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ENGINEERING OPERATIONS REPORT

TPA Equations and Logic Used
in System Digital Codes

(Project 100)

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INTRODUCTION

This report presents a detailed procedure which explains the method in which the turbine performance curves (generated by the TPA Department) are used in NETAP. This report input will be employed in completing Action Item #5 from the Turbopump Pre-Detail Design Review at ANSC, July 12 through July 15, 1971. (Also see TD 71-21.)

SUMMARY

This report contains the logic, equations, and inputs which describe the TPA in the systems analysis digital computer codes. Included are both the current method and equation set used in NETAP and CDM as well as a revised equation set and logic for the turbine that will be used in the future. These new turbine equations require only a slight modification to the current CDM computer code, and will bring it in line with the revised form of TPA data now being generated by the TPA section. It should be noted that all the TPA data stored in the program is in three-dimensional tables; however, in some cases, only two dimensions are required. For example, the turbine parameter

$\frac{w_r \sqrt{T_{T_i}}}{P_{T_i} A}$ is shown as a function of P_r , $\frac{N}{\sqrt{T_{T_i}}}$ and N ; however, for the present TPA, Reynolds number effects are considered small and the N dimension is not used. The present three-dimension tables in the computer code will be retained in the code to add flexibility.

The turbopump is mathematically modeled using (1) pump work equation, (2) turbine work equation, and (3) the energy balance on the turbopump assembly. These are:

(1) Pump work required is the flow times the isentropic enthalpy difference across the pump divided by the pump efficiency.

$$W_{P \text{ act}} = \frac{w_p \Delta h_{(s)}}{\eta_p}$$

where $W_{P \text{ act}}$ = pump work

w_p = pump flow rate

$\Delta h_{(s)}$ = isentropic enthalpy difference across pump

η_p = pump efficiency

Performance curves of $\eta_p = f(Q/N, N, \frac{HSVP}{N^2})$, $\epsilon = f(Q/N)$, and $\frac{\Delta H}{N^2} = f(Q/N, N, \frac{HSVP}{N^2})$ are used in calculating the pump work where Q = volumetric flow rate, N = speed, ΔH = pump head rise, and ϵ is bearing friction parameter, $HSVP$ = difference between pump inlet pressure and vapor pressure at the same temperature.

(2) Turbine work is found by calculating the isentropic turbine work and multiplying it by a turbine efficiency factor.

$$W_{T \text{ act}} = \eta_t [w_T (h_{11} - h_{12})_{\text{isen}}]$$

where $W_{T \text{ act}}$ = turbine work

η_t = turbine efficiency

w_t = turbine flow rate

h_{11} = turbine inlet enthalpy

$h_{12(s)}$ = turbine exit enthalpy

Performance curves of $\eta_t = f(\frac{U_m}{C_o}, N, N/\sqrt{T_{ti}})$ and $\frac{w_n \sqrt{T_{ti}}}{P_{ti} A_{SI}} = f(P_r, N, N/\sqrt{T_{ti}})$

are used in calculating the turbine work where

U_m = mean blade speed

C_o = isentropic spouting velocity

T_{ti} = turbine inlet temperature

N = speed

W_{ti} = turbine inlet flow

P_{ti} = turbine inlet pressure

A_{SI} = turbine first stage nozzle area

(3) Energy Balance on the Turbopump Assembly is calculated from the rotational energy differences that result between pump and turbine work.

$$\frac{d(I\alpha^2)}{dt} = W_{P \text{ act}} - W_{T \text{ act}}$$

where I = moment of inertia

α = angular velocity

t = time

$W_{P \text{ act}}$ = Actual pump work required

$W_{T \text{ act}}$ = Actual turbine work delivered

Converting to finite-difference form, solving for $W_{P \text{ act}}$, rearranging terms yields the equation

$$W_{P \text{ act}}(i+1) = -W_{P \text{ act}}(i) - \left[\frac{I}{12g \Delta t} \left(\frac{2\pi}{60} \right)^2 \left(\frac{N_{i+1}^2 - N_i^2}{778} \right) \right] + W_{T \text{ act}}(i+1) + W_{T \text{ act}}(i)$$

where i and $i+1$ represent the TPA parameters and time i and new time $i+1$, respectively.

Δt = Time Grid or $t_{i+1} - t_i$

N = TPA speed
I = Moment of Inertia
 $W_{P \text{ act}}$ = Actual pump work required
 $W_{T \text{ act}}$ = Actual turbine work delivered

I. Pump Equations and Logic

The definitions to be used in the following pump equations set are:

Subscripts and superscripts used are:

o indicates stagnation conditions
s indicates isentropic conditions
(1) pump inlet
(2) Pump discharge
v vapor condition of fluid
ta indicated table look up valve

Definition of terms:

h = Enthalpy BTU/lb
s = Entropy BTU/lb °R
P = Pressure #/ft²
T = Temperature °R
 $g_c = \text{Constant } \frac{\text{ft} \cdot \text{lb}}{\text{# sec}^2}$
J = Conversion Factor ft-#/BTU
N = Pump Speed R.P.M.
 η_p = Pump Efficiency

w = Pump Flow Rate
 ρ = Density lb/ft³
H = Head ft
Q = Flow Rate gal/min

1. From the engine program we obtain $T^\circ, P^\circ, h^\circ, w, N_{\text{guess}}, S_1^\circ$.

2. Compute $\text{HSVP} = P_1^\circ - P_v$

Note ρP_v is the vapor pressure of the fluid at temperature T_1°

3. A guess of pump outlet density, ρ_2 , is made by the program.

4. Compute $Q = \frac{w}{\rho_2} \left(\frac{60}{231} \right)$

60/231 is a conversion factor = $\frac{\text{gal-sec}}{\text{in}^3\text{-min}}$

5. Compute Q/N

6. Look up from TPA input Table

$$\left[\frac{\Delta H}{N^2} \right]_{\text{ta}} = f \left(\frac{Q}{N}, \text{HSVP}, N \right)$$

ta = stands for table look up value

7. Compute pump outlet head

$$H_2 = H_1 + \left[\frac{\Delta H}{N^2} \right]_{\text{ta}} N^2$$

8. Compute pump outlet enthalpy

$$h_2 \textcircled{\text{S}} = \frac{H_2}{J}$$

9. Since the NBS properties deck contains no h-s entry the computer code guesses a P_2 and with h_{2s} as well as the NBS properties deck looks up $S_2 = f(P_2, h_{2s})$. A P_2 is assumed and a S_2 is founded, $S_2 = F(P_2, h_{2s})$.
10. This S_2 is compared to S_1° , Step (1) and P_2 is changed until S_2 and S_1° agree within some predetermined value.

NOTE: A special interpolation routine, along with the fine grid NBS properties deck and an extremely tight closure tolerance on S are required to assure the answer is accurate and repeatable.

11. Look up pump theoretical efficiency from TPA input curves

$$\eta'_p = f\left(\frac{Q}{N}, \text{HSVP}, N\right)$$

12. Look up bearing friction parameter

$$\epsilon = f\left(\frac{Q}{N}\right)$$

13. Compute actual turbine efficiency

$$\eta_p = \frac{\eta'_p}{1 + \frac{\epsilon}{2\rho_2(1728)N^2}}$$

14. Compute actual pump discharge enthalpy

$$h_2 = h_1^\circ + \frac{[h_{2s} - h_1^\circ]}{\eta_p}$$

15. Using NBS properties deck, look up actual pump outlet density

$$\rho_2 = f(P_2, h_2)$$

16. Compare ρ_2 Step (15) vs. ρ_2 guess Step (3). Change ρ_2 Guess and redo calculations Step (4) through (16) until the densities agree within some predetermined value.
17. Compute actual pump work $W_{P \text{ act}} = w [h_2 - h_1^o]$
18. The speed guess N Step (1) is closed by comparing $W_{P \text{ act}}$ from step 17 with $W_{P \text{ act}}$ computed by equation

$$W_{P \text{ act}}(i+1) = -W_{P \text{ act}}(i) - \left[\frac{I}{2g \Delta t} \left(\frac{2\pi}{60} \right)^2 \left(\frac{N_{i+1}^2 - N_i^2}{778} \right) \right] + W_{T \text{ act}}(i+1) + W_{T \text{ act}}(i)$$

This equation was shown on Page 2 of this write-up. This closure takes place after the turbine power for time $i+1$ is computed.

II. Turbine Equations and Logic

Below is a simple flow schematic and definitions of terms to be used in this write-up.

Subscripts Used:

- (1) Inlet to Turbine Inlet Line and Turbine Bypass Line
- (2) Turbine Inlet
- (3) Turbine Discharge
- (4) Outlet of Turbine Discharge Line and Bypass Line
- s Indicates isentropic conditions
- o Indicates stagnation conditions
- ta Indicates table look up value

Definition of Terms:

- h = Enthalpy BTU/lb
- S = Entropy BTU/lb^{°R}

P	=	Pressure #/ft ²
T	=	Temperature °R
k	=	Ratio of Specific Heats
R	=	Gas Constant ft-#/lb-°R
g _c	=	Conversion factor ft-lb _f /lb _m - sec ²
C _p	=	Specific Heat at constant pressure BTU/lb°R
J	=	Conversion factor $\frac{\text{ft} \cdot \text{#}}{\text{BTU}}$
C ₁	=	Conversion factor TPA speed (N) to tip speed (V)
U	=	Turbine tip speed
N	=	Turbine speed R.P.M.
A _t	=	Turbine psuedo area for choke
η _T	=	Turbine efficiency
w _T	=	Turbine flow rate lb/sec
P _T	=	Turbine pressure ratio

1. Other parts of the program provide turbine inlet conditions including P₂[°], h₂[°], N, w_{Guess}.

2. From the NBS Para Hydrogen properties deck, look up R₂, k₂ and T₂[°] = f(P₂[°]h₂[°]).

3. A guess is made by the program of turbine outlet pressure, P₃[°].

4. From this, an initial value of P_R is computed.

$$P_R = \frac{P_2^\circ}{P_3^\circ}$$

5. Next, compute $N/\sqrt{T_2^\circ}$
6. Look up from the turbopump curve is made on:

$$\left[\frac{w_T \sqrt{T}}{P A_t} \right]_{ta} = f(P_R, N/\sqrt{T_2^\circ}, N)$$

7. An area is computed using:

$$A_t = \left[\frac{w_T \sqrt{T}}{P} \right]_{ta} \sqrt{\frac{k g_c}{R} \left(\frac{2}{k+1} \right)^{\frac{k+1}{k-1}}}$$

NOTE: A_t here is not the actual area of the turbine but is only a psuedo area for choke.

8. Again, look up R_2 , k_2 , T_2° , S_2° , C_{p_2} , using appropriate properties deck; i.e., Para Ortho, Equilibrium, or some percentage of Ortho to Para Hydrogen.

NOTE: Either the NBS fine grid deck or the generating codes is used for all TPA properties look ups.

$$9. \text{ Compute } P_2^{\circ'} = \frac{w_T \sqrt{T_2^\circ}}{A_t} \sqrt{\frac{k_2 g_c}{R_2} \left(\frac{2}{k_2+1} \right)^{\frac{k_2+1}{k_2-1}}}$$

$$\text{and } P_3^\circ = P_3^\circ / P_R$$

NOTE: This A_t is calculated in Step (7). This gives a method analogous to that shown in Section III below of this report. This method, however, uses parahydrogen properties from the deck as the reference gas instead of those listed on the turbine curves.

10. Compare P_3° vs. P_3° and if they are not equal go back to Step (1) and try a new P_3° until they are within a prescribed tolerance.
11. Compute $T_3^\circ = T_2^\circ \left(\frac{1}{P_R} \right)^{\frac{R_2}{C_{p2} J}}$. This is only used to get a better first guess for h_3° .
12. Use T_3° and P_3° to look up S_3°, h_3° from appropriate NBS properties look up.
13. Compare S_3° vs. S_2° and if not equal, guess new h_3° and look up $S_3^\circ = f(P_3^\circ, h_3^\circ)$. Continue until $S_3^\circ = S_2^\circ$.
14. $\Delta h_{\textcircled{S}} = h_2^\circ - h_3^\circ$
15. Compute $C_o = \sqrt{2g_c J \Delta h_{\textcircled{S}}}$
16. Compute $U/C_o = \frac{C_1 N}{C_o}$
17. Look up $\eta_t = f\left(N/\sqrt{T_2^\circ}, N, U/C_o\right)$
18. $\Delta h = \eta_t \Delta h_{\textcircled{S}}$
19. $h_3^\circ = h_2^\circ - \Delta h$
20. Look up $S_3^\circ, T_3^\circ, R_3^\circ$, etc. = $f(P_3^\circ, h_3^\circ)$ from appropriate NBS properties deck.
21. Use standard $\Delta P, \Delta h$ relationships on the turbine discharge line to compute fluid conditions at 4.

22. Compare P_4° (Step 21) vs. P_4° computed from the bypass line.

If they differ, go back to Step 1 and guess a new w . Continue iteration until these two pressures are equal.

23. Compute turbine power output $W_T = w_T \Delta h$

24. This work is compared to that required by the pump (including acceleration), and TPA speed (N) is changed to effect a closure.

This completes the main program logic. There are other program options for NERVA, however, such as fixing turbine bypass flow percentage and then computing the turbine area required to obtain this. Hydrogen properties across the turbine can be obtained from para, ortho, equilibrium, or any input fixed percentage of ortho to para hydrogen. This latter option has been used for the hydrogen conversion analysis.

Future TPA curves will be supplied with new formats, and this will require some modification to the computer code. This change can be easily made and the new program logic is as follows:

III. New Turbine Equations

(Refer to previous schematic and name list.)

1. From other parts of the program we get P_2° , h_2° , N , w_{guess} , A_t .
2. Using NBS properties (appropriate deck, i.e., Para, Ortho, Equilibrium, or some percentage of Para to Ortho Hydrogen), look up R_2 , k_2 , S_2° ,
 $T_2^\circ = f(P_2^\circ, h_2^\circ)$.

3. 3. Guess P_3°
4. Compute $P_R = \left(\frac{P_2^\circ}{P_3^\circ} \right)$
5. Compute $\theta_c = \left(\frac{k_2}{k_2+1} \right) R_2 T_2^\circ (6.19497 \times 10^{-5})$
6. Compute $N/\sqrt{\theta}$
7. Look up $\left[\frac{\alpha w \sqrt{\theta}}{A_t \beta} \right]_{ta} = f(P_R, N/\sqrt{\theta}, N)$ from TPA curves input
8. Compute $\alpha_c = \frac{1}{k_2} \left(\frac{k_2 + 1}{2} \right)^{\frac{k}{k-1}} (.528282)$
9. Compute $\beta_c = P_2^\circ (.068046)$

NOTE: The constants shown in equations 8, 9, and 5 above come from the parameters for air at standard conditions, i.e.

$$R = 53.35045 \frac{\text{ft} \cdot \#}{\text{lb}^\circ \text{R}}$$

$$k = 1.4$$

$$P = 14.696 \text{ #/in}^2$$

$$T = 518.688^\circ \text{R}$$

$$10. \text{ Compute } \beta_c^{-1} = \frac{w \alpha_c \sqrt{\theta_c}}{\left[\frac{\alpha w \sqrt{\theta}}{A_t \beta} \right]_{TA} A_t}$$

11. Check β_c^1 vs. β_c computed in Step (9) and change P_o^3 Guess

Step (3) until a predetermined closure is obtained.

12. Guess h_3° and look up $S_3 \textcircled{S}^\circ = f(P_3^\circ, h_3 \textcircled{S}^\circ)$ from appropriate NBS properties deck.

13. Compare $S_3 \textcircled{S}^\circ$ vs. S_1° , and change $h_3 \textcircled{S}^\circ$ until they are equal.

14. Compute $\Delta h \textcircled{S} = (h_2^\circ - h_3 \textcircled{S}^\circ)$

15. Compute $C_o = \sqrt{2g_c J \Delta h \textcircled{S}}$

16. Compute $U/C_o = \frac{C_1(N)}{C_o}$

17. Look up $\eta_t = f\left(\frac{U}{C_o}, N, N/\sqrt{\theta}\right)$ from TPA curves

18. $\Delta h = \eta_t \Delta h_s$

19. $h_3^\circ = h_2^\circ - \Delta h$

20. Look up $S_3^\circ, T_3^\circ, R_3^\circ$, etc. = $f(P_3^\circ, h_3^\circ)$ from appropriate NBS properties deck.

21. Use standard $\Delta P, \Delta h$, relationships on the turbine discharge line to compute fluid conditions at 4.

22. Compare P_4° , Step 21, vs. P_4° computed from bypass line. If these are not equal, go back to Step (1) and change w_{guess} until the closure tolerance is met.

23. Compute turbine power output $W_T = w_T \Delta h$.

24. This work is compared to that required by the pump (including acceleration) and TPA speed (N) is changed to effect a closure.