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THE CALCULATION OF COOLANT CIRCUITS FOR TURBULENT FLOW CONDITIONS

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INTRODUCTION

In the design of cyclotron dees, liners, high current magnet coils, etc., the coolant passages assume many geometric shapes such as round, square, and flattened tubing, and extrusions. Often the most easily fabricated coolant circuit is obtained by brazing flattened tubing to the surface from which the heat is to be removed. Good heat transfer is obtained by this method because of the large contact area. In addition, the thin sections thus formed lend themselves to design flexibility and space savings.

The calculation of such circuits is not difficult but it is indeed a laborious task. This report presents the necessary material for such calculations in a workable form along with all necessary curves and equations so that problems of this type can be solved without reference to other sources of information. In addition, this paper includes several specialized equations that have been derived and presented as an aid to simplification of calculations.

The sample calculation are provided to familiarize the reader with the use of the equations and the manipulation of the units used. It is to be noted that the material included is entirely generalized.

PART I: CIRCULAR PASSAGES

All flow calculations for circular passages can be made with standard flow equations. It is often necessary, however, to make several trial calculations before a suitable solution is obtained. In order to ease this task, these equations are simplified wherever possible.

The temperature rise of the coolant can be written as:

$$\Delta T_{\circ F} = \frac{kw \times 3414 \frac{\text{Btu}}{kw-hr}}{Q \frac{\text{ft}^3}{\text{sec}} \times 3600 \frac{\text{sec}}{\text{min}} \times 62.4 \frac{\text{lb}}{\text{ft}^3}}$$

Now, because 1 lb of water raised 1°F is equivalent to 1 Btu, we may substitute in the above equation and reduce it to the following:

$$\Delta T_{\circ F} = \frac{kw(0.0152)k}{Q} \quad \text{Eq. 1}$$

where Q = Flow rate (ft³/sec)
 kw = Power input (kw)
 ΔT = Increase in coolant temperature (°F)

Ordinarily k in Eq 1 is taken equal to unity. For problems requiring a higher degree of accuracy, use $k = \frac{62.4}{w}$ and take w from Chart 5, for the exit temperature of the coolant.

General Eq* 2, 3, and 4 are given below for reference.

* Vennard, Elementary Fluid Mechanics.

Head Loss Equation

$$h = f \frac{L}{d} \frac{V^2}{2g} \quad \text{Eq 2}$$

where h = Head of water (ft)
 f = Friction factor (dimensionless)
 L = Passage length (ft)
 d = Hydraulic dia* (ft)
 V = Velocity of coolant (ft/sec)
 g = Acceleration due to gravity (ft/sec²)

Flow Equation

$$Q = AV \quad \text{Eq 3}$$

where Q = Flow rate (ft³/sec)
 A = Area of passage (ft²)
 V = Velocity of coolant (ft/sec)

Reynolds Number Equation

$$N_R = \frac{V d \rho}{\mu} \quad \text{Eq 4}$$

where V = Velocity of flow (ft/sec)
 d = Hydraulic equivalent dia (ft)
 ρ = Density $\frac{\text{lb sec}^2}{\text{ft}^4}$ (Chart 5)
 μ = Viscosity of water $\frac{\text{lb sec}}{\text{ft}^2}$ (Chart 4)

Example

Heat to be removed	40 kw
Coolant path length	100 ft
Inlet coolant pressure	65 psi
Back pressure on coolant system	4 psi
Inlet coolant temperature	60 °F
Allowable increase in coolant temperature	50 °F

* The hydraulic diameter d is defined as $\frac{4 \text{ (flow area)}}{\text{wetted perimeter}}$. For round and square tubes it is identical, and the ID or the width is used. For other geometric shapes it must be calculated from the above formula before use in Eq 2 and 4.

Find: The ID (d) of tube required to meet the above conditions of cooling. For the solution to the problem stated above use the following method:

- (1) With Eq 1 find the flow required.
- (2) Using Eq 2 express the velocity in terms of d , assuming $f = 0.025$.*
- (3) Using Eq 3 and the answer from step (1) and step (2) above, find d in ft.
- (4) With answer from step (2) and the d from step (3) find the velocity V .
- (5) Using Eq 4 and the velocity V from step (4) find the Reynolds number and check f on Chart 3. If the f found is different than the one assumed, use it and recalculate steps (1) through (5).

The error in f converges very fast and usually one recalculation is sufficient. At any rate, recalculations should be continued until the assumed f and the f found on Chart 3 are very close. When this occurs the last d found is correct. As a final check substitute the values obtained for d and V in Eq 3 to see if the Q is the same as the Q required in step (1).

* This is always a very good value to assume for f .

Calculation of the Example

The flow required may be readily found from Eq 1.

$$(1) \quad 50^\circ\text{F} = \frac{40(0.0152)}{Q}$$

$$Q = 0.0122 \frac{\text{ft}^3}{\text{sec}}$$

With Q known we may now solve for the ID of the tube.

- (2) Assume the friction factor to be 0.025 and find V in terms of d . **

$$h = f \frac{L}{d} \frac{V^2}{2g}$$

$$V = \sqrt{\frac{2g h}{f L}} (d)^{1/2} = \sqrt{\frac{64.4 \times 140}{0.025 \times 100}} (d)^{1/2} = 60(d)^{1/2}$$

When h is given in lb/in^2 divide by $\frac{w}{144} \frac{\text{lb/ft}^3}{\text{in}^2/\text{ft}^2}$ to get ft of head.

- (3) From Eq 3 with Q known to be 0.0122, solve for d .

$$Q = AV \quad A = \frac{\pi}{4} (d)^2 \quad V = 60(d)^{1/2}$$

$$0.0122 = \frac{\pi}{4} (d)^2 (60)d^{1/2}$$

$$d^{5/2} = 0.000259$$

$$d = 0.0367 \text{ ft}$$

Since f was assumed in part 2 of this calculation, it must now be checked by use of the Reynolds number and the friction factor, Chart 3.

** Errors will always be reduced if L and d are kept in ft.

- (4) From d and by using the solution in (2), find the velocity of the flow.

$$V = 60(d)^{1/2} \quad \text{where } d = 0.0367 \text{ ft}$$

$$V = 60 \sqrt{0.0367} = 11.5 \text{ ft/sec}$$

- (5) From Eq (4) find N_R . (Use Charts 4 and 5 for values of ρ and μ .)

$$d = 0.0367 \text{ ft}$$

$$\rho_{110^\circ F} = 1.926 \frac{(\text{lb sec}^2)}{\text{ft}^4}$$

$$\mu_{110^\circ F} = 1.148 \times 10^{-5} \frac{(\text{lb sec})}{\text{ft}^2}$$

$$N_R = \frac{V d \rho}{\mu} = \frac{11.5 \times 0.0367 \times 1.926}{1.148 \times 10^{-5}} = 7.05 \times 10^4$$

Reference to Chart 3 shows that the f corresponding to $N_R = 7.05 \times 10^4$ is approximately 0.019 instead of the assumed value of 0.025. The procedure now is to use the new value of f and recalculate the problem. This seldom needs to be done more than once since the error in f converges very rapidly with each recalculation.

- (6) V from step (2) now becomes

$$V = \sqrt{\frac{0.025}{0.019}} (60)d^{1/2} = 68.75(d)^{1/2}$$

- (7) and step (3) becomes

$$Q = AV, \text{ where } A = \frac{\pi}{4} (d)^2 \text{ and } V = 68.75(d)^{1/2}$$

$$0.0122 = \frac{\pi}{4} (d)^2 (68.75) (d)^{1/2}$$

$$d = (0.000226)^{0.4} = 0.0348 \text{ ft}$$

(8) A recheck of $\underline{f}$ now gives: (step 4)

$$V = 68.75(d)^{1/2} = 68.75 \sqrt{0.0348} = 12.85 \text{ ft/sec}$$

(9) N_R is now: (step 5)

$$N_R = \frac{V d \rho}{\mu} = \frac{12.85 \times 0.0348 \times 1.926}{1.148 \times 10^{-5}} = 7.5 \times 10^4$$

A check of Chart 3 now shows that $\underline{f}$ for this N_R is very close to the 0.019 used, and further calculation is unnecessary. However, it is advisable at this point to check on the arithmetic by using the flow area and the velocity to see if the Q thus obtained is the same as the original Q required. (Part I).

$$Q = AV \text{ where } A = \frac{\pi}{4} (0.0348)^2 \text{ and } V = 12.85 \text{ ft/sec}$$

$$Q = \frac{\pi}{4} (0.0348)^2 (12.85) = 0.0122 \text{ ft}^3/\text{sec}$$

This value of Q is the same as the one required and indicates the absence of arithmetical errors.

Thus, the $\underline{d}$ found in step (7) is the correct one.

PART II: SQUARE PASSAGES

Heat to be removed	40 kw
Coolant path length	100 ft
Inlet coolant pressure	65 psi
Back pressure on coolant system	4 psi
Inlet coolant temperature	60 °F
Allowable increase in coolant temperature	50 °F

Find: The dimension of the square passage required. (Fig. 1b)

In Part I we found the correct diameter of round tube for the above problem. Let us examine for a moment how the solution would have changed had we been asked to supply a coolant circuit with a square passage of dimension d instead of the round one. This question is often asked, since when the tubes are stacked, as in a coil, they both require the same winding space. (Fig 1b)

FIG. 1

Since all flow rates are affected by the hydraulic diameter of the passage, we must now examine the square passage and relate it to the round one. Let d in Fig 1a be equal numerically to d in Fig. 1b. Now write the hydraulic diameter for each as follows:

$$\text{Hydraulic diameter } d = \frac{4(\text{Flow area})}{\text{Wetted perimeter}}$$

For the round tube

$$d \equiv d_a = \frac{4 \frac{\pi}{4} d_a^2}{\pi d_a} = d_a$$

For the square tube

$$d \equiv d_b = \frac{4 d_b^2}{4 d_b} = d_b$$

We find the hydraulic diameter to be the same for each. This shows that the velocity (Eq. 2) will remain the same for each tube provided f does not change. An examination of the Reynolds number (Eq. 4) shows that if V and d remain unchanged, there will be no change in f . In going from the round tube to the square tube, the area increased by 27.4%, and since the V remains unchanged there must be a change in flow. From Eq (3) it can be seen that Q has increased by 27.4%. A glance at Eq. (1) confirms the following: In going from the round to the square passage 27.4% more heat will be carried away if ΔT is held constant; if kw is held constant ΔT will decrease by 27.4%.

Our problem at the moment, however, is to find the size of square passage which will produce $\Delta T = 50^\circ F$.

There is only one Q which will produce a $\Delta T = 50^\circ F$ and we have already found it to be $Q = 0.0122$ ft/sec, (in Part I).

From $Q = AV$ we see that the two variables A and V each can be expressed as follows:

$$\text{Area for square passage} \quad A_b = d_b \times d_b = (d_b)^2$$

$$\text{Velocity for square passage} \quad V = \sqrt{\frac{2g h}{f L}} (d)^{1/2}$$

d is again the hydraulic diameter in ft and has been shown to be the same as the length of a side d_b . Then:

$$Q = AV = d_b^2 \sqrt{\frac{2g h}{f L}} (d_b)^{1/2}$$

or

$$d_b = \left(\frac{Q}{\sqrt{\frac{2g h}{f L}}} \right)^{2/5} \quad (5)$$

We now assume $f = 0.025$ and solve for d_b .

$$d_b = \left(\frac{0.0122}{\sqrt{\frac{64.4 \times 140}{0.025 \times 100}}} \right)^{0.4} = \left(\frac{0.0122}{60} \right)^{0.4} = 0.034 \text{ ft.}$$

f must now be checked to see if the assumed value of 0.025 is correct.

$$v = \sqrt{\frac{2g h}{f L}} (d_b)^{1/2} = 60(0.034)^{1/2} = 11.05 \text{ ft/sec}$$

$$N_R = \frac{v d \rho}{\mu} = \frac{11.05 \times 0.034 \times 1.926}{1.148 \times 10^{-5}} = 6.3 \times 10^4$$

From Chart 3, f for this $N_R = 0.019$.

Since the f was assumed as 0.025 and found to be by trial calculation equal to 0.019, d must be recalculated with the new value of f .*

$$d = \left(\frac{0.0122}{60 \sqrt{\frac{0.025}{0.019}}} \right)^{0.4} = 0.0318 \text{ ft}$$

* Errors in f converge rapidly and usually one recalculation is sufficient to correct it.

f must again be checked:

$$V = \sqrt{\frac{2g h}{f L}} (d)^{1/2}$$
$$= 68(0.0318)^{1/2} = 12.1 \text{ ft/sec}$$

$$N_{R(\text{new})} = N_{R(\text{old})} \times \frac{V(\text{new})}{V(\text{old})} = 6.3 \times 10^4 \times \frac{12.1}{11.05}$$
$$= 6.63 \times 10^4$$

For this N_R , $\underline{f} = 0.0189$ and is close enough to the assumed value of 0.019.

Thus the d found (0.0318 ft) is the correct d. As a check on arithmetic it is always advisable to prove that the V and A found actually give the required Q.

$$Q = AV \quad \text{where } A = \frac{(d)^2}{V} = 12.1 \text{ ft/sec}$$

$$Q = (0.0318)^2(12.1) = 0.01225 \text{ ft/sec}$$

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PART III: FLATTENED PASSAGES

We have thus far considered the round and square passage cases. Let us now examine a more common type of passage such as shown in Fig. 2b. Our problem here is to choose a tube of diameter D (Fig. 2a) such that when flattened as shown, for example, to $y = 0.312$ inches will flow the same amount of water, Q , as the round tube found in Part I and will therefore satisfy all the cooling requirements of the problem as stated on page 3.

Hydraulically a flattened tube is very different from a round one. As a tube is flattened the area decreases; the friction factor f increases; the hydraulic diameter decreases; and the velocity decreases. However, for each round tube there always can be found a flattened tube which with the same pressure head will flow the same amount of water, Q . The relationships which are necessary for the calculation of these circuits are given below.

FIG. 2

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When a round tube (Fig. 2a) is flattened as shown in Fig. 2b, the cross sectional flow area decreases. We may write the new flow area A_w , as a function of the original inside diameter D , and the new passage height y , as follows:

$$C = p = p_1 = \pi y + 2x$$

$$x = \frac{\pi}{2} (D-y)$$

$$A_w = \frac{\pi y^2}{4} + yx$$

Substitute for x in A_w ,

$$A_w = \frac{\pi y^2}{4} + y \pi \frac{(D-y)}{2}$$

$$= \frac{\pi y}{2} \left(D - \frac{y}{2} \right)$$

Eq 6

The change in geometric shape of the tube also brings about a change in the hydraulic characteristics of the passage which are based on the hydraulic radius R as follows:

$$R = \frac{\text{Flow area}}{\text{Wetted perimeter}} \quad \text{and, } d = 4R$$

We may write:

$$d = 4R = \frac{4 A_w}{p}$$

Substitute:

$$d = \frac{4 \left(\frac{\pi y}{2} \right) \left(D - \frac{y}{2} \right)}{\pi D}$$

Thus

$$d = \frac{2y}{D} \left(D - \frac{y}{2} \right)$$

Eq 7

or

$$D = \frac{y^2}{2y - d}$$

Eq 8

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The flattened width,

$$W = x + y + 2t$$

The rate of flow, Q , of any coolant circuit is always equal to the cross sectional area times the velocity of flow, i.e.:

$$Q = AV$$

By substituting into this equation the mathematical expressions obtained from the geometry of the flattened tubing we have

$$Q = \frac{\pi y}{2} \left(D - \frac{y}{2}\right) \sqrt{\frac{2gh}{fL} \frac{2y}{D} \left(D - \frac{y}{2}\right)} \quad \text{Eq 9}$$

Assuming all quantities except D are known, the above equation becomes a cubic in D , for which a solution is somewhat tedious. Since the object of this paper is to present a simplified solution for such flow circuits another approach must be found.

By writing the flow equation, Eq 3, in a slightly different form we have

$$Q = A_w(d)^{1/2} \sqrt{\frac{2gh}{fL}}$$

or

$$A_w(d)^{1/2} = \sqrt{\frac{Q^2}{\frac{2gh}{fL}}} \quad \text{Eq 10}$$

(always use A in ft^2 , d in ft , and L in ft)

Now by use of Eq 6 and 7 a family of curves of $A(d)^{1/2}$ vs D as a function of y has been plotted for all normally used values of D and for practical values of y from $y = 0.250$ inches to $y = 0.75$ inches in steps of $1/32$ inch. By use of these curves the problem now becomes a simple one:

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Heat to be removed	40 kw
Coolant path length	100 ft
Inlet coolant pressure	65 psi
Back pressure on coolant system	4 psi
Inlet coolant temperature	60 °F
Allowable increase in coolant temperature	50 °F

Find: The inside diameter, D, of a round tube which when flattened to $y = 0.312$ inch will produce the $\Delta T = 50^\circ\text{F}$ required above.

$$Q = \frac{KW(0.0152)}{\Delta T} \quad \text{Eq. (1)}$$

$$= \frac{40(0.0152)}{50} = 0.0122 \text{ ft/sec}$$

Assume friction factor = 0.025.*

$$A(d)^{1/2} = \frac{Q}{\sqrt{\frac{2gh}{fL}}} = \frac{0.0122}{\sqrt{\frac{(64.4)(140)}{(0.025)(100)}}} = 0.00024 \text{ or } 0.204 \times 10^{-3}$$

From Chart I for $y = 0.312$ and $A(d)^{1/2} = 0.000204$ we find the required ID of the unflattened tube to be, $D = 0.491$ inches. Since f was assumed it must now be checked because the solution is never complete until the assumed f and the calculated f agree.

* For turbulent flow this is a good assumption.

To check $\underline{f}$, $\underline{d}$ and V will be required and can be readily obtained from

$$\begin{aligned} d &= \frac{2y}{(D)} \left(\frac{D-y}{2} \right) \text{ inches} & (\text{Eq. 7}) \\ &= \frac{2(0.312)}{0.491} \left(\frac{0.491 - 0.312}{2} \right) = 0.426 \text{ inches} \end{aligned}$$

$$\text{Required } \underline{d} \text{ in ft} = \frac{0.426}{12} = 0.0354 \text{ ft}$$

and

$$\begin{aligned} V &= \sqrt{\frac{2gh}{fL}} (d)^{1/2} & (\text{Eq. 2}) \\ &= 60(0.0354)^{1/2} = 11.3 \text{ ft/sec} \end{aligned}$$

(Note - Always use $\underline{d}$ and L in ft)

The Reynolds number can now be found from

$$N_R = \frac{V d \rho}{\mu}$$

where

$$\rho_{110^\circ\text{F}} = 1.926 \text{ (Chart 5)}$$

$$\mu_{110^\circ\text{F}} = 1.148 \times 10^{-5} \text{ (Chart 4)}$$

$$N_R = \frac{(11.3)(0.0354)(1.926)}{1.148 \times 10^{-5}} = 6.7 \times 10^4$$

For this N_R , $f = 0.019$, hence, to correct for the assumed value of $f = 0.025$, a recalculation is necessary using the new value of $\underline{f}$.*

$$A(d)^{1/2} = \frac{0.0122}{\sqrt{\frac{(64.4)(140)}{(0.019)(100)}}} = 0.000177 \text{ or } 0.177 \times 10^{-3}$$

from Chart 1, $D = 0.443$ inches.

* Errors in $\underline{f}$ converge rapidly with each recalculation.

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A check of $\underline{f}$ now follows.

$$\begin{aligned} d &= \frac{2y}{D} \left(D - \frac{y}{2} \right) \\ &= \frac{2(0.312)}{0.443} \left(0.443 - \frac{0.312}{2} \right) = 0.402 \text{ inches} \end{aligned}$$

The required d in ft = $\frac{0.402}{12} = 0.0335$ ft.

$$\begin{aligned} v &= \sqrt{\frac{2gh}{fL}} (d)^{1/2} \\ &= 69(0.0335)^{1/2} = 12.6 \text{ ft/sec} \end{aligned}$$

$$\begin{aligned} N_R &= \frac{v d \rho}{\mu} \\ &= \frac{(12.6)(0.0335)(1.926)}{1.148 \times 10^{-5}} = 7.07 \times 10^4 \end{aligned}$$

From Chart 3 for $N_R = 7.07 \times 10^4$ the $\underline{f} = 0.019$ which is the same as the last value of $\underline{f}$ used, and the calculation is now complete - the last D found being the correct one. As an added step it is always a good idea to find the flow produced as a check on arithmetic errors.

Thus:

$$\begin{aligned} A_w^* &= \frac{\pi y}{2} \left(D - \frac{y}{2} \right) \text{ in}^2 \\ &= \frac{\pi(0.312)}{2} \left(0.443 - \frac{0.312}{2} \right) = 0.141 \text{ in}^2 \end{aligned}$$

$$\text{the required area ft}^2 = \frac{0.141}{144} = 0.000975 \text{ ft}^2$$

* Chart 2 may be used to find this area.

Now from $Q = AV$

$$Q = (0.000975)(12.6) = 0.0123 \text{ ft}^3/\text{sec}.$$

The Q found necessary by Eq 1 in the first step of this problem was $Q = 0.0122 \text{ ft}^3/\text{sec}$ so the values of Q compare favorably and helps prove the absence of arithmetic errors.

NOMENCLATURE

- A_w = Cross sectional flow area
- C = Circumference
- d = Hydraulic diameter (to avoid errors always use in ft)
- D = I.D. of round tube which when shaped as required will flow the proper Q . (to avoid errors always use in ft)
- f = Friction factor (dimensionless)
- g = Gravitational acceleration (32.2 ft/sec^2)
- h = Pressure head (ft)
- kw = Power to be removed by coolant (kilowatts)
- L = Length of passage (to avoid errors always use in ft)
- N_R = Reynolds number (dimensionless)
- p = Wetted perimeter of round tube
- p_1 = Wetted perimeter of odd shaped passage
- Q = Flow rate of coolant (ft^3/sec)
- ρ = Density $\frac{(\text{lb sec}^2)}{\text{ft}^4}$
- R = Hydraulic radius, $\frac{(\text{cross sectional area})}{\text{wetted perimeter}}$
- t = Wall thickness of tube
- μ = Viscosity of coolant $\frac{\text{lb sec}}{\text{ft}^2}$
- V = Velocity of coolant (ft/sec)
- W = Overall width (for flattened tube)
- x = Center to center distance of circular ends (flattened tube)
- y = Inside height of coolant passage (in.)

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Chart 2.

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Chart 3

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Chart 5

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