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CALCULATION OF
CONTROL ROD WORTH
IN APPR-1

by

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For The Atomic Energy Commission

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SUMMARY

Seven control rods will be used in the APPR-1 instead of five as originally planned. Critical experiments evaluating the rod worth for the APPR-1 have been performed at ORNL and calculations have been carried out by ALCO interpreting these experimental results. The calculations indicate that the capability of any four of the original five rods to shut down the cold reactor at mid-life is marginal. The calculations are summarized in this memorandum.

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INTRODUCTION:

With the available data from the APPR-1 Critical Experiment, the control rod requirements for the APPR-1 have been re-evaluated.

Calculations have been made using the cold clean experimental critical mass of 10.4 Kg U-235 as a basis (1). Two general methods were used to adjust two group theory to fit the critical experiment. One method was to adjust the core age, and the other was to adjust the absorptions in the stainless steel. Preliminary investigations of heterogeneity in the critical assembly indicate that therein lies the reason for the critical mass being higher than predicted (2)(3)(4).

Since it was necessary to make a decision on the number of control rods required in the APPR-1 before heterogeneity effects could be investigated experimentally and evaluated, it was necessary to base the calculations on the critical mass reported in (1).

Experiments planned at ORNL should permit determination of the absolute self-shielding in the critical experiment and the APPR-1. It will then be possible to determine the initial loading with more confidence than by the present methods*. Adjusting the age or stainless steel absorptions will predict a higher initial loading than that actually required since the APPR-1 is more homogeneous than the clean critical assembly. It was believed, however, that a good evaluation could be made of the control rod worth obtained from the critical experiment and the control rod requirements as determined by the two methods adjusted to fit the critical experiment.

All calculations were based on the straight two group theory. The reactivity as a function of core life was based on a uniform burnup of 7.5 Kg of U-235.

* In addition programming on an IBM-650 machine of a ten region two group problem should be available to investigate the effects of non-uniform burnout.

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I. ADJUSTING TWO GROUP THEORY TO FIT THE CRITICAL EXPERIMENT:

A critical mass of 10.4 Kg U-235 was reported for the cold clean critical assembly (1). Two methods were used to fit the experiment using a two group, two region model. One method commonly used was by adjusting the core age (4). There will be poor agreement between calculated and experimental fluxes and probably an over estimate of the hot critical mass. The other method was by adjusting the absorptions in the stainless steel. This method probably gives a truer indication of hot to cold changes. A self-shielding factor within the fuel element was calculated using $F = 1 - .321 \Sigma_a t$ (2) where t is the uranium foil thickness and Σ_a is its thermal absorption cross section. It was assumed that resonance escape was unity and that there was no fast multiplication.

A. Adjusted Age Method:

Originally a core age of 36 cm^2 was used to predict a mass of 8.7 Kg U-235 in a modified two group model (3), while an age of 40 cm^2 predicted a mass of 9.0 Kg U-235 in a straight two group model (4). Ages were adjusted to fit the results of three critical assemblies. An age of 45 cm^2 was found for a stainless steel fraction of 0.104 and an age of 50 cm^2 was found at fractions of 0.184 and 0.200. An age of 50 cm^2 was subsequently used in the APPR-1 calculations by the adjusted age method. Group constants are listed in Table I evaluated from cross-sections listed in Table III.

B. Adjusted Σ_a^{ss} Method:

In the adjusted stainless steel method, a core age of 36 cm^2 (believed to be best estimate) was used, and the absorptions were increased in the stainless steel. An effective cross-section 1.289 times that used in the adjusted age method ($\Sigma_a^{ss} = .2185 \text{ cm}^{-1}$) was needed to fit the experiment. This factor was subsequently used in the APPR-1 calculations by the adjusted stainless steel method. Group con-

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stants are listed in Table I. These were evaluated from the data listed in Table III.

TABLE I
TWO GROUP TWO REGION CONSTANTS AT 68°F

		<u>Core</u>		<u>Reflector</u>
		<u>Age Method</u>	<u>Σ_a^{ss} Method</u>	
L^2	cm^2	1.0623	0.9934	8.12
τ	cm^2	50	36	31.4
Σ_s	cm^{-1}	0.1694	0.1812	0.01955
Σ_f	cm^{-1}	0.0242	0.0336	0.0436
D_s	cm	0.1800	0.1800	0.1588
D_f	cm	1.21	1.21	1.37
K_{∞}^{th}		1.3512	1.2536	
K_g	U-235	10.4	10.4	
$s_r(\infty)$	cm		6.7	
$s_z(t)$	cm		6.5	
B^2	cm^{-2}		0.00683	

where "s" and "f" refer to slow and fast groups.

C. Hot to Cold Change

Microscopic cross-sections for uranium were evaluated at 68°F and 450°F using BNL-325. All other absorption cross sections were evaluated at 68°F using BNL 325 and corrected to 450°F assuming a $\frac{1}{v}$ dependence and a density correction on the water. The age in the core and reflector at 450°F was computed using the Marshak equation (4).

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TABLE II
TWO GROUP TWO REGION CONSTANTS AT 450° F

		<u>Core</u>		<u>Reflector</u>
		<u>Age Method</u>	<u>Σ_a^{ss} Method</u>	
L^2	cm ²	1.8030	1.7602	19.173
τ	cm ²	68	50	45.7
Σ_s	cm ⁻¹	0.1424	0.1457	0.01234
Σ_f	cm ⁻¹	0.0206	0.0280	0.0365
D_s	cm	0.2568	0.2564	0.2366
D_f	cm	1.40	1.40	1.67
K_{∞}^{th}		1.4408	1.3263	-
Kg	U-235	12.55	12.05	-
$s_r(\infty)$	cm		8.3	
$s_z(t)$	cm		8.0	
B^2	cm ⁻²		0.00625	

where "s" and "f" refer to slow and fast groups.

The adjusted age method gives a hot to cold change of 2.15 Kg U-235, and the adjusted stainless steel method gives a hot to cold change of 1.65 Kg U-235.

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II. CALCULATION OF MAXIMUM REACTIVITY IN APPR-1:

The two group equation for an equivalent bare reactor was used in all the calculations.

$$K_{\text{eff}} = \frac{K_{\infty}^{\text{th}}}{(1 + L^2 B^2)(1 + \tau_B^2)}$$

The geometric buckling, both hot and cold, was taken from the two group, two region calculations for the critical experiment. The buckling was taken to be independent of the loading. Some experimental verification of this is presented in (5). The macroscopic cross-sections as well as other pertinent data are tabulated in Table III.

TABLE III
MACROSCOPIC CROSS SECTIONS, AGE $\neq B^2$

	<u>68° F</u>	<u>450° F</u>
$\Sigma_a^u \text{ cm}^{-1}$	26.78	19.42
$\Sigma_a^{ss} \text{ cm}^{-1}$ (age method)	0.2185	0.1664
$\Sigma_a^{ss} \text{ cm}^{-1}$ (stainless steel method)	0.2817	0.2145
$\Sigma_a^{H_2O} \text{ cm}^{-1}$	0.01955	0.01234
$\Sigma_{tr}^u \text{ cm}^{-1}$	0.4775	0.4775
$\Sigma_{tr}^{ss} \text{ cm}^{-1}$	0.8524	0.8524
$\Sigma_{tr}^{H_2O} \text{ cm}^{-1}$	2.0997	1.4089
$\tau \text{ cm}^2$ (age method)	50	68
$\tau \text{ cm}^2$ (stainless steel method)	36	50
$B^2 \text{ cm}^{-2}$	0.00683	0.00625

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The self-shielding within the fuel element was calculated at each loading using the self-shielding factor (2), $F = 1 - 0.321 \Sigma_a t$ where t is the meat thickness and Σ_a is the absorption cross-section of the meat. In the APPR-1 the meat consists of UO_2 , stainless steel, B_4C , and fission products. The percent of each of these will depend on the initial loading and the past history of the reactor. Since the meat is approximately 50% of the metal fraction, the meat fraction is approximately 10% of the core.

A breakdown of the metal fraction in the core is :

Volume fraction stainless steel in clad, end plates, etc.	0.099
Volume fraction stainless steel in meat	0.087
Volume fraction UO_2	0.0175
Volume fraction B_4C	<u>0.0005</u>
Total	0.204

The volume fraction of stainless steel in the core was kept at 0.186 throughout the core lifetime. The initial loading of the reactor includes uranium for: burnup (7.5 Kg U-235), cold to hot change, override of Xe and Sm, fission products, and residual boron at the end of life, as well as boron to give $K_{eff} = 1.02$ at $450^\circ F$.

A. Xenon and Samarium

Equilibrium Xe and Sm were based on equations in (6). The basic data used were:

Xe cross-section at $450^\circ F$ averaged over a Maxwellian = $2.59 \times 10^{-18} \text{ cm}^2$.

Fractional yield of 135 chain = 0.065

Decay constant of Xe-135 = $2.09 \times 10^{-5} \text{ sec}^{-1}$

Fractional yield of Sm-149 = 0.014

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B. Fission Products

Fission products except xenon and samarium were assumed to have a cross section of 50 barns/fission at 68°F (38 barns/fission at 450°F). The fission product cross section in the reactor after 15 MWYR is 0.00403 cm^{-1} at 450°F.

The equation governing the fission product buildup is

$$\Sigma_{fp}^{(Y)} = 0.00403 \times \frac{Y}{15}$$

where Y = MWYR of core life.

C. Burnout Poison:

The amount of boron added to balance the uranium added for burnup was calculated as a $\Sigma_{\text{reactor}}^{\text{B10}}$. At constant power the uranium will burn out at a uniform rate such that

$$\Sigma_a^u(Y) = \Sigma_a^u(0) (1 - kY)$$

where Y = megawatt years and k = constant based on a burnup of 500 grams uranium per megawatt year.

The boron burns out at a faster rate and is described by

$$\Sigma_a^{\text{B10}}(Y) = \Sigma_a^{\text{B10}}(0) \left(1 - kY \frac{\sigma_a^{\text{B10}}}{\sigma_a^u}\right)$$

where $\frac{\sigma_a^{\text{B10}}}{\sigma_a^u} = \frac{2707}{433.3} = 6.25 \text{ (at 450°F)}$

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D. Variation of K_{eff} During Lifetime

The reactor composition was determined so that $K_{eff} = 1.02$ at 450°F initially and at the end of the cycle for operational flexibility. A plot of the variation of K_{eff} during the lifetime is shown in Figure 1 for both the adjusted age and adjusted stainless steel methods. The initial loadings are 23.75 Kg U-235 for the age method and 22.5 Kg U-235 for the stainless steel method. Uranium and boron were burnout uniformly hot for 15 MWYR. K_{eff} was then evaluated at 68°F without xenon. The peak reactivities occurred at 7 MWYR and are given in Table IV.

TABLE IV

CORE LOADING AND EXCESS REACTIVITY COLD AT 7 MWYR

	<u>Adjusted Age</u>	<u>Adjusted SS</u>
Kg U-235	20.25	19.00
Excess reactivity	0.130	0.135

These are the maximum reactivities that must be overridden by the control rods. These compare with an excess reactivity of 0.138 based on a clean critical mass of 7.26 Kg of U-235 (2).

III. CONTROL ROD EVALUATION:

The critical experiments of prime interest for the shutdown of the APPR-1 are those with the center line and three eccentric rods.* Critical loadings have been found with these rods fully inserted in a clean and a slightly poisoned reactor. These loadings are listed in Table V. For comparison the U-235 content at 7 MWYR core life (point of maximum reactivity) is given in Table VI. The actual loading in the 7 MWYR reactor with the four rods fully inserted is also given in Table VI.

*Eccentric rods at ORNL position (2)

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Comparison of Tables V and VI indicate that the rod measurements were made at loadings slightly below those that will exist in the reactor at 7 MWYR. The rod worths at 7 MWYR were found by extrapolating the two existing measurements to the midlife loading. The rod worths in Kg U-235 as measured and as extrapolated to 7 MWYR core life are listed in Table VII.

TABLE V

ORNL CRITICAL EXPERIMENTS WITH CENTER LINE AND THREE ECCENTRIC RODS

Actual loading in the reactor (with four rods in)		Uniform loading	
Kg U-235	Gram Boron	Kg U-235	Gram Boron
15.29	0	16.78	0
16.73	31.15	18.36	34.17

TABLE VI

U-235 CONTENT AT 7 MWYR CORE LIFE WITH AND WITHOUT FOUR CONTROL RODS

	Actual loading in the reactor (7 MWYR with four rods in)	Uniform Loading
	Kg U-235	Kg U-235
Age Method	18.45	20.25
Stainless S. Method	17.31	19.00

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TABLE VII

WORTH OF FOUR RODS IN KG U-235 MEASURED AND AT 7 MWYR LOADING

	Actual loading		Uniform loading	
	Kg U-235 in core	Rod Worth Kg U-235	Kg U-235 in core	Rod Worth Kg U-235
Measured	15.29	4.89	16.78	6.38
Measured	16.73	4.97	18.36	6.60
7 MWYR-SS	17.31	5.00	19.00	6.69
7 MWYR- Age	18.45	5.06	20.25	6.86

A. Calculation of Rod Worth

Several methods are available for interpretation of rod worth in the critical experiment. In addition the worth of the rods may be based on two different reactor configurations, which are shown below.

3 eccentric & center line
rods full in

x Kg U-235
y Gm Boron

$$K_{eff} = 1$$

Critical Experiment Configuration

(a)

rods out
uniform loading

$\frac{45}{41}$ x Kg U-235

$\frac{45}{41}$ y Gm Boron

$$K_{eff} > 1$$

Equivalent uniform reactor

(b)

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Each configuration may be interpreted in terms of excess reactivity or an equivalent uniform poison. The reactivity the rods will control in (a) is found by calculation of K_{eff} with the loading actually in the reactor and the rods considered not to be present. In (b) K_{eff} is calculated for the uniform loading. The equation used to find the equivalent uniform poison is:

$$K_{eff} = \frac{\eta}{(1 + L^2 B^2)(1 + \tau B^2)} \cdot \frac{\sum_a^U}{\sum_a^U + \sum_a^{SS} + \sum_a^{FP} + \sum_a^B + \sum_a^{H_0} + \sum_a^{CR}}$$

In configuration (a) the $\sum_a^{CR}$ necessary for $K_{eff} = 1$ corresponds to the absorptions in the rod. In configuration (b) the uniform poison actually includes the effect of displacing fuel from the core.

B. Shutdown of the APPR-1 at 7 MWYR:

From the calculations in Section II, the composition of the APPR-1 at midlife is established and the reactivity known for the cold reactor. The calculation in Section II treats only the uniform reactor.

The shutdown of this reactor can be checked by use of the uniform reactor (b) or the reactor with 41/45 of its fuel, boron, and fission products displaced, configuration (a). The reactivity of the rods in configuration (a) or (b) can be compared with the cold reactor at midlife. The alternate approach is to use the equivalent uniform poison found for configuration (a) or (b) and insert this into the APPR-1 at 7 MWYR. A shutdown reactivity can then be calculated.

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The reactivity values and homogeneous poisons for the control rods are considered invariant in the transformation from the critical experiment to the APPR-1 at midlife.

When rod worths are evaluated at the same loading in the critical experiment as is in the 7 MWYR core, the reactivity and equivalent poison methods give approximately the same shutdown reactivity. For this reason, only the reactivity comparison will be reported here .

The rod worth in reactivity versus core loading is given in Tables VIII and IX, as well as the reactivity comparison for both methods. The critical masses are for the clean, boron poisoned and midlife cores in configurations (a) and (b).

TABLE VIII

ADJUSTED AGE METHOD

Actual loading in the reactor (with four rods in)			Uniform Loading		
Kg U-235 in core	Rod Worth Kg U-235	ρ_{rod}	Kg U-235 in core	Rod Worth Kg U-235	ρ_{rod}
15.29	4.89	0.1141	16.78	6.38	0.1357
16.73	4.97	0.1055	18.36	6.60	0.1253
18.45	5.06	0.1034	20.25	6.86	0.1213

TABLE IX

ADJUSTED STAINLESS STEEL METHOD

Actual loading in reactor (with four rods in)			Uniform Loading		
Kg U-235 in core	Rod Worth Kg U-235	ρ_{rod}	Kg U-235 in core	Rod Worth Kg U-235	ρ_{rod}
15.29	4.89	0.1278	16.78	6.38	0.1521
16.73	4.97	0.1239	18.36	6.60	0.1461
17.31	5.00	0.1202	19.00	6.69	0.1416

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C. Results:

The excess reactivity of the cold midlife reactor and the reactivity the rods will override are given in Table X. Both methods (age, and stainless steel) and both configurations are given.

TABLE X

	<u>Adjusted Age</u>		<u>Adjusted Stainless Steel</u>	
	(a)	(b)	(a)	(b)
ρ_{excess}	0.1137	0.1300	0.1159	0.1353
ρ_{rods}	0.1034	0.1213	0.1202	0.1416
$\Delta\rho_{\text{rods in}}$	+ 0.0103	+ 0.0087	- 0.0043	- 0.0063

At 7 MWYR core life the adjusted age method gave a positive reactivity of $\sim 1\%$ with 3 eccentric and the central rods in the core at 68 deg. °F. The adjusted stainless steel method gave a negative reactivity of $\sim -0.5\%$. From these calculations the ability of the reactor to be shut down cold at 7 MWYR life with 80% of the rods is marginal. The uncertainties of adding additional uranium to extend the core life to 15 MWYR to take care of non-uniform burn-up, and of predicting the initial loading, give a questionable ability of 4 rods to control the cold reactor at midlife. For this reason seven rods are to be installed in the APPR-1.

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FIGURE 1

K_{EFF} VS. CORE LIFE

UNIFORM BURNOUT

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CORE LIFE MIN

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