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THERMAL STRESS ANALYSIS OF
CYLINDRICAL SHELLS

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THERMAL STRESS ANALYSIS OF CYLINDRICAL SHELLS

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PREFACE

This report is an extension of the method described in (1)¹, and it is written to be used in conjunction with the tables in Section 2 of (1). Part I describes the analysis of a cylindrical shell subjected to a temperature variation in the axial direction. This part is divided into two sections. Section 1 gives the formulas and the procedure for their application. In Section 2, the derivations of the cylindrical shell formulas are given. A cylindrical shell subjected to a linear temperature variation through the thickness is considered in Part II. Part III contains some sample problems which require the methods of Parts I and II for their solutions. Although this part demonstrates the use of the formulas derived, a familiarity with (1) is a prerequisite to an understanding of the material presented.

The author is indebted to Dr. F. J. Stanek who served in an advisory capacity in the writing of this report.

¹ Numbers in parentheses refer to similarly numbered references in bibliography at end of paper.

CONTENTS

	Page
Preface	
Part I: Cylindrical Shells Subjected to Temperature Variation in the Axial Direction	
Section 1	
Introduction	3
Symbols and Sign Convention	4
Cylindrical Shell Formulas	6
Procedure	10
Section 2	
Derivation of the Cylindrical Shell Formulas	11
Part II: Cylindrical Shells Subjected to a Linear Temperature Variation Through the Thickness	
Discussion and Comments	19
Part III: Illustrative Examples	23
Bibliography	30

PART I

CYLINDRICAL SHELLS SUBJECTED TO TEMPERATURE
VARIATION IN THE AXIAL DIRECTION

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1

2

3

40

SECTION 1

INTRODUCTION

A cylindrical shell, loaded with any combination of axisymmetrical loads and subjected to a temperature variation in the axial direction, as shown in Fig. 1, may be analyzed by the method presented in this report. It is assumed that the temperature is constant through the thickness of the wall. Since the analysis is based on small deflection theory, the maximum radial displacement, other than free thermal expansion, must be small relative to the radius. The temperature function, F_c , is assumed to be either of the form

$$F_c = T_1 + T_2x + T_3x^2 + T_4x^3$$

or

$$F_c = T_0 e^{-bx}$$

where x is the axial coordinate of the cylinder and the T_i 's and b are constants. The constants T_0 and T_1 represent the temperature at $x = 0$ above the base temperature.

The independent variable, x , is combined with a shell parameter to give the dimensionless coordinate y which is defined by

$$y = \beta x,$$

where β is the characteristic length of the cylinder. The cylindrical shell formulas are thus expressed as functions of y , whose numerical values are found in Section 2 of (1). The principle of superposition may be applied to the given formulas.

SYMBOLS AND SIGN CONVENTION FOR CYLINDRICAL SHELLS

Fig. 1

x = axial coordinate of cylindrical shell (in.)

a = radius of middle surface of cylindrical shell (in.)

h = thickness of cylindrical shell (in.)

L = length of cylindrical shell (in.)

$$\beta = \left[\frac{3(1-\mu^2)}{a^2 h^2} \right]^{\frac{1}{4}} = \left[\frac{Eh}{4a^2 D} \right]^{\frac{1}{4}} \quad \text{characteristic length of the cylinder (in.)}^{-1}$$

$y = \beta x$ - dimensionless coordinate of cylindrical shell

w = radial displacement of middle surface of cylindrical shell, positive inward (in.)

- u = axial displacement of middle surface of cylindrical shell, positive in positive x direction (in.)
 p = uniform internal pressure (psi)
 Q = shearing force in cylindrical shell
 See Fig. 1 for positive direction (lb/in.)
 $D = \frac{Eh^3}{12(1-\mu^2)}$ - flexural rigidity of an axial strip of the cylinder
 N = normal membrane force in cylindrical shell, tension positive (lb/in.)
 M = bending moment in cylindrical shell, positive when tensile stresses in outside surface (lb-in./in. = lb)
 E = Young's modulus of elasticity of the material of the cylindrical shell (psi)
 μ = Poisson's ratio of the material of the cylindrical shell (dimensionless)
 C_n = the four constants of integration, C_1, C_2, C_3 , and C_4 (lb/in.)
 C_5 = another constant of integration (lb/in.)
 α = coefficient of thermal expansion of the material of the cylinder (in./in.-°F)
 F_c = temperature function (positive above base temperature)
 T_i = constants, $i = 0, \dots, 4$
 b = constant (in.)⁻¹
 $d = \frac{b}{\beta}$ - temperature parameter (dimensionless)
 $\gamma = \frac{4\alpha T_0}{d^4 + 4}$ (in.)

CYLINDRICAL SHELL FORMULAS

In the formulas listed below

$$\sum_{n=1}^4 C_n(\dots)_n = \sum_{n=1}^4 C_n(\dots)_n \quad \text{where } (\dots) \text{ is some function of } y \text{ whose}$$

numerical values are tabulated in Section 2 of (1).

Shell Constants

$$(i) \quad \beta^4 = \frac{3(1-\mu^2)}{a^2 h^2} = \frac{Eh}{4a^2 D}$$

$$= \frac{2.73}{a^2 h^2}$$

$$(ii) \quad \beta^2 = \frac{1.6523}{ah}$$

for $\mu = 0.3$

$$(iii) \quad D = \frac{Eh^3}{12(1-\mu^2)}$$

Coordinate Transformation

$$(iv) \quad y = \beta x$$

Temperature Functions (Temperature above base temperature)

$$(va) \quad F_c = T_1 + T_2 x + T_3 x^2 + T_4 x^3$$

or

$$(vb) \quad F_c = T_0 e^{-bx}$$

Case A: $F_c = T_1 + T_2 x + T_3 x^2 + T_4 x^3$

Internal Bending Moments

$$(1a) \quad M_x = \frac{1}{4a\beta^2} \sum C_n M_n - 2Da\alpha(T_3 + 3T_4 \frac{y}{\beta})$$

$$(2a) \quad M_\theta = \mu M_x$$

Internal Membrane Forces

$$(3a) \quad Q = \frac{1}{4a\beta} \sum C_n Q_n + 6Da\alpha T_4$$

$$(4a) \quad N_x = \frac{(\text{axial force})}{2\pi a} = \text{a constant}$$

$$(5a) \quad N_\theta = - \sum C_n N_n + ap$$

Displacement of Middle Surface

$$(6a) \quad w = \frac{a}{Eh} (\sum C_n N_n + \mu N_x - ap) - a\alpha \left[T_1 + T_2 \left(\frac{y}{\beta}\right) + T_3 \left(\frac{y}{\beta}\right)^2 + T_4 \left(\frac{y}{\beta}\right)^3 \right]$$

$$(7a) \quad \frac{dw}{dx} = \frac{a\beta}{Eh} \sum C_n W'_n - a\alpha \left[T_2 + 2T_3 \left(\frac{y}{\beta}\right) + 3T_4 \left(\frac{y}{\beta}\right)^2 \right]$$

$$(8a) \quad u = \frac{1}{Eh\beta} \left[\frac{u}{4} \sum C_n Q_n + (N_x - \mu ap)y + C_5 \right],$$

Case B: $F_c = T_o e^{-bx}$

$$d \equiv \frac{b}{\beta}$$

$$\gamma \equiv \frac{4a\alpha T_o}{d^4 + 4}$$

Internal Bending Moments

$$(1b) \quad M_x = \frac{1}{4a\beta^2} \sum C_n M_n - Db^2 \gamma e^{-dy}$$

$$(2b) \quad M_\theta = \mu M_x$$

Internal Membrane Forces

$$(3b) \quad Q = \frac{1}{4a\beta} \sum C_n Q_n - Db^3 \gamma e^{-dy}$$

$$(4b) \quad N_x = \frac{(\text{axial force})}{2\pi a} = a \text{ constant}$$

$$(5b) \quad N_\theta = - \sum C_n N_n + ap - \frac{d^4}{d^4 + 4} E\alpha T_o e^{-dy}$$

Displacement of Middle Surface

$$(6b) \quad w = \frac{a}{Eh} (\sum C_n N_n + \mu N_x - ap) - \gamma e^{-dy}$$

$$(7b) \quad \frac{dw}{dx} = \frac{a\beta}{Eh} \sum C_n W_n' + b\gamma e^{-dy}$$

$$(8b) \quad u = \frac{1}{Eh\beta} \left[\frac{\mu}{4} \sum C_n Q_n + (N_x - \mu ap)y - aDb^3 \mu \gamma \beta e^{-dy} + c_5 \right]$$

Cases A and B:

Internal Bending Stresses

(+ outside surfaces, - inside surfaces)

$$(9) \quad \sigma_{\beta_x} = \pm \frac{6M_x}{h^2}$$

$$(10) \quad \sigma_{\beta_\theta} = \pm \frac{6M_\theta}{h^2}$$

Internal Membrane Stresses

$$(11) \quad \sigma_{m_x} = \frac{N_x}{h}$$

$$(12) \quad \sigma_{m_\theta} = \frac{N_\theta}{h}$$

Total or Principal Stresses (+ outside surface, - inside surface)

$$(13) \quad \sigma_x = \frac{N_x}{h} \pm \frac{6M_x}{h^2}$$

$$(14) \quad \sigma_\theta = \frac{N_\theta}{h} \pm \frac{6M_\theta}{h^2}$$

PROCEDURE

Numerical values of the sub-n functions are tabulated in Section 2 of (1) for the range of y values from 0 through 10 (in increments of 0.1).

It is usually more convenient to place the origin of the x coordinate at the loaded or restrained end of the cylindrical shell. Then the y values are in the range from 0 through βL . The exact value of $y_L (= \beta L)$ of the cylinder may not be one of the y values listed in the tables; in which case, the nearest value is selected and the value of L is adjusted slightly.

A set of four simultaneous equations is solved to evaluate the four constants of integration C_n when a short cylinder ($y_L < 10$) is analyzed. Each equation in this set is a mathematical statement of a "boundary condition" or "edge condition" which is specified for the cylinder. The four boundary conditions are usually obtained by specifying two of the four quantities, M_x , Q , w , and dw/dx at each end of the cylinder.

For long cylinders ($y_L > 10$) the constants of integration, C_3 and C_4 , are taken equal to zero, as the edge effects decay very rapidly as the coordinate x increases. Hence, only two boundary conditions are necessary at the "origin" end of the cylinder. If loads are applied at both edges of a long cylinder, two "origin" ends may be established and treated as if there are two long cylinders with loads at one edge of each.

The constant of integration C_5 appears in the formula for the axial displacement u only. Hence, its value is obtained by specifying the displacement u at some y coordinate value (usually at y_0 or y_L), which generally stipulates merely an axial displacement of the cylinder. Hence C_5 may usually be taken equal to zero.

SECTION 2

DERIVATION OF THE CYLINDRICAL SHELL FORMULAS

Using the symbols and sign convention established in Section 1, the basic fourth order differential equation for cylindrical shells may be derived from the following sets of equations:

Equations of Equilibrium

$$(1) \quad \frac{dN_x}{dx} = 0$$

$$(2) \quad N_\theta + a \frac{dQ}{dx} = ap$$

$$(3) \quad Q + \frac{dM_x}{dx} = 0$$

Force-Displacement Relations

$$(4) \quad N_x = \frac{Eh}{1-\mu^2} \left[\frac{du}{dx} - \mu \left(\frac{w}{a} + \alpha F_c \right) \right]$$

$$(5) \quad N_\theta = \frac{Eh}{1-\mu^2} \left[- \left(\frac{w}{a} + \alpha F_c \right) + \mu \frac{du}{dx} \right]$$

$$(6) \quad M_x = D \frac{d^2 w}{dx^2}$$

$$(7) \quad M_\theta = \mu M_x \quad \text{where } D = \frac{Eh^3}{12(1-\mu^2)}$$

The elimination of $\frac{du}{dx}$ from Eqs. 4 and 5 gives

$$(8) \quad N_\theta - \mu N_x = -Eh \left(\frac{w}{a} + \alpha F_c \right)$$

The elimination of Q from Eqs. 2 and 3 yields

$$(9) \quad N_{\theta} - a \frac{d^2 M_x}{dx^2} = ap$$

If N_{θ} is eliminated from Eqs. 8 and 9, then

$$(10) \quad \frac{d^2 M_x}{dx^2} + \frac{Eh}{a^2} w = \mu \frac{N_x}{a} - p - \frac{Eh\alpha F_c}{a}$$

On substituting from Eq. 6 and dividing by the coefficient of $\frac{d^4 w}{dx^4}$,

Eq. 10 becomes

$$(11) \quad \frac{d^4 w}{dx^4} + \frac{Eh}{a^2 D} w = \frac{1}{Da} (\mu N_x - Eh\alpha F_c - ap)$$

Letting $\beta^4 = \frac{Eh}{4a^2 D}$ and introducing $y = \beta x$, Eq. 11 becomes

$$(12) \quad \frac{d^4 w}{dy^4} + 4w = \frac{4a}{Eh} (\mu N_x - ap) - 4a\alpha F_c$$

The particular solution of Eq. 12 is determined by the function F_c .

Two cases are considered:

$$(A) \quad F_c = T_1 + T_2 x + T_3 x^2 + T_4 x^3$$

$$(B) \quad F_c = T_0 e^{-bx}$$

Hence the general solution of Eq. 12 is

$$(13) \quad w = \frac{a}{Eh} \left[e^{-y} (C_1 \sin y + C_2 \cos y) + e^y (C_3 \sin y + C_4 \cos y) + \mu N_x - ap \right] + kF_c,$$

where

$k = \text{constant.}$

If

$$\begin{aligned} N_1 &= e^{-y} \sin y \\ N_2 &= e^{-y} \cos y \\ N_3 &= e^y \sin y \\ N_4 &= e^y \cos y, \end{aligned}$$

then Eq. 13 becomes formula 6a for Case A and formula 6b for Case B.

In formula 6a,

$$k = -a\alpha$$

and in 6b,

$$k = -\frac{4a\alpha}{d^4 + 4}.$$

Formulas 7a and 7b are derived, respectively, by differentiating Eqs.

6a and 6b and defining

$$W'_n = \frac{dN_n}{dy}$$

Formulas 1a and 1b are derived by differentiating formulas 6a and 6b

$$M_n = \frac{d^2 N_n}{dy^2} = \frac{dW'_n}{dy}.$$

Equation 7 is formulas 2a and 2b.

Formulas 3a and 3b are obtained by differentiating formulas 1a and 1b, substituting in Eq. 3 and defining

$$Q_n = \frac{-dM_n}{dy}.$$

Formulas 5a and 5b result from substituting formulas 6a and 6b into Eq. 8.

Formulas 8a and 8b are derived in the following manner. If N_θ is eliminated from Eqs. 2 and 5, then

$$(14) \quad a \frac{dQ}{dx} = ap - \frac{Eh}{1-\mu^2} \left[-\left(\frac{w}{a} + F_c\right) + \mu \frac{du}{dx} \right]$$

From Eq. 11, with $\beta^4 = \frac{Eh}{4a^2 D} = \frac{3(1-\mu^2)}{a^2 h^2}$

$$(15) \quad w = \frac{1}{4\beta^4} \left(\frac{\mu N_x}{aD} - \frac{p}{D} - \frac{Eh\alpha}{Da} F_c - \frac{d^4 w}{dx^4} \right)$$

Substituting Eq. 15 into Eq. 14 yields

$$(16) \quad a \frac{dQ}{dx} = ap - \frac{Eh}{1-\mu^2} \left\{ - \left[\frac{1}{4\beta^4 a} \left(\frac{\mu N_x}{aD} - \frac{p}{D} - \frac{Eh\alpha}{Da} F_c - \frac{d^4 w}{dx^4} \right) + \alpha F_c \right] + \mu \frac{du}{dx} \right\}$$

which in turn yields

$$(17) \quad Eh \frac{du}{dx} = -\frac{a}{\mu} \left[(1-\mu^2) \frac{dQ}{dx} + D \frac{d^4 w}{dx^4} \right] + N_x - \mu ap.$$

Equations 3 and 6 yield

$$(18) \quad Q = -D \frac{d^3 w}{dx^3}$$

Hence by substituting Eq. 18 into Eq. 17, integrating and solving for u,

$$(19) \quad u = \frac{1}{Eh} \left[a\mu Q + (N_x - \mu a p)x + \frac{C'_5}{\beta} \right]$$

where $\frac{C'_5}{\beta}$ is the constant of integration. Formulas 8a and 8b follow by

substituting formulas 3a and 3b, respectively, and y into Eq. 19. In 8a,

$$C_5 = C'_5 + 6Da^2\alpha\beta\mu T_4 = \text{constant}$$

and in 8b,

$$C_5 = C'_5.$$

PART II

CYLINDRICAL SHELLS SUBJECTED TO A LINEAR
TEMPERATURE VARIATION THROUGH THE THICKNESS

DISCUSSION AND COMMENTS

Let F_i and F_o be the temperature functions (above the base temperature) at the inside and outside surfaces, respectively, of a cylindrical shell and let the variation of the temperature through the thickness be linear. Let the shell be subjected to an axisymmetrical load as discussed in (1). If $\Delta t = F_i - F_o$, then from (2), this superposes upon formula (1) in Section 1 of (1) the function $\frac{E\alpha(\Delta t)h^2}{12(1-\mu)}$ and upon formula 2 the function $\frac{E\alpha(\Delta t)h^2}{12}$. Thus

$$(1') \quad M_x = \frac{1}{4a\beta^2} \sum C_n M_n + \frac{E\alpha(\Delta t)h^2}{12(1-\mu)}$$

$$(2') \quad M_\theta = \mu M_x + \frac{E\alpha(\Delta t)h^2}{12}$$

and the other formulas remain unchanged.

It should be noted that these formulas are applicable at the edges of the cylinder as well as at points large distances from the edges. Example 4 of Part III illustrates this.

To use Part I of this report

$$F_c = \frac{F_i + F_o}{2}.$$

PART III

ILLUSTRATIVE EXAMPLES

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ILLUSTRATIVE EXAMPLES

Examples of axisymmetrical loading may be found in Section 3 of (1). These adequately demonstrate the general procedure. The examples that follow are set forth to demonstrate the use of the material contained in this report.

Example 1: A cylinder with a uniform thickness of $1/4$ in. has a radius of its middle surface of 4 in. A radial outward shearing force of 100 lbs/in. is applied at one edge of the cylinder, while the other edge is rigidly fixed. The length of the cylinder is 2 in. and Poisson's ratio is equal to 0.3. If the cylinder is subjected to a temperature variation, $F_c = 50x$ measured from the edge with the shearing force, find the circumferential membrane stress at the edge where the shearing force is applied and the axial bending stress at the fixed edge ($\alpha = 1.03 \times 10^{-5}$, $E = 2.65 \times 10^7$).

Solution:

Since $ah = 1$, Eq. ii yields

$$\beta^2 = 1.6523$$

$$\beta = 1.2854$$

$$\text{Thus } y_L = (1.2854)2 = 2.5708$$

The nearest value appearing in the tables of (1) is 2.6, and hence the adjusted value of L is 2.0227.

When the origin of the x coordinate is at the end of the cylinder with the shearing force, the boundary conditions are

$$\begin{array}{ll} \left. \begin{array}{l} 1. \quad M_x = 0 \\ 2. \quad Q = 100 \\ 3. \quad w = 0 \end{array} \right\} & \text{at } y = y_0 = 0 \\ \left. \begin{array}{l} 4. \quad \frac{dw}{dx} = 0 \end{array} \right\} & \text{at } y = y_L = 2.6 \end{array}$$

Formulas 1a, 3a, 6a, and 7a, together with the tables of Section 2 of (1), yield the following set of simultaneous equations:

$$\begin{array}{rclclcl}
 - 2.0000 C_1 & & + 2.0000 C_3 & & = 0 \\
 - 2.0000 C_1 & - 2.0000 C_2 & - 2.0000 C_3 & + 2.0000 C_4 & = 2056.6 \\
 0.038288 C_1 & - 0.063644 C_2 & + 6.9406 C_3 & - 11.537 C_4 & = 6901.3 \\
 - 0.10193 C_1 & + 0.025356 C_2 & - 4.5963 C_3 & - 18.477 C_4 & = 2654.3
 \end{array}$$

with solutions

$$\begin{array}{ll}
 C_1 = 510.38 & C_2 = -2325.7 \\
 C_3 = 510.38 & C_4 = -276.62
 \end{array}$$

From formula 5a,

$$N_{\theta} = 2602.3 \quad \text{at } y = 0$$

Thus formula 12 yields

$$\sigma_{m_{\theta}} = 10409$$

From formula 1a

$$M_x = -304.51 \quad \text{at } y = 2.6$$

Hence from formula 9,

$$\sigma_{\beta_x} = -29233 \quad \text{at outside surface}$$

Example 2: The temperature function above base temperature of a cylindrical shell with an axisymmetrical load is found to be

$$F_c(x) = R_1 e^{-x} - R_2 e^{-bx} + R_3 x + R_4 x^2$$

where the R_i 's and b are constants. What are the equations for the bending moments, radial displacement, $\frac{dw}{dx}$, and the shearing force?

Solution: By the principle of superposition, the radial displacement, w , is the sum of the displacements given by the axisymmetrical load and each of the temperature functions, $R_1 e^{-x}$, $-R_2 e^{-bx}$, and $R_3 x + R_4 x^2$. Similar statements can be made about the other formulas. Hence,

$$M_x = \frac{1}{4a\beta^2} \sum C_n M_n - D \left[\gamma_1 e^{-d_1 y} + b^2 \gamma_2 e^{-d_2 y} + 2a\alpha R_4 \right]$$

$$M_\theta = \mu M_x$$

$$w = \frac{a}{Eh} (\sum C_n M_n + \mu N_x - ap) - \gamma_1 e^{-d_1 y} - \gamma_2 e^{-d_2 y} - a\alpha \left[R_3 \left(\frac{y}{\beta}\right) + R_4 \left(\frac{y}{\beta}\right)^2 \right]$$

$$\frac{dw}{dx} = \frac{a\beta}{Eh} \sum C_n W_n' + \gamma_1 e^{-d_1 y} + b\gamma_2 e^{-d_2 y} - a\alpha \left[R_3 + 2R_4 \left(\frac{y}{\beta}\right) \right]$$

$$Q = \frac{1}{4a\beta} \sum C_n Q_n - D\gamma_1 e^{-d_1 y} - D\gamma_2 e^{-d_2 y}$$

where

$$d_1 = \frac{1}{\beta}$$

$$d_2 = \frac{b}{\beta}$$

$$\gamma_1 = \frac{4a\alpha R_1}{d_1^4 + 4}$$

$$\gamma_2 = \frac{-4a\alpha R_2}{d_2^4 + 4}$$

Example 3: The temperature variations at the inside and outside surfaces, respectively, of a cylindrical shell are given by

$$F_i = 400 e^{-2x}$$

$$F_o = 200 e^{-2x}$$

and the base temperature is 0°F .

(a) What are the equations for w , $\frac{dw}{dx}$, Q and M_θ if the cylinder is restrained at 300°F ? (b) What are these equations if it is restrained at 0°F ?

Solution: $\Delta t = 200 e^{-2x}$

and
$$F_c = \frac{F_i + F_o}{2} = 300 e^{-2x}$$

(a) If the cylinder is restrained at 300°F , two boundary conditions* are

$$\left. \begin{aligned} w &= -300 a\alpha \\ \frac{dw}{dx} &= 0 \end{aligned} \right\} \text{ at } x = 0.$$

Hence, by applying the principle of superposition to the formulas of Parts I and II

$$w = \frac{a}{Eh} (\sum C_n N_n + \mu N_x - a p) - \gamma e^{-dy}$$

$$\frac{dw}{dx} = \frac{a\beta}{Eh} \sum C_n W'_n + 2\gamma e^{-dy}$$

$$Q = \frac{1}{4a\beta} \sum C_n Q_n - 8D\gamma e^{-dy}$$

* Additional information is necessary to set up the other two boundary conditions.

$$M_{\theta} = \mu M_x + \frac{50 E \alpha h^2 e^{-dy}}{3}$$

where

$$M_x = \frac{1}{4a\beta^2} \sum C_n M_n - 4 D \gamma e^{-dy} + \frac{50 E \alpha h^2 e^{-dy}}{3(1-\mu)}$$

(b) If the cylinder is restrained at 0°F , two boundary conditions are

$$\left. \begin{array}{l} w = 0 \\ \frac{dw}{dx} = 0 \end{array} \right\} \text{ at } x = 0$$

The formulas for w , $\frac{dw}{dx}$, Q and M_{θ} are the same as in Part a; however, the different boundary conditions give different solutions for the C_i 's, $i = 1, \dots, 4$. In Parts a and b above

$$d = \frac{2}{\beta}$$

$$\gamma = \frac{1200 \alpha x}{d^4 + 4}$$

Example 4: A long cylindrical shell has a radius of 1 in. and a uniform thickness of 0.1 in. The inside temperature is 300°F and the outside temperature is 200°F . (a) Find w , $\frac{dw}{dx}$, and M_{θ} at one edge of the cylinder. (b) Write the general expression for M_x and M_{θ} . ($\alpha = 1.03 \times 10^{-5}$, $E = 2.65 \times 10^7$, $\mu = 0.3$).

Solution: (a) Eq. ii yields

$$\beta = 4.065$$

F_c is zero since the cylinder is unrestrained and the temperature is constant.

Also $\Delta t = 100$,

thus

$$\frac{E\alpha(\Delta t)h^2}{12(1-\mu)} = 32.49$$

If the origin is taken at the edge in question, then

$$\left. \begin{array}{l} M_x = 0 \\ Q = 0 \end{array} \right\} \text{at } y = 0$$

Formula 1' becomes

$$0 = -2C_1 + 2148 \quad \text{at } y = 0$$

and formula 3a becomes

$$0 = -2C_1 - 2C_2 \quad \text{at } y = 0$$

Hence,

$$C_1 = 1074$$

$$C_2 = -1074$$

By formulas 6a, 7a and 2', respectively,

$$w = -4.053 \times 10^{-4} \text{ in.}$$

$$\frac{dw}{dx} = 3.295 \times 10^{-3}$$

and

$$M_\theta = 22.75$$

at $y = 0$.

(b) From formulas 1' and 2'

$$M_x = 16.25(M_1 - M_2) + 32.49$$

$$M_\theta = 4.875(M_1 - M_2) + 32.49$$

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