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A GENERAL METHOD FOR COMPARING THERMAL PERFORMANCE
OF FUEL ELEMENT GEOMETRIES AND COOLANTS
FOR NON-BOILING REACTORS

by

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NOMENCLATURE AND ABBREVIATIONS

A_{fl}	Average cross-sectional flow area of core	ft^2
A_{fr}	Frontal area of core	ft^2
A_t	Heat transfer area of core	ft^2
c_p	Specific heat of coolant	$Btu/(lb)(F)$
D	Equivalent hydraulic diameter of flow passage = $4r_h$	ft
d_c	Diameter of core	ft
E_f	Unit flow-friction power	$ft\text{-}\#/(hr)(ft^2)$ or hp/ft^2 of surface area
#	Denotes pounds force in distinction to lb for pounds mass	
F	Denotes functional relationship	
f	Fanning friction factor	dimensionless
G	Mass velocity	$lb/(hr)(ft^2)$
g_c	Reciprocal proportionality in Newton's Second Law = $4.17 \times 10^8 (lb/\#)$	(ft/hr^2)
h	Heat transfer coefficient	$Btu/(hr)(ft^2)(F)$
ℓ_c	Length of core	ft
k	Thermal conductivity of coolant	$Btu/(hr)(ft^2)(F/ft)$
NP_r	Prandtl Number $\left(\frac{c_p/\mu}{k} \right)$	dimensionless
N_R	Reynolds Number $\left(\frac{DG}{\mu} \right)$	dimensionless
N_{St}	Stanton Number $\left(\frac{h}{G c_p} \right)$	dimensionless
NTU	Number of transfer units	dimensionless
P_f	Flow friction power of core	$kw, Btu/hr, hp$

P_q	Power Rating of core	kw, Btu/hr, hp
p	Porosity of Fuel element matrix	dimensionless
r_h	Hydraulic radius	ft
Δt_f	Bulk Temperature rise of coolant	F
V_c	Core volume	ft ³
W	Coolant mass flow rate	lb/hr
α	Surface area density of fuel element matrix	ft ² /ft ³
ρ	Density of coolant	lb/ft ³
θ	Local film temperature drop	F
θ_{avg}	Average film temperature drop in core	F
μ	Viscosity of coolant	lb/(hr)(ft)
$\underline{\underline{\Delta}}$	Denotes defining equation	

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A GENERAL METHOD FOR COMPARING THERMAL PERFORMANCE OF FUEL ELEMENT GEOMETRIES AND COOLANTS FOR NON-BOILING REACTORS

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ABSTRACT

A method is presented for comparing the influence of various fuel element geometries on reactor core shape for different specifications of thermal performance. These include reactor power, ideal pumping power as a fraction of reactor power, coolant used, and the average approach of coolant to fuel element surface temperature.

Illustrative examples are given for water, diphenyl, and carbon dioxide (at 450 psi) as coolants, and plate-channels (as in MTR), crossed wires, and sphere beds as fuel element geometries.

The method, in its most simple form, is useful primarily for survey-type thermal design studies and, as such, supplements rather than replaces the conventional, detailed methods currently employed in final core designs.

I. INTRODUCTION

The evaluation of thermal performance of a nuclear reactor cannot be carried out effectively using only heat transfer coefficient and flow pressure drop as criteria. Thermal performance must include a specification of pumping power. Moreover, core volume, or, conversely, power density in kw/liter, is not a sufficient criterion of suitability since the physics requirements demand core geometries approaching a sphere, a cube, or a right circular cylinder with length about equal to diameter.

This report presents a general method for calculating core dimensions as a function of pumping power for a given reactor, starting with the fluid properties and with the basic heat transfer and friction relationships of the fuel element heat transfer surface geometry. Illustrative examples are given for water, diphenyl, and carbon dioxide as coolants, and for plate channels (as in MTR), crossed wires and sphere beds, as fuel element

geometries. The effects on over-all geometry of such reactor variables as reactor power, and bulk fluid temperature rise, and such fuel element variables as hydraulic radius are also examined.

This method in its most simple form, is useful for survey-type studies primarily and, as such, supplements, rather than replaces, the conventional detailed thermal design methods currently employed in final core designs.

II. METHOD OF ANALYSIS

A. Development of Method

1. Geometrical Parameters

It is first necessary to define some parameters descriptive of the detailed geometry of the surface and then to relate these to the over-all core geometry.

The "surface" parameters are: hydraulic radius (r_h), area density (α), and porosity (p).

The core parameters are: core length (ℓ_c), frontal area (A_{fr}), flow area (A_{fl}), and heat transfer area (A_t).

The hydraulic radius for tube-type geometries is normally defined by:

$$r_h = \frac{A_{fl}}{\text{Wetted periphery}} \quad (1)$$

Multiplying both sides of Eq. (1) by ℓ_c we obtain a more general definition of r_h :

$$r_h = \frac{A_{fl} \ell_c}{A_t} \quad (1a)$$

which is applicable to matrix-type surfaces where flow-wetted periphery would be an ambiguous term.

Defining an average flow area as:

$$A_{fl} \triangleq p A_{fr} \quad (2)$$

Equation (1a) becomes:

$$r_h = \frac{p A_{fr} \ell_c}{A_t} \quad (1b)$$

However,

$$\alpha \triangleq \frac{A_t}{V_c} = \frac{A_t}{A_{fr} \ell_c} \quad (3)$$

and further substitution into Eq. (1b) yields

$$r_h = \frac{p}{\alpha} \quad (1c)$$

which will apply equally well to matrix-type and channel- or tube-type surfaces.

Rearrangement of Eq. (1a) yields

$$\frac{\ell_c}{r_h} = \frac{A_t}{A_{fl}} \quad (1d)$$

which relates the "slenderness ratio" of the flow passages, ℓ_c/r_h , to the core dimensions of total heat transfer area and average flow area.

2. Heat Transfer and Flow Friction Characteristics

a. Core Heat Transfer

A typical variation of fluid and surface temperature for an average flow path is shown in the illustration.

In a differential element of transfer area, dA_t , the differential heat transfer rate, dP_q , results in a coolant temperature rise, dt_f . The rate equation for this heat transfer is

$$dP_q = h\theta dA_t \quad (4)$$

and the energy balance is

$$dP_q = Wc_p dt_f \quad (5)$$

Note that the local flux, ϕ , is related to the heat transfer conditions by

$$\begin{aligned} \phi &\propto \frac{dP_q}{dA_t} = h\theta \\ &\propto \frac{dt_f}{dA_t} \end{aligned}$$

Combining these yields

$$\int_{t_f(\text{in})}^{t_f(\text{out})} \frac{dt_f}{\theta} = \int_0^{A_t} \frac{h}{Wc_p} dA_t \quad (6)$$

The left side of Eq. (6) is a measure of the heat transfer performance of the core. For the case of uniform heat generation together with constant h , the film temperature drop, θ , is constant and the fluid temperature, t_f , is linear with A ; thus

$$\int_{t_f(\text{in})}^{t_f(\text{out})} \frac{dt_f}{\theta} = \frac{t_f(\text{out}) - t_f(\text{in})}{\theta} = \frac{\Delta t_f}{\theta} \quad (7)$$

For the general case

$$\int_{t_f(\text{in})}^{t_f(\text{out})} \frac{dt_f}{\theta} = \frac{\Delta t_f}{\theta_{\text{avg}}} \quad (7a)$$

where this equation defines θ_{avg} , the mean approach of the coolant to fuel element surface temperature, or mean film temperature drop.

The grouping on the right side of Eq. (6) is termed the number of heat transfer units, NTU, a nondimensional expression of the "size" of the core from a heat transfer point of view:

$$\int_0^{A_t} \frac{h}{Wc_p} dA_t \triangleq \text{NTU} \quad (8)$$

This parallels the terminology used in chemical engineering design of packed towers for mass transfer processes.

In general, from Eqs. (8), (7a), and (6), since

$$\text{NTU} = \frac{\Delta t_f}{\theta_{\text{avg}}} \quad (9)$$

a large NTU means a small average film temperature difference, θ_{avg} , for a specified Δt_f . The latter, in turn, is related to the core power rating, P_q , and the coolant flow, W , by an integration of the energy equation (Eq. (5)):

$$P_q (\text{Btu/hr}) = Wc_p \Delta t_f \quad (10)$$

From the definition of NTU, one can develop the following.

$$\text{NTU} = \int_0^{A_t} \frac{h}{Wc_p} dA_t = \left(\frac{h}{Gc_p} \right) \left(\frac{A_t}{A_{fl}} \right) \quad (11)$$

Substitution from Eq. (1d) yields

$$\text{NTU} = N_{St} \frac{l_c}{r_h} \quad (11a)$$

where the Stanton Number is a function of N_R and N_{Pr} and is established by experiment.*

*The correlation of N_{St} versus N_{Pr} and N_R constitutes the heat transfer characteristic of the surface just as the correlation of the Fanning friction f versus N_R constitutes the experimentally established friction characteristic.

Rearranging Eq. (11a):

$$\frac{N_{St}}{r_h} = \frac{NTU}{\ell_c} \quad , \quad (11b)$$

where the left side is a function of the surface geometry and N_R only for a given fluid, and the right side depends only on the core flow length and the desired thermal performance or "heat transfer size" of the core.

b. Core Friction

The ideal flow friction for the reactor core will be defined so as to include only the frictional effects associated with flow through the core; that is, flow exit and entrance losses are excluded, as well as plenum losses and other losses identified with bringing the coolant into the reactor vessel. Then from the Fanning equation:

$$\frac{\Delta p}{\rho} = f \frac{\ell_c}{r_h} \left[\frac{G^2 / \rho^2}{2g_c} \right] \quad , \quad (12)$$

where the bracketed term is recognized as the velocity head for the flow through the fuel element.

One can multiply both sides by W/A_t to get the flow friction power (E_f) in ft-#/ft² of surface. Since

$$\frac{W}{A_t} = G \frac{A_{fl}}{A_t} = G \frac{r_h}{\ell_c} \quad , \quad (13)$$

the result is

$$E_f = f \frac{G^3 / \rho^2}{2g_c} \quad . \quad (12a)$$

The flow friction power per cubic foot of core is simply:

$$E_f \alpha = (p/r_h) E_f \quad . \quad (14)$$

For the entire core, the flow friction power is

$$P_f = E_f A_t = E_f \alpha \ell_c A_{fr} \quad , \quad (15a)$$

which may be arranged to:

$$E_f \alpha = P_f / V_c \quad . \quad (15b)$$

Here, in parallel with Eq. (11b), it can be recognized that the left-hand side is a characteristic depending on surface geometry and the N_R for a given fluid [see Eq. (12a)], and the right-hand side depends on the design-imposed limit on pumping power together with core geometry.

c. Surface Heat Transfer and Flow Friction

Basic design data for heat transfer and friction are ordinarily reported as:

$$\left(N_{St} \right) \left(N_{Pr} \right)^{0.67} = F_h \left(N_R \right) \quad (16)$$

and

$$f = F_f \left(N_R \right) \quad (17)$$

These data, determined by experiment, are available for channel plates and crossed-wire matrices^{1,2} and are plotted in Fig. 1.

Equation (16) can be rearranged to give the heat transfer coefficient, h , in terms of surface properties, fluid properties, and the Reynolds Number influence:

$$h = \left[\frac{1}{4r_h} \right] \left[\frac{c_p \mu}{(N_{Pr})^{0.67}} \right] \left[N_R F_h(N_R) \right] \quad (18)$$

Surface Fluid Reynolds No.

Combination of Eqs. (12a) and (17), while putting the results in terms of Reynolds Number instead of mass velocity, yields the following equation for E_f in terms of surface and fluid properties and the Reynolds Number influence:

$$E_f = \left[\frac{1}{2g_c} \right] \left[\frac{1}{4r_h} \right]^3 \left[\frac{\mu^3}{\rho^2} \right] \left[N_R^3 F_f(N_R) \right] \quad (19)$$

Surface Fluid Reynolds No.

One can then plot h vs E_f corresponding to particular Reynolds Numbers and show the effect of surface geometry and of coolant (see Fig. 2). Points of interest to note from Fig. 2 are: (1) the ratios of heat transfer coefficients - water to diphenyl to carbon dioxide

¹W. H. McAdams, "Heat Transmission" (New York: McGraw-Hill Book Co. Inc.).

²L. S. Tong, "Heat Transfer and Friction Characteristics of Screen Matrices at High Reynolds Numbers," Stanford University Technical Report No. 28, Office of Naval Research (NR-065-104), (April, 1956).

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FIG. 1
 FRICTION AND HEAT TRANSFER CHARACTERISTICS FOR
 PLATE-CHANNELS AND CROSSED WIRE FUEL ELEMENTS

FIG. 2
HEAT TRANSFER COEFFICIENT VS FLOW FRICTION POWER

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(at 30 atm) - are 33 to 5 to 1, respectively; (2) the crossed-wire matrix shows a marked advantage over the plate-type fuel element in the case of diphenyl and CO_2 (60-70% higher heat transfer coefficient for the same friction power); for pressurized water the gain is minor.

Since core dimensions are not involved, these comparisons are useful primarily for purposes of conceptual designs. However, Eqs. (11b) and (15b) may be used to relate the characteristics of the surface and the coolant to the core geometry and the specification of thermal and friction performance. This procedure is illustrated in the following section.

B. Calculation Procedure

The basic problem considered is the comparison of core geometry (ℓ_c, d_c) for a specified service ($W, \Delta t_f, \text{NTU}$), and a specified pumping power ($P_q = E_f A_t$), using different fuel element geometries of specified dimensions (r_h, α, p). If one makes this comparison for more than one reactor coolant, it is possible to show the influence of fluid properties on core geometry for various fuel element geometries.

The pumping power and core size are both idealized. The pumping power does not include pump inefficiency or pressure drop losses associated with plenum chambers or other parts of the pressure vessel. The core size assumes 100% heat transfer volume with no allowance for dead side plates, control rods, etc. A solution such as this used for purposes of comparison further assumes that the hot spot factors associated with each fuel element geometry are the same.

The problem in specific forms is stated as follows:

Given: $P_q, W, \Delta t_f, \theta_{\text{avg}}$; Surface geometry, α, r_h, p

To determine: $\ell_c, A_{fl}, A_{fr}, d_c$, vs P_f/P_q

Solution:

Basic Heat Transfer and Flow Friction Characteristics
[Eqs. (16) and (17)]:

$$(N_{St})(N_{Pr})^{0.67} = F_h(N_R) \quad (A)$$

$$f = F_f(N_R) \quad (B)$$

Energy Balance and Heat Transfer [Eqs. (10), (9), (11b)]:

$$P_q = W c_p \Delta t_f \quad (C)$$

$$NTU = \frac{\Delta t_f}{\theta_{avg}} \quad (D)$$

$$\frac{N_{St}}{r_h} = \frac{NTU}{\ell_c} \quad (E)$$

Geometry:

$$V_c = A_{fr} \ell_c \quad (F)$$

$$d_c^2 = (4/\pi) A_{fr} \text{ (cylindrical core)} \quad (G)$$

Continuity:

$$W = GA_{fl} = G(pA_{fr})$$

and then directly from the definition of Reynolds Number

$$A_{fr} = \left(\frac{4W}{\mu \alpha} \right) \left(\frac{1}{N_R} \right) \quad (H)$$

Friction Power [Eqs. (15b), (19)]:

$$E_f \alpha = \left(\frac{1}{2g_c} \right) \left[\frac{\alpha}{4r_h^3} \right] \left(\frac{\mu^3}{\rho^2} \right) (f N_R^3) \quad (J)$$

$$E_f \alpha = P_f / V_c$$

A work sheet, with instructions, is included as Table I for convenience in calculating the various parameters. Results can then be plotted as A_{fr} (or d_c) and ℓ_c versus the ratio of ideal friction power divided by reactor power, P_f/P_q , as in Figs. 3 to 6, inclusive.

After such a curve is prepared for a given fixed fuel geometry, and a given coolant at a fixed average temperature, it can be used to calculate the core size of a reactor with a different power or NTU. To establish a basis for this procedure, it is necessary to examine the effect of these variables on core size and to formulate a set of affinity relationships.

Table I
CALCULATION WORK SHEET

Conditions

Reactor
 $P_q = 100 \text{ mw}$
 $\Delta t_f = 40\text{F}$
 $NTU = 4$

Coolant
 $H_2O \text{ at } 550\text{F}$

Fuel Element
Plate; 0.060 in. thick
 $p = 0.50$
 $\alpha = 200 \text{ ft}^2/\text{ft}^3$
 $r_h = 0.0025 \text{ ft}$

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
N_R	N_{St}	f	W	A_{fr}	d_c	NTU/l_c	l_c	$E_f \alpha$	V_c	P_f	P_f/P_q
40,000	0.00266	0.0052	6.81×10^6	14.5	4.51	1.064	3.76	0.186	54.6	10.2	0.000102
100,000	0.00217	0.0043	6.81×10^6	5.79	2.72	0.868	4.61	2.33	26.7	62.3	0.000623
200,000	0.00189	0.00374	6.81×10^6	2.90	1.92	0.756	5.29	16.2	15.3	248	0.000248

Instructions:

- Column No:
- (1) Arbitrary values in design range.
 - (2) From Eq. (A) (Fig. 1) and Col. (1).
 - (3) From Eq. (B) (Fig. 1) and Col. (1).
 - (4) From Eq. (C) and fluid properties.
 - (5) From Eq. (H), Col. (1), and fluid properties.
 - (6) From Eq. (G) and Col. (5).
 - (7) From Eq. (F) and Col. (2).
 - (8) From Col. (7) and known NTU.
 - (9) From Col. (1) and fluid properties.
 - (10) From Eq. (F), Col. (5), and Col. (8).
 - (11) From Eq. (K), Col. (9), and Col. (10).
 - (12) From Col. (11) and known P_q .

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FIG. 3
COMPARISON OF CORE SIZE FOR VARIOUS FUEL
ELEMENTS WITH WATER COOLANT AT 550 F;
 $P_q = 100$ mw; NTU = 4; $\Delta t_f = 40$ F

FIG. 4
COMPARISON OF CORE SIZE FOR VARIOUS FUEL ELEMENTS WITH
DIPHENYL COOLANT AT 520 F; $P_q = 100$ mw; $NTU = 4$; $\Delta t_f = 80$ F

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FIG. 5
COMPARISON OF CORE SIZE FOR VARIOUS FUEL
ELEMENTS WITH CO_2 COOLANT AT 450 psi;
 $P_q = 30$ mw; $\text{NTU} = 4.55$; $\Delta t_f = 274$ F

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C. Influence of Variables on Core Geometry

The influence of certain variables on core geometry and pumping power can be readily determined in terms of the following affinity relationships.

1. NTU

With surface geometry, fluid properties, power rating, and fluid temperature rise constant:

$$\ell_c \propto \text{NTU} \quad [\text{Eq. (E)}]$$

$$A_{fr} \text{ is constant} \quad [\text{Eq. (H)}]$$

$$P_f/P_q \propto \text{NTU} \quad [\text{Eqs. (F), (J), (K)}]$$

2. Power Rating (P_q)

With surface geometry, fluid properties, Δt_f , and NTU constant:

$$\ell_c \text{ is constant} \quad [\text{Eq. (E)}]$$

$$A_{fr} \propto P_q \quad [\text{Eqs. (C), (H)}]$$

$$d_c \propto \sqrt{P_q} \quad [\text{Eq. (G)}]$$

$$P_f/P_q \text{ is constant} \quad [\text{Eqs. (J), (K)}]$$

3. Fluid Temperature Rise (Δt_f)

With surface geometry, fluid properties, P_q and P_f , and NTU constant:

$$\ell_c \propto \frac{1}{N_{St}} \quad [\text{Eq. (E)}]$$

$$A_{fr} \propto \frac{1}{\Delta t_f N_R} \quad [\text{Eqs. (C), (H)}]$$

$$V_c \propto \frac{1}{f N_R^3} \quad [\text{Eqs. (J), (K)}]$$

It can be shown further that

$$N_R \propto \left(\sqrt{\Delta t_f} \right) \left(\sqrt{\frac{N_{St}}{f}} \right) .$$

For a moderate N_R range, N_{St}/f is approximately constant; then

$$A_{fr} \propto \frac{1}{(\Delta t_f)^{1.5}} .$$

For a small range of N_R (say 1.5 to 1), f can be considered approximately constant. Therefore, for a Δt_f range of about 2 to 1, ℓ_c is approximately constant and

$$V_c \propto \frac{1}{(\Delta t_f)^{1.5}} .$$

4. Fluid Properties

With surface geometry, P_q , P_f , Δt_f , and NTU constant, the effect of varying fluid properties is the following:

$$\ell_c \propto \frac{1}{N_{St}} \quad [\text{Eq. (E)}]$$

$$A_{fr} \propto \frac{1}{c_p \mu N_R} \quad [\text{Eqs. (H), (C)}]$$

It can be shown further that:

$$N_R \propto \left(\frac{\rho}{\mu} \right) \left(\sqrt{\frac{c_p}{N_{Pr}^{0.67}}} \right) \left(\sqrt{\frac{N_{St} (N_{Pr})^{0.67}}{f}} \right) .$$

For channels:

$$\frac{N_{St} (N_{Pr})^{0.67}}{f} \approx K \approx \frac{1}{2} \quad (\text{see Fig. 1}).$$

For crossed wires:

$$\frac{N_{St} (N_{Pr})^{0.67}}{f} \propto (N_R)^{-0.275} \quad (\text{see Fig. 1}).$$

For channels:

$$f \propto (N_R)^{-0.2} \quad (\text{see Fig. 1}).$$

For crossed wires with $N_R > 10,000$:

$$f \propto (N_R)^{-0.1} \quad (\text{see Fig. 1}).$$

One can then establish that:

$$V_c \propto \frac{(\mu)^{0.73}}{(\rho)^{0.8} (k)^{0.93} (c_p)^{0.47}}$$

for channels, and

$$V_c \propto \frac{(\mu)^{0.4}}{(\rho)^{0.55} (k)^{0.85} (c_p)^{0.43}}$$

for crossed wires.

D. Application of Affinity Relationships

If results are plotted as d_c and ℓ_c versus P_f/P_q for a given P_q , NTU, and Δt_f , it is relatively easy, based on the foregoing "affinity relationships," to obtain from the same plot, ℓ_c and d_c for other specifications of P_q , NTU, and Δt_f .

For a changed reactor power, P_q , with other operating conditions remaining the same, the core length, ℓ_c , is constant, but d_c is proportional to $\sqrt{P_q}$; therefore, if the plot is for a 100-megawatt reactor, d_c for a 25-megawatt reactor will be one-half the value on the plot.

For a changed Δt_f and W over a small range, other operating conditions being held constant, P_f/P_q is constant by definition, and ℓ_c is approximately constant. Since

$$d_c \propto (\Delta t_f)^{-0.75}$$

the core diameter can be obtained by multiplying by the ratio of Δt_f 's to this power; e.g., decreasing Δt_f by 2, increases d_c by $(1/2)^{-0.75}$ or 1.68.

For a changed NTU, d_c is constant; but both ℓ_c and (P_f/P_q) are proportional to NTU. If NTU is doubled, so are ℓ_c and (P_f/P_q) . If (P_f/P_q) is to be held constant, the dimensions are read at one-half of the desired (P_f/P_q) value, and ℓ_c is doubled. Thus, if the plot is for NTU = 4, and one wishes the dimensions for NTU = 8 at $(P_f/P_q) = 0.004$, the dimensions are read on the plot at $(P_f/P_q) = 0.002$ and the NTU = 4 value of length is doubled to get the length for NTU = 8.

III. TYPICAL RESULTS

Results are plotted as core dimensions versus fraction ideal pumping power. Figures 3, 4, and 5 give comparisons of plates and crossed wires in water, diphenyl, and carbon dioxide; sphere bed matrices are also shown in Fig. 5. Figure 6 shows the effect of porosity on core dimension for crossed-wire matrices of a fixed diameter. Each plot is calculated for a fixed set of reactor conditions: P_q , Δt_f , and NTU. A nominal pumping power design range is indicated. If the ℓ_c and d_c curves intersect within this range, a one-pass core is feasible. If ℓ_c is considerably greater than d_c , a two-or multiple series pass core is indicated. If d_c is greater than ℓ_c , a split flow or parallel pass arrangement is required.

As outlined previously, it is possible to select values of ℓ_c and d_c for other power levels, NTU, etc. This has been done to provide the results in Tables II, III, and IV.

The plotted figures and the tables show some significant trends. On a volume basis, the poorer the coolant, i.e., the lower the film coefficient at a given pumping power, the greater the advantage of the crossed-wire matrix-type surface. This can be illustrated further for water and diphenyl by the affinity relationships for V_c as discussed previously. If one neglects density, which is about the same for both liquids, the higher viscosity, lower heat capacity, and thermal conductivity of diphenyl all tend to a larger core volume. Since for plates these properties all enter with a higher power in the proportionality, the result is a greater effect due to fluid properties for the case of plates. The calculated core volume ratios are 12.8 and 8 for plates and wires, respectively. Therefore, although plates and wires (porosity = 0.75) are comparable in water, there is over 50% advantage to wires in diphenyl.

It is significant that reduction in heat transfer coefficient at a fixed E_f from water to diphenyl (see Fig. 2) by a factor of ~ 6 reflects itself in almost a factor of 13 in power density for a plate core. This means that to obtain reasonable power densities with diphenyl, one should not adopt pressurized water reactor design practice with respect to NTU and Δt_f . By increasing Δt_f while reducing NTU, i.e., using a larger θ_{avg} , reasonable power densities can be realized for the organic reactor core.

Diphenyl with its high Prandtl Number (5.72 compared to 0.886 and 0.80 for water and CO_2 , respectively, at the conditions examined) yields relatively longer cores. A high Prandtl Number means a low Stanton Number and it has been shown earlier that

$$\ell_c \propto \frac{1}{N_{St}}$$

Table II

DIMENSIONS OF VARIOUS IDEALIZED REACTOR CORES
WITH PLATES AND WITH CROSSED WIRES

Coolant: Water at 550F
 Δt_f : 40F

Geometry	P_q , mw	NTU	P_f/P_q	Core Dimensions, ft				Volume, ft ³	ϵ , F	Power Density, kw/liter
				l_c	$l_c(TP)^*$	d_c	$d_c(TP)^*$			
Plates	100	4	0.002	5.18	2.59	2.04	2.89	16.9	10	209
Crossed Wires										
$p = 0.75$				1.75	-	3.06	-	12.9		274
$p = 0.60$				0.92	-	3.21	-	7.45		474
Plates	100	4	0.006	5.8	2.9	1.52	2.15	10.52	10	336
Crossed Wires										
$p = 0.75$				2.1	-	2.4	-	9.53		371
Plates	40	4	0.002	5.18	2.59	1.20	1.83	6.76	10	209
Crossed Wires										
$p = 0.75$				1.75	-	1.94	-	5.16		274
$p = 0.60$	40	8	0.004	1.84		2.03		5.96	5	237

* $l_c(TP)$ and $d_c(TP)$ are the length and diameter, respectively, of a two-pass core, Dimensions of a two-pass core are calculated by halving l_c and doubling A_{fr} (i.e.), increasing d_c by 1.41).

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Table III

DIMENSIONS OF VARIOUS IDEALIZED REACTOR CORES
WITH PLATES AND WITH CROSSED WIRES

Coolant: Diphenyl at 520F

Δt_f : 80F

Geometry	P_q , mw	NTU	P_f/P_q	Core Dimensions, ft				Volume, ft ³	θ , F	Power Density, kw/liter
				l_c	$l_c(TP)$	d_c	$d_c(TP)$			
Plates	100	4	0.002	12.9	6.45	2.8	3.96	79.5	20	44.5
Crossed Wires										
$p = 0.75$				3.72	-	3.72	-	40.3		87.7
$p = 0.60$		6	0.006	3.34	-	3.55	-	32.1	13.3	110
Plates	100	2	0.002	6.95	3.48	2.35	3.32	30.2	40	117
Crossed Wires										
$p = 0.75$				2.34	-	2.78	-	14.2	40	249
Plates	40	4	0.002	12.9	-	1.77	-	31.8	20	44.5
Crossed Wires										
$p = 0.75$				3.72	-	2.35	-	16.1		87.7
Plates	40	2	0.002	6.95	3.48	1.49	2.11	12.1	40	117
Crossed Wires										
$p = 0.75$				2.1	-	2.03	-	6.76		209
$p = 0.60$		5	0.004	2.6	-	2.36	-	11.5	16	123

Table IV

DIMENSIONS OF VARIOUS IDEALIZED REACTOR CORES
WITH PLATES, CROSSED-WIRES, AND SPHERES

Coolant: CO₂ at 450 psi Δt_f : 274F

Geometry	$P_{q'}$ mw	NTU	P_f/P_q	Core Dimen- sions, ft		Volume, ft ³	t_f , F	Power Density, kw/liter
				ℓ_c	d_c			
Plates	30	4.55	0.01	3.95	3.49	37.8	60	28.0
Wires								
p = 0.60				0.575	4.42	8.84		120.0
p = 0.75				1.12	4.51	17.95		59.0
Spheres								
D = 0.125				0.26	6.75	9.3		114.0
D = 0.25				0.623	6.85	23.0		46.0
D = 0.50				1.53	7.01	59.1		17.9
Plates			0.02	4.25	3.00	30.0		35.3
Wires								
p = 0.60				0.64	3.82	7.34		144.0
p = 0.75				1.25	3.90	14.95		71.0
Spheres								
D = 0.125				0.292	5.70	7.45		142.0
D = 0.25				0.695	5.80	18.4		57.5
D = 0.50				1.70	5.95	47.3		22.4
Plates	10	4.55	0.01	3.95	2.01	12.6	60	28.0
Wires								
p = 0.60				0.575	2.55	2.94		120.0
p = 0.75				1.12	2.60	5.98		59.0
Spheres								
D = 0.125				0.26	3.89	3.09		114.0
D = 0.25				0.623	3.95	7.68		46.0
D = 0.50				1.53	4.05	19.7		17.9
Plates	30	9.10	0.02	7.90	3.49	75.6	30	14.0
Wires								
p = 0.60				1.15	4.42	17.68		60.0
p = 0.75				2.24	4.51	35.9		29.5
Spheres								
D = 0.125				0.52	6.75	18.6	30	57.0
D = 0.25				1.25	6.85	46.0		23.0
D = 0.50				3.06	7.01	118.2		8.9
Plates	30	2.28	0.02	2.30	2.60	11.28	120	94.0
Wires								
p = 0.60				0.355	3.3	3.04		348.0
p = 0.75				0.705	3.36	6.25		170.0

if only the fluid properties are allowed to change. This is a further advantage to the wire-matrix cores which tend to "pancake," especially for the lower porosities. (Figure 6 indicates d_c is independent of porosity, but that ℓ_c decreases with decreasing porosity.) Similarly, low power levels help the low-porosity wire cores since reduction in power level means a decrease in diameter with no change in length; hence a more slender core.

If a plate and a crossed-wire core have about the same diameter, but the crossed-wire core is shorter for the same reactor parameters, lengthening the crossed-wire core results in a higher NTU. This is illustrated in the last comparison listed in both Table II and Table III. Higher NTU means a lower θ_{avg} which can reflect a higher steam pressure or lower primary coolant temperature and pressure, both advantageous in reactor design.

IV. CONCLUSIONS AND RECOMMENDATIONS

A method has been developed which permits simplified comparison of reactor thermal performance. It is useful primarily for survey-type parametric studies on fuel element geometries and reactor coolants and, as such, is a supplement to conventional methods used in detailed thermal design studies.

In general, the poorer the coolant, the greater is the incentive to choose matrix-type rather than plate-type fuel elements.

Typical results indicate that no one fuel geometry is likely to be superior for all applications. Therefore basic heat transfer and friction data on many fuel element geometries are needed to provide the reactor designer with the best geometry for his particular application.

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