COMPUTER SIMULATION OF ANTIPROTON RF STACKING IN THE
THE PRECOOLER RING

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\$ 1 Introduction

It has been recently proposed a scheme [1] to produce a high-intensity antiproton beam in a pre cooler ring with functions of both stochastic cooling and accumulating. According to this scheme 13 batches from $\bar{p}$ -target are stacked and precooled on the stacking orbit every Main Ring cycle. After that the precooled beam is transported in a second strage ring (the Accumulator) and cooled for several hours together with previous $\bar{P}$ batches. As alternative it has also been proposed to decelerate each pre cooler pulse down to 200 MeV at which energy it can be transferred in a modified Electron Cooling Ring for storage and further cooling.

The sequence of stacking operations in the Pre cooler consists of (1) injecting a $\bar{P}$ beam pulse onto the injection orbit, capturing it by standing-by RF buckets, rotating the bunch in the stationary RF buckets, and reducing the bucket height so that the stationary bucket will surround tightly the bunch, (2) decelerating to the stacking orbit and (3) finally turning off the RF adiabatically, allowing the beam to debunch. This sequence is repeated 13 times to build up a complete stack every Main Ring cycle. The above steps require fairly complicated manipulations of RF system parameters. The 13 $\bar{P}$ -batches are separated by 100 msec from each other for target heating considerations.

In the present study, the above sequence of stacking is investigated by numerical simulation.

In section 2-a the mathematical equations used in the present numerical experiments are exposed. The general character of the present stacking method is described in 2-b whereas the optimization of the r.f. parameters to minimize beam dilution is studied in 2-c. Finally the method and results of the numerical simulation of the r.f. stacking in the Precooler ring are presented in § 3. The stacking efficiency and final energy spread for this Precooler ring are given.

Parameters of the antiproton beam from the target which has been used in the present study are shown in Table 1. Machine parameters of the Precooler ring are listed in Table 2.

Table 1

Kinetic energy of $\overline{P}$	E	4.5	GeV
Momentum spread	$\Delta P/P_0$	1 %	full (flat distribution)
Bunch length (r.m.s.)	σ_z	0.21	m
Longitudinal phase area (width $\pm 2\sigma_z$, 95% of beam)	S	0.15	eV-s

table 2

Average radius	R	75	m
Transition energy	γ_T	10	
Harmonic number	h	84	
Number of injected $\overline{P}$	n	13	
pulses every M.R. cycle			

\$ 2 RF Stacking

2-a Synchrotron Oscillations

The theory of the longitudinal phase motion, which describes the energy and phase oscillation that occur when a particle passes repetitively through one or more "accelerating cavity" situated at localized points around the accelerator ring, is well known. Since these oscillations normally are of a relatively low frequency, it is often legitimate as well as convenient to analyze them theoretically with differential equations derived by spreading the accelerating field uniformly around the orbit. In reality the energy changes experienced by a particle are better represented by step equations and depend on the sine of the electrical phase angle ϕ at which the particle traverses the cavity. The corresponding equations of motion are therefore non-linear and discrete.

We consider here both the case of a stationary bucket (

stable phase angle, $\phi_s = \pi$) and the case of a moving bucket. To obtain the working transformation, we consider a short RF cavity system operating at a harmonic number h and an angular frequency ω_{rf} . The quantities denoted E_n and ϕ_n are respectively the energy and the electrical phase angle with which a particle enters the cavity at the time of transit.

Then the non-linear transformations may be written as follows :

$$E_{n+1} = E_n + eV \cdot \sin \phi_n \quad (1-1)$$

$$\phi_{n+1} = \phi_n + \omega_{rf} \cdot \frac{2\pi}{\omega(E_{n+1})} \quad (1-2)$$

where eV is the energy gain per turn and the revolution period $2\pi/\omega(E_{n+1})$ is expanded in the form :

$$\frac{2\pi}{\omega(E_{n+1})} = \frac{C_0 \left(1 + \alpha(E_{n+1} - E_0)/\beta_0^2 E_0 \right)}{C \sqrt{1 - (m_0 c^2/E_{n+1})^2}} \quad (2)$$

with C_0 -- length of the closed orbit corresponding to the injection energy E ,

β_0 -- relativistic β of the injected particle,

α -- momentum compaction factor,

c -- light velocity,

$m_0 c^2$ -- rest mass of antiproton.

The synchronous particle can be defined by the equations :

$$\omega_{rf} = h \cdot \omega((E_s)_{n+1}) \quad (3-1)$$

$$(E_s)_{n+1} = (E_s)_n + e V \sin \phi_s \quad (3-2)$$

where E_s is the energy of the synchronous particle at the $(n + 1)$ - th transit. Therefore $E = \text{const}$ and $\phi_s = \pi$, for the case of a stationary bucket, requires $\omega_{rf} = \text{const}$. On the other hand ω_{rf} in the case of a moving bucket must be modulated with time.

The solution of the system Eqs. (1-1) and (1-2) with constant V and ω_{rf} is well known. But in our case we need to change V and ω_{rf} to perform the three steps we have mentioned before, of capturing, stacking, and debunching. The r.f. manipulation involved is therefore very important and it is not possible to have an analytical analysis of them. Our goal is to simulate the flow in the three dimensional phase space (E, ϕ, t) of test particles which follow the transformation Eqs. (1-1) and (1-2), as seen in Fig. 1.

Fig. 1

2-b Stacking Process

The stacking process consists of the following six phases :

1. RF Capture. The injected $\bar{P}$ particles are captured by standing by RF voltage at a constant frequency harmonically related to the revolution frequency of particles with the kinetic energy of 4.5 GeV. This creates a region of stable phase oscillations known as a " stationary bucket ", as seen in Fig.(2-b-1).
2. Bunch Rotation. After a quarter of a phase oscillation , the bunch is rotated by 90 degrees as seen in Fig.(2-b-2).
3. Rapid Suppress of Bucket Height. The bucket height is rapidly reduced so that the stationary bucket will surround tightly the bunch as seen in Fig.(2-b-3).
4. Alteration. An adiabatic alteration of the bucket parameters is necessary in order to convert the " stationary bucket " into a " moving bucket " capable of changing the mean energy of the particles trapped inside as seen in Fig. (2-b-4).
5. Deceleration. A period of deceleration follows, employing a moving bucket of constant area and of constant stable phase angle ϕ_s , as seen in Fig.(2-b-5).
6. Adiabatic Debunching. The RF is turned off adiabatically after have been transformed again into a stationary bucket of the same area from the moving bucket as seen in Fig.(2-b-6).

This series is repeated 13 times to build up a " stack at the top ", in which successive pulses are decelerated to the same

stacking orbit corresponding to the same energy E_t , at which the RF voltage is adiabatically switched off.

Process 1 through Process 6, except for Process 3, take place adiabatically. Process 3 must happen fast with the purpose to minimize particle losses and bunch dilution. Moreover the minimum moving bucket area is required to reduce the effect on the stack width.

The detailed behavior of the particle distribution in the phase plane (ϕ , E) after Process 4 is governed by two basic features. The first important feature arises from the presence of the unstable fixed point of the moving bucket. During Process 5 a small fraction of a moving bunch is scattered outside the moving bucket in the vicinity of the unstable fixed point. This effect is caused by the fact that the motion in proximity of the unstable fixed point is non-adiabatic.

Secondly, as a consequence of Liouville's theorem, the phase area transported downwards in energy by a decelerating RF bucket must be accompanied by the upwards transport of an equal phase area in the region outside the bucket, a property known as phase displacement which is seen in Fig.(2-b-7). However there is really a dilution of the beam area due to the highly non-linear motion in this region. The phenomenon is the most remarkable when the bucket is moving through the edge of the stack. This dilution is cumulative as more pulses enter the stack. At the end of the

stacking cycle one has attained a final density distribution in E which has a peak at E_t approaching the original bunch density, and a steady fall-off for higher values of E as seen in Fig.(2-b-8).

2-c Optimization of RF Parameters for High Stacking Efficiency and Small Momentum Dispersion

One can easily assume that neglecting the dilution effect, the final energy spread of a stacked beam ΔE is given by $n \cdot \delta E$, where n is the number of stacked pulses and $(2\pi \cdot \delta E)$ is the phase area of a moving bucket. Therefore to make the final energy spread as narrow as possible, we must choose the minimum bucket in Process 3 surrounding the rotated $\overline{P}$ -bunch. From these circumstances we shall assume a reduced stationary bucket and a moving bucket with the same phase area of 0.15 eV-s.

Taking dilution into account, one can define stacking efficiency η with respect to an idealized stack of n pulses obtained by perfect adiabatic debunching, as the ratio of the number of particles actually obtained with n pulses inside the ideal energy width $n \cdot \delta E$ to the number that would have been obtained for the perfect stack i.e. the total number of particles inside the total stack.

The fundamental effect which limit this stacking efficiency are miss-matching of a bunch into the RF bucket through Process 1 to 4, spill-out of particles during Process 5 which results from imperfect adiabaticity near the unstable fixed point, and dilution

in the stacked region. Now defining $f(E)$ as the normalized final distribution in Energy and η_0 as the capture efficiency by RF bucket in Process 4, and ϵ is the ratio of spilling out of the moving bucket through Process 5, the stacking efficiency η can

be written as follows,

$$\eta = \frac{\int_{E_{ideal}}^{E_{ideal} + n \cdot \delta E} n \cdot (1 - \epsilon) \cdot \eta_0 \cdot N_0 \cdot f(E) dE}{n \cdot N_0}$$

$$= (1 - \epsilon) \cdot \eta_0 \int_{E_{ideal}}^{E_{ideal} + n \cdot \delta E} f(E) dE \quad (4)$$

where n = number of stack pulses,

N_0 = particle number in an injected pulse,

$E_{ideal} = E_t - \delta E/2$,

and $f(E) \neq 0 \quad (E)_{min} \leq E \leq (E)_{min} + \Delta E$

(E_{min} is the minimum energy in the final stacked beam,

ΔE is the real energy spread)

Three above mentioned effects on the stacking efficiency are well represented in Eq.(4). Thus we must optimize r.f. parameters of the stationary or moving buckets to increase the stacking efficiency η as possible.

The first effect may be measured as the capture efficiency η_0 by the moving bucket in Process 4. The capture efficiency results from the geometrical relation between the extension, in the phase space, of a rotated bunch and the phase area of the reduced stationary bucket and of the subsequent moving bucket.

The geometrical extension, in the phase space, of a rotated bunch is dependent on the choice of r.f. parameters, mainly the voltage V_{r0} , during Process 1. Therefore it is interesting to show the capture efficiency η_0 by the same moving bucket area as the function of the rotation voltage V_{r0} . For this purpose the behavior of a injected bunch, which consists of 10^4 test particles with the gaussian phase spread $\pm 3\sigma$ ($\sigma = 13^\circ$) and the uniform momentum spread $\Delta P/P_0 = \pm 0.5\%$, is simulated for different rotation voltages through Process 1 to 4. The r.f. parameters of the stationary bucket in Process 1, the reduced stationary bucket in Process 2 and the moving bucket in Process 3 are shown in Table 3. Fig.(2-c-1) summarizes simulation results and shows that the maximum capture efficiency η_0 of $\pm 2\sigma$ beam is 96.1% when the rotation voltage V_{r0} is 400 KV. This result can be well understood by inspecting Fig. (2-c-2) which compares the beam

phase areas with the reduced stationary bucket.

The spill-out of particles in the vicinity of the unstable fixed point cannot be avoided because the variation of the RF frequency ω_{rf} is done within a short period of time. But this effect is really negligible. According to our results, the loss rate ϵ is less than 1% during Process 5.

Table 3

		Stational bucket in Process 1	Reduced bucket in Process 2	Moving bucket in Process 3
RF peak voltage	V (KV)	300--650	19	33
RF harmonic number	h	84	84	84
Transition energy	γ_T	10	10	10
Synchronous phase	ϕ_s	180	180	180-7
Bucket area	S (eVs)	.61--.90	.154	.154
Bucket height	H (MeV)	25.0-36.8	6.3	9.57
Phase oscill. fre.	f (KHz)	2.36-3.34	.595	.777-.788

The third effect also cannot be avoided, because there are regions with different particle density in the stack. It is

obvious that the final momentum spread of a stacked beam with all the dilutions taken into account should strongly depend on r.f. parameters of the moving bucket during Process 5. Therefore we must optimize r.f. parameters of the moving bucket in order to minimize the dilution caused to the stack. In the Appendix we have investigated in detail the beam dilution occurring when the phase region occupied by the beam is traversed by a moving bucket. The obvious result is that the effect is the smallest for the most flattened moving bucket. But it takes very long time for stacking by such a flat moving bucket. Since we require that each pulse should be stacked within 100 msec, the time at which $\bar{P}$ -batches come apart, we concluded that it is proper to choose the synchronous phase of 7° .

As conclusion of all these considerations, we have attained the following r.f. parameters :

Rotation voltage $V = 400 \text{ KV},$

Synchronous phase $\phi_s = \pi - 7^\circ .$

\$ 3 Results---Application to the Precooler Ring---

The parameters of numerical stacking are assumed as follows :

n (stack pulses) $= 13,$

N (number of injected test particles every cycle) $= 200.$

E (energy variation) = 227 MeV

Other r.f. parameters have already been shown in section 2-c. Further the stacking diagram with detail progresses is schematically shown in Fig.3-1. Of course it has the same processes in 2-b.

Results of numerical experiments are shown in Fig.3-2. It represents the energy distribution of stacked particles every stacking cycle, we obtain the finalfull energy spread of 2.1 % , the overall stacking efficiency of 88.%, and the final capture efficiency of 89.% for $\pm 3\sigma$ beam (99.7%), (See Fig.3-3).

It took the computation times of 63 hours on CYBER 175 to do the above numerical experiment of RF stacking.

Appendix

Let us assume a beam distributed uniformly over 4.45 GeV $\pm (\Delta E)_{\text{initial}}/2$ in the phase space (E, ϕ) where $(\Delta E)_{\text{initial}}$ is 10 MeV.

At the location corresponding to the injection energy of 4.5 GeV the stationary bucket with area A_0 (.154 eV-s) is slowly excited and is adiabatically transformed into the vacant moving bucket with the same bucket area. The peak r.f. voltage V_m of a moving bucket is derived from the following equation :

$$V_s^{\frac{1}{2}} = V_m^{\frac{1}{2}} \cdot \alpha(\Gamma), \quad \Gamma = \sin \phi_s \quad (\text{A-1})$$

where V_s is the peak voltage of the stationary bucket, $\alpha(\Gamma)$ is the area ratio^[6], and ϕ_s is the synchronous phase.

Next the vacant moving bucket traverses from 4.4 GeV to 4.4 GeV on the phase plane and is adiabatically transformed into the stationary bucket. Lastly RF is slowly turned off. The energy spread $(\Delta E)_{\text{final}}$ and dispersion $(\sigma)_{\text{final}}$ of the assumed beam are calculated at the last time. The ratio of $(\Delta E)_{\text{final}}$ and $(\sigma)_{\text{final}}$ to each initial value are shown as the function of the synchronous phase ϕ_s as seen in Fig.2-c-3-(f).

The series of processes can be known schematically in Figs.2-c-3-(a) ~ 2-c-3-(e).

References

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Figure Captions

Figs.2-b-1 ~ 2-b-8 show the phase flow through RF stacking so as to know the essential feature of the present RF stacking. But for the purpose of saving computational time, r.f. parameters of these figures are different from the ultimately chosen parameters for the Precooler ring. But any characteristic features of the present RF stacking method will not be lost. The r.f. parameters and a number of representative points chosen from Fig.2-b-1 through 2-b-8 are shown on Fig.2-b-1.

Fig.2-b-1 RF Capture

The figure shows a initial $\bar{P}$ bunch captured inside the stationary bucket, which has the gaussian distribution in phase and uniform distribution in energy. The under figure shows the particle distribution in E.

Fig.2-b-2 Bunch Rotation

The figure shows a $\bar{P}$ bunch after quarter phase angle which corresponds to the particle revolution times of 60 . The distortion in edges of a bunch is considered to result from nonlinearity in the boundary of the stationary bucket. The under figure shows the particle distribution in E.

Fig.2-b-3 Supprss

The figure shows a $\bar{P}$ bunch after abruptly reducing the r.f. peak voltage. This reducing must be done within five revolution times, because after more times the the stationary bucket becomes to be

not able to surround tightly the rotated $\overline{P}$ bunch.

Fig.2-b-4 Alteration

The figure shows the $\overline{P}$ bunch captured in the moving bucket. The transition from the stationary bucket into the moving bucket is done adiabatically ; it takes 500 revolution times which is enough long time compared with the phase oscillation period. The tail by which the moving bucket is like to be accompanied consists of particles having failed to be surrounded by the reduced stationary bucket to be left.

Fig.2-b-5 Deceleration

The figure shows the $\overline{P}$ bunch captured in the moving bucket and the particles left in the former steps. The swirl inside the moving bucket results from the non-linear motion of the representative particles. Namely the oscillation frequency of particles in the boundary slips out of one of the linear oscillation.

Fig.2-b-6-(a) Adiabatic Debunching

The figure shows the $\overline{P}$ bunch captured in the stationary bucket into which has been adiabatically exchanged from the moving bucket. It takes 500 revolution times altering. And the mean energy of the bunch corresponding to the stacked orbit is 4.327 GeV.

Fig.2-b-6-(b)

The figure shows the stacked beam after r.f. voltage has been adiabatically turned off. The under histogram represents the

energy distribution of the stacked beam per one stacking cycle.

Fig.2-b-7-(a)

The figure shows the new decelerated $\overline{P}$ bunch and the already stacked beam.

Fig.2-b-7-(b)

The figure shows the situation that the moving bucket is passing through the region occupied by the stacked beam. From the Liouville's theorem the stacked beam is seen to move upwards.

Fig.2-b-7-(c)

The figure shows the situation that the new moving bucket has just arrived at the stacking orbit, namely, at the top of stack.

Fig.2-b-8

The figure shows the stacked beam which completed the second stacking cycle. We can know that the energy distribution corresponding to the new stacked part is same as that in Fig. 2-b-6-(b), while the energy distribution corresponding to the former stacked parts has the steady fall-off for higher and lower values of E .

Fig. 2-b-1

E

4.5 GeV

0 360

$P(E)$

Fig. 2-b-5

$$\begin{aligned} \phi_s &= \pi - 18^\circ \\ V \text{ (peak voltage)} &= 59 \text{ KV} \\ H \text{ (bucket height)} &= 7.56 \text{ MeV} \end{aligned}$$

III

E

Fig. 2-b-7-(a)

E

4. 326 GeV

E

Fig. 2-b-7-(b)

Fig. 2-b-7-(c)

4. 326 GeV

Fig. 2-b-8

V (peak voltage) = 0 KV

Fig. 2-c-1

Capture Efficiency η_0 vs. Rotation Voltage V_{rot}
by a moving bucket

Fig. 2-c-2

$V = 300 \text{ KV}$

4.5 GeV

$V_{ro} = 400 \text{ KV}$

4.5 GeV

$V_{ro} = 650 \text{ KV}$

4.5 GeV

Fig. 2-c-3-(a)

Fig. 2-c-3-(b)

[illegible]

LE

Fig. 2-c-3-(e)

E

.....

$(\Delta E)_{\text{final}}$

4.45 GeV

0

360

$\rho(E)$

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E

Fig. 2-c-3-(f)

Dilution vs. Synchronous Phase of Moving Bucket

Fig. 3-1

Diagram of RF Stacking

Fig. 3-2

Cycle = 1 ϕ_s (synchronous phase) = $\pi - 7^\circ$
 V (rotation voltage) = 400

Cycle = 2

Cycle = 3

Cycle = 4

Cycle = 6

Cycle = 7

Cycle = 9

Cycle = 10

Cycle = 11

Cycle = 12

Cycle = 13

Fig. 3-3