} &= -65 n & + 65 n \delta_D \\ \dot{T} &= n - P \sqrt{T} \\ \dot{c} &= 0.1 n - 0.1 c \\ \dot{P} &= -0.4 \frac{P^2}{\sqrt{T}} & + 0.4 P \sqrt{T} V\end{aligned}$$

Now to simplify, modify additional inputs by a gain of:

$$\begin{aligned}\text{New input} &= \delta_{DD} \quad \text{Gain} = \frac{1}{n} \\ \text{New input} &= V_D \quad \text{Gain} = \frac{1}{P \sqrt{T}}\end{aligned}$$

$$\text{Then: } \dot{n} = 65 n + 65 \delta_{DD}$$

$$\dot{T} = n - P \sqrt{T}$$

$$\dot{c} = -0.1 n - 0.1 c$$

$$\dot{P} = 0.4 P^2 / \sqrt{T} + 0.4 V_D$$

$$\text{Now since } n = \frac{\delta_{DD}}{\frac{S}{65} + 1} \approx \delta_{DD} \text{ (for 6.82 response)}$$

New system:

$$\dot{T} = -P \sqrt{T} + \delta_{DD}$$

$$\dot{c} = -0.1 c + 0.1 \delta_{DD}$$

$$\dot{P} = -0.4 P^2 \sqrt{T} + 0.4 V_D$$

Now feedback:

$$P \sqrt{T} - K_D T \text{ to } \delta_{DD}$$

$$\text{and } P^2 / \sqrt{T} - K_D P \text{ to } V_D$$

where K_D is desired root location.

Call input to δ_{DD} and V_D (reference inputs) δ_{DDr} and V_{Dr}

Then:

$$\dot{T} = -K_D T + \delta_{DDr}$$

$$\dot{c} = -0.1 c + 0.1 (P \sqrt{T} - T + \delta_{DDr})$$

$$\dot{P} = -K_D P + V_{Dr}$$

K_D here was chosen as 6.28

This design was mechanized on an analog computer along with the non-linear engine model and tested. The response near the 100% level (where the linear design was done) is shown in Figure 4. As can be seen, by comparison with Figure 2, the response of T and P to steps are the same for the non-linear and linear

FIGURE 4
STEPS NEAR 100% WITH
NONLINEAR MODEL AND CONTROL

designs (i.e., approximately 1 hertz) with essentially complete decoupling of T and P. Figure 5 shows the results of steps at mid-scale showing the same basic response in T and P. The major changes in plant characteristics, however, can be seen from the much larger V and δ_D excursions required to achieve this response. Figure 6 shows a step in temperature near the bottom of the range. Again the response of T is nearly the same as before, however, one of the inputs (δ_D) is moving a great deal further and in fact reached saturation for a very short time. These results show that the non-linear control indeed gives the designed response over the range of operation (within the limits of input saturation) even though large changes are occurring in the plant.

A question existed as to the stability sensitivity of the non-linear control design to changes in system gains. Each feedback gain of the system was varied $\pm 20\%$ and step responses measured. No significant tendency toward instability was noted during these tests, some offset in operating point did of course occur. An outer integral loop on each of the prime parameters would be added in any final system to eliminate any offsets due to gain mismatches.

Transients (typical of the NERVA engine) were run with the non-linear control to evaluate velocity errors and general performance. Figure 7 shows a transient from the low end of operation to near 100%. As may be seen from the error (Te and Pe) channels very tight control is maintained. The velocity constants indicated for both P and T are approximately 5. An integral loop was added around the non-linear compensated system which was the integral of the difference between T and T demand. A transient run similar to the one just discussed for Figure 7 was then made with this integral loop active. The results of the run are shown in Figure 8. As can be seen the velocity errors in T are reduced to essentially zero and the minor mismatch of gain is

FIGURE 5

STLPS MID SCALE
NONLINEAR MODEL AND CONTROL

FIGURE 6

STEPS NEAR ZERO

NONLINEAR MODEL AND CONTROL

FIGURE 7

TRANSIENT RUN NONLINEAR MODEL

AND CONTROL - NO INTEGRAL LOOPS

FIGURE 8

TRANSIENT RUN NONLINEAR MODEL AND
CONTROL WITH EXTERNAL INTEGRAL LOOP ON T_e

eliminated. Peak T error, due to acceleration, is approximately 0.1%. The achievement of this "type two" response is considered very significant even for this fairly simple fourth order non-linear system.

Further effort is required to generalize the technique and to develop computer programs to handle the mathematics of higher order systems. In addition, a technique needs to be developed for system simplification to eliminate the need for using all system variables. Since only a limited number of types of systems have presently been evaluated, it is possible that some system types may present a problem for the technique.