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Engineering Experiment Station
University of Illinois

October 31, 1953

Technical Report No. 11

COLLECTION OF AEROSOLS BY FIBER MATS

by

J. B. Wong, Research Assistant

and

H. F. Johnstone, Technical Director

Contract No. AT(30-3)-28

U. S. Atomic Energy Commission

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I

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COLLECTION OF AEROSOLS BY FIBER MATS

by

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Engineering Experiment Station
University of Illinois
Urbana, Illinois

October 31, 1953

ABSTRACT

The inertial impaction of homogeneous sulfuric acid aerosol particles on two platinum wires, 29.0 and 82.6 microns diameters, and two tungsten wires, 53.1 and 105.7 microns diameters, respectively, was measured at several velocities of flow past the wires. The Reynolds Number, N_{Re} , ranged from 13 to 330. The experimental results verify the prediction that the efficiency of inertial impaction is a function of the inertia parameter Ψ and N_{Re} only. For values of $\sqrt{\Psi}$ less than about 1.4, the experimental efficiencies are in good agreement with those predicted by Langmuir and Blodgett based on the potential flow of an ideal fluid around a cylinder and those calculated by Landahl and Herrmann from Thom's equation of flow for $N_{Re} = 10$, when the effect of the Reynolds Number is taken into account. The experimental values are higher than the values calculated by these authors for $\sqrt{\Psi}$ greater than 1.4. A critical value of $\sqrt{\Psi}$ exists at approximately 0.25, below which inertial impaction does not occur.

Experimental results on the collection of sulfuric acid aerosols by glass fiber mats with fiber diameters of 3.51, 6.24, and 9.57 microns show that the collection efficiency is a function of Ψ , N_{Re} , and R . Inertial impaction ceases to be important for values of $\sqrt{\Psi}$ below about 0.4. No maximum value of $\sqrt{\Psi}$ above which inertial impaction no longer exerts any additional effect was found. The results show that Davies' equation overpredicts the mutual interference of the fibers on the collection efficiency.

An equation is proposed for the correlation of the collection efficiencies of fiber mats on the basis of an effective efficiency of the fibers in the mat. This equation predicts correctly the effects of mat thickness, fiber diameter, and the fiber volume fraction.

The results on the pressure drop across fiber mats show that the function of the Reynolds Number in the equations proposed by Kozeny-Carman, Davies, and Langmuir is inadequate and that in Iberall's equation is approximately correct. An equation is proposed for the correlation of the pressure drop across fiber mats on the basis of an effective drag coefficient of the fibers in the mat. This equation predicts correctly the effects of mat thickness, fiber diameter, and velocity through the mat. By means of the proposed equation, the present data have been shown to agree with those obtained by Blasewitz (2), First (12), and Ramskill and Anderson (30), on glass fiber mats, in a plot of the effective fiber drag coefficient versus the Reynolds Number with the fiber volume fraction as the parameter.

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INTRODUCTION

A major class of aerosol filters consists of mats of individual fibers. The efficiency of collection and the pressure drop are the important practical considerations in the design of these fibrous filters. An understanding of the mechanisms by which particles are collected on isolated cylinders and the flow pattern around the cylinders is fundamental in the design.

The mechanisms of collection include gravity settling, inertial impaction, interception, electrostatic attraction, and Brownian diffusion. Collection by settling results from a particle in a slowly moving stream colliding with a fiber due to the influence of gravity. Inertial impaction takes place when a particle approaching a fiber crosses the streamlines because of its inertia and strikes the surface of the fiber. In the case of interception, the trajectory of the center of mass of a relatively large particle, while not intersecting the surface of the fiber, passes close enough for its surface to touch the fiber. Electrostatic attraction may cause the particle to strike the surface of the fiber because of actual or induced electrostatic forces between the particle and the fiber. In the case of Brownian diffusion, a relatively small particle, by virtue of its random motion, tends to strike the surface of the fiber.

In view of the several mechanisms of particle collection, it seems best to investigate each individually in order to understand in what range

of operation each may affect the efficiency of collection of aerosols by a filter mat. In the present work, the emphasis in the experimental study was placed on the mechanism of inertial impaction. The mechanisms of interception and gravity settling are also treated in the theoretical discussion. Electrostatic attraction and Brownian diffusion are beyond the scope of the present work but they are being studied in separate investigations.

The work is divided into two parts. Part I deals with the impaction of aerosol particles on single cylinders (metallic wires) with axes perpendicular to the direction of flow. Part II deals with the collection efficiency and the pressure drop of fiber mats.

PART I

IMPACTION OF AEROSOLS ON SINGLE CYLINDERS (METALLIC WIRES)

The theory of impaction of aerosol particles on circular cylinders perpendicular to the direction of flow has been studied by Sell (34), Albrecht (1), Langmuir and Blodgett (27), Landahl and Herrmann (25), and Davies (7). Davies indicates that the inertial impaction efficiency should be a function of the inertial parameter ψ and the Reynolds Number based on the diameter of the cylinder. Albrecht, and Langmuir and Blodgett predict that a critical value of ψ exists below which inertial impaction does not occur. Albrecht gives 0.09 whereas Langmuir and Blodgett give 0.0625 for the critical value. The other authors do not indicate such a critical value.

Since experimental verification of the theoretical conclusions is lacking, the present work was undertaken with the purpose of ascertaining the correct function of the efficiency of inertial impaction and the existence or nonexistence of a critical value of ψ .

I. THEORETICAL DEVELOPMENT

1. Mathematical Statement of the Problem of Impaction

Consider an aerosol stream moving in the x-direction towards a circular cylinder which is perpendicular to the direction of flow and effectively infinite in length. The streamlines tend to spread when approaching the cylinder. Consider also the aerosol particles moving with the stream. The motion of the suspended particles will not be the same as that of the gas because they are subjected to several forces which tend to impact them onto the surface of the cylinder.

Application of the law of motion to the movement of the particles gives

$$-\frac{(\vec{u} - \vec{v})}{Z} + \vec{F} = \frac{d(m\vec{u})}{dt} \quad (1)$$

The velocity of the particle relative to the gas is represented by $(\vec{u} - \vec{v})$. The mobility Z is defined in such a way that the frictional forces of the gas opposing the particle motion give a resultant force of $-(\vec{u} - \vec{v})/Z$. $\vec{F}$ represents the vector sum of all extraneous forces, such as gravitational and electrostatic attractions, which may be acting on the particle. In accordance with Newton's Second Law, these forces cause a change in the motion of the particle represented by $d(m\vec{u})/dt$.

The mobility can be stated explicitly in terms of the drag coefficient of the particle as

$$Z = \frac{2}{\rho_w A_p C_R} \quad (2)$$

Before Equation 1 can be solved, the velocity of the gas, $\vec{v}$, and the drag coefficient of the particle, C_R , must be expressed in terms of the space coordinates and time. Functions of $\vec{v}$ and C_R for limiting cases of flow conditions have been derived in classical hydrodynamics. A brief review of some aspects of this subject will be given to point out the

assumptions and limitations of certain theoretical results that are applicable to the present problem.

2. Forces on a Fluid Element

Application of the law of motion to a fluid element (19) gives

$$\begin{aligned}\rho \frac{dv_x}{dt} &= \rho F_x + \frac{\partial p_{xx}}{\partial x} + \frac{\partial p_{yx}}{\partial y} + \frac{\partial p_{zx}}{\partial z} \\ \rho \frac{dv_y}{dt} &= \rho F_y + \frac{\partial p_{xy}}{\partial x} + \frac{\partial p_{yy}}{\partial y} + \frac{\partial p_{zy}}{\partial z} \\ \rho \frac{dv_z}{dt} &= \rho F_z + \frac{\partial p_{xz}}{\partial x} + \frac{\partial p_{yz}}{\partial y} + \frac{\partial p_{zz}}{\partial z}\end{aligned}\quad (3)$$

The terms on the left represent the acceleration of the fluid element resulting from the several forces acting on it as given by the terms on the right. Equation 3 has not been solved for the flow of a real fluid around a sphere or a cylinder.

3. Potential Flow of an Ideal Fluid Around a Circular Cylinder

In order to obtain solutions for Equation 3, an ideal fluid is considered from classical hydrodynamics which is described by the following relations:

$$\begin{aligned}p_{xx} &= p_{yy} = p_{zz} = -p \\ p_{xy} &= p_{xz} = p_{yx} = p_{yz} = p_{zx} = p_{zy} = 0\end{aligned}\quad (4)$$

The first relation states that the normal stresses on the three planes passing through the center of the fluid element are equal to the static pressure at the center. The second relation states that all tangential stresses are absent, or in other words, the fluid has no internal friction.

For the flow of an ideal fluid, Equation 3 reduces to

$$\begin{aligned}\frac{dv_x}{dt} &= F_x - \frac{1}{\rho} \frac{\partial p}{\partial x} \\ \frac{dv_y}{dt} &= F_y - \frac{1}{\rho} \frac{\partial p}{\partial y} \\ \frac{dv_z}{dt} &= F_z - \frac{1}{\rho} \frac{\partial p}{\partial z}\end{aligned}\quad (5)$$

In addition to the equation of motion, there is available the equation of continuity based on the conservation of fluid mass:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} + \frac{\partial(\rho v_z)}{\partial z} = 0 \quad (6)$$

For the flow around an infinite circular cylinder perpendicular to the direction of flow in the absence of an extraneous force and pressure gradient in the z direction, the problem becomes a two-dimensional one.

In two dimensions, if the flow is irrotational, i.e., $\partial v_y / \partial x = \partial v_x / \partial y$, there exists a potential function Φ such that

$$v_x = \frac{\partial \Phi}{\partial x} ; \quad v_y = \frac{\partial \Phi}{\partial y} \quad (7)$$

This potential function Φ is analogous to the potentials encountered in the theories of attraction, electrostatics, magnetism, and the steady flow of heat. If the flow is incompressible, there exists a stream function Γ such that

$$v_x = \frac{\partial \Gamma}{\partial y} ; \quad v_y = - \frac{\partial \Gamma}{\partial x} \quad (8)$$

The physical significance of the stream function is that it represents the volume of fluid passing any line connecting an arbitrary point (e.g., the origin) and the point (x,y) per unit time.

For the steady-state two-dimensional motion of an ideal, irrotational, and incompressible fluid around an infinite cylinder perpendicular to the direction of flow, in the absence of extraneous forces, Equation 5 has been solved to give the following potential and stream functions (20):

In rectangular coordinates,

$$\Phi = v_o x \left(1 + \frac{r_c^2}{x^2 + y^2} \right) \quad (9)$$

$$\Gamma = v_o y \left(1 - \frac{r_c^2}{x^2 + y^2} \right)$$

In polar coordinates,

$$\begin{aligned} \Phi &= v_o r \left(1 + \frac{r_c^2}{r^2} \right) \cos \theta \\ \Gamma &= v_o r \left(1 - \frac{r_c^2}{r^2} \right) \sin \theta \end{aligned} \quad (10)$$

The following are the components of the fluid velocity at any point:

In rectangular coordinates,

$$\begin{aligned} v_x &= \frac{\partial \Phi}{\partial x} = \frac{\partial \Gamma}{\partial y} = v_0 \left[1 + \frac{r_c^2 (y^2 - x^2)}{(x^2 + y^2)^2} \right] \\ v_y &= \frac{\partial \Phi}{\partial y} = \frac{\partial \Gamma}{\partial x} = -2v_0 \frac{r_c^2 xy}{(x^2 + y^2)^2} \end{aligned} \quad (11)$$

In polar coordinates,

$$\begin{aligned} v_r &= \frac{\partial \Phi}{\partial r} = \frac{1}{r} \frac{\partial \Gamma}{\partial \theta} = v_0 \left(1 - \frac{r_c^2}{r^2} \right) \cos \theta \\ v_\theta &= \frac{1}{r} \frac{\partial \Phi}{\partial \theta} = -\frac{\partial \Gamma}{\partial r} = -v_0 \left(1 + \frac{r_c^2}{r^2} \right) \sin \theta \end{aligned} \quad (12)$$

This velocity distribution is expected to hold at high values of the Reynolds Number which is defined by

$$N_{Re} = \frac{D_c v_0 \rho}{\mu} \quad (13)$$

4. Navier-Stokes Equation for the Flow of a Viscous Fluid

For the flow of fluid around spheres or cylinders at relatively low values of the Reynolds Number, (i.e., of the order of unity or smaller) the internal friction of the fluid becomes the predominating factor. The velocity field of the fluid around the object now depends primarily on the tangential stresses which are absent in the ideal fluid. Making appropriate assumptions and substitutions for the normal and tangential stresses in Equation 3, the following equation, usually attributed to Navier and Stokes, has been obtained (21):

$$\begin{aligned} \rho \frac{dv_x}{dt} &= \rho F_x - \frac{\partial p}{\partial x} + \frac{1}{3} \mu \frac{\partial \Delta}{\partial x} + \mu \Delta^2 v_x \\ \rho \frac{dv_y}{dt} &= \rho F_y - \frac{\partial p}{\partial y} + \frac{1}{3} \mu \frac{\partial \Delta}{\partial y} + \mu \Delta^2 v_y \\ \rho \frac{dv_z}{dt} &= \rho F_z - \frac{\partial p}{\partial z} + \frac{1}{3} \mu \frac{\partial \Delta}{\partial z} + \mu \Delta^2 v_z \end{aligned} \quad (14)$$

where $\Delta = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$

$$\Delta^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Equation 14 is an exact mathematical statement of the dynamic conditions within a viscous fluid. It is applicable to every type of viscous flow provided the correct boundary conditions are introduced.

5. Stokes' Law for the Drag of a Rigid Sphere

Stokes made the following simplifications:

- a) no extraneous forces act on the fluid, i.e., $F_x^i = F_y^i = F_z^i = 0$;
- b) the fluid is incompressible, i.e., the equation of continuity is $\nabla = 0$; and
- c) the inertial terms are negligible comparing to terms representing viscous forces, i.e.,

$$\frac{\partial v_x}{\partial t} = \frac{\partial v_y}{\partial t} = \frac{\partial v_z}{\partial t} = 0,$$

and solved Equation 14 for the steady-state flow around a rigid sphere to obtain the following important equation known as Stokes' Law, for the viscous drag on a sphere (37):

$$F_R = 3\pi\mu D_p w \quad (15)$$

Stokes' Law is not expected to hold for values of the Reynolds Number, N_{Re} , greater than about 1 because the assumption that inertial terms are negligible no longer applies for these values (28). N_{Re} is defined by

$$N_{Re} = \frac{D_p w \rho}{\mu} \quad (16)$$

6. Flow of a Viscous Fluid Around a Circular Cylinder

With the same simplifications as those made by Stokes for his treatment on the rigid sphere, Equation 14 yields solutions for the steady-state flow around an infinite circular cylinder perpendicular to the direction of flow which do not satisfy all the boundary conditions. Following an approach attributed to Oseen, Lamb partially takes into account the inertial terms by writing $v_0 + v_x$ for v_x and neglecting terms of the second order in v_x , v_y , and v_z to obtain the equation of motion in the following form (22):

$$\begin{aligned}\rho v_0 \frac{\partial v_x}{\partial x} &= - \frac{\partial p}{\partial x} + \mu \nabla^2 v_x \\ \rho v_0 \frac{\partial v_y}{\partial x} &= - \frac{\partial p}{\partial y} + \mu \nabla^2 v_y \\ \rho v_0 \frac{\partial v_z}{\partial x} &= - \frac{\partial p}{\partial z} + \mu \nabla^2 v_z\end{aligned}\quad (17)$$

Equation 17 has been solved by Lamb to obtain the following velocity profile of the fluid and the drag force on the cylinder for the transverse motion of an infinite circular cylinder with constant velocity v_0 through a still fluid:

$$v_x = C_L \left[\delta - \frac{1}{2} + \ln(kr/2) + \frac{1}{2} (r^2 - r_c^2) \frac{\partial^2}{\partial x^2} \ln r \right] \quad (18)$$

$$v_y = \frac{C_L}{2} (r^2 - r_c^2) \frac{\partial^2}{\partial x \partial y} \ln r$$

$$F_D = 4\pi\mu C_L v_0 \quad (19)$$

where

$$C_L = \frac{v_0}{(1/2 - \delta - \ln(kr_c/2))} \approx \frac{v_0}{2.002 - \ln N_{Re}}$$

$\delta = 0.577$ Euler's constant, dimensionless

$$k = v_0 \rho / (2\mu), \text{ cm.}^{-1}$$

F_D = drag force per unit length of the cylinder, dynes/cm.

Lamb's equation (Equation 18), when modified by impressing a unidirectional velocity in the positive x-direction to give the transverse motion of a fluid near the surface of a stationary cylinder, becomes

In rectangular coordinates,

$$\begin{aligned}v_x &= C_L \left[\ln \frac{(x^2 + y^2)^{\frac{1}{2}}}{r_c} + \frac{1}{2} (x^2 + y^2 - r_c^2) \frac{(y^2 - x^2)}{(x^2 + y^2)^2} \right] \\ v_y &= - C_L (x^2 + y^2 - r_c^2) \frac{xy}{(x^2 + y^2)^2}\end{aligned}\quad (20)$$

$$\text{for } x \leq 0 ; x^2 + y^2 \geq r_c^2$$

In polar coordinates,

$$\begin{aligned}v_r &= C_L \left[\ln\left(\frac{r}{r_c}\right) - \frac{1}{2} \left(1 - \frac{r_c^2}{r^2}\right) \right] \cos \theta \\ v_\theta &= - C_L \left[\ln\left(\frac{r}{r_c}\right) + \frac{1}{2} \left(1 - \frac{r_c^2}{r^2}\right) \right] \sin \theta\end{aligned}\quad (21)$$

$$\text{for } \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2} ; r \geq r_c$$

Equations 20 and 21 are expected to hold for low values of the Reynolds Number, i.e., N_{Re} of the order of unity or smaller for regions close to the surface of the cylinder. For regions far removed from the cylinder, these equations are no longer valid.

Davies has reviewed Lamb's solution of Equation 17 (8). After criticizing some of Lamb's simplifying assumptions in which certain terms in the solution were neglected, he resolved Equation 17 to obtain velocity components of the fluid near the surface of the cylinder which he considers to be more accurate than those given by Lamb. Davies also obtained Equation 19 for the drag force per unit length of the cylinder. The equations for the velocity distribution of the fluid given by Davies are much more complicated than those given by Equations 20 and 21.

7. Impaction Efficiency of Aerosol Particles Which Follow Stokes' Law

In the light of the foregoing treatment of classical hydrodynamics, the problem of the impaction of aerosol particles on the cylinder can now be considered in a quantitative manner. If the aerosol particles can be assumed to be rigid spheres and if the values of the Reynolds Number, N_{Re} , are of the order of 1 or smaller, Stokes' Law of resistance given by Equation 15 is applicable. The drag coefficient of the particles, as defined by the equation

$$C_R = \frac{2F_R}{\rho w^2 A_p} \quad (22)$$

is simply

$$C_R = \frac{24}{N_{Re}} \quad (23)$$

and the particle mobility becomes

$$Z = \frac{1}{3\pi\eta D_p} \quad (24)$$

When the particles have diameters of the order of the mean free path of the gas molecules, they have a tendency to slip between the molecules giving a higher particle mobility than indicated by Equation 24. Cunningham (6), has introduced a correction factor given by

$$C = 1 + k_m \frac{\lambda_m}{D_p} \quad (25)$$

Millikan, from precise measurements in gas with particles of the order of one micron diameter, obtained results for the Cunningham factor which can be represented by the empirical equation (28)

$$C = 1 + \left(1.644 + 0.552 e^{-0.656 D_p / \lambda_m} \right) \lambda_m / D_p \quad (26)$$

In air at ordinary temperatures and pressures, $C \approx 1 + 0.16 \times 10^{-4} / D_p$.

In the absence of gravity, electrostatic and other extraneous forces, the Stokes - Cunningham Law for the motion of an aerosol particle of uniform and constant density becomes

$$-(\vec{u} - \vec{v}) = \frac{C \rho_p D_p^2}{18\mu} \frac{d\vec{u}}{dt} \quad (27)$$

The particle at a point upstream of the cylinder far removed from the effect of the cylinder will follow the gas stream with $\vec{u} = \vec{v}$. As the cylinder is approached, $\vec{v}$ changes markedly to allow the gas to spread in a manner depending on the nature of the flow. The particle will follow a trajectory given by Equation 27. Whether the particle will reach the surface of the cylinder depends on the nearness of the upstream position of the particle to the axis of flow through the center of the cylinder, the manner in which the gas spreads around the cylinder, the mass and diameter of the particle, and the diameter of the cylinder.

From a theoretical standpoint, the efficiency of impaction of aerosol particles on a collector is defined as the ratio of the cross-sectional area of the original aerosol stream from which particles are removed by the collector to the projected area of the collector in the direction of flow. This definition implies that in any plane perpendicular to the

direction of flow, the number concentration of aerosol particles in the stream is uniform.

In order to deduce some general information on the efficiency of impaction of aerosol particles on an infinite circular cylinder, Equation 27 is reduced to dimensionless form by the following substitutions of variables:

$$\begin{aligned} u_x &= \tilde{u}_x v_0 & x &= \tilde{x} D_c / 2 \\ u_y &= \tilde{u}_y v_0 & y &= \tilde{y} D_c / 2 \\ v_x &= \tilde{v}_x v_0 & t &= \tilde{t} D_c / 2(2 v_0) \\ v_y &= \tilde{v}_y v_0 \end{aligned}$$

Thus,

$$2\psi \frac{d^2 \tilde{x}}{d\tilde{t}^2} + \frac{d\tilde{x}}{d\tilde{t}} - \tilde{v}_x = 0 \quad (28)$$

$$2\psi \frac{d^2 \tilde{y}}{d\tilde{t}^2} + \frac{d\tilde{y}}{d\tilde{t}} - \tilde{v}_y = 0$$

where

$$\psi = \frac{C_{p,p} v_0 D_p^2}{18\mu D_c}$$

In general, the parameter ψ is not a function of the reduced independent variables $\tilde{x}$, $\tilde{y}$ and $\tilde{t}$. For solving Equation 28, ψ can be treated as a constant.

Consider the velocity field of the fluid to be in a steady state and a function of $\tilde{x}$, $\tilde{y}$ and N_{Re} only. The starting conditions for the particle at $\tilde{t} = 0$ and $\tilde{x}_0 = -\infty$ are $\tilde{y} = \tilde{y}_0$, $\tilde{u}_x = d\tilde{x}/d\tilde{t} = 1$, and $\tilde{u}_y = d\tilde{y}/d\tilde{t} = 0$. A solution of Equation 28 with the boundary conditions will give the position of the particle at any time $\tilde{t}$ in the form

$$\tilde{x} = \tilde{x}(\tilde{y}_0, \tilde{t}, \psi, N_{Re}) \quad (29)$$

$$\tilde{y} = \tilde{y}(\tilde{y}_0, \tilde{t}, \psi, N_{Re})$$

Eliminating $\tilde{t}$ from Equation 29 will give

$$\tilde{y} = \tilde{y}(\tilde{x}, \tilde{y}_0, \psi, N_{Re}) \quad (30)$$

An explicit function in $\tilde{y}$ from Equation 30 will give the $\tilde{y}$ position of a particle at any $\tilde{x}$, $\tilde{y}_0$, ψ and N_{Re} , or in other words, the point of intersection

of the particle trajectory with any line perpendicular to the $\tilde{x}$ axis. Theoretically at least, it is possible to transform Equation 30 into an explicit function of $\tilde{y}_0$ and to solve for a particular value $\tilde{y}_0^*$ for the trajectory which is tangent to the surface of the cylinder for any values of Ψ and N_{Re} . This limiting trajectory, starting from $\tilde{x} = -\infty$ and $\tilde{y} = \tilde{y}_0^*$, is dependent only on the values of Ψ and N_{Re} .

Assume that the aerosol stream is homogeneous, i.e., all aerosol particles have the same mass and diameter, then particle trajectories will not cross each other. Assume also that all particles which strike or barely touch the surface of the cylinder will be retained on the surface. With these two assumptions, consider presently a hypothetical situation in which the aerosol particles are infinitesimal in size, insofar as impaction on the surface of the cylinder is concerned. In this case, $\tilde{y}_0^*$ for the limiting trajectory is identical with the efficiency of impaction as defined previously. This efficiency will be referred to as the efficiency of inertial impaction, η_I , to distinguish from the total efficiency, η , and the efficiency of interception, η_C , which are discussed in the following paragraphs. For the present case on inertial impaction, the aerosol particles are assumed to have finite sizes only in determining the resistance to their motion relative to the gas, and a finite mass only in determining their resulting acceleration. Important conclusions drawn from the above considerations are, a) the efficiency of inertial impaction, η_I , is a function of the parameter Ψ and the Reynolds Number N_{Re} only, and b) η_I , for flow fields geometrically and dynamically similar, is a function of the parameter Ψ only. Ψ will be referred to as the inertial parameter.

In an actual situation, an aerosol particle has a finite size, and can touch the surface of the cylinder when the particle trajectory passes the surface of the cylinder within a distance $D_p/2$. Theoretically it is also

possible to find the value $\tilde{y}_0^*$ for this limiting trajectory from Equation 30, and arrive at the total efficiency, i.e., $\eta = \tilde{y}_0^*$. η is a function not only of Ψ and N_{Re} , but also of D_p/D_c , which will be referred to as the interception parameter R . The fact that D_p/D_c enters the picture rather than just D_p is because of the use of reduced coordinates.

The difference between η and η_I is η_c , the efficiency of interception. Although the separation of η_I and η_c from η is purely arbitrary, it serves the purpose of distinguishing two mechanisms of aerosol collection which do not depend on the same variables. The efficiency of interception is a function of R , N_{Re} and, to a lesser extent, of Ψ . The efficiency of interception for the limiting case of $\Psi = 0$ will be discussed later in REVIEW OF PREVIOUS WORK.

The above conclusions may now be applied to the impaction of aerosol particles on cylinders when the gas flow around the cylinders can be represented by equations derived from classical hydrodynamics. If the gas is in viscous flow so that Equations 20 and 21 are applicable, η is a function of Ψ , R and N_{Re} , and η_I is a function of Ψ and N_{Re} only. If the gas flow approximates the potential flow of an ideal fluid so that Equations 11 and 12 are applicable, η is a function of Ψ and R , and η_I is a function of Ψ only.

8. Impaction Efficiency of Aerosol Particles Which Do Not Follow Stokes' Law

Stokes' Law holds approximately up to $N_{Re}^0 = 1$. For higher values of N_{Re}^0 , experimental values of the drag coefficient C_R must be used in the equation of motion. Equation 28 can be written in the form

$$\begin{aligned} \frac{48\Psi}{C_R N_{Re}^0} \frac{d^2 \tilde{x}}{d\tilde{t}^2} + \frac{d\tilde{x}}{d\tilde{t}} - \tilde{v}_x &= 0 \\ \frac{48\Psi}{C_R N_{Re}^0} \frac{d^2 \tilde{y}}{d\tilde{t}^2} + \frac{d\tilde{y}}{d\tilde{t}} - \tilde{v}_y &= 0 \end{aligned} \quad (31)$$

which reduces to the form given by Equation 28 when Stokes' Law is applicable.

Considerations of Equation 31 similar to those given to Equation 28 will lead to the following general conclusions for the case when all extraneous forces are negligible:

- a) If the gas around the cylinder is in viscous flow so that Equations 20 and 21 are applicable, the total efficiency of impaction, η , is a function of $(\frac{\psi}{C_R N_{Re}})$, R , and N_{Re} , and the efficiency of inertial impaction, η_I , is a function of $(\frac{\psi}{C_R N_{Re}})$ and N_{Re} only.
- b) If the gas flow approximates the potential flow of an ideal fluid such that Equations 11 and 12 are applicable, η is a function of $(\frac{\psi}{C_R N_{Re}})$ and R , and η_I is a function of $(\frac{\psi}{C_R N_{Re}})$ only.

9. Physical Interpretations of the Inertial Parameter ψ

Consider a spherical particle with uniform density moving in a quiescent fluid with an initial unidirectional velocity v_o . The force acting on the particle in the opposite direction necessary to stop the particle in a distance $D_c/2$ is

$$F = - \frac{\pi D_p^3 \rho_p}{6} \frac{\int_{v_o}^0 u du}{D_c/2} = \frac{\pi \rho_p v_o^2 D_p^3}{6 D_c} \quad (32)$$

When the Stokes-Cunningham Law is applicable, the fluid resistance on the particle moving with a relative velocity v_o is

$$R_F = \frac{v_o}{Z} = \frac{3\pi\mu v_o D_p}{C} \quad (33)$$

Dividing Equation 32 by Equation 33 gives

$$\frac{F}{R_F} = \frac{C \rho_p v_o D_p^2}{18\mu D_c} = \psi \quad (34)$$

Hence, the inertial parameter ψ represents the ratio of the force necessary to stop a particle initially travelling at a velocity v_o in a distance $D_c/2$ to the fluid resistance on the particle moving according to the Stokes-Cunningham Law with a relative velocity v_o .

The stopping distance, S , of a particle is defined as the distance the particle will travel in a still fluid given an initial velocity v_o . For a spherical particle with uniform density moving according to the Stokes-

Cunningham Law with an initial velocity v_0 , the equation of motion is

$$\frac{\pi D_p^3 \rho_p}{6} \frac{du}{dt} = - \frac{3\pi\mu D_p}{C} \quad (35)$$

Integrating Equation 35 with limits v_0 to u for du and 0 to t for dt gives

$$u = \frac{dx}{dt} = v_0 e^{-\frac{18\mu}{C\rho_p D_p^2} t} \quad (36)$$

Integrating a second time with limits 0 to S for dx and 0 to ∞ for dt gives

$$S = \frac{C\rho_p v_0 D_p^2}{18\mu} \quad (37)$$

The relation between S and D_c can now be stated as

$$\frac{S}{D_c} = \frac{C\rho_p v_0 D_p^2}{18\mu D_c} = \Psi \quad (38)$$

Thus, the inertial parameter Ψ also represents the ratio of the stopping distance of a particle moving according to the Stokes-Cunningham Law with an initial velocity v_0 to the diameter of the cylinder.

When Stokes' Law is not applicable and the drag coefficient has to be used, similar considerations will give the relation

$$\frac{F}{R_F} = \frac{S}{D_c} = \frac{24\Psi}{C_R N_{Re}} \quad (39)$$

10. Effect of Gravity Settling

Heretofore extraneous forces have been considered negligible.

The case in which the force of gravity is important but all other extraneous forces are negligible will be considered presently. When the density of the aerosol particle is large compared with that of the gas, the equation of motion for the particle takes the form

$$- \frac{(\vec{u} - \vec{v})}{Z} + m\vec{g} = \frac{d(m\vec{u})}{dt} \quad (40)$$

If the particle is spherical, Equation 40 can be reduced to the following dimensionless form by the substitutions of variables given on Page 11:

$$\frac{48\Psi}{C_{RRe}N_{Re}} \frac{d^2\tilde{x}}{dt^2} + \frac{d\tilde{x}}{dt} - \tilde{v}_x = 0 \quad (41)$$

$$\frac{48\Psi}{C_{RRe}N_{Re}} \frac{d^2\tilde{y}}{dt^2} + \frac{d\tilde{y}}{dt} - \tilde{v}_y + G = 0$$

where

$$G = \frac{4}{3} \frac{C_{pp} D_p^2 g}{\mu C_{RRe} N_{Re} v_o} = \left(\frac{24\Psi}{C_{RRe}N_{Re}} \right) \left(\frac{g D_c}{v_o^2} \right)$$

Considerations of Equation 41 similar to those given in Equation 28 lead to the following general conclusions for the case in which gravity settling is important:

- a) If the gas around the cylinder is in viscous flow so that Equations 20 and 21 are applicable, the total efficiency of impaction, η , is a function of $\left(\frac{\Psi}{C_{RRe}N_{Re}} \right)$, R , G , and N_{Re} , and the efficiency of inertial impaction, η_I , is a function of $\left(\frac{\Psi}{C_{RRe}N_{Re}} \right)$, G , and N_{Re} only.
- b) If the gas flow approximates the potential flow of an ideal fluid such that Equations 11 and 12 are applicable, η is a function of $\left(\frac{\Psi}{C_{RRe}N_{Re}} \right)$, R , and G , and η_I is a function of $\left(\frac{\Psi}{C_{RRe}N_{Re}} \right)$, and G only.

When Stokes' Law is applicable to the particle, $C_{RRe}N_{Re}$ is a constant, i.e., $C_{RRe}N_{Re} = 24$, and the efficiencies of impaction become functions of Ψ instead of the group $\left(\frac{\Psi}{C_{RRe}N_{Re}} \right)$.

11. Physical Interpretations of the Settling Parameter G

The parameter G which characterizes the effect of gravity settling will be referred to as the settling parameter. Consider a homogeneous aerosol with spherical particles moving transverse to an infinite circular cylinder. Assume the direction of flow and the axis of the cylinder are both in the horizontal position. If the particles do not interfere with each other and the density of the particles is large compared with that of the gas, all particles will fall at the same terminal velocity

$$v_t = \frac{4}{3} \frac{C_p D_p^2 g}{\mu C_{R Re}} \quad (42)$$

The ratio of v_t to v_o is

$$v_o = \frac{4}{3} \frac{C_p D_p^2 g}{\mu C_{R Re} v_o} = G \quad (43)$$

Hence, the settling parameter G is the ratio of the terminal velocity of the particles to the upstream velocity of the aerosol stream.

The efficiency of impaction has been defined as the area of the original aerosol stream from which particles are removed to the projected area of the collector. For a unit length of the cylinder, the area of the original aerosol stream from which particles are removed by settling onto the cylinder is the terminal velocity, v_t , multiplied by the time D_c/v_o . The projected area of the cylinder is D_c . Hence the apparent efficiency due to settling, η_s , is

$$\eta_s = v_t/v_o = G \quad (44)$$

Since the apparent efficiency of impaction due to settling is of the order of unity when $G = \left(\frac{24 \psi}{C_{R Re}} \right) \left(\frac{g D_c}{v_o^2} \right)$ is of the order of unity, the mechanism of gravity settling will be more important than the mechanism of inertial impaction when the group $\left(\frac{g D_c}{v_o^2} \right)$ is greater than the group $\frac{24 \psi}{C_{R Re}}$, and vice versa.

II. REVIEW OF PREVIOUS WORK

1. Numerical Solutions for the Efficiency of Inertial Impaction

The work of Sell (34), Albrecht (1), and Langmuir and Blodgett (27) on the impaction of aerosol particles on circular cylinders has been primarily of a theoretical nature and considers inertia as the only force involved. Landahl and Herrmann (25) measured the efficiencies of impaction of atomized liquid droplets on cylinders and found poor agreement with the predictions of Sell. They calculated an impaction-efficiency curve based on the flow field obtained by Thom (40) for $N_{Re} = 10$. Davies (7), using his own viscous flow equation (8), calculated an impaction-efficiency curve for $N_{Re} = 0.2$. Figure 1 shows the calculated efficiencies of inertial impaction, together with the experimental curve obtained in the present work.

Sell was the first to show that the efficiency of inertial impaction should be a unique function of the inertial parameter Ψ . He determined experimentally the velocity profile of water flowing past a circular cylinder 10 cm. in diameter with its axis transverse to the flow by introducing a solution of a red dye through small holes and photographing the resulting red lines after steady-state flow had been reached. Based on these velocity distributions he calculated the limiting trajectories for different values of Ψ using a stepwise integration, and arrived at the theoretical efficiency curve for inertial impaction shown in Figure 1. In order to check his calculated curve, Sell atomized India ink droplets into a stream of air which was directed to pass transversely the 10 cm. cylinder, and photographed the limiting trajectories. With the experimental efficiencies thus obtained, he read off corresponding values of Ψ from his calculated curve and found the corresponding particle diameters. Since the calculated diameters were approximately the same as those obtained in a similar manner for the air

stream carrying India ink droplets passing a flat plate and a streamlined trap chamber, Sell considered his theoretical efficiency curve to be verified. Unfortunately, there were no independent means at the time for checking the diameters of the ink droplets. While Sell's experimental results did agree surprisingly well with his theory, they could not be considered as direct verification of his theory.

The work of Albrecht on this subject was purely theoretical. He used the potential flow equation for an ideal fluid (Equation 11) for his step-by-step calculations of limiting particle trajectories to determine the efficiencies of inertial impaction. His efficiency curve agrees with that of Sell for values of $\sqrt{\Psi}$ greater than 0.4. For values of $\sqrt{\Psi}$ smaller than 0.4, however, Albrecht's curve indicates a much greater slope and an intercept at $\sqrt{\Psi} = 0.3$, whereas Sell's curve is asymptotic to the origin at $\eta_I = \sqrt{\Psi} = 0$. His critical value of $\sqrt{\Psi}$ at 0.3 has been a point of disagreement among workers in this field and it will be discussed later in more detail.

Langmuir and Blodgett employed the Differential Analyser to establish the efficiency curves both for the case in which Stokes' Law is valid and for the case in which the varying drag coefficient, C_R , must be used. Their curve for the Stokes' Law case is shown in Figure 1. Theoretically, their curve should be identical with that calculated by Albrecht since the potential flow of an ideal fluid and Stokes' Law were assumed in the derivations leading to both curves. The discrepancy is probably due to the fact that, a) Albrecht introduced the aerosol particle at the point $\tilde{x} = -3$ and assumed that the particle velocity at that point is the same as that of the free stream, whereas Langmuir and Blodgett introduced the particle at $\tilde{x} = -4$ and corrected the particle velocity by approximations; and b) Albrecht had to resort to larger increments for his step-by-step calculations

whereas Langmuir and Blodgett used smaller differential increments by means of the Differential Analyser. Langmuir and Blodgett compared their efficiencies with those calculated by Glauert (Restricted British Report) and give explanations similar to those just mentioned for their differences. Both Langmuir and Blodgett and Glauert give the critical value of $\sqrt{\Psi} = 0.25$ below which inertial impaction does not take place.

Landahl and Herrmann used the flow field calculated by Thom for $N_{Re} = 10$ for their step-by-step calculations of the theoretical efficiencies. They introduced the aerosol particle at $\tilde{x} = -6$ and assumed the particle to be at the free stream velocity at that point. They fitted the resulting efficiencies with an empirical equation. The curve shown in Figure 1 has been calculated from their equation. Although these authors did not find a critical value of $\sqrt{\Psi}$, their empirical equation indicates that the efficiency of inertial impaction is less than 0.001 for $\sqrt{\Psi}$ less than 0.25.

Davies (7) also performed the step-by-step calculations based on his viscous flow equation for $N_{Re} = 0.2$ to obtain an inertial impaction-efficiency curve. As shown in Figure 1, his curve indicates much lower efficiencies than those of the other authors. The efficiency of inertial impaction is a function not only of Ψ , but also of N_{Re} when the flow is viscous. A comparison of the curves of Davies, Landahl and Herrmann, and Langmuir and Blodgett leads to the conclusion that the more viscous the flow is, the lower is the efficiency of inertial impaction. It is to be noted that for the filtration of aerosol particles by fiber mats, the Reynolds Number is nearly always less than unity as will be shown in PART II. For values of the Reynolds Number of the order of unity or less, the viscous-flow equation must be used and only Davies' efficiency curve is applicable.

In addition to the work of Langmuir and Blodgett, Kentrovitz

(Restricted Report, National Advisory Committee for Aeronautics) and Brun and Vasseur (3) have published theoretical papers on the inertial impaction of particles on cylinders at high values of N_{Re} where Stokes' Law no longer applies. This case is of importance in aerodynamics, e.g., the impaction of ice particles on airfoils, but is of little interest in the filtration problems. Aerosol particles, by virtue of their small diameters, do not move with high relative velocities such that N_{Re} is larger than unity except under very unusual situations.

Although as shown in Figure 1, the theoretical studies on the inertial impaction of aerosol particles on circular cylinders perpendicular to the direction of flow have been extensive, experimental studies have been scanty. The early work of Sell was conducted with India ink droplets of unknown size. Langmuir and Blodgett tried to interpret the experimental data of Cunningham by their theoretical curves without much success. What is more, Cunningham's data contain no information on the particle sizes. Landahl and Herrmann give experimental data on the impaction of atomized liquid droplets on wires. Their data, however, indicate a wide range of droplet sizes and distributions which make any comparison with their calculated efficiency curve of doubtful value. Hence, there has been no experimental verification of any of the theoretical curves.

2. Effect of Interception

If the potential flow of an ideal fluid is assumed, a massless particle with finite size in the absence of extraneous forces will follow fluid streamlines given by the stream function Γ of Equation 9. The limiting streamline, Γ_0 , that passes the point $x = 0, y = r_c + (D_p/2)$ is

$$\Gamma_0 = v_0 r_c (1 + R) \left[1 - \frac{1}{(1 + R)^2} \right] \quad (45)$$

This particular streamline at $x = -\infty$ is

$$r_o = v_o y_o^* \quad (46)$$

Substituting Equation 46 into Equation 45 will give

$$y_o = r_c(1 + R) \left[1 - \frac{1}{(1 + R)^2} \right] \quad (47)$$

By definition, the total efficiency of impaction is y_o^*/r_c . But having assumed the particle to be massless, hence,

$$\eta_c = (1 + R) - \frac{1}{(1 + R)} \quad (48)$$

Equation 48 is shown in Figure 2 together with curves for efficiencies of interception in the viscous-flow region to be discussed later. Equation 48 represents the minimum total efficiency of impaction as the value of ψ approaches zero. For large values of R compared with unity, η_c approaches $(1 + R)$, which is the same limit as that when ψ becomes infinite, i.e., when particle trajectories become parallel to the direction of fluid flow.

Landahl and Herrmann have assumed arbitrarily that the efficiency of interception is equal to R and is directly additive to the efficiency of inertial impaction. Rodebush gives the same relation for the interception effect, but states that it is applicable only in the limiting case in which the diameter of the cylinder approaches zero (33).

Consider presently the case in which the fluid is in viscous flow so that Equation 21 is applicable. The tangential velocity at the plane $\theta = \pi/2$ is

$$\left[v_\theta \right]_{\theta = \pi/2} = -C_L \left[\ln \frac{r}{r_c} + \frac{1}{2} \left(1 - \frac{r_c^2}{r^2} \right) \right] \quad (49)$$

The volumetric flow rate of the fluid, Q_1 , passing the plane $\theta = \pi/2$ bounded by $r = r_c$ and $r = r_c + (D_p/2)$ per unit length of the cylinder is

$$Q_1 = - \int_{r_c}^{r_c + (D_p/2)} \left[\frac{v_\theta}{r} \right]_{\theta = \pi/2} dr = r_c C_L \left[(1+R) \ln(1+R) - \frac{R(2+R)}{2(1+R)} \right] \quad (50)$$

If the particles are assumed to be massless so that they follow the fluid streamlines, the ratio of Q_1 to $v_0 r_c$, the volumetric flow rate which passes a plane representing one half of the projected area of a unit length of the cylinder, is the efficiency of interception. Hence,

$$\eta_c = \frac{1}{(1/2) - \gamma - \ln(\pi r_c/2)} \left[(1+R) \ln(1+R) - \frac{R(2+R)}{2(1+R)} \right] \\ \approx \frac{1}{2.002 - \ln N_{Re}} \left[(1+R) \ln(1+R) - \frac{R(2+R)}{2(1+R)} \right] \quad (51)$$

This equation was first deduced by Ranz (31) from Langmuir's viscous-flow equation (26).

Curves of η_c versus R for $N_{Re} = 0.01, 0.1$, and 1.0 calculated from Equation 51 are given in Figure 2. It is of interest to note that the effect of interception is considerably smaller in viscous flow than in the potential flow of an ideal fluid. The efficiency of interception varies directly with the Reynolds Number based on the cylinder diameter. Since the efficiency of inertial impaction likewise varies with the Reynolds Number, it is expected that the total efficiency of impaction, η , increases with increasing values of N_{Re} .

Davies (7) also calculated a curve for the efficiency of interception at $N_{Re} = 0.2$. While he does not give an equation, efficiencies read from his curve for values of R between 0.75 and 1.5 are almost identical with those calculated from Equation 51 for $N_{Re} = 0.2$. For values of R less than about 0.7 , his efficiencies are higher than those given by Equation 51.

Davies also performed stepwise calculations for the limiting particle trajectories using his flow equation to obtain the total efficiency of impaction for $N_{Re} = 0.2$. His curves have been plotted in terms of η

versus $\sqrt{\psi}$ with R as the parameter in Figure 3. An interesting point to be noted is that the effect of interception increases as ψ is increased. This shows that the efficiency of interception is a function not only of R and N_{Re} , but also of ψ . The limiting value of η_c at $\psi = 0$ when directly added to the efficiency of inertial impaction would give an efficiency less than the correct total efficiency.

Davies gives the following curve-fitting equation for the total efficiency of impaction:

$$\eta = 0.16 \left[R + (0.5 + 0.8R) \psi - 0.1052 R \psi^2 \right] \quad (52)$$

III. EXPERIMENTAL

1. Apparatus

Figure 4 shows the experimental equipment used to establish the curve for the efficiency of inertial impaction of aerosol particles on circular cylinders. One-mil and 3-mil platinum wires and 2-mil and 4-mil tungsten wires were used as the cylinders. The diameters of the wires were measured accurately under the microscope. Table 1 shows the arithmetic averages and maximum deviations from the average.

Table 1. Wire Diameters

Wire	Average Diameter, Microns	Maximum Deviation from Average, Percent
1-mil platinum	29.0	8.6
3-mil platinum	82.6	3.6
2-mil tungsten	53.1	6.1
4-mil tungsten	105.7	0.87

The assembly consisted of a brass drum from which the wire was unwound, a horizontal Venturi-shaped nozzle made of Lucite through which the aerosol stream passed, and an 8-mm. Pyrex-glass wash tube into which the wire which had been exposed could be lowered and washed with conductivity water. The drum was attached to the center of a circular brass plate. The winding and unwinding of the wire was accomplished by manually rotating the drum in the appropriate direction. The drum was enclosed in a Lucite casing to prevent the wire from being contaminated.

The Lucite nozzle was installed with its circular cross-section in a vertical plane directly below the brass drum. The throat of the nozzle was 0.357 cm. in diameter and approximately 0.6 cm. in length. The cone-shaped convergent and divergent sections were both tapered at an angle of 60° cm. in length. Small holes were drilled through the upper and lower

walls of the nozzle throat to allow the wire to pass down into the wash tube. Since the diameters of the holes were large compared with the diameters of the wires, there was little chance for the plumbed wire to touch the edge of the hole leading to the wash tube.

The 8-mm. Pyrex wash tube, which was about four feet in length, was connected to the Lucite nozzle by means of a rubber stopper. To facilitate the introduction and removal of wash water, a side arm was attached near the top of the tube and a stopcock was connected to the lower end. In order to keep the wire straight and in a vertical position, a brass weight, ranging from 1 to 4 g. depending on the wire diameter, was attached to the end of the wire.

The homogeneous aerosol generator was similar to that designed by Sinclair and LaMer (36). The air for the generator was supplied at 90 lbs. per sq. in. pressure, saturated with water vapor, and filtered through a Fulflo filter. It was then passed through a pressure regulator and an Army Chemical Corps aerosol filter canister (Filter, Special, Particulate E12R3) to two rotameters. One rotameter delivered air to the salt nuclei generator while the other delivered air to the bubbler of the aerosol generator. The air flow through the rotameters in the experiments ranged from 0.3 to 4 liters per minute at 70°F. and 10 lbs. per sq. in. gage pressure, depending on the particle size desired.

Condensation nuclei were generated by heating a mass of sodium chloride held by a spiral Nichrome ribbon in a glass flask by means of an electric current. The current was controlled manually by a Variac in conjunction with an ammeter and a voltage regulator.

The aerosol generator was inclosed in an insulated box. The two glass flasks, one containing the aerosol material and the other serving as

the reheater, were placed in metallic tanks filled with liquid paraffin heavy oil. The oil baths were heated electrically by means of heaters installed directly below them. The temperatures in the baths were regulated by mercury thermoregulators.

Concentrated sulfuric acid, C.P. 95.5 percent, was used as the aerosol material. The generator produced aerosol particles of nearly uniform diameters in the range from 0.6 to 1.4 microns at a rate of 1 to 5 mg. per minute.

The sulfuric acid aerosol was pulled through the equipment by means of a rotary vacuum pump. The aerosol could also be discharged through a line to a water aspirator while the generator was coming to equilibrium. It was mixed with filtered air to obtain the desired flow rates through the nozzle. From the mixing chamber, it flowed through a two-way quick-opening valve. When the valve was in one position, the aerosol passed to the Lucite nozzle, in the other position, the aerosol passed through one of the following paths: a) through a line leading directly to the filter before the vacuum pump; b) through a high velocity impactor for the determination of acid concentration of the particles; or c) through the Owl for the determination of particle diameters.

The aerosol stream, after passing the Lucite nozzle, passed through a filter train and a Lucite Venturi meter before reaching the vacuum pump. The filter train consisted of three separate filters, each tightly packed with one inch of No. 800 Pyrex Brand wool filtering fiber. These filters could be removed and weighed separately. Complete removal of the particles was assumed if nearly all the weight increase appeared in the first two filters. The flow rate through the nozzle was determined by two Lucite Venturi meters with different ranges of flow rates. These were calibrated against a set of Fischer and Porter rotameters. In the calibration

and use of these meters, the equation for the weight rate of discharge recommended by the A.S.M.E. Special Research Committee on Fluid Meters was used (9). A mercury thermometer was installed at the downstream end of the Venturi meter. The temperature indicated by the thermometer in the Venturi meter was always nearly the same as the room temperature and was taken as the temperature at the nozzle throat.

The acid concentration of the aerosol particles was determined by collecting a weighable quantity of particles in the cup of a high velocity impactor (32) and analyzing the content in the cup by titrating with 0.1 N sodium hydroxide solution.

The diameter of the aerosol particles was measured with a calibrated Owl (No. G-2) (35) obtained from the U.S. Army Chemical Corps. When additional air of nearly the same moisture content was mixed with the aerosol rising from the generator to give the necessary velocity through the throat of the nozzle, the particle diameter remained substantially the same as that before the air was added.

A precision conductivity bridge built according to the design of Luder (29) was used to analyze the quantity of acid particles impacted on the wire. A dip conductivity cell with platinized platinum electrodes with a cell constant of approximately 0.1 was employed. The bridge was capable of indicating down to 0.02 percent of the cell resistance. A Simpson meter was used for balancing the bridge. The dip cell was calibrated at 30°C. using 10^{-6} to 10^{-2} N sulfuric acid solution made by diluting 10^{-1} N standard acid solutions.

2. Procedure

The aerosol generator was operated for about four hours before the experimental runs were started. This allowed the operating conditions to be adjusted and steady state to be established. In the first part of this

period, the aerosol was discarded through the line connected to the water aspirator. During the later part, the vacuum pump was started and samples of the aerosol were taken in the Owl. Particle size measurements were made every five to ten minutes until the same size was indicated in successive measurements.

As soon as the particle size had been established, the two-way valve, was turned to allow the aerosol stream to pass through the nozzle for measuring the impaction of the particles on the vertical wire hung across the nozzle throat. The needle valve regulating the input of additional air and the valves regulating the input to the vacuum pump were adjusted until the desired flow rate through the nozzle was obtained. The pressure on the upstream of the nozzle was maintained equal to that outside the system. The circular plate attached to the brass drum was rotated manually to lower the wire at a predetermined rate so that enough particles would be collected on the wire for accurate analysis but not enough to cause reentrainment of particles from the surface of the wire. The time for approximately 4 feet of the wire to pass the nozzle throat ranged from 3 to 19 minutes depending on the estimated rate of aerosol generation and the efficiency of impaction for that test run. The duration of most runs was 4 to 6 minutes.

At the completion of a run, the aerosol was allowed to pass the Owl so that the particle size could be rechecked. In most test runs, the particle size did not change appreciably during the run. The quantity of aerosol which escaped impaction was determined by weighing the filters in the filter train before and after the run.

The acid collected on the wire was washed with a known quantity of conductivity water (45 cc. for most runs). The solution was withdrawn through the bottom of the tube, and the washing procedure repeated twice with the same solution. The tube was then washed with fresh conductivity

water. The solution was divided into three tubes and placed in the thermostat at 30°C. The dip conductivity cell was thoroughly washed in the solution in one of the test tubes, and the concentrations of the solutions in the other two tubes were measured. For most runs, the concentrations of the two solutions agreed with each other within the accuracy of the calibration curve for the cell.

For each series of runs made under one set of conditions in the aerosol generator, the concentration of aerosol particles was determined by collecting the particles in an impactor cup, using a velocity sufficient to give 100 percent removal efficiency (32). The density of the particles was obtained from tables.

3. Sample Calculations

Table 5 in the APPENDIX is a compilation of the experimental data and the results for the impaction of sulfuric acid aerosol particles on the four single wires. The following are illustrative calculations of the efficiency of impaction on the 4-mil tungsten wire (Test Run No. T4-15) with the equipment operating under a given set of conditions:

Absolute pressure on upstream of Venturi meter	= 943 cm. H ₂ O
Pressure drop across Venturi meter	= 8.0 cm. H ₂ O
T ₀ , temperature of aerosol passing Venturi meter and nozzle	= 300.5° K.
Mass rate of air passing nozzle according to meter	= 29.6 g./sec.
p ₀ , absolute pressure of nozzle	= 992 cm. H ₂ O
ρ, density of air at nozzle [*]	= 1.129 x 10 ⁻³ g./cc.
Diameter of nozzle throat	= 0.357 cm.
Area of nozzle throat	= 0.1002 cm. ²
v ₀ , superficial velocity of aerosol passing nozzle throat	= 2615 cm./sec.
μ, viscosity of air at nozzle ^{**}	= 1.854 x 10 ⁻⁴ poise

D_c , diameter of wire	= 105.7 microns
$N_{Re} = D_c v_o \rho / \mu$, Reynolds Number based on wire diameter	= 168
Weight of acid collected in cup of high velocity impactor	= 0.0366 g.
Volume of sodium hydroxide solution required for titration	= 4.40 cc.
Normality of sodium hydroxide solution	= 0.0996
Concentration of acid in aerosol particles	= 58.7 %
ρ_p , density of aerosol particles ***	= 1.478 g./cc.
D_p , particle diameter according to Owl	= 0.97 micron
C , Cunningham factor (Equation 26)	= 1.178
$\Psi = \frac{C \rho_p v_o D_p^2}{18 \mu D_c}$, inertial parameter	= 1.21
$\sqrt{\Psi}$	= 1.10
$R = D_p / D_c$, interception parameter	= 0.0090
Weight of aerosol particles collected in filter train	= 0.0305 g.
Weight of 100 percent sulfuric acid collected in filter train	= 0.0179 g.
Volume of conductivity water used to wash wire	= 45.0 cc.
Concentration of wash solution according to dip cell	= $16.0 \times 10^{-5} \text{ N}$
Weight of 100 percent sulfuric acid impacted on wire	= 0.000353 g.
Ratio of projected area of wire to area of nozzle throat	= 0.0376
η , efficiency of impaction	= 0.514

* The density of air was calculated by the perfect gas equation assuming the air has a density of 1.200×10^{-3} g./cc. at 70° F. and 1 atm. pressure.

★★ The viscosity of air was based on values given by Hirschfelder, J. O., Bird, R. B., and Spotz, E. L., J. Chem. Phys., 16, 968 (1948); Chem. Rev., 44, 205 (1949); Trans. Am. Soc. Mech. Engrs., 71, 921 (1949).

★★★ The density of the sulfuric acid aerosol particles was based on values given in "Chemical Engineers' Handbook", 3rd ed., p. 184, New York, McGraw-Hill Book Co., Inc., 1950.

IV. DISCUSSION OF RESULTS

1. Characteristics of the Experimental Impaction Efficiency Curve

Figure 5 shows the experimental impaction efficiencies of sulfuric acid aerosol particles on four diameters of wires. ^{*} The ranges of variables represented are: diameter of aerosol particles, 0.56 to 1.40 microns; superficial velocity of aerosol stream passing wires, 399 to 5090 cm./sec.; diameters of wires, 29.0, 53.1, 82.6, and 105.7 microns; and Reynolds Number based on the measured wire diameters, 13.0 to 330. The acid concentration of the aerosol averaged around 58.5 percent sulfuric acid with a maximum deviation from the average of less than 6 percent. The density of the aerosol particles was substantially constant with an average of 1.48 g./cc.

In the range of particle diameter and aerosol stream velocity employed in the experiment, collection due to Brownian diffusion was negligible. Collection by electrostatic forces was improbable since both the aerosol particles and the wires were uncharged. Gravity settling should also be unimportant with the wires in the vertical position. The values of the interception parameter, R , ranged from 0.022 to 0.038 for the 29.0 micron wire, from 0.013 to 0.023 for the 53.1 micron wire, from 0.007 to 0.017 for the 82.6 micron wire, and from 0.005 to 0.012 for the 105.7 micron wire. The effect of interception was considered to be negligible for the two large wires. For the 29.0 and 53.1 micron wires, impaction efficiencies recorded were, respectively, 0.21 and 0.074 or higher so that small effects of interception would not appreciably affect the shape and position of the

^{*} Some preliminary experimental impaction efficiencies of sulfuric acid aerosol particles on a 3-mil platinum wire have been published in a paper entitled "Impaction of Dust and Smoke Particles on Surface and Body Collectors" by W. E. Ranz and J. B. Wong in Ind. Eng. Chem., 44, 1371 (1952). It was discovered later that the wire-holding assembly used to collect these preliminary data was of a poor design and the Owl used to measure the particle diameter did not give correct readings for particle diameters below about 0.4 micron. For these reasons the early data are not as accurate as those presented in this report.

impaction efficiency curve. The curve drawn through the points in Figure 5, therefore, represents the experimental efficiencies of inertial impaction.

The experimental curve is S-shape, characteristic of the inertial impaction mechanism on surface and body collectors. It indicates a critical value of $\sqrt{\Psi}$ of approximately 0.25, below which impaction does not occur. At higher values of the inertial parameter, the curve appears to be asymptotic to the value of $\eta_I = 1$. Because of difficulties involved in the control of experimental variables, especially the diameter of aerosol particles, many test runs were necessary to establish the position of the curve. The impaction efficiencies were reproducible in terms of $\sqrt{\Psi}$, in view of the ten-fold variation in velocity, the three-fold variation in particle size, and the insensitivity of the Owl for detecting small variations in the particle size. The accuracy and reliability of the results depend largely on the homogeneity of the particle size and the accuracy of the particle size measurements.

2. Effect of the Reynolds Number

A careful examination of the results plotted in Figure 5 and the tabulated values of the Reynolds Number, N_{Re} , in Table 5 in the APPENDIX shows the effect of N_{Re} on the impaction efficiency. For the two platinum wires, for which the values of N_{Re} ranged from 13.7 to 32.8 for the 1-mil wire and up to 248 for the 3-mil wire, the impaction efficiency in all cases is higher for higher values of the Reynolds Number with the same value of $\sqrt{\Psi}$. The same observation can be made on the two tungsten wires, for which N_{Re} ranged from 13.0 to 58.5 for the 2-mil wire and up to 330 for the 4-mil wire. This agrees with the conclusion drawn previously.

Comparisons of the data for the 1-mil platinum wire with those for the 2-mil tungsten wire, and the data for the 3-mil platinum wire with those for the 4-mil tungsten wire, however, show an opposite effect of the

Reynolds Number, i.e., the impaction efficiencies on the 3-mil wire at lower values of N_{Re} are generally higher than those on the 4-mil wire for corresponding values of $\sqrt{\psi}$. The explanation of this is not apparent. When these wires were observed under the microscope, it was noticed that the surface of the platinum wire was much smoother than that of the tungsten wire. Possibly the aerosol particles adhered to the surface of the platinum better than to the tungsten wire.

3. Comparisons with Theoretical Curves

Figure 1 is a plot of the experimental inertial impaction efficiency curve together with the various theoretical curves proposed. The experimental curve agrees closely with that calculated by Landahl and Herrmann based on $N_{Re} = 10$, up to $\sqrt{\psi} = 1.4$. For values of $\sqrt{\psi}$ between 0.4 and 1.2, the experimental data indicate impaction efficiencies somewhat smaller than those shown by the curve of Langmuir and Blodgett and considerably smaller than the values of Sell, and of Albrecht. This is to be expected since the curves of Langmuir and Blodgett, and Albrecht were based on the potential flow of an ideal fluid, and that of Sell was based on a flow pattern at large values of N_{Re} on a 10 cm. cylinder. In the experimental flow conditions, viscous forces enter to some extent, thus decreasing the impaction efficiencies.

For values of $\sqrt{\psi}$ greater than about 1.4, the experimental efficiencies are higher than those according to the curves of Langmuir and Blodgett, and Landahl and Herrmann. The reason for this discrepancy is not entirely clear. One point to be noted is that, for higher efficiencies of inertial impaction, i.e., efficiencies approaching unity, the particle trajectories must be nearly parallel to the direction of flow and the particle must cut across the streamlines upstream of the wire where the streamlines begin to spread. In stepwise calculations of particle trajectories, it is not

practical to start the calculation more than a few diameters of the collector upstream. This method was followed by both Langmuir and Blodgett, and Landahl and Herrmann. It is possible that errors introduced by this could cause the calculated impaction efficiencies in the high efficiency range to be lower than the correct values.

The critical value of $\sqrt{\psi}$ at approximately 0.25 shown by the experimental impaction efficiencies agrees with the values of 0.3 and 0.25 explicitly stated by Albrecht, and Langmuir and Blodgett, respectively. The curve of Landahl and Herrmann also implies that the efficiency of inertial impaction is negligible at values of $\sqrt{\psi}$ less than about 0.25.

No comparison can be made with Davies' theoretical curve since it is based on viscous flow at $N_{Re} = 0.2$, far below the range attainable with the method used in the experiment. Furthermore, the impaction efficiencies predicted for such a low Reynolds Number could not be verified with the experimental technique well because of the analytical difficulties.

PART II

COLLECTION EFFICIENCY AND PRESSURE DROP OF FIBER MATS

The theory of the collection of particles on fibrous filters has been studied by Albrecht (1), Langmuir (26), and Davies (7). Albrecht's theory is based on the potential flow of an ideal fluid, a condition very different from viscous flow which ordinarily exists in these filters. Langmuir's theory takes into account only two mechanisms of collection, interception and Brownian diffusion. The experimental data of LaMer (23), and of Ramskill and Anderson (30) show that the mechanism of inertial impaction also plays an important part in the collection efficiency of these filters. These authors, however, have not evaluated this mechanism quantitatively. Davies' theory takes into account all of the major mechanisms of particle collection. He proposed an equation derived on theoretical grounds for the efficiency of the fibers in the filter. The present work was conducted with the purpose of evaluating quantitatively the mechanism of inertial impaction and testing the validity of Davies' equation.

The pressure drop across fibrous media has been studied on the basis of the hydraulic radius concept of Kozeny (18) and Carman (5). Davies (7) studied the problem using dimensional analysis. Ilerall (15) and Langmuir (26) derived theoretical equations for the pressure drop. The conclusions of these authors are not in good agreement. Another object of the present work was to test the proposed equations experimentally.

I. THEORETICAL DEVELOPMENT

1. Collection Efficiency of Fiber Mats

Consider an idealized mat to consist of uniform circular fibers which are everywhere perpendicular to the direction of the flow of the aerosol stream. If the mat is so porous that no fiber is influenced by neighboring fibers as far as the collection of aerosol particles is concerned, the situation will be identical with that of the impaction of particles on single cylinders treated previously in PART I. It is recalled that in cases where electrostatic attraction and Brownian diffusion are negligible, the total efficiency of impaction is a function of the inertial parameter, the interception parameter, the settling parameter, and the Reynolds Number based on the cylinder diameter. Thus the function of the collection efficiency of the fibers in the mat is established. The relation between the single fibers in the mat and the physical constants of the mat is obtained in the following manner: Consider L as the length of fibers per unit volume of the idealized mat. The volume of fibers per unit volume of the mat is then $\pi D_f^2 L/4$. This fiber volume fraction, denoted by α , is also obtainable as ρ_b/ρ_f . Hence,

$$L = \frac{4\alpha}{\pi D_f^2} = \frac{4(\rho_b/\rho_f)}{\pi D_f^2} \quad (53)$$

Consider a homogeneous aerosol stream flowing toward the mat, carrying N_0 particles per unit volume of gas. If N is the number concentration of particles at any plane of the mat perpendicular to the direction of flow, and $-dN$ is the incremental decrease of the number concentration in passing through the thickness of the mat, then $-dN/N$ represents the fraction of aerosol particles the aerosol stream loses in passing dh . Assuming the mat to be uniform, the length of the fibers which occupy the volume Adh is

$LAdh$, and the projected area of the fibers in the direction of flow is $D_f LAdh$. The ratio of this projected area to the area of the face of the mat multiplied by the total efficiency of the single fiber, $D_f L \eta dh$, is the function of aerosol particles passing dh that is collected by the fibers. This fraction must be equal to $-dN/N$ if electrostatic attraction and Brownian diffusion are negligible, and if no particle is evaporated. Hence, substituting the value of L from Equation 53 gives

$$-\frac{dN}{N} = \frac{4\alpha \eta dh}{\pi D_f} \quad (54)$$

Integrating Equation 54 between the limits from N_0 to N_h for dN and from 0 to h for dh , where N_h is the number concentration after the aerosol stream has passed the mat, gives

$$N_h/N_0 = e^{-\frac{4\alpha \eta h}{\pi D_f}} \quad (55)$$

N_h/N_0 is the fraction of aerosol particles penetrating the mat. The collection efficiency of the mat is

$$\eta_{mat} = 1 - (N_h/N_0) = 1 - e^{-\frac{4\alpha \eta h}{\pi D_f}} \quad (56)$$

Equation 56 contains the physical constants of the mat that are ordinarily available, i.e., α , D_f , and h , and the total efficiency of the single fiber, η , which is directly related to the variables of the aerosol stream, e.g., the velocity and particle diameter.

A second and perhaps more realistic derivation of the collection efficiency of a fiber mat may be obtained for comparison as follows. Consider an idealized mat to consist of n layers of uniform-diameter fibers which are everywhere perpendicular to the direction of flow. If these fibers are distributed evenly through the mat, a parallelepiped with two perpendicular edges equal to h/n and the third edge at unit length will contain on the average a volume of fibers equal to $\pi D_f^2/4$. This fiber volume divided by the volume of the parallelepiped, $(h/n)^2$, is equal to

the fiber volume fraction, α . Hence,

$$n = (4\alpha/\pi)^{\frac{1}{2}} h/D_f \quad (57)$$

If the mat is porous so that the fibers do not overlap one another, the ratio of projected area of the fibers to the area of the face of the mat for any one layer is $D_f/(h/n)$ and the collection efficiency of that layer is $D_f\eta/(h/n)$. This collection efficiency is the same for all layers of the mat if a homogeneous aerosol stream and perfect mixing of the stream between layers are assumed. The fraction of aerosol particles penetrating each layer is $1 - D_f\eta/(h/n)$ and the fraction of the particles of the original aerosol stream penetrating the entire mat is $\left[1 - D_f\eta/(h/n)\right]^n$. Thus,

$$N_h/N_o = \left[1 - D_f\eta/(h/n)\right]^n \quad (58)$$

Combining Equations 58 and 57, the fraction penetration is given by

$$N_h/N_o = \left[1 - (4\alpha/\pi)^{\frac{1}{2}} \eta\right]^{(4\alpha/\pi)^{\frac{1}{2}} h/D_f} \quad (59)$$

The collection efficiency of the mat is

$$\eta_{mat} = 1 - (N_h/N_o) = 1 - \left[1 - (4\alpha/\pi)^{\frac{1}{2}} \eta\right]^{(4\alpha/\pi)^{\frac{1}{2}} h/D_f} \quad (60)$$

The collection efficiency given by Equation 60 can be compared with that given by Equation 56 by expanding both into infinite series of the following functions:

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots \quad (61)$$

$$(1 - y)^m = 1 - my + \frac{m(m-1)}{2!} y^2 - \frac{m(m-1)(m-2)}{3!} y^3 + \dots$$

for $-1 < y < 1$

If m is large compared with unity, the two series become identical when

$x = my$. Letting $x = \frac{4\alpha\eta h}{\pi D_f}$; $y = (4\alpha/\pi)^{\frac{1}{2}} \eta$; and $m = (4\alpha/\pi)^{\frac{1}{2}} h/D_f$, it can

be shown that the collection efficiency given by Equation 60 is identical with that given by Equation 56, if $(4\alpha/\pi)^{\frac{1}{2}}\eta < 1$ and $(4\alpha/\pi)^{\frac{1}{2}} h/D_f \gg 1$. Consider first the group $(4\alpha/\pi)^{\frac{1}{2}}\eta$. If η is unity, α will have to be higher than 0.78 in order to make the group unity. Since this high value of the fiber volume fraction is not attainable with fiber mats, it can be assumed safely that the group $(4\alpha/\pi)^{\frac{1}{2}}\eta$ is always less than unity. Consider now the group $(4\alpha/\pi)^{\frac{1}{2}} h/D_f$. The minimum value of this group in fiber mats ordinarily used as aerosol filters can be estimated by taking $\alpha = 0.005$, $h = 0.1$ cm., and $D_f = 0.001$ cm. (10 microns). With this set of values, the group is equal to 8.0. It is true that in special cases, the value of D_f is greater than 0.001 cm., but in such cases, the value of h usually has to be more than proportionally greater than 0.1 cm. Thus, the minimum value of the group $(4\alpha/\pi)^{\frac{1}{2}} h/D_f$ can be assumed to be of the order of 8.0 for fiber mats. Since this minimum value is large compared with unity, it can be concluded that the two equations derived for the collection efficiency of fiber mats are identical. Equation 56, being in a simpler form, will be used throughout this report.

Equation 56 has been derived on the simple assumptions that a) all fibers are perpendicular to the direction of flow; b) the fibers do not interfere with each other; c) the ends of fibers have a negligible effect; and d) the total efficiency of the single fiber perpendicular to the direction of flow is applicable. In actual fiber mats, none of these assumptions is completely justified. It is conceivable that an equation can be derived assuming a certain fraction of the fibers of the mat to be parallel to the direction of flow. Such an equation, while based on a more realistic assumption, will contain the efficiency of the fibers parallel to the direction of flow and the length of the fibers. Since the functional

dependency of the efficiency of impaction of particles on single fibers parallel to the flow has not been investigated, and since the length of the fibers in a fiber mat is difficult to measure, such an equation would be of little practical value.

The approach to be followed in the present work is to use Equation 56 as the basis for correlating experimental data, incorporating all the effects which have not already been taken into account into an effective fiber efficiency, η_e , i.e.,

$$\eta_{mat} = 1 - (N_h/N_o) = 1 - e^{-\frac{4\alpha\eta_e h}{\pi D_f}} \quad (62)$$

2. Pressure Drop of Fiber Mats

The idealized fiber mat consisting of uniform fibers everywhere perpendicular to the direction of flow can also be used to arrive at an equation for the pressure drop of a fiber mat. Consider the idealized mat to have a face area A and a thickness h as before. The total length of the fibers making up the mat is LAh , and the projected area in the direction of flow is $LAhD_f$. The drag coefficient of a circular cylinder perpendicular to the direction of flow is

$$F_T = \frac{\rho v_o^2 A_c C_D}{2} \quad (63)$$

Application of Equation 63 to the fibers of the mat gives

$$\Delta P A = \frac{\rho v_o^2 LAhD_f C_D}{2} \quad (64)$$

Substituting the value of L given by Equation 53 into the above equation gives

$$\Delta P = \frac{2\rho v_o^2 \alpha h C_D}{\pi D_f} \quad (65)$$

Unlike the total efficiency of impaction, which is an incompletely defined function of several parameters and the Reynolds Number, the drag coefficient of the cylinder, C_D , has been found to be a function of the

Reynolds Number alone. For the fluid in viscous flow, Lamb's theoretical equation (Equation 19) gives

$$C_D = \frac{8\pi CL}{N_{Re} v_0} \approx \frac{8\pi}{N_{Re}(2.002 - \ln N_{Re})} \quad (66)$$

Finn (31) has recently made measurements of the fluid drag on tungsten and chromel wires ranging from 4 to 13 microns diameter at Reynolds Number from 0.06 to 6.0 and verified Equation 64 up to Reynolds Number of about 1.0. His data indicate that the equation overpredicts the drag coefficient for values of Reynolds Number greater than about 1.0.

Equation 65 contains the measurable physical constants of the met, α , h , and D_f , and the drag coefficient of the fiber, C_D , which is completely defined, at least in the region of viscous flow, in terms of the Reynolds Number. This equation will be used as a basis for correlating pressure drop data with an effective fiber drag coefficient, C_{De} , which incorporates the effects of non-perpendicular fibers, fiber interference, ends of fibers, and non-uniform fiber distributions, i.e.,

$$\Delta P = \frac{2\rho v_0^2 \alpha h C_{De}}{\pi D_f} \quad (67)$$

II. REVIEW OF PREVIOUS WORK

1. Collection Efficiency of Fiber Mats

Albrecht (1) has obtained a theoretical equation for the collection efficiency of a fibrous filter. He states, however, that his equation holds only for the case of potential flow of a non-viscous fluid in the spaces between the fibers. He predicts that for values of the inertial parameter, Ψ , below 0.09, particles are not collected by a filter.

Langmuir (26) has revised some of Albrecht's calculations using flow lines around the cylinder from Lamb's theory of flow of a viscous fluid and found that the critical value of Ψ at which inertial impaction begins is 0.27. Assuming this critical value to be the correct one, he calculated that for a gas flow of 3.5 cm./sec., a fiber diameter of 8.8 microns, oleic acid particles (with density of 0.89 g./cc.) could be collected by inertial impaction only if they have diameters greater than 5.0 microns. Experimental data available at the time showed that the penetration of particles through a filter increases when the velocity is increased, an effect that is directly opposite to the case where inertial impaction is important. Reasoning from this experimental finding and his critical value of Ψ , Langmuir concluded that the inertial impaction mechanism could not be important for the filtration of fine smoke particles and that Albrecht's theory is not applicable to aerosol filtration.

Proceeding from fundamental principles, Langmuir then derived a theory of mechanical filtration, i.e., filtration which does not involve electrostatic forces, which includes only two mechanisms of collection, interception and Brownian diffusion. His final equations are

$$\frac{\Delta P}{a_{ch}} = \frac{4.05 \times 10^{-4} D_v}{(\epsilon_o r_f + 9 \times 10^{-6})(\epsilon_o r_f - r_p)^2} \quad (68)$$

$$\frac{r_f \delta}{AR_p} = 1760 \epsilon_o r_f (\epsilon_o r_f + 18 \times 10^{-6}) \quad (69)$$

Using the equation

$$D_v = k_0 TC / (3\pi\eta D_p) \quad (70)$$

Langmuir calculated the values of the diffusion coefficient of aerosol particles of different radii. These are tabulated in Table 2. It is to be noted that Equation 70 is identical with Einstein's equation (10) when $C = 1$.

Table 2. Calculated Diffusion Coefficients for Aerosol Particles in Air at 20°C. and 1 Atmosphere ($\lambda_m = 1.31 \times 10^{-5}$ cm.)

r_p , microns	D_v , cm. ² /sec.	r_p , microns	D_v , cm. ² /sec.
0.05	6.80×10^{-6}	0.65	2.05×10^{-7}
0.1	2.21×10^{-6}	0.7	1.89×10^{-7}
0.15	1.23×10^{-6}	0.8	1.63×10^{-7}
0.2	8.35×10^{-7}	0.9	1.435×10^{-7}
0.25	6.28×10^{-7}	1.0	1.280×10^{-7}
0.3	5.03×10^{-7}	1.2	1.053×10^{-7}
0.35	4.18×10^{-7}	1.5	8.33×10^{-8}
0.4	3.56×10^{-7}	2	6.16×10^{-8}
0.45	3.10×10^{-7}	3	4.05×10^{-8}
0.5	2.76×10^{-7}	5	2.41×10^{-8}
0.55	2.47×10^{-7}	10	1.193×10^{-8}
0.6	2.24×10^{-7}		

Langmuir's theory predicts that for a particular filter operating at a particular velocity there is a size of aerosol particles which would have a maximum penetration through a filter mat. This prediction comes about from the fact that Brownian diffusion increases as the particle diameter is decreased and interception decreases as the particle diameter is decreased.

Since the time Langmuir's theory appeared in 1942, a wealth of experimental data have become available. In the light of some of these data, the adequacy of Langmuir's theory, particularly the existence of the optimum particle size for penetration, has been seriously questioned. Experimental data of Freundlich (14), LaMer and Sinclair (34), and Burton (4) indicate that an optimum particle size exists whereas the data of Knudson and White

(16), LaMer (23), and Ramskill and Anderson (30) show no such an optimum size. Based on his latest extensive investigation of aerosol filtration, LaMer has concluded that inertial impaction persists to much smaller particle sizes than stated by Langmuir and this is the reason that no optimum particle size was observed. Ramskill and Anderson think that the face velocity of the aerosol stream passing the filter of 3.5 cm./sec. on which Langmuir derived his theory is perhaps confusing the entire filtration picture. They reconsidered the three mechanisms of collection, i.e., inertial impaction, interception, and Brownian diffusion, with the face velocity as the primary variable. These authors give a qualitative picture of the three collection mechanisms in a graph in which the percent penetration of particles through the filter is plotted against the face velocity. The graph indicates that the penetration increases with increasing velocity for Brownian diffusion and is independent of the velocity for interception, both of which are indicated in a quantitative manner in Langmuir's theory. The filtration picture of Ramskill and Anderson indicates that for a particular filter and particle size, the inertial impaction mechanism starts at some critical velocity v_c , and this mechanism no longer exerts any additional effect at some velocity v_m . This is equivalent to saying that there is a critical value of ψ below which inertial impaction does not occur and one above which no additional inertial impaction takes place.

Ramskill and Anderson performed a series of experiments using homogeneous aerosols of dioctylphthalate (DOP) and sulfuric acid with particle diameters ranging from 0.2 to 1.0 micron. They measured the penetrations of particles through various types of filters with the N.R.L. E-3 Smoke Penetration Meter described by Knudson and White (16). They present their data in graphs in which the percent penetration is plotted against

the linear velocity through the filter. The critical velocity v_0 for each curve was estimated to be the velocity at which the curve indicates a maximum in percent penetration. The velocity v_m was estimated to be the velocity at which the percent penetration drops to zero. From the results obtained on a particular type of filter (the 3-micron diameter glass-filter fiber) they calculated the lower critical value of ψ to be 0.04, and the upper limit of ψ to be 0.28 when inertial impaction has no additional effect. Assuming these two values of ψ to be correct, they calculated equivalent diameters for other filters using estimated values of v_0 and v_m from the curves. The equivalent diameters so obtained agreed fairly well with the measured diameters for the 1- and 2-micron glass fiber filters. For other types of fiber filters, the equivalent diameters calculated were many times smaller than the average measured values. For example, in one case, the equivalent diameter of the fiber based on v_0 was 0.5 micron, that based on v_m was 0.8 micron, and the average measured value was 15 microns. From these findings, these authors concluded tentatively that when the inter-fiber distance is larger than the fiber diameter, the equivalent diameter is a direct measure of the fiber diameter and when the interfiber distance is less than the fiber diameter, the equivalent diameter is a direct measure of the interfiber distance. They also found no optimum particle size for penetration. They concluded that the inertial impaction mechanism definitely exists and plays an important role in the mechanical filtration of aerosols.

A more recent theory on mechanical filtration has been advanced by Davies (7) who also included all three mechanisms of particle collection, i.e., inertial impaction, interception, and Brownian diffusion. His equation for the fraction of aerosol particles penetrating the filter is

$$N_h/N_0 = e^{-\frac{4\alpha \eta h}{\pi D_f(1-\alpha)}} \quad (71)$$

When α is small compared with unity, Equation 71 reduces to Equation 55. Davies gives the following functional relation for the total efficiency of the individual fibers in the filter:

$$\eta = \eta(P_t, N_{Re}/(1 - \alpha), R) \quad (72)$$

where $P_t = P_i + P_d =$ a general particle parameter

$P_i = 2 / (1 - \alpha) =$ Davies' inertial parameter

$P_d = 2D_v(1 - \alpha)/(v_o D_f) =$ Davies' diffusion parameter

In order to obtain a quantitative relation for the total efficiency η that is directly applicable to Equation 71, Davies combined his equation for the total efficiency of impaction of particles on single cylinders, as given by Equation 52, with an exact periodic solution to the Navier-Stokes equations in two dimensions of the flow of a viscous fluid passing a grid of cylinders given by Kovasnay (17) to arrive at the following equations:

$$\eta = \left[R + (0.25 + 0.4R)P_t - 0.0263 R P_t^2 \right] (0.16 + 10.9 \alpha - 17\alpha^2) \quad (73)$$

Equation 73 is a curve-fitting equation fitting two sets of curves, one set of which is obtained by stepwise calculation of the limiting particle trajectories passing a cylinder at $N_{Re} = 0.2$ in the form graphically represented in Figure 3, while the other set is obtained by estimating the lateral displacement of streamlines passing a two-dimensional grid. This elaborate curve-fitting process made on two sets of approximate results puts Equation 73 on rather doubtful grounds. In addition, Davies has assumed that P_d , the diffusion parameter, and P_i , his inertial parameter, have the same effect on the total collection efficiency of the filter.

For the case where gravity settling is important, Davies gives the following equation for the fraction of particles penetrating the filter:

$$N_h/N_o = e^{-\frac{4\alpha h}{\pi(1-\alpha)D_f} \left[\eta + \frac{v_t(1-\alpha)}{\pi v_o} \right]} \quad (74)$$

Equations 73 and 74 represent Davies' theory of mechanical filtration. This theory takes into account all the measurable physical constants of the filter, and all the mechanisms of particle collection encountered in mechanical type of filters. The accuracy of the theory depends heavily on the accuracy of Equation 73, which in turn depends on the correct representation of the general particle parameter, P_t . It also depends on whether the interference factor on particle collection, i.e., $(0.16 + 10.9\alpha - 17\alpha^2)/0.16$, estimated from the streamlines passing a two-dimensional grid, can be applied to a filter. All these factors must be determined experimentally.

Davies' theory also indicates an optimum particle size for penetration. This comes about mainly from the assumption that P_i and P_d have identical effects on the collection efficiency of the filter. Assuming that the diffusion coefficient can be expressed as $D_v = (8 \times 10^{-16})/D_p^2$, Davies plotted P_i and P_d versus $D_p^2 v_o/(1-\alpha)$ on the same graph with D_f as the parameter. From this plot, he estimates that the optimum particle size for penetration is given by

$$\frac{D_p^2 v_o}{4(1-\alpha)} = 10^{-9.4} \quad (75)$$

Davies' table showing the optimum particle sizes for penetration at varying mean velocities inside the filter is shown in Table 3.

From this brief review, it appears that the knowledge of the efficiency of particle collection of mechanical filters is far from complete. In the light of the recent data of LaMer, and of Ramskill and Anderson, it can be concluded that Langmuir's filtration theory is inadequate insofar as

Table 3. Davies' Optimum Particle Sizes for Penetration

Mean air velocity in filter, $v_o/(1 - \alpha)$, cm./sec.	Optimum particle diameter for penetration microns
100	0.04
16	0.1
4	0.2
1.8	0.3
1.0	0.4
0.64	0.5
0.44	0.6
0.25	0.8
0.16	1.0

it neglects the inertial impaction mechanism. Both LaMer, and Ramskill and Anderson present their data in plots of percent of the particles that penetrate the filter versus the linear velocity through the filter, or versus the particle diameter. With inadequate information about the physical constants of the filters they employed, they have not evaluated the mechanisms of collection quantitatively. The arbitrary assumption made by Ramskill and Anderson that there exists a value of the inertial parameter above which inertial impaction no longer exerts any additional effect does not appear to be theoretically sound, since all theoretical investigations on the inertial impaction of aerosol particles on single cylinders have indicated that the inertial parameter must be infinitely large before the inertial impaction efficiency reaches unity. Referring to Figure 1, if the theoretical curve of Davies for $N_{Re} = 0.2$ is approximately correct for the inertial impaction efficiency of the fibers in the filter (as will be shown to be true from experimental data), the inertial parameter ψ would have to be as large as 4 before the efficiency would reach 0.4, which is far from unity. Ramskill and Anderson, however, claim that they obtained the value of $\psi = 0.28$ above which inertial impaction ceases to be important. These authors actually

obtained this particular value of ψ for only one type of filter. For the other filters, they assumed this value to be correct and calculated equivalent diameters for the fibers in these filters. Reasoning objectively, if these authors had taken one of the filters which they calculated to have an equivalent fiber diameter less than $1/15$ of the measured fiber diameter and used it to obtain the value of the inertial parameter above which inertial impaction ceases to be important, then the value would turn out to be more than 15 times 0.28. One of the purposes of the present experimental work was to ascertain whether such a value of the inertial parameter above which inertial impaction ceases to be important does exist, and if so, whether the value given by Ramskill and Anderson is correct.

2. Pressure Drop of Fiber Mats

Several approaches have been followed in the study of the pressure drop across fibrous media. If the fibrous media is considered to be in the form of multiconnected channels one obtains D'Arcy's equation for the flow of water through sand

$$\Delta P/h = K v_o \mu \quad (76)$$

A theoretical treatment by Kozeny (18), who introduced the hydraulic radius concept, and subsequent modification of Kozeny's equation by Carman (5), enables the constant in Equation 76 to be related to the physical constants of the porous bed in the so-called Kozeny-Carman equation

$$\frac{\Delta P}{h} = K_1 v_o \mu \frac{s_o^2 (1 - \epsilon)^2}{\epsilon^3} \quad (77)$$

For granular materials of low porosity, the constant K_1 has been found to be about 5.

The Kozeny-Carman equation, either in the form given in Equation 77, or in some other modified form has been applied to fibrous media by Wiggins, Campbell and Meass (42), Fowler and Hertel (13), Sullivan and Hertel

(39), Sullivan (38), and First, et al. (12). Sullivan and Hertel found experimentally, for the flow of air through samples of glass-fiber medium, that K_1 is approximately 3.0 when the glass fibers are oriented parallel to the direction of flow, the constant is 6.0 when the fibers are perpendicular to the direction of flow, and 4.5 when the fibers are oriented random. Sullivan finally gives the resistance of the fiber bed as

$$\Delta P/h = \frac{K_0 v_0 \mu S_0^2 (1 - \epsilon)^2}{H \epsilon^3} \quad (78)$$

From hydrodynamic considerations, it has been shown that the orientation factor H is unity for the case where the fibers are parallel to the direction of flow, and $1/2$ for fibers perpendicular to the flow. Sullivan found in later experiments that the shape factor, K_0 , increases with the porosity from about 3 at a porosity of 0.8 to more than 20 at a porosity of 0.98. Equation 78 is not expected to hold for values of the porosity higher than about 0.88, and, for smaller values of porosity, the value of 3 for K_0 would give the approximately correct resistance for fibrous media.

Davies (7) thinks that the Kozeny-Carman equation can only be adjusted to fit the experimental data for low values of the porosity and will not give better agreement for higher values of the porosity by merely altering the constant because the function of the equation itself is incorrect. From dimensional analysis, assuming D'Arcy's equation holds, Davies has deduced the following functional relation:

$$\frac{\Delta P D_f^2}{v_0 \mu h} = f(\alpha) \quad (79)$$

Measurements on actual pads made on a wide variety of materials with values of α ranging from about 0.006 to about 0.3 show that the following empirical equation fits the data satisfactorily:

$$\frac{\Delta P D_f^2}{v_o \mu h} = 64 \alpha^{1.5} (1 + 56 \alpha^3) \quad (80)$$

Iberall (15) has followed another approach in which the pressure drop across a fiber mat is the total drag force on the fibers in a unit face area of the mat. He assumed an equipartition of the fibers in the three perpendicular directions, one of which is in the direction of flow. From Emersleben's paper (11) on the viscous drag of a fluid on a special array of cylinders, Iberall estimates the drag force on a circular cylinder with axis parallel to the direction of flow to be

$$F_D = 4\pi\mu v_o \quad (81)$$

For the drag force on the fibers with axes perpendicular to the direction of flow, he uses Lamb's equation (Equation 19) which he has written in the form

$$F_D = \frac{4\pi\mu v_o}{2 - \ln N_{Re}} \quad (82)$$

By summing the total drag force on the fibers of the mat, Iberall derives the following equation:

$$\Delta P = \frac{16\mu v_o \alpha h}{3D_f^2 (1 - \alpha)} \left\{ \frac{4 - \ln \left(\frac{N_{Re}}{(1 - \alpha)} \right)}{2 - \ln \left(\frac{N_{Re}}{(1 - \alpha)} \right)} \right\} \quad (83)$$

The mean velocity inside the mat, $v_o/(1 - \alpha)$, was used in the derivation.

Still another approach to the problem was advanced by Langmuir (26) at the time that he put forth his theory for mechanical filtration. He utilized the viscous drag concept to calculate the flow resistance as a function of the pressure drop across the filter in a manner analogous to the electrical resistance as a function of the potential drop given by Ohm's law. He considers a hypothetical filter model which has evenly spaced cylindrical fibers parallel to the direction of flow. Dividing the model filter into honey-comb hexagonal prisms each of which contains one coaxial

fiber, and then replacing each prism by a cylinder of the same cross-section, he calculates the resistance to the longitudinal flow of gas through the space between two coaxial cylinders of different diameters. Langmuir's final equation for the pressure drop across a fiber mat is

$$\Delta P = 16 B \mu v_o a h \mathcal{P} / D_f^2 \quad (84)$$

where $\mathcal{P} = 1/(-\ln \alpha + 2\alpha - \alpha^2/2 - 3/2)$

The effect of transverse flow and that of non-uniform distribution of fibers affect the value of B. On examination of the theory of the movement of ellipsoids through fluids, Langmuir concluded that, even if all fibers were transverse to the flow, B cannot be greater than 2. The effect of random non-uniform distributions of fibers is to lower B. This comes about from the stranding of the fibers which causes the effective D_f^2 to increase more than $\alpha \mathcal{P}$. The net result is a decrease in ΔP .

Langmuir has reported that values of B determined with glass fiber filters lie between 0.5 and 1 and are apparently independent of α . He concludes that the lower values of B indicate greater non-uniformities of fiber distribution.

Using Equation 67 as the definition of the effective drag coefficient, C_{De} , for the single fibers in a filter, the theoretical and semi-empirical equations for the pressure drop across a fiber filter can be reduced to the following dimensionless forms:

$$\text{Kozeny-Carman,} \quad C_{De} = \frac{8\pi K_o}{N_{Re} H} \frac{\alpha}{(1 - \alpha)^3} \quad (78a)$$

$$\text{Davies,} \quad C_{De} = \frac{32\pi}{N_{Re}} \alpha^{0.5} (1 + 56\alpha^3) \quad (80a)$$

$$\text{Iberall,} \quad C_{De} = \frac{8\pi}{3N_{Re}(1 - \alpha)} \left\{ 4 - \left[\ln \frac{N_{Re}}{(1 - \alpha)} \right] \right\} \left\{ 2 - \left[\ln \frac{N_{Re}}{(1 - \alpha)} \right] \right\} \quad (83a)$$

Langmuir,

$$C_{De} = \frac{8\pi B}{N_{Re}} \quad \phi$$

(84a)

This brief review has indicated that several different approaches can be followed in the correlation of the pressure drop across a fiber filter. The correlation based on the Kozeny-Carman type of equation appears to be applicable only for low porosities, a situation seldom encountered in aerosol filtration. Davies' equations, based on dimensional analysis, do not have a strong theoretical basis. Iberall's approach appears to be fundamentally sound. There is, however, one assumption in Iberall's derivation that is contrary to fact. He assumed that the drag forces for the parallel and transverse fibers to be those given, respectively, by Equations 81 and 82. Equation 82 is Lamb's theoretical result which has been shown to be valid by Finn's experimental data. Equation 81 is a deduction from Emersleben's theoretical paper and has not been verified by experiment. A comparison of Equations 81 and 82 will show that for values of the Reynolds Number smaller than unity, the drag force per unit length of fiber is at least twice as large for the fiber parallel to the flow than for the fiber transverse to the flow, e.g., for $N_{Re} = 0.01$, the drag force on the fiber parallel to the flow is approximately 6.6 times that on the fiber perpendicular to the flow. This does not appear to be logical, particularly in view of the results of Sullivan and Hertel who have shown that the drag on parallel fibers is only one half of that on transverse fibers. Iberall's derivation also implies that the effect of fiber interference can be accounted for by using the mean air velocity inside the fiber filter. Langmuir's theoretical equation for the pressure drop has not taken into account the effect of fiber interference.

Comparisons of the alternate forms of the pressure drop equations show that, with the exception of Iberall's equation, the effective drag

coefficient, C_{De} , is inversely proportional to the Reynolds Number, and hence, a plot of C_{De} versus N_{Re} on log-log paper will give straight lines with a slope of -1 with the positions of the lines dependent primarily on the value of α . Iberall's equation, however, shows that such a plot will give a non-linear curve. These equations will be tested critically with the present experimental data with this criterion as the basis.

III. EXPERIMENTAL

1. Fiber Mats and Mat-holding Assembly

The fiber mats used in the present work were formed by Arthur D. Little, Inc., Cambridge, Massachusetts, from three types of glass fibers manufactured by Glass Fibers, Inc., Toledo, Ohio: unbonded "B" fibers, "450" yarns, and "150" yarns. The diameters of the fibers were measured under the microscope. For the yarns, the measurements were made after the starch binder had been burned off at 400°C. Table 4 gives the arithmetic averages and maximum deviations from the average.

Table 4. Glass-fiber Diameters

Type of Fiber	Average Diameter, Microns	Maximum Deviation from Average, Percent
Unbonded "B" Fibers	3.51	78
"450" Yarns	6.24	34
"150" Yarns	9.57	27

The mat-holding assembly consisted of a Lucite holder and two Lucite connectors. The mat holder was essentially a coupling with an inside diameter of 1.9 cm. made out of a piece of Lucite rod 3 inches in diameter. One section of the holder had a collar 1/2 inch deep and 1 3/4 inches outside diameter made to fit exactly into the depression of the other section. On each of the two adjoining faces of the holder, a shallow depression 1 1/8 inches in diameter was made and a piece of 16-mesh Saran-wire screen about the same diameter was attached to the depressed surface. The depression had a depth slightly greater than the thickness of the screen so that when the two sections of the holder were coupled together, there was a clearance of about 1/32 inch between the two screens. When a mat of a certain thickness was to be placed in the mat holder, one or more ring-shaped rubber gaskets cut for the purpose were placed between the adjoining rims of the

holder so that the distance between the two Saran screens would be just equal to the thickness of the mat. Four screws were used to keep the holder in a coupled position. In this way, the outer rim of the mat was always under compression, and bypassing of the aerosol stream around the outer rim of the mat was kept at a minimum. The purpose of the two screens was to keep the mat in a stable position. The wires of the screens were so large compared with the diameters of the fibers in the mat that they had a negligible effect on the collection efficiency and pressure drop of the mat.

After the fiber-mat holder was assembled, it could be connected with its axis in a horizontal position and parallel to the direction of the aerosol flow to the two Lucite connectors by means of screws and rubber gaskets. The two Lucite connectors which had the same inside diameter as that of the mat holder, were installed permanently in the equipment. There was a pressure tap on each of the connectors for measuring the pressure drop across the mat. The connector on the upstream of the mat holder was connected to a Lucite tube which also had the same inside diameter as that of the mat holder. This led directly to the two-way quick-opening valve. The connector on the downstream of the mat holder was attached to a copper tube leading to the filter train.

2. Auxiliary Equipment

Figure 6 shows the experimental equipment used for determining the collection efficiency and pressure drop of fiber mats. The equipment was essentially the same as that used for determining the impaction efficiency of aerosol particles on single wires with the mat-holding assembly replacing the wire-holding assembly. Concentrated sulfuric acid, 95.5 percent, was also used here as the aerosol material. The particle diameter was measured with the Owl (No. G-2) as before.

During a test run, the aerosol stream after passing horizontally through the two-way valve first reached the fiber mat on which a fraction of the aerosol particles was collected and then passed through the filter train where the remaining particles were collected. From the filter train, the clean air passed the Lucite Venturi meter where the flow rate was determined and finally passed through the filter leading to the vacuum pump.

The three filters of the filter train were filled with No. 800 Pyrex Brand wool filtering fiber for the runs made with relatively large aerosol particles and with the unbonded "B" fibers, or a combination of unbonded "B" fibers and "Fiberfrax" ceramic fiber made by the Carborundum Company, Perth Amboy, New Jersey, for the runs made with relatively small aerosol particles. The "B" fibers and ceramic fibers were used instead of the No. 800 fibers because the former, by virtue of their smaller diameters, gave complete collection with less weight. The ceramic fibers were found to give a negligible increase to the conductance of the high-grade conductivity water used in connection with the precision conductivity bridge.

The two Lucite Venturi meters used in the experiments on single cylinders were also used here to indicate flow rates.

For the first series of test runs, the acid concentration of the aerosol particles was determined in two ways: a) by collecting a weighable quantity of particles in the cup of the high velocity round-jet impactor and analyzing the contents by titrating with 0.1 N sodium hydroxide solution; and b) by observing the gain in weight of the fiber mat and the filters in the filter train and analyzing their contents whenever these were sufficient to be titrated accurately with 0.05 or 0.1 N sodium hydroxide solution. Since substantially the same acid concentration for the aerosol particles was obtained by the two methods, collection by the high velocity impactor

was discontinued for the later test runs and the acid concentrations of the aerosol particles was based on the analysis of the fiber mat and/or the filters in the filter train. The average acid concentration of the aerosol particles for a series of test runs made under the same operating conditions for the aerosol generator was used as the acid concentration for all the runs in the series. It was thought that this average minimized personal errors in the analysis and tended to give a better comparison of the data. The acid concentration of the aerosol particles for the runs made on fiber mats ranged from 40 to 60 percent by weight of sulfuric acid. This represented a wider variation than the acid concentration observed for the runs on single wires. The variation here was partly due to the wider variation of the room temperature in the period during which the test runs were made and partly to the wide variation of operating conditions for the aerosol generator so as to be able to generate a greater particle size range.

3. Procedure

The fiber mats were cut into circular disks 4.12 cm. in diameter with a cork boring tool. Each disk was placed between two pieces of copper screens in a specially made ring-type brass coupling. After the brass coupling had been heated in a muffle furnace maintained at approximately 400° C. for 30 minutes, it was inverted and heated another 30 minutes. This heating period was necessary to ensure that the binding agent in the mat had been completely burned off. The thickness and diameter of the binder-free mats were measured with a micrometer caliper. The particular mat chosen for the test run was then weighed and placed in the Lucite mat holder which was then installed in the equipment.

The aerosol generator was operated approximately four hours before experimental data were taken. During the latter part of this period, the

aerosol was directed to the Owl. As soon as the particle size was established, the aerosol stream was directed through the fiber mat, the filter train, the Lucite Venturi meter, and the filter of the vacuum pump. The valves were adjusted for the desired flow rate. The manometers indicating the pressure drop and those indicating the flow rate were recorded at the beginning and the end of the run and at about every minute during the run. The time for each run was estimated before the run so as to have enough aerosol particles collected on the mat and on the filters in the filter train for accurate analysis but not enough to cause appreciable clogging of the fiber mat. The clogging of the mats could be noted by the increase of the pressure drop with time. In no case was this increase allowed to exceed 10 percent of the average pressure drop. The duration of most of the test runs was from 3 to 5 minutes.

At the end of the run, the aerosol stream was directed to the Owl where the particle diameter was again measured. The mat was carefully removed and its gain in weight recorded. The filters were removed from the filter train and their weight increases were recorded. In case the quantities of acid particles collected on the mat and in the filters were large enough to be titrated accurately, they were washed with distilled water and the solutions were titrated with 0.05 or 0.1 N sodium hydroxide solution. When these quantities were too small to be titrated, they were dissolved in known quantities of conductivity water (ranging from 50 to 100 cc.) and the resulting solutions were analyzed in the dip cell by the precision conductivity bridge. Removable Lucite bottoms were installed on each filter of the filter train to facilitate the removal of fibers from the filters. Care was taken in handling the mat and the filter fibers so that at no time was there any chance for these materials to be contaminated. From the quantities of acid collected on the mat and in the filters, the

collection efficiency of the mat could be calculated.

In the first series of test runs, the acid concentration of the aerosol particles was also determined by the high velocity round-jet impactor. The procedure for the impactor run was the same as that followed in the experiments on single wires.

4. Sample Calculations

Table 6 in the APPENDIX is a compilation of the experimental data and the results for the collection efficiency and pressure drop of fiber mats formed from the three sizes of glass fibers. The following are illustrative calculations of the collection efficiency and pressure drop of a fiber mat formed from the unbonded "B" fiber (Test Run No. B-62):

Absolute pressure on upstream of Venturi meter	= 810 cm. H ₂ O
Pressure drop across Venturi meter	= 34.0 cm. H ₂ O
T ₀ , temperature of aerosol passing Venturi meter and mat	= 306.4°K.
Mass rate of air passing mat according to meter	= 0.575 g./sec.
p ₀ , absolute pressure upstream of mat	= 1021 cm. H ₂ O
ρ, density of air upstream of mat	= 1.141 x 10 ⁻³ g./cc.
Diameter of mat exposed to aerosol	= 1.90 cm.
Area of mat exposed to aerosol	= 2.84 cm. ²
v ₀ , superficial velocity of aerosol passing mat	= 177.2 cm./sec.
μ, viscosity of air on upstream of mat	= 1.881 x 10 ⁻⁴ poise
D _f , average diameter of fibers in mat	= 3.51 microns
N _{Re} = D _f v ₀ ρ/μ, Reynolds Number based on fiber diameter	= 0.377
Mass of mat	= 0.4138 g.
Area of mat	= 13.33 cm. ²
h, thickness of mat	= 0.20 cm.
ρ ₀ , bulk density of mat	= 0.1550 g./cc.

ρ_f , density of fiber in mat	= 2.54 g./cc.
α , fiber volume fraction	= 0.0610
ΔP , average pressure drop across mat	= 20.1 cm. H_2O
C_{De} , effective fiber drag coefficient based on Equation 62	= 24.85
Weight of acid particles collected on mat	= 0.0186 g.
Volume of sodium hydroxide solution required for titration	= 3.41 cc.
Normality of sodium hydroxide solution	= 0.0559
Weight of 100 percent sulfuric acid collected on mat	= 0.00933 g.
Concentration of acid in aerosol particles	= 50.1 %
ρ_p , density of aerosol particles	= 1.386 g./cc.
D_p , particle diameter according to Owl	= 0.92 cm. $\times 10^{-4}$
C , Cunningham factor according to Equation 26	= 1.199
$\Psi = \frac{C \rho_p v_o D_p^2}{18 \mu D_f}$, inertial parameter	= 1.975
$\sqrt{\Psi}$	= 1.402
$R = D_p/D_f$ = interception parameter	= 0.263
Volume of conductivity water used to dissolve acid in filters	= 75 cc.
Concentration of resulting solution according to dip cell	= 6.25×10^{-6} <u>N</u>
Weight of 100 percent sulfuric acid collected in filters	= 0.0000230 g.
N_h/N_o , fraction of particles penetrating mat	= 0.00246
$\eta_{mat} = 1 - (N_h/N_o)$ = collection efficiency of mat	= 0.9975
η_e = effective fiber efficiency based on Equation 56	= 0.1361

IV. DISCUSSION OF RESULTS

1. Collection Efficiency of Fiber Mats

Figure 7 shows the experimental collection efficiencies of the three types of glass fiber mats, correlated on the basis of the effective fiber efficiency defined by Equation 62. The ranges of variables represented are: diameter of aerosol particles, 0.43 to 1.30 microns; superficial velocity of aerosol stream passing mats, 17.7 to 257 cm./sec.; average diameters of fibers, 3.51, 6.24, and 9.57 microns; Reynolds Number based on fiber diameter, 0.038 to 1.39; fiber volume fraction, 0.045 to 0.098; and thickness of mats, 0.13 to 0.40 cm. The density of the aerosol particles was substantially constant with an average of 1.4 g./cc. The collection efficiency on the mats ranged from 0.039 to 0.998.

In the ranges of particle diameter and aerosol stream velocity employed in the experiments, collection due to the Brownian diffusion was negligible. Collection by electrostatic forces was improbable since both the aerosol particles and the mats were uncharged. The values of the settling parameter, G , was never larger than 4×10^{-4} showing that the effect of gravity was negligible. The effective fiber efficiencies shown in Figure 7, therefore, represent the total efficiencies of impaction, which include the inertial impaction and interception efficiencies, and the effects of fiber interference, fiber ends, and non-uniformity on the total efficiency of impaction.

The interception parameter, R , ranged from 0.049 to 0.313. Curves representing the effective fiber efficiencies for the values of R equal to 0.1, 0.2, and 0.3 are shown. It was not possible to draw these curves through all the points representing these values of the interception parameter because the points cover a more than 30-fold variation of the Reynolds Number. The shapes of the curves are similar to that of the

experimental curve for the inertial impaction efficiencies on single wires. A careful examination of the results plotted and the tabulated values of the Reynolds Number, N_{Re} , in Table 6 in the APPENDIX shows that the higher the Reynolds Number, the higher the effective fiber efficiency when other variables are approximately constant. This agrees with the conclusion drawn previously. Variation of the Reynolds Number in the range from 0.5 to 1.0 had a greater effect on the effective fiber efficiency than the same variation in the range less than 0.5.

The variation of the fiber volume fraction from 0.045 to 0.098 in the experiments had no additional effect on the effective fiber efficiency than that accounted for in Equation 62. The thickness of single mats ranged from 0.13 to 0.22 cm. Double and triple mats up to 0.4 cm. thick were also used. The effect of the thickness of the mat was found to be that predicted by Equation 62. Mats with greater thickness were not employed because they gave too high a collection efficiency for accurate analysis. It is concluded, therefore, that Equation 62 represents correctly the effect of the measurable variables of the mat, i.e., the fiber diameter, D_f , the fiber volume fraction, α , and the mat thickness, h , and can be used to correlate the collection efficiency on the basis of the effective fiber efficiency.

2. Comparison of Collection Efficiencies with Previous Work

The experimental data definitely indicate a critical value of the inertial parameter below which inertial impaction ceases to be important as shown by the flattening of the curves at small values of $\sqrt{\Psi}$. The exact value of this critical $\sqrt{\Psi}$ is difficult to determine because of the scatter of the data. Apparently Langmuir's value of the critical $\sqrt{\Psi}$ of 0.52 is somewhat high and the critical value of $\sqrt{\Psi}$ of 0.4 should be more correct. No maximum value of Ψ was found above which inertial impaction ceases to

be important. At the value of $\sqrt{\psi}$ equal to 0.53 which Ramskill and Anderson claimed to be the maximum, the present data indicate that inertial impaction is just starting to be the controlling mechanism of collection.

Figure 8 is a comparison of the present data with the data of Ramskill and Anderson, and the theoretical curves of Davies, all plotted on the basis of the effective fiber efficiency. The data of Ramskill and Anderson are those on the inertial impaction of dioctylphthalate (DOP) aerosol particles on their I and J filters, which consisted of glass fibers with 3- and 2-micron diameters, respectively. The values of α were 0.186 and 0.097 for the I and J filters, respectively. The value of R for their data ranged from 0.10 to 0.15. Their data agree fairly well with the present data. Some data of these authors not presented here on the impaction of DOP and sulfuric acid aerosols on Viscose fiber mats indicate somewhat higher effective fiber efficiencies. This is probably due to the effect of non-circular cross-section of the Viscose fibers. The data of Ramskill and Anderson when presented on the basis of the effective fiber efficiency do not indicate a maximum value of ψ for inertial impaction.

The curve for $R = 0$, and $\alpha = 0$ shown in Figure 8 is Davies' theoretical curve for inertial impaction efficiency for single cylinders shown previously in Figure 1. The other theoretical curves were calculated by means of Equation 73. The experimental curves indicate efficiencies much lower than those predicted by the Davies equation when the fiber volume fraction is taken into consideration. Portions of the experimental curves are even lower than the curve predicted by Davies for the efficiencies of single cylinders. It is concluded, therefore, that Davies' equation overpredicts the effect of the fiber interference on the impaction efficiency of the fibers in the mat. Davies' efficiency curve for single cylinders predicts approximately the fiber efficiencies. It is to be noted that Davies'

curves are all based on a Reynolds Number of 0.2. The comparison is justified since the majority of the data were obtained in the range of the Reynolds Number from 0.1 to 0.5 and the effect of the Reynolds Number on the effective fiber efficiency is small in this range.

For small values of $\sqrt{\Psi}$, the effective fiber efficiencies appear to be independent of $\sqrt{\Psi}$. The reason that the efficiencies do not approach zero is due to the efficiency of interception. The effective interception efficiencies can be obtained by extrapolating the experimental curves to $\sqrt{\Psi} = 0$. The limiting effective interception efficiencies so obtained are higher than the interception efficiencies shown in Figure 2 for single cylinders at Reynolds Numbers of 1 or less, but they are much lower than those predicted by Davies' equation. This indicates that there is some effect due to fiber interference accounting for the higher interception efficiencies, but the effect is much less than Davies predicted.

3. Pressure Drop across Fiber Mats

Figure 9 shows the pressure drop across the fiber mats correlated on the basis of the effective fiber drag coefficient defined by Equation 67. The ranges of variables are the same as those described for the collection efficiencies since the measurements were taken simultaneously. The pressure drop across the three types of glass fibers ranged from 0.33 to 29.9 cm. of water. Equation 67 predicts correctly the effects of mat thickness, the fiber diameter, and the velocity through the mat. The fiber volume fraction, α , has a marked effect on the effective fiber drag coefficient. The higher the value of α , the higher is the drag coefficient. This means that the higher the fiber interference, the higher is the drag per unit length of the fibers.

4. Comparison of Pressure Drop Data with Previous Work

Figure 9 also shows the theoretical and semi-theoretical equations

proposed for the pressure drop across fibrous filters. These curves were calculated for $\alpha = 0.06$, the average fiber volume fraction for the present data. The curve for the Kozeny-Carman equation was calculated based on $K_1 = K_0/H = 4.5$, found by Sullivan and Hertel for glass fibers oriented randomly. The curve for Langmuir's equation was based on $B = 1$. Of all these equations, only the Kozeny-Carman approximately predicts the experimental data, and then only for values of N_{Re} below about 0.3. All the other equations overpredict the pressure drop. For values of N_{Re} greater than about 0.3, the Kozeny-Carman equation underpredicts the pressure drop.

The experimental curves drawn through the points representing values of 0.04 and 0.08 are linear up to $N_{Re} = 0.6$. The slope of these curves is less than -1. This indicates that the pressure drop equations of Kozeny-Carman, Davies, and Langmuir, while approximately applicable to a limited range of the Reynolds Number, do not describe adequately the dependency on N_{Re} . For values of N_{Re} greater than 0.6, the experimental curves exhibit a curvature. Iberall's equation, although predicting too high a pressure drop, appears to have the approximately correct functional relation of N_{Re} .

Lamb's equation (Equation 66) for the drag coefficient of single cylinders gives values lower than the data for N_{Re} less than 0.2. For higher values of N_{Re} , however, Lamb's curve is higher than some of the data. This indicates that the lower the Reynolds Number, the higher the effect of fiber interference, in agreement with a conclusion reached by White from his data on the drag of circular cylinders in containers where the wall effect is controlling (41). The fact that some of the data lie below Lamb's curve is probably due to a fraction of the fibers in the mat which were oriented parallel to the flow.

Figure 10 is a plot of some of the data on pressure drop across glass fiber mats obtained by Blasewitz, et al. (2), First, et al. (12), and Ramskill and Anderson (30) together with the two experimental curves obtained in the present work. The present curves agree surprisingly well with the data of these authors at corresponding values of α . It is to be noted that the data of Blasewitz are in poor agreement with those of First at low values of α , i.e., of the order of 0.01 or less. The discrepancy is probably due to the poor distribution of the fibers in the mats at low fiber volume fractions.

NOMENCLATURE

- A = face area₂ of the mat measured perpendicular to the direction of flow, cm.²
- A_c = projected area of the cylinder in the direction of flow, cm.²
- A_p = projected area of the aerosol particle in the direction of flow, cm.²
- a = a numerical constant of the order of unity in Langmuir's filtration theory
- B = a numerical factor of the order of unity in Langmuir's pressure drop equation to account for the effect of transverse position of the fibers in the mat
- C = $1 + (1.644 + 0.552 e^{-0.656 D_p/\lambda_m}) \lambda_m/D_p$ = Cunningham factor based on empirical correlation of Millikan's data
- C_D = drag coefficient of the cylinder or fiber, dimensionless
- C_{De} = effective fiber drag coefficient as defined by Equation 67, dimensionless
- C_L = $\frac{v_o}{(1/2 - \delta - \ln(kr_c/2))} \approx \frac{v_o}{2.002 - \ln N_{Re}}$, cm./sec.
- C_R = drag coefficient of the aerosol particle, dimensionless
- D_c = diameter of the cylinder, cm.
- D_f = diameter of the fiber, cm.
- D_p = diameter of the aerosol particle, or sphere, cm.
- D_v = diffusion coefficient of aerosol particles of radius r_p , cm.²/sec.
- F = force acting on the aerosol particle as given by Equation 32, dynes
- $\vec{F}$ = sum of extraneous forces acting on the aerosol particle, dynes
- F_D = drag force per unit length of the cylinder or fiber, dynes/cm.
- F_R = drag force on the aerosol particle or sphere, dynes
- F_T = total drag force on the cylinders or fibers, dynes
- F_x^f, F_y^f, F_z^f = components of extraneous forces acting on the fluid element in the x, y, z directions, respectively, per unit mass of the fluid, dynes/g.
- G = $\frac{4C_p D_p^2 g}{3\mu C_{RRe} v_o} = (\frac{24\Psi}{C_{RRe}})(\frac{g D_c}{v_o^2}) = (\frac{24\Psi}{C_{RRe}})(\frac{g D_f}{v_o^2})$ = settling parameter, dimensionless

- g = absolute value of g , cm./sec.²
- $\vec{g}$ = local acceleration of gravity, cm./sec.²
- H = orientation factor for the fibers in the mat, dimensionless
- h = thickness of the mat measured perpendicular to the direction of flow, cm.
- K = a numerical constant in D'Arcy's equation
- K_1 = a numerical constant in the Kozeny-Carman equation
- K_0 = shape factor which should be the same for geometrically similar fiber mats, dimensionless
- k = $v_0 \rho / (2\mu)$, cm.⁻¹
- k_m = proportional constant in Equation 25, dimensionless
- k_0 = 1.38×10^{-16} erg/°K. = Boltzmann's constant
- L = length of fibers per unit volume of mat, cm.⁻²
- m = mass of the aerosol particle, g.
- N = number concentration of aerosol particles inside the mat at any plane perpendicular to the direction of flow, number/cc.
- N_h = number concentration of aerosol particles downstream of the mat, number/cc.
- N_0 = number concentration of aerosol particles upstream of the mat, number/cc.
- $N_{Re} = D_c v_0 \rho / \mu = D_f v_0 \rho / \mu$ = Reynolds Number based on the diameter of the cylinder or fiber
- $N'_{Re} = D_p w \rho / \mu$ = Reynolds Number based on the diameter of the aerosol particle or sphere
- n = number of layers of fibers in the mat
- ΔP = pressure drop across the fiber mat, dynes/cm.²
- $P_d = 2D_v(1 - \alpha)/(v_0 D_f)$ = Davies' diffusion parameter, dimensionless
- $P_i = 2\psi/(1 - \alpha)$ = Davies' inertial parameter, dimensionless
- $P_t = P_i + P_d$ = a general particle parameter in Davies' filtration theory, dimensionless

p = static pressure at the center of the fluid element, dynes/cm.²;
subscript $_o$ refer to nozzle, or mat

P_{xx}, P_{xy}, P_{xz} = components of stress on a plane perpendicular to the x axis passing the center of the fluid element in the x, y, z directions, respectively, dynes/cm.²

P_{yx}, P_{yy}, P_{yz} = components of stress on a plane perpendicular to the y axis passing the center of the fluid element in the x, y, z directions, respectively, dynes/cm.²

P_{zx}, P_{zy}, P_{zz} = components of stress on a plane perpendicular to the z axis passing the center of the fluid element in the x, y, z directions, respectively, dynes/cm.²

Q = flow rate of the aerosol stream passing the mat, cc./sec.

Q_1 = volumetric flow rate of the fluid as given by Equation 50, cc./sec.

$R = D_p/D_c = D_p/D_f$ = interception parameter, dimensionless

R_F = fluid resistance to the aerosol particle as given by Equation 33, dynes

$R_p = \Delta P/Q$ for ΔP small compared with the absolute pressure upstream of the mat, (dynes/cm.²)/(cc./sec.)

r, θ = polar coordinates, cm. and radians, respectively

r_c = radius of the cylinder, cm.

r_f = radius of the fiber in Langmuir's filtration equations, microns

r_p = radius of the aerosol particle in Langmuir's filtration equations, microns

S = stopping distance of the aerosol particle, cm.

S_o = specific surface, surface exposed to the fluid per unit volume of solid material making up the bed, cm.⁻¹

T = absolute temperature of the aerosol stream, °K.; subscript $_o$ refers to nozzle, or mat

t = time, sec.

$\tilde{t}$ = reduced time as defined on p. 11, dimensionless

$\vec{u} = \vec{u}(x, y, z, t)$, velocity of the aerosol particle, cm./sec.

u_x, u_y = velocity components of the aerosol particle in the x, y directions, respectively, cm./sec.

$\tilde{u}_x, \tilde{u}_y$ = reduced velocity components of the aerosol particle in the x, y directions, respectively, as defined on p. 11, dimensionless

$\vec{v} = \vec{v}(x, y, z, t)$, velocity of the gas or fluid, cm./sec.

v_c = a critical velocity at which inertial impaction begins to be important in the filtration picture of Ramskill and Anderson, cm./sec.

v_m = a velocity above which inertial impaction no longer exerts any additional effect in the filtration picture of Ramskill and Anderson, cm./sec.

v_o = unidirectional velocity of the gas or fluid on the upstream removed from the effect of the cylinder or fibers, cm./sec.

v_r, v_θ = radial and tangential components of the velocity of the gas or fluid, respectively, cm./sec. and radians/sec.

v_t = terminal velocity of the aerosol particle, cm./sec.

v_x, v_y, v_z = velocity components of the gas or fluid in the x, y, z directions, respectively, cm./sec.

$\tilde{v}_x, \tilde{v}_y$ = reduced velocity components of the gas in the $\tilde{x}, \tilde{y}$ directions, respectively, as defined on p. 11, dimensionless

$w = |\vec{u} - \vec{v}|$ = absolute value of the relative velocity of the aerosol particle or sphere with respect to the gas or fluid, cm./sec.

x, y, z = position coordinates, cm.

$\tilde{x}, \tilde{y}$ = reduced position coordinates as defined on p. 11, dimensionless

$\tilde{x}_o, \tilde{y}_o$ = original $\tilde{x}, \tilde{y}$ position of the aerosol particle at $\tilde{t} = 0$

y_o^* = limiting value of y_o for the aerosol particle to impact on the cylinder

$\tilde{y}_o^*$ = limiting value of $\tilde{y}_o$ for the aerosol particle to impact on the cylinder

$Z = Z/(\rho_w A_p C_R) =$ mobility of the aerosol particle, cm./(dyne)(sec.)

$\alpha = \rho_b/\rho_f =$ fiber volume fraction, i.e., volume of fibers per unit volume of mat, dimensionless

Γ = stream function for the incompressible flow of a fluid, cm.²/sec.

Γ_o = limiting streamline for interception, cm.²/sec.

$\gamma = 0.577 \dots$ = Euler's constant, dimensionless

$\epsilon = 1 - \alpha =$ porosity of the medium, ratio of the volume of void space to the total volume of mat, dimensionless

$\epsilon_o =$ ratio of the effective minimum thickness of layer from which particles can diffuse to the radius of the fiber, dimensionless

η = total efficiency of impaction on the cylinder or fiber

$\eta_c =$ efficiency of interception on the cylinder or fiber 116 079

η_e = effective fiber efficiency as defined by Equation 62

η_I = efficiency of inertial impaction

η_{mat} = collection efficiency of the mat

η_s = apparent efficiency due to gravity settling = G

$$\nabla = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

λ_m = mean free path of the gas molecules, cm.

μ = coefficient of viscosity of the gas or fluid, poise

ρ = density of the gas or fluid, g./cc.

ρ_b = bulk density of the mat, g./cc.

ρ_f = density of the fiber, g./cc.

ρ_p = density of the aerosol particle, g./cc.

σ = filtering action = $-\ln(N_h/N_o)$, dimensionless

Φ = potential function for the irrotational flow of a fluid, cm.²/sec.

$\mathcal{P} = 1/(-\ln\alpha + 2\alpha - \alpha^2/2 - 3/2)$, dimensionless

$$\Psi = \frac{C\rho_p v_o D_p^2}{18\mu D_c} = \frac{C\rho_p v_o D_p^2}{18\mu D_f} = \text{inertial parameter, dimensionless}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

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APPENDIX

Table 5

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EXPERIMENTAL DATA AND RESULTS
ON IMPACTION OF AEROSOL PARTICLES ON SINGLE WIRES D_c , measured diameter of wires:

P1 (platinum - 1 mil), 29.0 microns;

P3 (platinum - 3 mil), 82.6 microns;

T2 (tungsten - 2 mil), 53.1 microns;

T4 (tungsten - 4 mil), 105.7 microns

Diameter of nozzle throat: 0.357 cm.

Run No.	T_o °K.	P_o cm, H ₂ O	v_o cm./ sec.	N_{Re}	Acid Conc. in Aero- sol, %	D_p microns	$\sqrt{\psi}$	Wt. of 100% Acid in Filter mg.	Wt. of 100% Acid on Wire mg.x10 ²	η
P1-1	299.4	1021	1050	19.2	58.3	0.77	1.07	9.74	4.90	0.49
P1-2	299.0	1006	873	15.7	58.3	0.71	0.91	4.90	1.56	0.31
P1-3	299.0	1000	827	14.9	58.3	0.68	0.86	7.23	1.58	0.21
P1-4	298.7	1005	758	13.7	58.3	0.65	0.79	7.52	2.14	0.27
P1-6	299.5	1010	1337	24.2	58.3	0.73	1.16	9.32	4.90	0.51
P1-7	299.8	1012	1342	24.4	58.3	0.74	1.17	9.21	5.32	0.56
P1-8	301.2	1008	1080	19.3	59.1	0.85	1.20	10.2	6.21	0.59
P1-9	300.8	1010	1221	21.9	59.1	0.80	1.20	9.35	4.94	0.51
P1-10	300.6	1007	889	15.9	59.1	0.78	1.00	9.06	3.71	0.39
P1-11	300.6	1012	1368	24.6	59.1	0.91	1.43	9.59	6.69	0.67
P1-12	300.6	1012	1586	28.5	59.1	1.00	1.68	9.65	7.65	0.76
P1-13	299.2	1015	1411	25.7	58.5	1.20	1.88	10.9	9.70	0.85
P1-14	300.1	1012	1815	32.8	58.5	1.22	2.17	9.78	9.65	0.95
P1-15	300.1	1012	1526	27.5	58.5	1.26	2.05	9.13	8.84	0.93
P1-16	300.1	1013	1140	20.6	58.5	1.23	1.73	9.71	8.58	0.85
P1-19	299.2	1002	1520	27.3	57.4	1.00	1.65	8.95	7.44	0.80
P1-20	299.1	1000	1735	31.0	57.4	1.10	1.92	11.1	9.43	0.82
P1-21	299.2	1001	1643	29.6	57.4	1.04	1.78	10.3	9.41	0.87

Table 5 (Continued)

Run No.	T ₀ °K.	P ₀ cm. H ₂ O	v ₀ cm./ sec.	N _{Re}	Acid Conc. in Aero- sol, %	D _p microns	$\sqrt{\psi}$	Wt. of 100 % Acid in Filter mg.	Wt. of 100 % Acid on Wire ² mg.x10 ²	η
P1-22	299.5	1002	1580	28.4	59.4	0.80	1.37	9.39	6.97	0.71
P1-23	299.8	1025	1379	25.3	59.4	0.80	1.28	10.5	6.75	0.62
P1-24	300.4	1023	962	17.5	59.4	0.78	1.04	7.84	3.75	0.33
P1-25	298.7	1021	1550	28.5	59.7	0.96	1.61	13.4	10.7	0.77
P1-26	299.5	1021	748	13.7	59.7	0.94	1.10	11.8	6.94	0.57
P1-27	299.8	1020	875	16.0	59.7	0.95	1.19	12.1	6.69	0.53
P3-1	301.2	1020	439	22.6	56.9	0.72	0.39	5.29	0.73	0.05
P3-2	301.4	1006	630	32.0	56.9	0.74	0.47	8.19	1.09	0.05
P3-3	299.8	997	626	31.8	56.9	0.74	0.47	14.2	1.08	0.03
P3-4	301.2	1061	480	25.8	56.9	0.74	0.41	8.42	0.99	0.04
P3-5	301.1	1011	1224	62.7	59.1	0.93	0.82	16.2	18.2	0.38
P3-6	300.9	1008	1151	58.7	59.1	0.98	0.84	13.5	15.6	0.39
P3-7	300.7	1010	834	42.6	59.1	0.97	0.70	13.9	11.6	0.28
P3-8	300.5	1012	1469	75.9	57.9	1.18	1.02	13.0	21.8	0.56
P3-9	299.5	1012	1680	86.7	57.9	1.07	1.09	11.2	21.1	0.63
P3-10	299.5	1013	1109	57.4	57.9	1.12	0.93	9.03	14.0	0.52
P3-11	299.2	1016	856	44.4	57.9	0.99	0.73	9.15	11.0	0.40
P3-12	299.2	1009	1329	68.4	57.9	1.04	0.95	9.60	15.2	0.53
P3-13	301.8	1009	1625	82.4	59.3	0.82	0.84	7.59	11.5	0.40
P3-14	300.5	1009	1590	81.2	59.3	0.85	0.86	9.78	13.9	0.48
P3-15	300.3	1009	1198	61.1	59.3	0.83	0.73	9.30	8.48	0.31
P3-16	298.0	1027	476	25.1	61.8	0.60	0.35	14.0	1.23	0.03
P3-17	298.2	1028	474	25.0	61.9	0.59	0.34	24.0	1.04	0.02
P3-18	298.2	1011	518	26.9	61.6	0.59	0.36	24.6	1.08	0.02
P3-19	298.6	1009	800	41.2	61.8	0.59	0.45	20.6	1.96	0.03
P3-20	298.6	1011	1081	55.9	61.8	0.60	0.53	11.9	2.85	0.08
P3-21	298.2	1011	1018	52.7	61.8	0.58	0.49	14.5	3.17	0.07
P3-22	300.4	1011	1530	78.5	59.1	0.85	0.84	10.3	14.8	0.48
P3-23	301.2	1011	1337	68.1	59.1	0.88	0.82	9.05	10.5	0.39

80 Table 5 (Continued)

Run No.	T ₀ °K.	P ₀ cm. H ₂ O	v ₀ cm./ sec.	N _{Pe}	Acid Conc. in Aero- sol, %	D _p microns	$\sqrt{\psi}$	Wt. of 100 % Acid in Filter mg.	Wt. of 100 % Acid on Wire mg.x10 ²	η
P3-24	301.4	1011	1218	62.0	59.1	0.89	0.79	9.76	11.8	0.41
P3-25	301.7	1011	1118	56.9	59.1	0.86	0.73	9.35	11.0	0.39
P3-26	301.6	1011	829	42.2	59.1	0.89	0.65	9.17	7.64	0.28
P3-27	301.3	1011	815	41.5	59.1	0.90	0.65	8.46	5.46	0.22
P3-28	301.2	1011	493	25.1	59.1	0.85	0.48	8.88	2.09	0.08
P3-29	302.6	1024	506	25.9	59.6	0.97	0.55	10.5	3.04	0.10
P3-30	303.2	1002	538	27.0	59.6	0.97	0.56	8.95	3.82	0.15
P3-31	303.2	1002	551	27.6	59.6	0.89	0.53	9.00	2.50	0.09
P3-32	303.5	1002	641	32.1	59.6	0.96	0.61	10.5	4.45	0.14
P3-33	303.7	1001	774	38.8	59.6	0.88	0.62	9.79	4.96	0.17
P3-34	304.1	1001	1080	53.6	59.6	0.88	0.73	10.4	8.05	0.26
P3-35	301.6	1010	1974	100.0	58.4	1.26	1.38	13.4	27.4	0.68
P3-36	302.0	1010	1819	92.1	58.4	1.22	1.28	9.34	20.7	0.74
P3-37	302.6	1010	1715	86.5	58.4	1.19	1.22	9.40	19.8	0.70
P3-38	302.7	1010	1615	81.5	58.4	1.19	1.18	11.0	23.6	0.71
P3-39	301.8	1013	1960	100.0	58.4	1.21	1.32	12.9	29.8	0.77
P3-40	301.8	1012	1960	100.0	58.4	1.26	1.37	11.3	26.5	0.78
P3-41	300.9	1012	3920	201	58.4	1.18	1.83	11.0	30.4	0.91
P3-42	301.0	1011	2790	143	58.4	1.22	1.59	11.3	29.8	0.87
P3-43	301.3	1011	3025	154	58.4	1.40	1.89	10.3	29.6	0.95
P3-45	299.2	999	3300	169	58.5	1.22	1.74	8.49	22.5	0.68
P3-46	298.8	998	3840	196	58.5	1.26	1.93	12.4	35.7	0.96
P3-47	298.6	1004	3985	206	58.5	1.05	1.59	14.0	35.7	0.85
P3-48	299.2	1001	3220	165	58.5	1.09	1.54	13.6	32.0	0.78
P3-49	299.1	1001	4850	248	58.5	1.06	1.84	11.7	32.9	0.93
T2-2	301.2	990	585	18.8	59.1	0.73	0.57	9.89	1.38	0.07
T2-3	302	1014	713	23.4	59.1	0.72	0.62	8.75	1.54	0.09
T2-4	302.3	1039	462	15.3	59.1	0.69	0.47	6.87	1.13	0.09

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Table 5 (Continued)

Run No.	T _o °K.	P _o cm. H ₂ O	v _o cm./ sec.	N _{Re}	Acid Conc. in Aero- sol, %	D _p microns	$\sqrt{\psi}$	Wt. of 100 % Acid in Filter mg.	Wt. of 100 % Acid on Wire 2 mg.x10	η
T2-6	301.5	1011	790	25.9	59.1	0.72	0.65	16.2	2.49	0.08
T2-7	302.3	1013	950	31.1	59.1	0.69	0.69	9.30	2.70	0.15
T2-9	301.5	1005	399	13.0	58.2	0.84	0.53	10.4	1.82	0.09
T2-10	302.3	1005	611	19.8	58.2	0.85	0.66	11.8	2.95	0.13
T2-11	302.6	1009	968	31.4	58.2	0.88	0.86	10.0	5.65	0.30
T2-12	303.2	1008	1111	35.9	58.2	0.85	0.89	10.8	6.10	0.30
T2-13	302.0	1012	1153	37.8	58.2	0.89	0.95	17.4	11.3	0.34
T2-14	302.9	1015	1380	45.0	58.2	0.86	1.00	13.5	11.0	0.43
T2-15	303.2	1014	1241	40.4	58.2	0.86	0.95	9.78	6.90	0.37
T2-16	300.9	996	748	24.2	57.8	1.01	0.86	13.5	7.11	0.28
T2-17	300.9	996	970	31.4	57.8	1.08	1.04	16.6	10.6	0.34
T2-18	301.3	995	1324	42.9	57.8	1.01	1.14	13.9	12.3	0.46
T2-19	301.4	996	1050	34.0	57.8	0.99	1.00	10.4	7.61	0.39
T2-20	301.5	999	544	17.6	57.8	0.98	0.71	9.19	3.13	0.18
T2-21	300.9	995	1270	40.9	58.0	1.24	1.36	19.7	21.8	0.58
T2-22	301.8	1010	1158	37.7	58.0	1.18	1.24	10.0	9.60	0.50
T2-23	301.6	995	971	31.3	58.0	1.18	1.13	11.8	11.4	0.51
T2-24	302.1	1002	863	27.9	57.0	1.00	0.91	9.58	7.44	0.41
T2-25	301.5	1002	1378	44.8	57.0	1.01	1.16	9.69	10.0	0.54
T2-26	300.9	1002	1361	44.4	57.0	0.94	1.08	8.72	7.55	0.45
T2-28	299.8	1002	1561	51.2	57.0	1.22	1.48	13.2	16.0	0.63
T2-34	299.8	1010	1766	58.5	57.8	1.14	1.48	8.44	9.70	0.60
T4-4	300.5	1005	3610	235	57.6	1.04	1.37	7.50	18.7	0.65
T4-5	300.9	1005	5090	330	57.6	0.97	1.53	7.26	20.4	0.73
T4-6	301.6	1002	2055	133	58.1	1.30	1.26	8.90	22.7	0.66
T4-7	302.3	1001	4115	264	58.1	1.12	1.57	10.3	28.9	0.73
T4-8	302.4	1001	4215	270	58.1	1.14	1.62	9.89	33.3	0.87
T4-10	300.8	1001	3460	224	58.1	1.04	1.35	9.01	24.0	0.69
T4-11	300.5	1001	3000	194	58.1	1.08	1.30	10.9	28.2	0.67

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82 Table 5 (Continued)

Run No.	T _o °K.	p _o cm. H ₂ O	v _o cm./ sec.	N _{Re}	Acid Conc. in Aero- sol, %	D _p microns	$\sqrt{\Psi}$	Wt. of 100 % Acid in Filter mg.	Wt. of 100 % Acid on Wire ² mg.x10	η
T4-12	299.9	1001	3010	196	58.1	1.08	1.30	9.89	25.1	0.66
T4-13	300.5	1001	3505	227	58.1	1.23	1.59	9.30	27.4	0.76
T4-14	300.5	1001	4600	298	58.1	1.14	1.70	9.48	29.8	0.81
T4-15	300.5	992	2615	168	58.7	0.97	1.10	17.9	35.3	0.51
T4-16	301.8	990	2180	139	58.7	0.86	0.90	11.2	18.3	0.43
T4-17	301.9	991	2725	174	58.7	0.84	0.98	10.0	18.3	0.48
T4-18	299.5	991	3755	242	58.7	0.84	1.16	12.9	29.6	0.60
T4-19	299.0	992	4470	290	58.7	0.82	1.24	11.0	26.3	0.62
T4-20	299.0	990	3085	200	58.7	0.83	1.04	10.0	20.1	0.52
T4-21	298.6	991	1680	109	58.7	0.86	0.80	10.1	13.6	0.35
T4-22	298.9	1006	1840	121	58.8	0.84	0.81	11.2	16.7	0.39
T4-23	300.4	1011	2195	144	56.1	0.74	0.78	6.01	7.55	0.33
T4-24	301.5	1011	1835	120	56.1	0.73	0.70	6.96	8.34	0.32
T4-25	302.7	1012	878	56.8	56.1	0.73	0.49	7.41	2.20	0.08
T4-26	302.5	1012	1315	85.0	56.1	0.71	0.58	9.43	6.05	0.17
T4-27	302.5	1015	1656	108	56.1	0.69	0.63	9.67	9.43	0.25
T4-28	299.8	1010	1402	92.1	60.5	0.69	0.60	16.2	9.00	0.15
T4-29	300.2	1009	890	58.2	60.5	0.67	0.46	11.9	1.72	0.04
T4-30	300.2	1015	534	35.1	60.5	0.56	0.30	13.5	1.30	0.03
T4-31	299.0	1018	555	36.9	60.5	0.65	0.36	11.9	1.24	0.03
T4-32	299.8	1010	1364	89.6	60.5	0.66	0.56	13.3	4.26	0.09
T4-33	300.2	1030	519	34.7	60.5	0.67	0.35	12.8	1.10	0.02
T4-34	300.4	1039	490	33.0	60.5	0.57	0.30	12.0	1.13	0.03
T4-35	300.4	1011	1461	95.6	60.5	0.78	0.68	8.30	7.68	0.24

Table 6

EXPERIMENTAL DATA AND RESULTS
ON COLLECTION EFFICIENCY AND PRESSURE DROP OF FIBER MATS

D_f , Average diameter of fibers in mat:
B (unbonded "B" fibers), 3.51 microns;
450 ("450" yarns), 6.24 microns;
150 ("150" yarns), 9.57 microns

P_f , density of fiber in mat: 2.54 g./cc.

Diameter of mat cross-section exposed to
aerosol stream: 1.90 cm.

Run No.	T_o OK.	P_o cm. H ₂ O	v_o cm./ sec.	N_{Re}	h cm.	α	ΔP cm. H ₂ O	C_{De}	Acid Conc. in Aero- sol, %	D_p microns	$\sqrt{\psi}$	$R =$ D_p/D_f	N_h/N_o	η_e
B-16	308.6	1005	105	0.217	0.13	0.0496	5.5	37.4	48.2	0.50	0.64	0.14	0.400	0.039
B-17	309.5	1005	116	0.238	0.13	0.0478	6.4	37.6	48.2	0.52	0.69	0.15	0.368	0.044
B-18	309.7	1005	186	0.383	0.13	0.0466	10.5	24.4	48.2	0.51	0.86	0.15	0.300	0.055
B-19	309.6	1005	143	0.294	0.14	0.0479	8.2	29.2	48.2	0.51	0.76	0.15	0.343	0.048
B-20	309.7	1010	66	0.136	0.13	0.0481	3.1	55.8	48.2	0.45	0.46	0.13	0.472	0.033
B-21	306.6	1017	61	0.129	0.13	0.0475	2.7	55.8	53.0	0.76	0.72	0.22	0.252	0.062
B-22	306.8	1019	105	0.222	0.13	0.0483	4.9	33.7	53.0	0.73	0.91	0.21	0.205	0.070
B-23	307.4	1019	137	0.288	0.13	0.0475	6.8	28.0	53.0	0.71	1.01	0.20	0.175	0.078
B-24	307.2	1020	171	0.360	0.14	0.0459	8.6	22.0	53.0	0.69	1.10	0.20	0.159	0.079
B-39	308.2	1013	140	0.295	0.14	0.0452	7.0	26.7	53.8	0.76	1.09	0.22	0.103	0.099
B-40	308.5	1016	193	0.403	0.14	0.0454	9.6	19.5	53.8	0.76	1.28	0.22	0.066	0.118
B-29	306.3	1019	74	0.156	0.14	0.0445	3.5	49.6	54.5	1.04	1.06	0.30	0.050	0.133
B-30	307.0	1020	114	0.240	0.13	0.0478	5.1	30.3	54.5	1.02	1.28	0.29	0.025	0.164
B-31	307.3	1018	132	0.278	0.14	0.0459	6.5	27.6	54.5	1.02	1.37	0.29	0.015	0.180
B-41	305.2	1012	192	0.406	0.17	0.0597	16.9	21.6	50.8	0.53	0.92	0.15	0.107	0.061
B-42	305.4	1012	138	0.283	0.17	0.0607	11.4	27.8	50.8	0.53	0.78	0.15	0.227	0.040
B-43	305.7	1014	106	0.225	0.16	0.0572	8.9	41.3	50.8	0.53	0.68	0.15	0.234	0.044
B-44	305.9	1011	69	0.146	0.17	0.0582	5.5	55.7	50.8	0.45	0.48	0.13	0.295	0.034

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Table 6 (Continued)

Run No.	T ₀ °K.	P ₀ cm. H ₂ O	v ₀ cm./ sec.	N _{Re}	h cm.	α	ΔP cm. H ₂ O	C _{De}	Acid Conc. in Aero- sol, %	D _p microns	$\sqrt{\Psi}$	R= D _p /D _f	N _h /N ₀	η_e
B-25	307.9	1019	64	0.134	0.17	0.0561	5.1	62.4	53.0	0.66	0.65	0.19	0.160	0.053
B-26	307.9	1019	100	0.209	0.17	0.0595	9.1	43.6	53.0	0.68	0.83	0.19	0.091	0.065
B-27	307.9	1020	141	0.296	0.16	0.0576	12.0	31.6	53.0	0.64	0.93	0.18	0.075	0.078
B-28	307.6	1019	184	0.386	0.17	0.0559	15.3	22.9	53.0	0.70	1.16	0.20	0.036	0.097
B-36	306.8	1013	86	0.180	0.17	0.0595	7.9	51.6	53.8	0.94	1.03	0.27	0.016	0.113
B-37	307.3	1015	116	0.243	0.16	0.0600	9.9	37.0	53.8	0.81	1.05	0.23	0.026	0.105
B-38	308.2	1015	151	0.315	0.17	0.0564	12.8	28.3	53.8	0.84	1.24	0.24	0.012	0.127
B-33	307.4	1020	66	0.140	0.17	0.0546	5.4	62.7	54.5	1.06	1.02	0.30	0.008	0.144
B-34	307.3	1020	121	0.256	0.17	0.0572	10.0	33.3	54.5	1.02	1.33	0.29	0.003	0.162
B-35	306.8	1019	139	0.293	0.16	0.0606	11.9	30.5	54.5	1.02	1.42	0.29	0.003	0.170
B-45	306.0	1011	76	0.160	0.20	0.0571	7.5	54.0	50.8	0.46	0.51	0.13	0.218	0.037
B-46	306.0	1012	113	0.239	0.20	0.0603	12.3	38.1	50.8	0.48	0.65	0.14	0.181	0.039
B-47	305.9	1016	136	0.278	0.20	0.0586	14.3	31.7	50.8	0.43	0.65	0.12	0.180	0.041
B-48	306.5	1012	169	0.356	0.20	0.0625	20.3	27.3	50.8	0.44	0.73	0.13	0.149	0.042
B-49	303.5	1019	65	0.140	0.20	0.0590	6.8	64.9	50.9	0.74	0.72	0.21	0.055	0.068
B-50	304.2	1022	115	0.247	0.20	0.0569	11.8	37.2	50.9	0.72	0.93	0.21	0.044	0.076
B-51	304.5	1022	148	0.318	0.20	0.0595	16.1	29.1	50.9	0.72	1.06	0.21	0.022	0.089
B-52	304.6	1022	214	0.459	0.18	0.0606	22.1	21.0	50.9	0.73	1.29	0.21	0.007	0.127
B-59	306.2	1017	66	0.140	0.20	0.0603	7.4	67.8	51.0	0.94	0.90	0.27	0.013	0.100
B-60	306.3	1016	97	0.206	0.20	0.0607	10.6	44.3	51.0	0.94	1.09	0.27	0.009	0.108
B-61	305.7	1024	148	0.317	0.20	0.0630	17.5	29.9	50.1	0.92	1.28	0.26	0.004	0.120
B-62	306.4	1021	177	0.377	0.20	0.0610	20.1	24.9	50.1	0.92	1.40	0.26	0.002	0.136
B-65	307.0	1024	187	0.397	0.21	0.0720	29.9	26.8	50.1	0.61	1.02	0.17	0.014	0.078
B-66	307.2	1021	126	0.267	0.22	0.0765	20.8	37.3	50.1	0.61	0.84	0.17	0.026	0.060
B-67	307.4	1021	100	0.212	0.21	0.0783	6.8	48.5	50.1	0.65	0.79	0.19	0.036	0.056
B-68	307.4	1021	69	0.146	0.22	0.0732	11.5	71.4	50.1	0.65	0.66	0.19	0.052	0.053
B-53	304.7	1022	190	0.408	0.21	0.0723	29.9	25.9	50.9	0.73	1.21	0.21	0.003	0.107
B-54	305.0	1017	77	0.163	0.22	0.0714	12.2	63.1	51.0	0.78	0.82	0.25	0.015	0.074
B-55	305.4	1018	117	0.249	0.21	0.0770	17.6	37.9	51.0	0.74	0.97	0.21	0.012	0.075
B-56	305.7	1018	154	0.328	0.21	0.0746	24.5	31.3	51.0	0.72	1.08	0.21	0.009	0.083

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Table 6 (Continued)

Run No.	T ₀ °K.	P ₀ cm. H ₂ O	v ₀ cm./ sec.	N _{Re}	h cm.	α	ΔP cm. H ₂ O	C _{De}	Acid Conc. in Aero- sol, %	D _p microns	$\sqrt{\psi}$	R _m D _p /D _f	N _h /N ₀	η_e
B-57	306.0	1018	163	0.345	0.21	0.0742	25.8	29.8	51.0	0.72	1.11	0.21	0.005	0.092
B-58	306.2	1018	164	0.347	0.21	0.0784	29.1	31.4	51.0	0.72	1.11	0.21	0.005	0.087
B-63	306.6	1022	75	0.160	0.22	0.0748	12.6	64.1	50.1	0.90	0.92	0.26	0.004	0.095
B-225	301.7	1020	21	0.047	0.13	0.0466	0.9	148	54.0	1.06	0.58	0.30	0.132	0.092
B-226	301.9	1020	26	0.057	0.13	0.0462	1.1	120	54.0	1.00	0.61	0.29	0.183	0.078
B-227	302.3	1020	38	0.082	0.13	0.0477	1.6	81.6	54.0	0.98	0.72	0.28	0.163	0.081
B-228	302.3	1023	57	0.123	0.13	0.0472	2.6	62.6	54.0	1.02	0.91	0.29	0.099	0.104
B-233	307.5	1021	18	0.038	0.13	0.0473	0.7	164	52.5	1.06	0.53	0.31	0.141	0.088
B-234	307.9	1020	19	0.039	0.13	0.0498	0.7	158	52.5	1.03	0.52	0.29	0.167	0.077
B-235	308.0	1020	29	0.060	0.13	0.0456	1.1	109	52.5	1.00	0.63	0.29	0.189	0.078
B-236	308.0	1019	45	0.095	0.13	0.0469	1.9	73	52.5	0.96	0.76	0.27	0.154	0.085
B-237	307.9	1019	64	0.135	0.13	0.0461	2.9	55.2	52.5	0.96	0.91	0.27	0.125	0.096
B-268	308.0	1019	45	0.095	0.17	0.0548	3.1	77.9	55.0	0.46	0.40	0.13	0.321	0.034
B-239	307.2	1017	52	0.110	0.17	0.0591	4.6	79.3	51.1	0.72	0.63	0.21	0.108	0.061
B-238	307.9	1020	30	0.063	0.17	0.0603	2.2	114	52.5	0.94	0.61	0.27	0.066	0.073
B-229	302.4	1021	18	0.040	0.17	0.0591	1.5	210	54.0	1.06	0.54	0.30	0.039	0.089
B-230	305.6	1023	18	0.038	0.17	0.0564	1.3	204	52.5	1.10	0.54	0.31	0.041	0.092
B-231	306.3	1020	31	0.065	0.17	0.0553	2.4	127	52.5	1.05	0.69	0.30	0.046	0.091
B-232	307.9	1023	50	0.105	0.17	0.0591	4.3	82.2	52.5	1.00	0.83	0.29	0.025	0.102
B-240	307.4	1017	50	0.104	0.19	0.0565	4.5	82.1	51.1	0.68	0.58	0.19	0.121	0.055
B-241	307.8	1017	39	0.081	0.19	0.0585	3.5	102	51.1	0.72	0.54	0.21	0.133	0.050
B-242	308.2	1020	31	0.066	0.19	0.0611	3.1	132	51.1	0.70	0.47	0.20	0.101	0.055
B-243	308.3	1020	20	0.041	0.19	0.0625	1.9	204	51.1	0.70	0.38	0.20	0.087	0.057
B-244	306.8	1017	20	0.042	0.22	0.0821	3.7	250	59.0	0.66	0.37	0.19	0.027	0.055
B-245	307.3	1017	25	0.053	0.22	0.0829	4.7	193	59.0	0.64	0.40	0.18	0.036	0.051
B-246	307.7	1014	36	0.075	0.22	0.0694	5.2	129	59.0	0.64	0.48	0.18	0.055	0.053
B-247	308.0	1016	50	0.104	0.22	0.0721	8.2	101	59.0	0.64	0.57	0.18	0.049	0.053

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992

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Table 6 (Continued)

Run No.	T ₀ °K.	P ₀ cm. H ₂ O	v ₀ cm./ sec.	N _{Re}	h cm.	α	ΔP cm. H ₂ O	C _{De}	Acid Conc. in Aero- sol, %	D _p microns	$\sqrt{\Psi}$	R= D _p /D _f	N _h /N ₀	η_e
450-80	305.4	1010	60	0.225	0.14	0.0550	1.2	38.1	55.7	0.52	0.33	0.08	0.890	0.007
450-81	305.8	1018	90	0.338	0.14	0.0596	2.0	24.5	55.7	0.52	0.47	0.08	0.870	0.008
450-82	306.0	1010	158	0.590	0.14	0.0494	3.4	16.9	55.7	0.54	0.65	0.09	0.873	0.010
450-83	306.4	1010	257	0.959	0.14	0.0546	7.8	13.2	55.7	0.52	0.80	0.08	0.668	0.026
450-84	306.4	1012	155	0.579	0.14	0.0639	4.4	17.5	55.7	0.52	0.62	0.08	0.800	0.012
450-79	307.5	1011	59	0.220	0.14	0.0584	1.3	40.1	40.6	0.83	0.55	0.13	0.690	0.022
450-69	306.9	1020	126	0.472	0.14	0.0512	2.7	20.4	48.1	0.92	0.90	0.15	0.430	0.058
450-70	307.6	1024	223	0.837	0.14	0.0541	6.8	15.3	48.1	0.92	1.20	0.15	0.140	0.127
450-71	307.9	1025	173	0.650	0.14	0.0532	4.7	17.8	48.1	0.92	1.06	0.15	0.258	0.089
450-72	308.0	1023	92	0.346	0.14	0.0546	2.2	28.6	48.1	0.90	0.76	0.14	0.605	0.032
450-93	303.9	1012	64	0.246	0.14	0.0526	1.3	34.9	49.5	1.22	0.85	0.20	0.383	0.064
450-94	304.4	1014	96	0.361	0.14	0.0586	2.6	29.0	49.5	1.14	0.97	0.18	0.291	0.074
450-95	305.2	1012	124	0.468	0.14	0.0552	2.6	18.5	49.5	1.16	1.12	0.19	0.198	0.103
450-96	305.4	1014	150	0.564	0.14	0.0600	3.9	17.6	49.5	1.14	1.21	0.18	0.147	0.112
450-89	304.1	1018	161	0.613	0.22	0.0550	7.9	21.1	53.9	0.51	0.58	0.08	0.681	0.016
450-90	304.3	1018	135	0.515	0.22	0.0477	4.9	21.5	53.9	0.51	0.53	0.08	0.779	0.012
450-91	304.2	1016	99	0.375	0.22	0.0531	4.1	30.2	53.9	0.51	0.46	0.08	0.770	0.011
450-92	304.1	1014	57	0.217	0.22	0.0492	2.0	48.1	53.9	0.51	0.35	0.08	0.777	0.011
450-85	303.6	1018	163	0.699	0.22	0.0522	7.1	19.5	53.9	0.71	0.83	0.11	0.534	0.027
450-86	304.2	1016	132	0.501	0.22	0.0501	5.3	23.4	53.9	0.71	0.75	0.11	0.649	0.019
450-87	304.0	1016	100	0.379	0.22	0.0518	3.9	29.0	53.9	0.72	0.66	0.12	0.657	0.018
450-88	304.1	1016	59	0.223	0.22	0.0559	2.5	49.5	53.9	0.70	0.49	0.11	0.696	0.014
450-73	308.0	1021	200	0.746	0.22	0.0551	9.9	17.3	48.1	0.82	1.02	0.13	0.097	0.095
450-74	308.2	1020	178	0.664	0.22	0.0487	7.6	18.9	48.1	0.82	0.97	0.13	0.196	0.075
450-75	306.7	1016	93	0.347	0.22	0.0556	3.6	28.3	40.6	1.02	0.83	0.16	0.226	0.060
450-76	306.5	1014	182	0.682	0.22	0.0615	8.9	16.9	40.6	1.02	1.17	0.16	0.087	0.088
450-77	307.1	1013	137	0.512	0.22	0.0526	5.1	19.9	40.6	1.02	1.01	0.16	0.133	0.085
450-78	307.4	1014	57	0.212	0.22	0.0565	2.4	50.2	40.6	1.02	0.65	0.16	0.446	0.032
450-97	305.5	1012	170	0.640	0.22	0.0514	7.3	18.9	49.5	1.14	1.29	0.18	0.019	0.173
450-98	305.8	1011	152	0.573	0.22	0.0498	6.1	20.5	49.5	1.14	1.22	0.18	0.039	0.145

Table 6 (Continued)

Run No.	T _o °K.	P _o cm. H ₂ O	V _o cm./ sec.	N _{Re}	h cm.	α	ΔP cm. H ₂ O	C _{De}	Acid Conc. in Aero- sol, %	D _p microns	$\sqrt{\Psi}$	R= D _p /D _f	N _h /N _o	η_e
450-111	304.1	1016	186	0.708	0.22	0.0614	10.5	18.9	58.3	0.60	0.78	0.10	0.485	0.026
450-112	304.4	1017	141	0.535	0.22	0.0613	7.5	23.7	58.3	0.58	0.66	0.09	0.580	0.020
450-113	304.4	1013	108	0.409	0.22	0.0660	5.9	29.3	58.3	0.58	0.58	0.09	0.620	0.016
450-114	304.4	1013	59	0.224	0.22	0.0571	2.5	48.4	58.3	0.58	0.43	0.09	0.708	0.014
450-99	302.4	1013	150	0.576	0.22	0.0607	7.6	21.2	46.3	0.93	1.00	0.15	0.125	0.077
450-100	303.0	1012	177	0.674	0.22	0.0615	9.6	17.5	46.3	0.93	1.08	0.15	0.112	0.079
450-101	303.1	1015	112	0.425	0.22	0.0560	5.1	27.8	46.3	0.93	0.86	0.15	0.289	0.050
450-102	303.0	1012	58	0.221	0.22	0.0590	2.5	48.5	46.3	0.90	0.60	0.14	0.504	0.026
450-115	300.9	1018	170	0.657	0.22	0.0615	9.1	19.4	46.2	1.24	1.39	0.20	0.010	0.167
450-116	301.3	1018	158	0.610	0.22	0.0605	8.2	20.7	46.2	1.24	1.39	0.20	0.016	0.153
450-117	301.0	1019	111	0.431	0.22	0.0625	6.0	29.3	46.2	1.20	1.09	0.19	0.073	0.093
450-118	301.3	1019	73	0.283	0.22	0.0600	3.3	39.6	46.2	1.12	0.83	0.18	0.253	0.051
450-107	302.5	1017	159	0.610	0.22	0.0837	11.5	20.7	58.3	0.62	0.74	0.10	0.341	0.029
450-108	303.2	1017	134	0.510	0.22	0.0835	10.3	26.5	58.3	0.62	0.68	0.10	0.445	0.022
450-109	303.6	1017	105	0.399	0.22	0.0982	10.4	37.1	58.3	0.60	0.58	0.10	0.410	0.020
450-110	303.9	1013	68	0.257	0.22	0.0800	5.1	53.7	58.3	0.60	0.47	0.10	0.611	0.014
450-103	303.5	1013	187	0.711	0.22	0.0808	14.3	19.4	46.3	0.90	1.08	0.14	0.044	0.087
450-104	303.6	1015	105	0.570	0.22	0.0788	10.3	22.3	46.3	0.93	0.99	0.15	0.083	0.070
450-105	303.8	1012	107	0.409	0.22	0.0792	7.8	32.9	46.3	0.90	0.82	0.14	0.178	0.049
450-106	303.8	1012	62	0.236	0.22	0.0780	4.2	53.2	46.3	0.88	0.61	0.14	0.435	0.024
450-119	301.6	1016	60	0.232	0.22	0.0875	5.0	60.0	46.2	1.10	0.74	0.18	0.130	0.052
450-120	301.7	1018	105	0.407	0.22	0.0830	7.9	32.7	46.2	1.06	0.94	0.17	0.088	0.065
450-121	301.6	1018	135	0.523	0.22	0.0888	11.5	27.0	46.2	1.02	1.03	0.16	0.035	0.085
450-122	301.9	1015	207	0.798	0.22	0.0801	16.7	18.5	46.2	1.08	1.35	0.17	0.006	0.143
450-258	307.9	1012	21	0.078	0.22	0.0548	0.8	131	58.4	0.90	0.38	0.14	0.599	0.021
450-259	308.1	1013	47	0.174	0.22	0.0528	1.9	62.2	58.4	0.86	0.54	0.14	0.577	0.023
450-260	308.2	1019	36	0.134	0.22	0.0493	1.3	76.1	58.4	0.84	0.46	0.14	0.631	0.021
450-269	308.2	1017	19	0.070	0.22	0.0587	0.8	147	55.0	0.45	0.19	0.07	0.711	0.013
450-270	308.2	1018	32	0.119	0.22	0.0677	1.7	96.2	55.0	0.45	0.25	0.07	0.698	0.012
450-271	308.2	1018	46	0.171	0.22	0.0651	2.5	69.9	55.0	0.45	0.30	0.07	0.750	0.010

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Table 6 (Continued)

Run No.	T ₀ °K.	P ₀ cm. H ₂ O	v ₀ cm./ sec.	N _{Re}	h cm.	α	ΔP cm. H ₂ O	C _{De}	Acid Conc. in Aero- sol, %	D _p microns	$\sqrt{\Psi}$	R= D _p /D _f	N _h /N ₀	η_e
450-248	308.2	1014	48	0.179	0.22	0.0821	3.3	66.3	59.0	0.60	0.40	0.10	0.632	0.013
450-249	308.5	1018	38	0.142	0.22	0.0824	2.7	85.6	59.0	0.60	0.35	0.10	0.587	0.014
450-250	308.8	1016	30	0.110	0.22	0.0855	2.2	114	59.0	0.60	0.31	0.10	0.571	0.015
450-251	308.8	1016	22	0.081	0.22	0.0770	1.3	135	59.0	0.57	0.25	0.09	0.601	0.015
450-216	305.3	1013	127	0.478	0.30	0.0525	5.0	16.7	48.4	0.45	0.48	0.07	0.792	0.007
450-207	303.0	1009	145	0.550	0.30	0.0559	7.1	17.1	48.4	0.89	0.94	0.14	0.145	0.057
450-208	303.5	1013	150	0.399	0.30	0.0524	3.7	18.1	48.4	0.87	0.79	0.14	0.416	0.027
450-209	304.0	1012	62	0.237	0.30	0.0523	2.2	30.2	48.4	0.87	0.61	0.14	0.572	0.018
450-204	305.9	1013	157	0.589	0.30	0.0578	7.3	14.6	40.0	1.22	1.28	0.20	0.018	0.113
450-205	306.2	1012	118	0.442	0.30	0.0702	7.6	21.9	40.0	1.24	1.13	0.20	0.018	0.095
450-206	306.4	1011	83	0.309	0.30	0.0624	4.1	27.4	40.0	1.16	0.89	0.19	0.011	0.058
450-214	305.0	1016	118	0.446	0.40	0.0489	7.1	22.0	48.4	0.48	0.49	0.08	0.640	0.011
450-215	305.2	1013	64	0.241	0.40	0.0446	2.9	33.9	48.4	0.47	0.36	0.08	0.746	0.008
450-210	304.3	1012	57	0.214	0.40	0.0603	4.2	45.8	48.4	0.84	0.56	0.14	0.389	0.019
450-211	304.8	1012	127	0.479	0.40	0.0603	10.5	22.9	48.4	0.80	0.80	0.13	0.205	0.032
450-212	304.8	1013	179	0.675	0.40	0.0534	11.4	14.1	48.4	0.78	0.93	0.13	0.187	0.039
450-213	304.9	1013	147	0.553	0.40	0.0595	11.2	18.5	48.4	0.87	0.93	0.14	0.092	0.049
450-202	304.8	1012	164	0.620	0.40	0.0626	13.5	17.0	40.0	1.20	1.29	0.19	0.005	0.104
450-203	305.7	1012	131	0.494	0.40	0.0614	10.9	21.9	40.0	1.22	1.17	0.20	0.011	0.091
150-155	304.4	1020	138	0.806	0.14	0.0471	1.1	11.7	58.3	0.51	0.47	0.05	0.961	0.005
150-156	304.9	1018	106	0.615	0.14	0.0493	0.9	15.0	58.3	0.51	0.41	0.05	0.959	0.005
150-157	305.2	1018	181	1.05	0.14	0.0476	1.8	10.4	58.3	0.51	0.54	0.05	0.949	0.006
150-158	305.2	1017	55	0.319	0.14	0.0635	0.6	27.0	58.3	0.51	0.30	0.05	0.946	0.005
150-125	301.5	1022	100	0.597	0.14	0.0568	1.1	17.0	53.0	0.86	0.63	0.09	0.876	0.013
150-126	301.5	1020	56	0.336	0.14	0.0655	0.6	27.1	53.0	0.80	0.44	0.08	0.906	0.008
150-123	301.3	1023	156	0.932	0.14	0.0575	1.8	11.4	53.0	0.94	0.85	0.10	0.694	0.034
150-124	301.3	1020	141	0.838	0.14	0.0664	1.9	13.0	53.0	0.93	0.80	0.10	0.751	0.023
150-153	304.4	1017	170	0.989	0.14	0.0450	1.8	12.8	54.3	1.02	0.96	0.11	0.466	0.091
150-154	304.7	1021	147	0.856	0.14	0.0515	1.6	12.9	54.3	1.02	0.89	0.11	0.590	0.055

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Table 6 (Continued)

Run No.	T ₀ °K.	P ₀ cm. H ₂ O	v ₀ cm./ sec.	N _{Re}	h cm.	α	ΔP cm. H ₂ O	C _{De}	Acid Conc. in Aero-sol, %	D _p microns	$\sqrt{\psi}$	R= D _p /D _f	N _h /N ₀	η_e
150-136	301.4	1024	63	0.379	0.14	0.0499	0.6	27.2	53.2	1.10	0.63	0.12	0.822	0.021
150-133	300.9	1026	201	1.21	0.14	0.0648	2.9	10.0	53.2	1.22	1.23	0.13	0.183	0.141
150-134	301.4	1026	154	0.922	0.14	0.0496	1.5	11.4	53.2	1.13	1.00	0.12	0.612	0.053
150-135	301.3	1027	116	0.693	0.14	0.0487	1.1	15.9	53.2	1.18	0.90	0.12	0.601	0.056
150-159	305.4	1020	191	1.11	0.20	0.0584	3.9	11.7	58.3	0.50	0.54	0.05	0.892	0.007
150-160	305.4	1017	61	0.354	0.20	0.0583	1.0	28.5	58.3	0.47	0.29	0.05	0.925	0.005
150-161	304.0	1021	120	0.704	0.20	0.0595	2.2	16.5	60.2	0.58	0.49	0.06	0.897	0.007
150-162	304.7	1024	138	0.805	0.20	0.0619	2.7	14.6	60.2	0.58	0.53	0.06	0.888	0.007
150-127	301.8	1019	173	1.03	0.20	0.0595	3.3	12.0	53.0	0.98	0.93	0.10	0.404	0.057
150-128	301.8	1020	135	0.799	0.20	0.0615	2.5	14.6	53.0	0.94	0.79	0.10	0.521	0.040
150-129	301.8	1022	62	0.369	0.20	0.0564	1.0	29.3	53.0	0.97	0.55	0.10	0.787	0.016
150-130	301.8	1019	97	0.579	0.20	0.0580	1.7	19.8	53.0	0.94	0.67	0.10	0.687	0.024
150-131	300.8	1027	154	0.921	0.20	0.0600	2.8	12.5	53.2	1.25	1.10	0.13	0.132	0.127
150-132	300.8	1027	200	1.20	0.20	0.0565	3.9	10.8	53.2	1.21	1.22	0.13	0.117	0.143
150-151	304.1	1020	180	1.06	0.20	0.0602	3.6	12.0	54.3	1.08	1.04	0.10	0.244	0.088
150-152	304.4	1017	143	0.832	0.20	0.0595	2.7	14.2	54.3	1.04	0.89	0.10	0.428	0.054
150-137	301.8	1023	105	0.625	0.20	0.0636	2.1	19.0	53.2	1.10	0.81	0.12	0.504	0.041
150-138	301.8	1023	57	0.340	0.20	0.0632	1.1	39.1	53.2	1.14	0.61	0.12	0.686	0.023
150-163	305.0	1024	161	0.938	0.22	0.0630	3.8	14.0	60.2	0.57	0.56	0.06	0.875	0.007
150-164	305.3	1024	114	0.666	0.22	0.0631	2.4	17.3	60.2	0.54	0.45	0.06	0.915	0.005
150-165	305.4	1021	58	0.339	0.22	0.0632	1.2	33.4	60.2	0.50	0.30	0.05	0.921	0.004
150-169	305.2	1023	190	1.11	0.22	0.0548	4.3	12.6	54.0	0.93	0.93	0.10	0.369	0.062
150-170	305.8	1020	157	0.910	0.22	0.0555	3.3	14.0	54.0	0.93	0.84	0.10	0.453	0.049
150-171	305.9	1021	103	0.600	0.22	0.0596	2.3	21.4	54.0	0.86	0.65	0.09	0.743	0.017
150-172	306.1	1019	70	0.405	0.22	0.0517	1.3	29.5	54.0	0.80	0.49	0.08	0.865	0.010
150-185	303.8	1018	156	0.916	0.22	0.0593	3.4	13.9	57.1	1.10	0.99	0.12	0.296	0.070
150-186	304.4	1018	130	0.759	0.22	0.0613	3.0	17.0	57.1	1.10	0.91	0.12	0.354	0.058
150-179	304.4	1018	77	0.450	0.22	0.0621	1.6	25.7	57.2	1.14	0.72	0.12	0.544	0.034
150-180	304.8	1018	99	0.576	0.22	0.0609	2.0	20.1	57.2	1.17	0.84	0.12	0.417	0.049
150-181	305.2	1021	147	0.858	0.22	0.0566	2.9	14.0	57.2	1.19	1.04	0.12	0.193	0.100

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Table 6 (Continued)

Run No.	T _o °K.	p _o cm. H ₂ O	v _o cm./ sec.	N _{Re}	h cm.	α	ΔP cm. H ₂ O	C _{De}	Acid Conc. in Aero- sol, %	D _p microns	$\sqrt{\Psi}$	R= D _p /D _f	N _h /N _o	η_e
150-182	305.3	1021	239	1.39	0.22	0.0540	5.1	9.7	57.2	1.14	1.27	0.12	0.070	0.168
150-166	305.5	1019	155	0.900	0.22	0.0789	5.0	15.7	60.2	0.48	0.47	0.05	0.843	0.007
150-167	305.4	1023	120	0.701	0.22	0.0722	3.4	19.4	60.2	0.55	0.47	0.06	0.848	0.008
150-168	305.8	1023	63	0.367	0.22	0.0760	1.8	35.9	60.2	0.49	0.31	0.05	0.878	0.006
150-173	306.3	1023	59	0.344	0.22	0.0781	2.0	42.4	54.0	0.77	0.44	0.08	0.796	0.010
150-174	306.3	1020	123	0.712	0.22	0.0740	3.8	19.8	54.0	0.77	0.63	0.08	0.653	0.020
150-175	306.4	1023	152	0.882	0.22	0.0714	4.7	16.7	54.0	0.74	0.67	0.08	0.614	0.023
150-188	305.0	1021	147	0.858	0.22	0.0769	5.3	18.7	57.1	0.94	0.83	0.10	0.359	0.046
150-187	304.9	1018	175	1.02	0.22	0.0714	5.1	13.6	57.1	1.06	1.01	0.11	0.200	0.077
150-183	305.0	1018	114	0.666	0.22	0.0766	3.7	21.6	57.2	1.13	0.87	0.12	0.253	0.061
150-184	305.0	1016	85	0.494	0.22	0.0708	2.2	25.3	57.2	1.08	0.72	0.10	0.545	0.029
150-177	303.6	1019	177	1.04	0.22	0.0843	6.8	15.0	57.2	1.26	1.20	0.13	0.024	0.152
150-178	304.3	1017	160	0.931	0.22	0.0821	5.5	15.6	57.2	1.30	1.17	0.14	0.044	0.130
150-263	306.3	1018	43	0.249	0.22	0.0738	1.0	42.0	55.0	0.55	0.28	0.06	0.900	0.005
150-264	306.9	1018	38	0.220	0.22	0.0791	1.0	49.7	55.0	0.49	0.24	0.05	0.900	0.005
150-265	307.2	1018	19	0.109	0.22	0.0801	0.5	100	55.0	0.47	0.16	0.05	0.856	0.007
150-254	306.9	1011	50	0.286	0.22	0.0568	0.9	36.5	58.4	0.97	0.50	0.10	0.826	0.012
150-255	307.3	1012	39	0.222	0.22	0.0579	0.7	46.6	58.4	0.98	0.45	0.10	0.832	0.011
150-256	307.4	1013	29	0.167	0.22	0.0626	0.5	56.8	58.4	0.94	0.37	0.10	0.858	0.008
150-257	307.7	1012	21	0.121	0.22	0.0566	0.3	76.9	58.4	0.92	0.31	0.10	0.867	0.009
150-252	308.9	1017	30	0.171	0.22	0.0768	0.8	69.9	59.0	0.51	0.22	0.05	0.858	0.007
150-253	309.0	1018	21	0.121	0.22	0.0741	0.6	98.4	59.0	0.51	0.18	0.05	0.847	0.008
150-261	308.1	1019	52	0.295	0.22	0.0751	1.5	44.4	58.4	0.52	0.29	0.05	0.882	0.006
150-262	308.2	1019	40	0.228	0.22	0.0722	1.1	54.6	58.4	0.52	0.26	0.05	0.866	0.007
150-199	306.3	1013	161	0.926	0.40	0.0662	7.1	13.4	54.8	0.56	0.54	0.06	0.702	0.010
150-200	306.3	1015	117	0.671	0.40	0.0617	4.3	16.5	54.8	0.56	0.46	0.06	0.800	0.007

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Table 6 (Continued)

Run No.	T ₀ °K.	P ₀ cm. H ₂ O	v ₀ cm./ sec.	N _{Re}	h cm.	α	ΔP cm. H ₂ O	C _{De}	Acid Conc. in Aero- sol, %	D _p microns	$\sqrt{\psi}$	R= D _p /D _f	N _h /N ₀	η_e
150-201	306.3	1012	62	0.357	0.40	0.0563	1.9	29.0	54.8	0.56	0.34	0.059	0.827	0.006
150-196	306.1	1016	160	0.923	0.40	0.0566	5.4	12.1	54.8	0.95	0.87	0.099	0.330	0.037
150-197	306.2	1016	132	0.758	0.40	0.0650	5.2	15.2	54.8	0.92	0.77	0.096	0.480	0.021
150-198	306.4	1020	94	0.543	0.40	0.0659	3.7	20.6	54.8	0.88	0.62	0.092	0.631	0.013
150-193	305.9	1016	178	1.03	0.40	0.0599	6.5	11.1	54.8	1.23	1.16	0.129	0.026	0.115
150-194	306.4	1016	139	0.800	0.40	0.0558	4.6	13.8	54.8	1.22	1.02	0.128	0.144	0.065
150-195	306.8	1017	120	0.691	0.40	0.0556	3.7	15.0	54.8	1.13	0.89	0.118	0.302	0.041

FIG. 1. COMPARISON OF THEORETICAL AND EXPERIMENTAL EFFICIENCIES OF INERTIAL IMPACTION ON CIRCULAR CYLINDERS

FIG. 2. THEORETICAL EFFICIENCY OF INTERCEPTION ON CIRCULAR CYLINDERS

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FIG. 3. CALCULATED TOTAL EFFICIENCY OF IMPACTION ON CIRCULAR CYLINDERS FOR $NR_e = 0.2$ (AFTER DAVIES)

FIG. 4. EXPERIMENTAL EQUIPMENT FOR DETERMINING IMPACTION EFFICIENCY OF CIRCULAR CYLINDERS

FIG. 5. EXPERIMENTAL EFFICIENCIES OF INERTIAL IMPACTION
ON CIRCULAR CYLINDERS

η_i , EFFICIENCY OF INERTIAL IMPACTION

$$\sqrt{\Psi} = (C_p v_0 / 18 \mu D_c)^{1/2} D_p$$

FIG. 6. EXPERIMENTAL EQUIPMENT FOR DETERMINING COLLECTION EFFICIENCY AND PRESSURE DROP OF FIBER MATS.

FIG. 7. EXPERIMENTAL EFFECTIVE FIBER EFFICIENCIES

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FIG. 8. COMPARISON OF EXPERIMENTAL EFFECTIVE FIBER EFFICIENCIES
AND DAVIES' THEORETICAL CURVES

FIG. 9. COMPARISON OF EXPERIMENTAL EFFECTIVE FIBER DRAG COEFFICIENTS WITH EQUATIONS