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OF SOME DOUBLE-SAMPLE TESTS

by

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1. Introduction. The exact power curves of the double-sample tests for hypotheses on the mean introduced in [1], if a normal population and known standard deviation are assumed, are shown in the accompanying graph. The τ -test procedure given in [1] is shown to be more powerful than one chosen from Bowker and Goode in [2], if the expected number of observations for the two procedures is equated. The τ -test procedure also yields a more powerful test except for extreme alternatives, when the expected number of observations is equated, if the second sample has the same number of observations as the first, as opposed to the second sample's having twice as many observations as the first.

2. Outline of the Testing Procedure. A test of the hypothesis that a lot has a proportion p or less of its items below a value L , against the alternative that more than a proportion p is below L , will be given. The necessary modification to change the test to an upper limit is given in [2]. The lot will be assumed to have a normal distribution and the significance level will be 0.05. The procedure is to take n_1 items from the lot, measure the characteristic X , and accept the lot if

$$\bar{x}_1 - k_a \sigma > L$$

and reject the lot if

$$\bar{x}_1 - k_r \sigma < L,$$

where

$$\bar{x}_1 = \frac{1}{n_1} \sum_{i=1}^{n_1} x_i$$

If neither inequality is satisfied, a second sample of $n - n_1$ additional observations is taken. The lot is now accepted if

$$\bar{x}_\tau - k_\tau \sigma \geq L$$

where

$$\bar{x}_\tau = \frac{1}{n} \sum_{i=1}^n x_i$$

and, if otherwise, the lot is rejected.

The quantities k_a , k_r , and k_τ are obtained from

$$k_a = H_p - a/\sqrt{n_1} ,$$

$$k_r = H_p - r/\sqrt{n_1} , \text{ and}$$

$$k_\tau = H_p - \tau/\sqrt{n} ,$$

where a , r , and τ are the critical constants which determine the procedure, and H_p is defined by

$$p = \frac{1}{\sqrt{2\pi}} \int_{H_p}^{\infty} \exp \left(-\frac{1}{2} t^2 \right) dt .$$

For the τ -test procedure of [1], if the second sample is twice the size of the first, the values of a , r , and τ are $a = -0.0595$, $r = 1.9588$, $\tau = 1.8261$, and $n = 3n_1$. For the τ -test procedure of [1], if the second sample is the same as the first, $a = 0.5415$, $r = 1.7847$, $\tau = 1.9546$, and $n = 2n_1$.

From Bowker and Goode [2], page 201, the test under sample-size letter 0 and AQL = 0.19 per cent was chosen arbitrarily for comparison. The constants are $n_1 = 22$, $n = 66$,

$$k_a = 2.78 = 2.8943 - 0.5361/\sqrt{22} ,$$

$$k_r = 2.52 = 2.8943 - 1.7556/\sqrt{22} , \text{ and}$$

$$k_\tau = 2.64 = 2.8943 - 2.0486/\sqrt{66} ;$$

i.e., a was taken equal to 0.5361, r taken equal to 1.7556, and τ taken equal to 2.0486. The powers of these tests are given by:

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{r+w\sqrt{n_1/n}} \exp\left(-\frac{1}{2}t^2\right) dt$$

$$+ \frac{\sqrt{n}}{2\pi\sqrt{n-n_1}} \int_{-\infty}^{r+w} \int_{r+w\sqrt{n_1/n}}^{a+w\sqrt{n_1/n}} \exp\left[\frac{-n}{2(n-n_1)}\left(u^2 - 2\sqrt{\frac{n_1}{n}}uv + v^2\right)\right] du dv,$$

where

$$w = \sqrt{n}(H_p - H_{p_1}),$$

p_1 is the true proportion of the lot below L , and H_{p_1} is defined analogously to H_p .

The expected sample sizes of these tests are given by

$$E(N) = n - \frac{n-n_1}{\sqrt{2\pi}} \int_{-\infty}^{r+w\sqrt{n_1/n}} \exp\left(-\frac{1}{2}t^2\right) dt - \frac{n-n_1}{\sqrt{2\pi}} \int_{a+w\sqrt{n_1/n}}^{\infty} \exp\left(-\frac{1}{2}t^2\right) dt$$

The expected sample sizes for the three procedures were computed and equated by solving the proportion $w_1^2/w_2^2 = E_1/E_2$ for w_2 in terms of fixed values for w_1 , where w_1 and E_1 are corresponding values of the alternative and the expected number of observations for one procedure, and w_2 and E_2 are the values for a second procedure. The powers were then computed and plotted against w_1 . The accompanying graph shows that the τ -test procedure with the second sample twice the first is everywhere a bit more powerful than the Bowker and Goode test chosen. Also, the τ -test procedure with the second sample the same size as the first is more powerful than the τ -test procedure where the second sample is twice the first, except for $w > 3.9$ and $w < -0.9$, i.e., the extreme alternatives. Since it would probably be less difficult to administer a procedure with equal-size samples, it would appear that more consideration should be given to equal-size samples, at least in the situation considered here. The differences between all these in power are, of course, small, the greatest difference being approximately 0.02.

For the τ -test procedure with equal-size samples at the two stages, the sample size for any given situation may be obtained from

$$n_1 \geq 5.2074 / (H_p - H_{p_1})^2 ,$$

where, if the proportion below L is p the lot is rejected 5 per cent of the time,
and if that proportion is p_1 , the lot is rejected 90 per cent of the time.

REFERENCES

- [1] D. B. Owen, "A Double Sample Test Procedure:", Ann. Math. Stat., Vol. 24, pages 449-457, 1953.
- [2] A. H. Bowker and H. P. Goode, Sampling Inspection by Variables, McGraw-Hill Book Co., 1952.

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