Annual Technical Progress Report III

THE STUDY OF DISLOCATION DYNAMICS ASSOCIATED WITH DUCTILE-BRITTLE
TRANSITION AND HYDROGEN EMBRITTLEMENT PHENOMENA OBSERVED IN BCC METALS

BY

NOTICE
This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

William W. Gerberich and David G. Atteridge
Department of Chemical Engineering and Materials Science
University of Minnesota

MASTER

March 1975

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

EB

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

TABLE OF CONTENTS

	<u>Page</u>
PROGRAM OBJECTIVES AND SUMMARY	i
OBJECTIVES	iii
SUMMARY	iv-v
I. ACOUSTIC STRESS-WAVE EMISSION TECHNIQUES AND DUCTILE FRACTURE	1
I-1 Analysis of Incremental Cracking	1
I-2 J-Integral Analysis, Ductile Rupture and Acoustic Emission	2
II. DEVELOPMENT OF STRESS-FIELD APPROACHES TO THE STUDY OF HYDROGEN-ASSISTED CRACKING	2
II-1 Analytical and Numerical Solutions for Crack Problems	3
II-2 Threshold Stress Intensity and Crack Growth Modelling	4
II-3 An Experimental Determination of the Crack Nucleation Site in Embrittled High Strength Steel	5
II-4 The Effect of Hydrogen Pressure on the Threshold Stress Intensity and Kinetics in 4340 Steel	8
III. DISLOCATION DYNAMICS AND THE DUCTILE-BRITTLE TRANSITION .	11
III-1 Strain-Rate Sensitivity: Comparison of Change of Strain Rate, Acoustic Emission and Stress Relaxation Methods	11
III-2 Stress Relaxation as a Measure of Dislocation Dynamics	15
III-3 Ductile-Brittle Transition: Preliminary Comparison to Yield and Strain-Rate Sensitivity Results	15
APPENDIX A-1 Analysis of Incremental Cracking by the Stress-Wave Emission Technique	
APPENDIX A-2 An Acoustic Emission Investigation of Microscopic Ductile Fracture	

	<u>Page</u>
APPENDIX B-1	Exact Solutions of Mode I, II, III Problems with Conformal Mapping
APPENDIX B-2	The Effect of Prior Austenitic Grain Size on Hydrogen Assisted Failure of Type 4340 Steel
APPENDIX C	A Theoretical Review of Stress Relaxation Testing

PROGRAM OBJECTIVES AND SUMMARY

OBJECTIVES

This program has three main objectives associated with the different types of microscopic fracture modes.

Ductile Fracture

We desire to understand the microscopic void coalescence process leading to crack initiation and growth.

Plastically-Induced Cleavage

Our aim is to tie together fracture mechanics, dislocation dynamics, solute softening and cohesive energy concepts into a comprehensive ductile-brittle transition theory.

Hydrogen-Assisted Intergranular Fracture

To verify what constitutes a viable embrittlement model, we are investigating both stress field effects as well as concentration and microstructural effects. Theoretical concepts of both crack growth kinetics and threshold conditions are being developed.

SUMMARY

Three significant developments in the study of flow and fracture pertain to dislocation dynamics, brittle fracture and hydrogen-assisted crack growth.

1. Dislocation Dynamics - Stress relaxation at small strains promises to connect up change of strain rate data at large strains to acoustic emission phenomena at very low strains. We have already found that logarithmic differentiation of the acoustic emission representation of mobil dislocation density gives an estimate of strain rate sensitivity from $\partial \ln \rho_m / \partial \ln \dot{\sigma} = \frac{1}{m}$. This compares favorably with strain rate sensitivity exponents, m' , taken from change of strain rate tests. The ramifications of this to the study of dislocation dynamics could be considerable.
2. Brittle Fracture - Ductile-brittle transition analysis points towards a relatively simple criterion for predicting behavior using strain-rate sensitivity, solute-softening, cohesive-energy and fracture mechanics concepts. Initial evaluation of impact and yield data from Fe, Fe-Ni and Fe-Si alloys indicates that a single parameter, $m_0 \ln \left[\frac{\sigma_f^* - 3\sigma_{ys}}{\sigma_u} \right]_{DBTT}$ controls cleavage. Here, m_0 is the strain-rate sensitivity exponent, and σ_f^* , σ_{ys} and σ_u are cleavage, yield and thermally-activated flow stress parameters. Thus far, this parameter separates impact data into the brittle and ductile fracture regimes.
3. Hydrogen Embrittlement - Tests in both cathodically-charged and gaseous hydrogen environments are providing evidence for a unified concept of hydrogen-induced slow crack growth. It is shown that increased

grain size(d) slows kinetics with growth rate (da/dt) being roughly proportional to $d^{-3/2}$. This tentatively supports theories based upon relatively long-range diffusion of hydrogen to prior austenite grain boundaries. Additional evaluation of cathodically-charged specimens has demonstrated that da/dt can decrease with increasing stress intensity during stage II growth. This is explained in terms of plastic deformation at the crack tip. Crack tip blunting can alter both the severity of the plastic constraint and the position of maximum triaxiality. This effect must be incorporated into all embrittlement theories which have stress-induced diffusion as a major component.

I. Acoustic Stress-Wave Emission Techniques and Ductile Fracture

As was indicated in last year's proposal, one goal of this research program was to use the quantitative relationship between stress-wave amplitude and microcrack extension to study the details of crack growth. In section II-3, it is indicated that this technique will be used to specify the very initial stages of crack nucleation associated with hydrogen-induced cracking. This will allow a more accurate determination of the initiation site. Prior to this, we had established that the technique can be used in a quantitative way to study the ductile-rupture process in ultra-high strength materials. We are presently attempting to see if this technique can be extended to lower yield strength, high toughness alloys as indicated below. Over the last year, three graduate students (Linda Thury, Joseph Lessar and Michael Stout) and one post-doctoral fellow (David Atteridge) have been involved in this phase of the research.

I-1 Analysis of Incremental Cracking

William W. Gerberich, Linda Thury, Joseph Lessar and David G. Atteridge

Most of these data were presented in the previous Annual Report II, and are now published (or are in press) as indicated in Appendices A-1 and A-2. One set of unfinished data analysis involved approximately 20 tests of a 0.40 wt. % carbon steel heat treated to a range of strength levels. Fracture toughness testing and acoustic emission data have been analysed and the work is presently being analysed with respect to how sensitive the technique is. Initial results indicate that the threshold energy for triggering an emission event is a function of strength level (tempering temperature). Thus, a careful analysis of different microcracking phenomena (e.g. the region of plastic instability)

associated with acoustic emission is required if this is to be a viable non-destructive testing technique.

I-2 J-Integral Analysis, Ductile Rupture and Acoustic Emission

Michael Stout and William W. Gerberich

As an ideal, we would like to have an intermediate strength level material that has extraordinary resistance to crack instability. In this way we could study slow crack growth processes at high stress intensity levels where linear elastic fracture mechanics is no longer valid. One group of materials which possess excellent strength/toughness relationships, produce stable ductile fractures and have controllable microstructures are superalloys. We will study Inconel 718 in various aged conditions since then, the volume fraction, precipitate sizes, and interparticle spacings, which control the ductile fracture process, can be varied considerably. In order to have a reasonable framework for studying the slow crack growth process, we will utilize the J-Integral Analysis. This allows an assessment of fracture toughness well into the plastic regime by integrating the plastic strain energy density associated with crack advance. In situ studies of cracking with the acoustic emission technique and post fracture studies with replica and scanning electron microscopy will characterize the ductile fracture process.

II. The Development of Stress-Field Approaches to the Study of Hydrogen-Assisted Cracking

It is essential that hydrogen adsorption, dissociation, recombination, absorption, diffusion and concentration be understood if an appropriate analysis

is going to emerge. Although we cannot possibly study all of these variables, we can study the diffusion and concentration aspects as associated with the fracture mechanics approach. Such variables as stress field, microstructure and environment drastically affect diffusion and concentration. If one is to adequately assess such variables, then theoretical studies of crack-tip stress fields and their interaction with embrittling species must be made. These will later be coupled with experimental studies of embrittlement that are presently in progress. As discussed in the program summary, this would have considerable ramifications for all types of energy related production systems from the drilling of deep sour gas wells where H_2S is a problem, to the containment of coal-gasification reactions where CH_4 embrittles the reactor. Over the last year three M.S. candidates and two Ph.D. candidates have been involved in these projects, with two candidates having finished their requirements for a Master's degree.

II-1 Analytical and Numerical Solutions for Crack Problems.

Elias C. Alfantis and William W. Gerberich

Within the last four years, there has been an increasing effort to quantify elastic interactions of crack-tip and/or dislocation stress fields with interstitial atoms so as to explain embrittlement phenomena. In practically all of these, a large number of details such as plastic deformation, crack-tip blunting, exact descriptions of the elastic field equations, etc., have been largely overlooked. By doing a conformal mapping analysis of the crack problem, we have shown that elastic solutions based upon a one-term singularity can be in error up to within five percent of the crack length. This may have a bearing

on embrittlement models if diffusion is very long range. On the other hand, it has implications on plasticity solutions as well as discussed in Appendix B-1.

II-2 Threshold Stress Intensity and Crack Growth Modelling.

Joseph Lessar, Yew-Tsung Chen and William W. Gerberich

As was indicated in last year's proposal, there are a number of parameters that affect threshold stress intensity, K_{TH} , and crack growth rate, da/dt . Grain size is one of these. It is important to kinetics if (i) the crack is growing a grain size at a time and hydrogen collects over a region proportional to the grain size or (ii) if elements such as Sb, Sn, S which segregate to grain boundaries, concentrate in different amounts and affect the critical condition for grain boundary fracture. In the former case, one might expect an increased grain size to retard crack growth while in the later, to accelerate growth. There was conflicting observations in the literature but the present observations clearly show that an increased grain size slows kinetics. This would seem to eliminate (ii) at least for one high strength steel tempered at 300°C. By normalizing a considerable amount of data, it is shown that at low stress intensity levels, da/dt is approximately proportional to $d^{-3/2}$, which is in accordance with the bulk diffusion model derived last year*. These findings are detailed in Appendix B-2.

Further development of initial threshold and kinetic models along with associated crack growth data have been presented and published as indicated in

* Annual Technical Progress Report II and, "Effect of Hydrogen on High-Strength and Martensitic Steels," Proc. Seven Springs Conference on Hydrogen in Metals, 1973.

Appendix B-3. In addition, one paper by Y. T. Chen and W. W. Gerberich had been submitted to Metallurgical Transactions (not shown in this report) where the effects of crack-tip plasticity are discussed in great detail. In this paper, crack-tip profiles, as observed via scanning electron and light microscopy, are discussed in detail. Effects on stage II crack growth kinetics are suggested. One set of crack-tip radii, Figure 1, and their dependence on stress intensity level, Figure 2, are shown to decrease the intensity of the stress gradient with increasing stress intensity. The result of this is to allow a decreasing crack growth rate during stage II even when the stress intensity is increasing.

Additional identification of the hydrogen-induced crack nucleation rate and associated kinetics are detailed in the following two studies.

II-3 An Experimental Determination of the Crack Nucleation Site In Embrittled High Strength Steel

Richard Page

Introduction:

Although the phenomena of hydrogen embrittlement has been extensively studied, controversy still exists concerning the mechanism involved. Three models have been developed in an effort to explain the experimental results. Zapffe⁽¹⁾ and Tetelman⁽²⁾ propose that hydrogen "precipitates" as a gas in a void. It is the added pressure of this gas which causes embrittlement. Troiano⁽³⁾ proposes that the hydrogen present in the lattice reduces the cohesive strength of the metallic bond. It is this reduction in bond strength that is believed to cause embrittlement. Williams and Nelson⁽⁴⁾ propose that

(a) 15μ

(b) 2μ

AISI 4340, Temp $T = 480^{\circ}\text{C}$, $B = 0.75\text{ in}$, $K = 33.4\text{ ksi-in}^{1/2}$

(c) $K = 57.6\text{ ksi-in}^{1/2}$ 10μ

(d) Same sample as (c) 5μ

(e) $K = 96\text{ ksi-in}$ 30μ

(f) $K = 107.5\text{ ksi-in}^{1/2}$ 30μ

AISI 4340, Temp $T = 550^{\circ}\text{C}$, $B = 0.75\text{ in}$

embrittlement is the result of hydrogen adsorbed at or near the crack tip.

The model receiving the most attention is the lattice embrittlement model proposed by Troiano. This model has been further developed by Oriani⁽⁵⁾ into a decohesion theory which implies crack nucleation at the tip of the crack and by Gerberich⁽⁶⁾ into a theory that implies that the crack will nucleate at the elastic-plastic boundary when a critical hydrogen concentration is reached. A determination of the crack nucleation site should help determine which of these models apply to hydrogen embrittlement and shed considerable light on crack growth kinetics. It is the purpose of this study to make such a determination.

Experimental Plan:

Crack nucleation site determinations will be performed on specimens with four different notch root radii, ranging from a machined notch of one tenth of an inch to a fatigue crack. These tests will be conducted in five different environmental conditions. One fifth of the specimens will be tested in a molecular hydrogen (H_2) atmosphere. One fifth of the specimens will be tested in distilled water. The remaining three fifths will be cathodically charged, to three concentrations, with atomic hydrogen, cadmium plated, and baked for a short time to eliminate any concentration gradients⁽⁷⁾.

Twenty samples of 4340 alloys steel will be used in the tests. The hydrogen atmosphere samples will be of the Double Cantilever Beam (DCB) design⁽⁸⁾. The DCB was chosen for the hydrogen atmosphere because it is self loading. The remaining samples will be of the Compact Tension design.

The specimen, equipped with accelerometers, will be slowly loaded. When the first signs of stress wave emission are detected the specimen will be rapidly unloaded. The specimen will then be sectioned and examined to determine the

location of the crack nucleation site. The best method for the location of the crack nucleation site has not yet been determined. Two methods now being studied are scanning electron microscopy and surface replication techniques.

References:

1. C. Zapffe, Trans. ASM, 1947, Vol. 39, p191.
2. A. S. Tetelman, Fracture of Solids, p. 671, John Wiley and Sons, New York, 1962.
3. A. R. Troiano, Trans. ASM, 1960, vol. 52, p. 54.
4. D. P. Williams and H. G. Nelson, Met. Trans., 1970, Vol. 1, pp. 63-68.
5. A. R. Oriani and P. H. Josephic, Scripta Met. 1972, Vol. 6, p. 681.
6. W. W. Gerberich, "Effect of Hydrogen on High-Strength and Martensitic Steels", Proceedings, Seven Springs Conference on Hydrogen in Metals, 1973.
7. R. H. Johnson, J. G. Morlet, and A. R. Troiano, Transactions of the Metallurgical Society of AIME, August, 1958.
8. M. V. Hyatt, Corrosion - Nace, Vol. 26, No. 11, November, 1970.

II-4 The effect of hydrogen pressure on the threshold stress intensity and kinetics in 4340 steel.

Jeffrey Garry

The effect of atmospheric hydrogen pressure on the threshold stress intensity (K_{TH}) and crack growth rate (da/dt) were studied, using side-grooved DCB specimens of 4340 steel. Stress wave emissions (SWE) associated with hydrogen cracking allowed the process to be monitored and gives a valuable insight into the actual cracking mechanism.

The double cantilever beam (DCB) specimens used in the study are shown in Fig. 1. Studies (1-3) have shown the necessity of side-grooving the specimens to prevent arm breakoff. The compliance of the DCB specimens was determined using pin-loaded specimens (4) in the Instron tensile testing machine.

The testing apparatus consisted of three systems. The atmosphere control system consists of a bell jar evacuated by a diffusion pump and a rotary pump. The hydrogen pressure was controlled by a cartesian manostat and measured with a precision mercury manometer. Ultra-pure hydrogen (dew point -96°F) was used. The loading system consisted of a vise to hold the specimen, a load cell and a manipulator. The stress wave monitoring system consists of an accelerometer attached to the specimen, a charge amplifier, a 3kHz to 40 kHz band pass filter, a decade amplifier and an oscilloscope for the observation of the stress waves.

The specimens were precracked, mounted and the chamber filled with hydrogen. The specimen was then loaded until stress waves were observed. The load was recorded as a function of time. After threshold had been achieved, as indicated by the absence of load drops and SWE, the final deflection was measured, the specimen was broken open and the initial and final crack lengths were measured (the hydrogen cracking zone was easily discernable due to the difference in

fracture surfaces). The stress intensity and crack growth rate were calculated as a function of time by a computer program.

Figure 2 shows a plot of K_{TH} versus atmospheric hydrogen pressure (P_{H_2}). The solid line represents a least-squares fit of the data. The Troiano model (5) proposes that hydrogen diffuses into the region in front of the crack assisted by the tensile stress field. When a critical hydrogen concentration C_{CR} is exceeded at the elastic-plastic boundary, a secondary crack nucleates and propagates to the main crack tip. This process is then repeated.

$$C_{CR} = C_0 \exp\left[\frac{2(1+\nu)\bar{V}_H K}{3\sqrt{2\pi r} RT}\right] \quad (1)$$

where C_0 is the initial hydrogen concentration, $\bar{V}_H$ is the partial molar volume of hydrogen in iron and r is the plastic zone radius. At threshold, C_0 will be the equilibrium concentration and may be represented by a Sieverts' law:

$$C_0 = sP_{H_2}^{\frac{1}{2}} \quad (2)$$

where s is the Sieverts' parameter. Combining equations (1) and (2), we get a relation of the form:

$$K_{TH} = C' - C' \ln P_{H_2} \quad (3)$$

where

$$C' = \left(\frac{3\sqrt{2\pi r} RT}{2(1+\nu)\bar{V}_H}\right) \ln(C_{CR}/s) \quad \text{and} \quad C' = \left(\frac{3\sqrt{2\pi r} RT}{4(1+\nu)\bar{V}_H}\right)$$

Figure 3 shows a plot of the least-squares data versus $\log P_{H_2}$, showing good agreement with this type of relation, except at low pressures.

Figure 4 shows the crack growth rate, da/dt , versus K for 500 Torr. It can be seen that in the low K region ($K \sim K_{TH}$), stage I kinetics ($da/dt \propto K$) are observed. In the higher K regions, stage III kinetics are observed

($da/dt \propto K^2$). Stage II kinetics ($da/dt \neq f(K)$) are not observed. Figure 5 shows least squares plots of da/dt versus K for four hydrogen pressures. It can be seen that increasing the hydrogen pressure increases the slope of the $da/dt - K$ curve. Williams and Nelson (6) found $da/dt \propto P_{H_2}^n$ at a given K in 4130 steel. Figure 6 shows a log-log plot of da/dt versus P_{H_2} which indicates that this relation is obeyed.

Stress wave emissions were continuously monitored during tests. Crack growth as indicated by measurable load drop was always accompanied by numerous SWE, which usually continued for a period after load drops had ceased. The magnitude of the emission and the strong correlation with crack growth as indicated by load drops lends credence to the theory that hydrogen cracking occurs in large, discontinuous steps.

REFERENCES

1. M. V. Hyatt, Corrosion, 1970, 26, no. 11, p. 487.
2. C. N. Freed and J. M. Krafft, J. of Mat'ls, 1966, 1, no. 4, p. 770.
3. S. Mostovoy, et al., J. of Mat'ls, 1967, 2, no. 3, p. 661.
4. J. F. Lessar, M. S. Thesis, University of Minnesota, Minneapolis, 1975.
5. A. R. Troiano, Trans. ASM, 1960, 52, p. 54.
6. D. P. Williams and H. G. Nelson, Met. Trans., 1970, 1, p. 63.

Figure 1: DCB Specimen - Full Scale

DCB Specimen - Full Scale

Figure 2: Threshold Stress Intensity as a Function of Hydrogen Pressure.

Figure 3: Semi-Log Fit of Sievert's Relationship

Figure 4: Crack-Growth Rate Data at 500 torr as a Function of Applied Stress Intensity.

Figure 5: Kinetic Data for all Partial Pressures

Figure 6: Log-Log Representation of Kinetics as a Function of Hydrogen Pressure.

III. Dislocation Dynamics and the Ductile-Brittle Transition

All of the Fe-Binary alloys have been evaluated in change of strain rate tests except for the Fe-Pt alloy. In addition most of the acoustic emission tests have been completed and are in the initial stages of evaluation. Some problems have been encountered at low temperatures where twinning leads to so much acoustic noise, that it is impossible to quantify any parameters which might be associated with mobile dislocation density. Thus, useful acoustic emission data associated with yielding processes will be relegated to temperatures about -50°C . All impact studies have been completed except for the Fe-Pt tests. Not shown in this report is a detailed analysis of electron microfractography, which evaluates the microscopic fracture transition associated with the DBTT in Fe, Fe-Ni and Fe-Si alloys. The conclusion is that the fracture surfaces of the two alloys and the parent metal exhibit identical fracture morphology changes as test temperature is increased through the DBTT. This is presently being submitted to Metallurgical Transactions for review.

III-1 Strain-Rate Sensitivity: Comparison of Change of Strain Rate, Acoustic Emission and Stress Relaxation Methods.

Yew Tsung Chen and William W. Gerberich

Preliminary change of strain rate data are given in Table III-1. These data were obtained by determining $(\partial \ln \dot{\epsilon} / \partial \ln \sigma)_{\epsilon_p}$ and then plotting these as a function of plastic strain. As shown in Figure 1, these give m_0 when extrapolated to zero strain, the implication being that $(\partial \ln p / \partial \ln \sigma)$ is zero at zero strain. If all strain rate sensitivity exponents are plotted versus yield strength, two groupings emerge as indicated in Figure 2. All Fe, Fe-Si and low

temperature Fe-Ni data fall on a single curve while the room temperature Fe-Ni data seem to behave in a completely different manner. A preliminary interpretation would be that thermally activated cross slip is much easier in the Fe-Ni alloys at room temperature while planar slip is much more pronounced at lower temperatures and in the Fe-Si alloys. Certainly, the lower values of m_0 would be associated with a trend toward brittle fracture.

To ascertain if there were parameters for correlating acoustic emission data to tensile or strain-rate change data, the information in Table III-2 were plotted in several ways. First, it was previously observed that acoustic emission count rate, $\dot{N}$, could be fit to a Gilman dislocation mobility equation, given by

$$\dot{N} = m_p \epsilon_p e^{-\phi \epsilon_p}$$

Here m_p is a multiplication coefficient, ϵ_p is the plastic strain and ϕ is a work hardening exponent. If m_p is a crude estimate of the proportionality between count rate and mobile dislocation density, then there might be a correlation between yield strength and m_p . Indeed, this is the case as shown in Figure 3 which simply indicates that higher yield strength material have higher dislocation densities. It is significant that the curve in Figure 3 would extrapolate to an m_p of 2.72×10^8 at a yield strength of 160,000 psi. This is very close to the observed value of 2.84×10^8 which was reported for 4340 heat treated to 160,000 psi⁽¹⁾. A second basis for correlating acoustic emission data involves the treatment of the dislocation dynamics equation derived in the last report⁽¹⁾. It was suggested that the acoustic emission count rate, $\dot{N}$, represented $dp_m/d\epsilon$ so that integration gave

$$\rho_m = \frac{\alpha m_p}{\dot{\epsilon}} \left[\frac{1}{\phi^2} - \frac{1}{\phi^2} e^{-\phi \epsilon_p} - \frac{1}{\phi} \epsilon_p e^{-\phi \epsilon_p} \right] \quad (1)$$

One can use this to compare to strain-rate sensitivity data if it is assumed that $\partial \ln \rho / \partial \ln \sigma$ is the main contributor to m' at very small strains.

First, take the natural log of equation (1) and differentiate it. With $\alpha, m_p, \dot{\epsilon}$ independent of plastic strain, this gives

$$\frac{\partial \ln \rho_m}{\partial \ln \sigma} = \frac{\phi^2 \epsilon_p^2 e^{-\phi \epsilon_p}}{1 - e^{-\phi \epsilon_p} - \phi \epsilon_p e^{-\phi \epsilon_p}} = n \frac{\partial \ln \rho_m}{\partial \ln \sigma} \quad (2)$$

The equality at the right stems directly from the exponential work hardening relationship where n is the macroscopic work hardening exponent. One can compare m' to equation (2) since differentiation of the Johnston-Gilman dislocation dynamics equation gives

$$m' = \frac{\partial \ln \dot{\epsilon}}{\partial \ln \sigma} = \frac{\partial \ln \bar{v}}{\partial \ln \sigma} + \frac{\partial \ln \rho_m}{\partial \ln \sigma} \quad (3)$$

Thus, one might expect some correlation between m_0 as taken from change of strain rate data and ϕ as taken from the acoustic emission data. Although the data are very rough and preliminary, there appears to be some correlation suggested in Figure 4. The anomalous point with the ? is the same one that is correlatable in Figure 2. This is presently being checked out with additional tests.

At this stage, we must treat such correlations as Figure 4 as pure speculation until more evidence is collected. The reason for saying that is that we are essentially correlating m' from large strain data, assuming ρ_m is constant, to m' from small strain data, assuming $\bar{v}$ is constant. To be precise,

we would be saying that

$$\left. \frac{\partial \ln \dot{\epsilon}}{\partial \ln \sigma} \right|_{\rho_m}^{\text{Large } \epsilon_p} = \left. \frac{\partial \ln \rho_m}{\partial \ln \sigma} \right|_{\bar{v}}^{\text{Small } \epsilon_p} \quad (4)$$

Of course there is no basis for saying that $\bar{v}$ should be constant at small strains except near a yield point where σ is barely changing. There is evidence that ρ_m may be constant at large plastic strains and this will be briefly discussed in the next section. To further demonstrate that such a correlation as Figure 4 needs considerable examination, we have calculated m' from Eqs. (2) and (3), from acoustic emission data and knowing that $n=0.182$ for pure Fe. This was then compared to m' data as taken from change of strain rate tests in Figure 5. Coincidentally, the extrapolated value of $m' \rightarrow m_0$ at zero strain on a linear plot does compare to the acoustic emission value of m' at very small strains. However, there is a minimum in between where the material seems to be extraordinarily strain rate sensitive. This was verified by some "anomalous" stress relaxation data as discussed in the next section. This is quite interesting since, at even lower strains, m' values from stress relaxation must get larger at some smaller strain so that a minimum should be predicted from two separate types of tests.

It is anticipated that further stress relaxation testing over a range of pre-strains will prove whether the acoustic emission approach is giving a valid measure of strain-rate sensitivity at small strains.

Figure 1: Strain-Rate Sensitivity versus Applied Strain for Fe-Ni Alloys

Figure 2: Relationship Between Yield Strength and Strain Rate Sensitivity.

Figure 3: Relationship Between Yield Strength and Acoustic Emission Coefficient.

Figure 4: Possible Correlation Between Strain Rate Sensitivity and Acoustic Emission Exponents.

Figure 5: Strain Rate Sensitivity as a Function of Plastic Strain for Three Types of Mechanical Tests.

III-2. Stress Relaxation as a Measure of Dislocation Dynamics

Elias C. Alfantis and William W. Gerberich

Preliminary results from stress relaxation testing have demonstrated that the conditions for using the traditional Johnston-Gilman approach at small strains are lacking. That is, stress relaxation is anomalously high at small strains in pure polycrystalline Fe. The stress relaxation data are inconsistent with the hypotheses that either $\rho_m = \text{const.}$ or $(\partial\sigma/\partial\ln\dot{\epsilon}_p) = \text{const.}$ Since we are concerned with the initial stages of yielding near the ductile-brittle transition, it is essential to measure the dislocation mobility parameters at small strain. For this reason, we will use stress-relaxation techniques where the dislocation mobility parameters can be evaluated at small pre-strains. This is detailed in Appendix C where a review and some initial data are presented. These data in conjunction with the acoustic emission and change of strain rate data should give us a more complete picture of dislocation dynamics as a function of strain. This does not minimize the change-of-strain rate data since at the DBTT, plastically-induced cleavage could be associated with fairly large strains at the notch or crack tip.

III-3 Ductile-Brittle Transition: Preliminary Comparison to Yield and Strain-Rate Sensitivity Results.

William W. Gerberich and Yew-Tsung Chen

Yield strength data have been evaluated at R.T. and -40°C for all Fe, Fe-Ni and Fe-Si materials. As seen in Figure 1, both systems exhibit solute softening at -40°C with the degree of softening being somewhat greater in the Fe-Ni system. However, trends in the DBTT cannot be predicted on the basis of

yield strength alone since Fe-Ni alloys with yield strengths greater than pure Fe had DBTT's up to 70°C lower. With respect to a plastic-constraint induced ductile-brittle transition, this is exactly the opposite of what one might expect.

This theme can be expanded by taking the structure, σ_{ys} , and strain-rate sensitive portions of the yield stress; apply a plastic constraint factor and obtain the cleavage fracture stress, σ_f^* . Taking the plastic constraint to be about three, this would be

$$\sigma_f^* \approx 3[\sigma_{ys} + k_y d^{-1/2}] + \sigma_u \left(\frac{\dot{\epsilon}}{\rho_m b v_o} \right)^{1/m_o} \quad (5)$$

If we associate the first term in brackets with the macroscopic yield under slow strain rate conditions, then under impact, one could say that

$$\left. \frac{\dot{\epsilon}}{\rho_m b v_o} \right|_{CR} = m_o \ln \left(\frac{\sigma_f^* - 3\sigma_{ys}}{\sigma_u} \right) \Bigg|_{DBTT} \quad (6)$$

represented a condition for fracture if $\rho_m \approx \text{constant}$. To ascertain whether this strain-rate sensitivity parameter could correlate the DBTT results as given previously⁽¹⁾, we took the Fe and Fe-Si data and interpolated the values of σ_{ys} and m_o that occurred at the DBTT. These were then put into Eq. (6) with a constant $\sigma_f^* = 180,000$ psi as taken from other sources^(2,3). A nearly constant value of $\dot{\epsilon}/\rho_m b v_o = 10$ emerged from Eq. (6). Considering a ρ_m of about $10^9/\text{cm}^2$, this value would correspond to a strain rate of about 250 sec^{-1} which could easily occur within several grains at the tip of a notch (crack) under impact conditions. Encouraged by this result, we next applied the same approach to the Fe-Ni data but found it impossible to correlate the data. The change that made most sense was to increase σ_f^* with increasing Ni

content. By constantly increasing σ_f^* from 180,000 to 245,000 psi, an $\dot{\epsilon}/\rho_m b v_o$ of 10 could be obtained. The overall result is shown in Figure 2. It would appear that one can explain the Fe-Si ductile-brittle transition data based upon solute softening, yield and strain-rate sensitivity parameters alone. On the other hand, using the same criterion for Fe-Ni alloys requires an increase of σ_f^* which increasing Ni which is exactly the magnitude of increase that has been observed experimentally^(4,5).

As a double check, we evaluated all data in Table III-1 and found those values of m_o and $\ln(\frac{\sigma_f^* - 3\sigma_{ys}}{\sigma_u})$ which resided in the latched region. Based upon the previous impact data, these were observed to produce brittle fracture. Similarly, those values of m_o and $\ln(\frac{\sigma_f^* - 3\sigma_{ys}}{\sigma_u})$ residing in the ductile fracture region produced ductile fracture. In all of the evaluation's thus far, only one data point has violated this criterion by a small margin. Further checks of the criterion will include additional Fe-Si data at higher test temperatures, the Fe-Pt data and actual measurements of σ_f^* .

III-4 REFERENCES

1. W. Gerberich and D. G. Atteridge, Annual Technical Progress Report II, AEC Contract AT(11-1)-2212, March 1974.
2. G. T. Hahn, B. L. Averbach, W. S. Owen, M. Cohen, in Fracture, ed. by B. L. Averbach, MIT Press, Cambridge, Mass., 1959.
3. P. J. Guest, "The Micromechanics of Fracture of Pearlitic Structural Steels," D. Eng. Thesis, University of California, Berkeley, 1969.
4. M. Gensamer, in Fracture of Engineering Materials, Am. Soc. for Metals, Metals Park, Ohio, 1959, p. 1.
5. J. Nunes and F. R. Larson, Trans. AIME, 227, 1963, p. 1369.

TABLE III-1: Tensile and Strain Rate Change Results

<u>Material</u>	<u>25°C</u>		<u>-40°C</u>	
	<u>Yield Strength (psi)</u>	<u>m</u>	<u>Yield Strength (psi)</u>	<u>m₀</u>
Fe	15,700	9	29,900	15
Fe-1%Si	20,600	40.9	26,660	19.5
Fe-2.5%Si	25,900	21.5	35,000	33
Fe-4%Si	39,200	41	42,220	36.5
Fe-1%Ni	14,530	47	22,950	10.5
Fe-2.5%Ni	20,410	50	29,880	27
Fe-4%Ni	28,800	52	35,106	34

TABLE III-2: Tensile and Acoustic Emission Results

<u>Material</u>	<u>Yield Strength (psi)</u>	<u>$\dot{N}_{max.}$</u>	<u>$m_p (\text{sec}^{-1})$</u>	<u>ϕ</u>
Fe	15,300	3.95×10^2	$.865 \times 10^7$	6,160
Fe-1%Si	20,600	4.5×10^2	1.0×10^7	7,070
Fe-2.5%Si	25,400	4.2×10^2	3.34×10^7	23,200
	26,400	9.4×10^2	2.03×10^7	18,200
Fe-4%Si	34,400	7.25×10^2	4.15×10^7	25,000
Fe-1%Ni	14,530	$.47 \times 10^2$	$.29 \times 10^7$	22,700
Fe-2.5%Ni	20,410	3.4×10^2	2.8×10^7	30,200
Fe-4%Ni	28,800	2×10^2	1.81×10^7	33,300

Figure 1: Yield Strength as a Function of Alloy Additions of Si or Ni at Two Test Temperatures.

Figure 2: Strain Rate Sensitivity Diagram for Separating Ductile and Brittle Fracture Regions.

A THRESHOLD STRESS INTENSITY CONCEPT FOR ENVIRONMENTAL CRACKING

W. W. Gerberich and Y. T. Chen

Department of Chemical Engineering and Materials Science

University of Minnesota

Minneapolis, Minnesota 55455 USA

tel: 612/373-3135

Although there has been considerable effort [1] to study the threshold stress intensity K_{TH} for stress-corrosion cracking (SCC) or hydrogen-assisted cracking (HAC), there has been little quantitative effort to characterize the state of stress effects on its magnitude. Based upon a simple pressure tensor model, the prediction and existence of HAC has been demonstrated for Mode I but not for Mode III loading. [2,3] Some other interesting qualitative aspects of SCC and HAC were also explained in terms of the stress fields. However, these models have dealt with elastic stress fields, whereas it is obvious that it is the plastic stress field at the crack tip that is controlling in most instances. The present contribution is a preliminary evaluation of how the plastic stress fields might contribute to such observed phenomena as yield strength and hydrogen concentration level effects on K_{TH} .

In steel, titanium, and aluminum alloys, the effect of stress intensity on time to failure is typically found to be that schematically shown in Figure 1. Below the threshold value, no crack growth occurs. How does one go about predicting a threshold stress intensity? In steel and very probably titanium, one makes an assumption that a critical amount of hydrogen is necessary for the critical cracking event. Then, the conditions under which this hydrogen will collect at the crack tip are derived. Given an initial concentration of hydrogen C_0 (either in the material or adsorbed at the crack tip surface), the critical amount of hydrogen C_{CR} must be at least achievable under equilibrium conditions or no cracking will occur. Thus, the condition for a threshold stress intensity is:

$$K_{\text{applied}} = K_{TH} \quad \left| \quad C_{eq} = C_{CR} \right.$$

Equilibrium just achieves the critical event.

$$K_{\text{applied}} > K_{TH} \quad \left| \quad C_{eq} > C_{CR} \right.$$

Equilibrium is not achieved and cracking proceeds under non-equilibrium conditions.

$$K_{\text{applied}} < K_{TH} \quad \left| \quad C_{eq} < C_{CR} \right.$$

Equilibrium does not achieve critical event and no cracking occurs even at long times.

Hence, C_{eq} is the equilibrium concentration of hydrogen collected at the crack tip under a triaxial stress field. In several previous in-

vestigations [2,3,4] it has been shown that the equilibrium concentration is given by

$$C_{eq} = C_o \exp [\sigma_{ii} V_H / 3RT] \quad (1)$$

where σ_{ii} is the sum of the principal stresses, V_H is the partial molar volume of hydrogen, R is Boltzman's constant times Avogadro's number, and T is absolute temperature. If one next uses Hill's [5] plasticity solution for a notch under constrained flow, puts the pressure tensor in terms of a plastic constraint factor (pcf), and uses Hahn and Rosenfield's observations on pcf factors for sharp cracks [6], the threshold criterion along with (1) leads to:

$$K_{TH} = [kT/2V_H] \ln [C_{CR}/C_o] - \sigma_{ys}/4 \quad (2)$$

where σ_{ys} is the yield strength of the material. We are presently evaluating cathodically-charged AISI 4340 steel and find that (2) qualitatively predicts the trend in data. In fact, however, we have determined that the best fit₂ is represented by eq. (2) with $C_{CR}/C_o = \beta/\sigma_{ys}$, and $\beta = 460,000 \text{ lb/in}^2$. One can make a strong argument as to why C_{CR}/C_o should be a function of yield strength. Experimental evidence shows that it either takes greater critical hydrogen concentration levels [7] to promote embrittlement at lower yield strengths or the solubility limit [8] is lower for lower yield strength levels. If we take our HAC model and now assume that the same condition applies to the stress corrosion cracking* threshold K_{TH} , the comparison to existing data in the literature is shown in Figure 2. We purposely included only low alloy steels with similar carbon concentrations since there is ample evidence to show that lower carbon steels and high alloy steels such as maraging steels could have higher threshold levels at comparable strengths; this simply implies a higher value of β . For the class of steels in Figure 2; however, a single value of $\beta = 460,000 \text{ lb/in}^2$ works well.

Although not discussed here, it is a simple matter to demonstrate that AISI 4340 steel [9] with increasing initial concentration levels of hydrogen, C_o , have decreasing K_{TH} values that follow the trends predicted by (2). Furthermore, it is possible to predict quantitative thickness effects on K_{TH} for SCC or HAC by interpreting how the stress tensor varies from plane strain to plane stress.

Acknowledgements: This work was supported by the United States Atomic Energy Commission.

*That is, one doesn't care what kind of reaction takes place at the crack tip as long as there is an excess of hydrogen available and there is sufficient time for it to diffuse to the fracture nucleation site. Of course, this model breaks down if adsorption rate, for example, is controlling.

REFERENCES

- [1] B. F. Brown, in *The Theory of Stress-Corrosion Cracking in Alloys*, ed. by J. C. Scully, NATO Brussels (1971) W. J. Marey, and Son, Ltd., Leeds, England, 186-204.
- [2] H. W. Liu, *Journal of Basic Engineering*, 92 (1970) 633-638.
- [3] C. St. John and W. W. Gerberich, *Metallurgical Transactions*, 4 (1973) 589-594.
- [4] R. A. Oriani, in *Fundamental Aspects of Stress Corrosion Cracking*, ed. by R. Staehle, NACE, Houston (1969) 32-49.
- [5] R. Hill, *The Mathematical Theory of Plasticity*, Oxford, London (1950).
- [6] G. T. Hahn and A. R. Rosenfield, *Transactions of the ASM Quarterly*, 59 (1966) 909-919.
- [7] K. Farrall and A. Quarrell, *Journal of the Iron and Steel Institute* 202 (1964) 1002-1011.
- [8] C. D. Kim and A. W. Loginow, *Corrosion*, NACE 24 (1968) 313-318.
- [9] J. G. Johnson and A. Troiano, *Transactions of the AIME*, 212 (1958) 528-536.

11 June 1973 (rev 3 July 1973)

Fig. 1. Schematic of threshold stress intensity dependence on yield strength

Fig. 2. K_{ISCC} Dependence on yield strength

The Effect of Thickness on Hydrogen-Induced Slow Crack Growth

by

W. W. Gerberich and Y. T. Chen

University of Minnesota
Dept. of Chemical Eng. &
Materials Science
Minneapolis, Minnesota 55455

(Received January 14, 1974)

Introduction

Four major variables have been identified as being the prime factors in hydrogen-induced crack growth kinetics — the material condition, the stress intensity magnitude, the environment and the geometry of the test configuration⁽¹⁾. A survey of existing experimental data on steel typically indicated⁽¹⁻⁵⁾ that

Material Condition* : $da/dt \propto \sigma_{ys}^2$: $n \gg 2$
 Stress Intensity : $da/dt \propto K^\beta$: $0 < \beta < 2$
 Test Temperature : $da/dt \propto \exp(-Q/RT)$: $Q \approx 9000$ cal/mol
 Concentration : $da/dt \propto C_0$

Geometry or State of Stress : ?

Here, σ_{ys} is the yield strength, K is the applied stress intensity, Q is the apparent activation energy and C_0 is the initial hydrogen concentration**.

* This apparent yield strength effect is not observed for maraging steels although large variations in da/dt are possible by varying the aging sequence. In fact, the observation for low alloy, high strength steels may be partially microstructural in origin as the yield strength is most often varied by altering the tempering temperature.

** One must be careful here in defining the initial hydrogen concentration. It should not be the total hydrogen that is in both traps and in the lattice. However, should it be the lattice concentration that is only soluble under stress-free conditions, that which is soluble in the material under stress (initially neglecting the high gradient near the crack tip) or that soluble at grain boundaries or interfaces? The scope of this paper cannot address this problem.

Consider the above observations in order. With regard to tempering effects on crack growth kinetics, a factor of two increase in the yield strength can produce a 10^3 increase in growth rate. Increasing the stress intensity or test temperature can increase the growth rate by one to two orders of magnitude. The concentration dependence is not nearly so well established but may be inferred from the change in failure time produced by varying charging and baking times. Alternatively, the gaseous hydrogen pressure dependence^(4,5,6) can be shown to yield $da/dt \propto C$ if there is no adsorption rate limiting step. Increasing the concentration (partial pressure) can increase the growth rate by three orders of magnitude^(3,4). Thus, it would appear that the major variables affect growth kinetics by one to three orders of magnitude.

Most of these experimental observations can be accounted for by a hydrogen-diffusion model which is simply a Cottrell drift velocity model applied to the stress field of a crack⁽¹⁾. This is given by

$$da/dt = \frac{D_A D_H \bar{V}_H t_{CR} dp/dx}{(C_{CR} - C_0) dRT} \quad (1)$$

where D_A is the apparent hydrogen diffusivity, $\bar{V}_H$ is the partial molar volume of hydrogen in Fe, t_{CR} is the crack jump size, dp/dx is the pressure tensor gradient, C_{CR} is the critical concentration for cracking, and d is the grain size. Both t_{CR} and dp/dx can be dependent on K such that $0 < \beta < 2$. The test temperature or activation energy dependence is included in D_A . If $C_{CR} \gg C_0$, then $da/dt \propto C_0$ is a good first approximation.

One test of this model is to change the state of stress so that the stress gradient is altered. It has been reported that there is a greater propensity for thick specimens to crack than thin ones⁽⁷⁾ and cracking is observed in tensile crack opening Mode I but not under torsional anti-plane strain Mode III⁽⁸⁾. Although these are supportive of Eq. (1), they represent no quantitative measure of da/dt dependence. The object of the present paper is to report some experimental verification that Eq. (1) is a reasonable approach to the kinetic aspects of hydrogen embrittlement.

TEST PROCEDURE, RESULTS AND DISCUSSION

From the same plate of 4340 material, 0.1 inch thick tensile specimens and 0.15 inch to 0.75 inch thick compact tension specimens⁽⁹⁾ for slow crack growth studies were machined. The plate had the following composition

Composition, wt. %

C	P	S	Mn	Si	Cr	Mo	Ni
0.04	0.035	0.04	0.80	0.21	0.80	0.25	1.90

All samples were cut longitudinally, austenitized at 850°C for one hour, and oil-quenched and tempered for either 300°C or 480°C, two hours. The tensile test characterization gave 0.2% offset yield strengths of 208,000 psi (1.43×10^9 N/m²) and 173,000 psi (1.19×10^9 N/m²); and ultimate strengths of 238,000 psi (1.64×10^9 N/m²) and 186,000 psi (1.28×10^9 N/m²) for 300°C and 480°C tempered conditions, respectively.

Prior to hydrogen charging, a pre-fatigue crack was introduced with a ΔK stress intensity range of 10,000 psi-in^{1/2} ($9.1 \text{ MN-m}^{-3/2}$). Cycling at $K_{\max}/K_{\min}$ equal to two allowed 0.08 inches of fatigue growth to be achieved in 70,000 cycles. The hydrogen charging procedure, as described in detail elsewhere⁽¹⁰⁾, involved charging, Cd-plating and baking. The initial uniformly distributed concentration was maintained at C_0 total ≈ 2.6 ppm, based upon hydrogen entry into 200,000 psi (1.38×10^9 N/m²) yield strength 4340 steel.

Over the entire thickness range studied, the experimental data as shown in Figure 1, could be expressed by

$$da/dt = mB^n \quad (2)$$

with

$$\begin{array}{ll} 300^\circ\text{C} \left\{ \begin{array}{l} m = 5.6 \times 10^{-3} \\ n = 3.73 \end{array} \right. & 480^\circ\text{C} \left\{ \begin{array}{l} m = 2 \times 10^{-4} \\ n = 3.75 \end{array} \right. \\ \text{temper} & \text{temper} \end{array}$$

Here, B is specimen thickness in inches and m has the appropriate units to give da/dt in inches/second. This demonstrates that the thickness effect, in the range where the plane stress to plane strain transition takes place, can account for a factor of nearly 10^3 in da/dt for each of these two conditions.

How does this compare to that predicted by Eq. (1)? In these and similar high-strength, low alloy steels, there is some evidence that grain by grain cracking is appropriate for the predominantly intergranular process observed⁽¹⁾. If this is the case, then one could assess the stress gradient and l_{CR} at a grain diameter distance from the crack tip. In fact, for a given material and heat treatment, charged identically, and tested under identical environmental conditions, Eq. (1) is appropriate for both plane stress and plane strain if the crack grows approximately a grain at a time. Thus, a first approximation would be

$$da/dt \approx A dp/dx \quad (3)$$

where A is a constant containing the other terms in Eq. (1). For the above described test conditions, it is seen that the pressure tensor gradient is the major variable left. From elasticity theory, one can differentiate the pressure tensor⁽⁸⁾ and obtain

$$\left. \frac{dp}{dx} \right|_{\substack{\text{elastic} \\ \text{plane strain}}} = \frac{2K}{15} (1+\nu)x^{-3/2} \quad (4)$$

From plasticity theory⁽¹⁾, one can similarly determine that

$$\left. \frac{dp}{dx} \right|_{\substack{\text{plastic} \\ \text{plane strain}}} = \sigma_{ys}/(\rho+x) \quad (5)$$

where ρ is the crack-tip radius. For $\rho \ll x$, $K = 35,000 \text{ psi} \cdot \text{in}^{1/2}$, $\nu = 0.3$ and $x \approx d = 10\mu$ for the steel under investigation, one finds

$$\left. \frac{dp}{dx} \right|_{\substack{\text{elastic} \\ \text{plane strain}}} \approx \left. \frac{dp}{dx} \right|_{\substack{\text{plastic} \\ \text{plane strain}}} \approx 6 \times 10^8 \text{ psi/in} \quad (6)$$

For thin plates under plane stress, the stress gradient is quite mild and from experimental evidence^(11,12), appears to be on the order of

$$dp/dx \approx M \sigma_{ys} \approx 2 \times 10^6 \text{ psi/in} \quad (7)$$

Here, M is a constant of about 10 in^{-1} . Considering Eqs. (3), (6) and (7), it is seen that the crack growth rate should decrease by about a factor of 300 as one transcends from plane strain (thick plate) to plane stress (thin sheet)

conditions*. This agrees reasonably well with the extremes of Figure 1.

The conclusion is that if about the same critical concentration of hydrogen is required for fracture under plane strain and plane stress conditions, then the difference in growth kinetics can be explained solely on the basis of long range hydrogen diffusivity. Of course, it is recognized that differences in the state of stress under plane stress and plane strain could require different amounts of hydrogen locally to promote fracture. Thus, the observed difference in growth kinetics could be considerably greater than that predicted by the above model alone. Whatever the balance between these two factors, it is obvious that thickness or state of stress is a major variable to be considered in any hydrogen embrittlement model.

REFERENCES

1. W. W. Gerberich, "Effect of Hydrogen on High Strength and Martensitic Steels," Seven Springs Conf. on Hydrogen in Metals, September, 1973.
2. H. H. Johnson, Fracture, Vol III, p. 705, Academic Press (1971).
3. G. E. Kerns and R. W. Staehle, Scripta Met., **6**, p. 631 (1972).
4. D. P. Williams and H. G. Nelson, Met. Trans., **1**, p. 63 (1970).
5. H. G. Nelson, D. P. Williams and A. S. Tetelman, Met. Trans., **2**, p. 953 (1971).
6. R. A. Oriani, Acta. Met., **18**, p. 147 (1970).
7. B. F. Brown, Stress Corrosion Cracking - A State of the Art, p. 3, ASTM STP 518, Philadelphia (1972).
8. C. St. John and W. W. Gerberich, Met. Trans., **4**, p. 589 (1973).
9. J. H. Underwood, J. L. Swedlow and D. P. Kendall, Eng. Fract. Mech., **2**, p. 183 (1971).
10. K. S. Grewal, V. Weiss, G. Sachs and J. Sessler, SURI Rep. 604-6012, Syracuse University, December (1968).
11. E. T. Wessel, Eng. Fract. Mech., **1**, p. 77 (1968).
12. W. W. Gerberich and Y. T. Chen, "Hydrogen-Controlled Cracking - A Model for Threshold Stress Intensities," Met. Trans., in press.

* If one wanted to use the mean value theorem and average the gradient over some finite grain boundary region, this difference would be somewhat larger.

APPENDIX B-1

Exact Solutions of Mode I, II, III Problems
with Conformal Mapping

by

E. C. Aifantis and W. W. Gerberich

EXACT SOLUTIONS OF MODE I, II, III PROBLEMS WITH CONFORMAL MAPPING

E. C. Aifantis and W. W. Gerberich

ABSTRACT

An exact linear elastic solution is obtained for the problem of a crack in a semi-infinite plate subjected, at infinity, to anti-plane stress (Mode III) loading. The use of the conformal mapping technique results in a convenient stress and displacement solution for each point of the semi-infinite domain. For completeness, the solution technique is extended to Mode I, II problems with center-cracks as well as the Mode III V-notch problem. It is shown that the limiting case of the V-notch collapses to the stress functions independently derived for the edge-cut. At distances equivalent to 10 percent of the crack length away from the crack tip, the exact solutions give stresses about 7.5 percent greater than the one-term results. Ramifications of the exact solutions to finite-element solutions and elastic-plastic problems are discussed.

INTRODUCTION

Sneddon (1) was the first to give stress-field expansions in the vicinity of the crack for two individual cases. Irwin (2,3) and Williams (4) extended the partial results of Sneddon to develop field equations for the most general case for an isotropic elastic body (3). The main attention of these investigators was concentrated in the description of the stress-displacement fields in the neighborhood of existing discontinuities in infinite linear elastic homogeneous, isotropic media. Their work results in convenient formulas for the stress and displacement fields when an arbitrarily "small region" around the end of the singularity is considered. Investigators in fracture mechanics widely use these formulas which in reality are approximations of the exact solutions.

Sneddon and Lowengrub (5), by means of fourier transforms, obtained solutions for each point of an infinite linear elastic homogeneous, isotropic plate. The three most interesting crack boundary value problems, Mode I, Mode II and Mode III, are extensively discussed and analytical solutions are given in terms of bipolar coordinates. The authors indicate that the well known approximate solutions are deduced from their solutions when an arbitrarily "small region" around the tip of the crack is considered.

The most direct approach to the approximate determination of the stress and displacement fields associated with each of the above modes, follows in the manner of Irwin (2,6) based on the method of Westergaard (7). At this point, we might mention that the results of Westergaard were partial cases of Muskhelishvili (8) and his school.

In the present paper an alternative exact solution for the mode III displacement is presented. The solution holds for each point of a semi-infinite cracked, linear elastic, homogeneous isotropic medium. An appropriate conformal transformation is investigated which results in a

convenient solution formula. Numerical calculations are then made for the comparison of exact stress distribution to the approximate one-term solution.

STATEMENT AND SOLUTION OF THE PROBLEM

We consider a semi-infinite domain, with a crack of length a cut out of one side as shown in Figure 1. At infinity the domain is subjected to the uniform shearing stress

$$\tau_{xz} = \tau_{\infty} \quad , \quad \tau_{yz} = 0$$

If it were not for the presence of the crack, there would be a uniform state of stress throughout the domain. Because of the presence of the crack the stress is defined by

$$\sigma_x = \sigma_y = \sigma_z = \tau_{xy} = 0 \quad (1)$$

$$\tau_{xz} = \tau_x(x,y) \quad , \quad \tau_{yz} = \tau_y(x,y)$$

The displacements and strains are given by

$$u_x = u_y = 0 \quad , \quad u_z = w(x,y)$$

$$\epsilon_x = \epsilon_y = \epsilon_z = \gamma_{xy} = 0 \quad (2)$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} \quad , \quad \gamma_{yz} = \frac{\partial w}{\partial y}$$

The only equilibrium equation to be satisfied is

$$\frac{\partial \tau_x}{\partial x} + \frac{\partial \tau_y}{\partial y} = 0 \quad (3)$$

This will be identically satisfied by introducing a stress function ψ such that

$$\tau_x = \frac{\partial \psi}{\partial y}, \quad \tau_y = -\frac{\partial \psi}{\partial x} \quad (4)$$

Invoking the Cauchy-Riemann equations and defining a complex analytic function, $\Omega(z)$, one may show that

$$\frac{d\Omega}{dz} = \frac{\partial \psi}{\partial y} + i \frac{\partial \psi}{\partial x} = \tau_x - i \tau_y \quad (5)$$

Therefore, the "complex conjugate vector" is

$$T = \tau_x - i \tau_y = \frac{d\Omega}{dz} \quad (6)$$

The prescribed boundary conditions are

$$\tau_x = \tau_\infty, \quad \tau_y = 0$$

at infinity and

$$T_{ij} n_j = 0 \quad \text{on} \quad G^* A^* B^* C^* D^*$$

where T is the stress tensor and n the outnormal unit vector on the finite boundary. In terms of Ω , these boundary conditions take the form

$$\Omega'(z) = \frac{d\Omega}{dz} = \tau_\infty \quad \text{at} \quad \text{infinity} \quad (D^* E^* F^* G^*) \quad (7)$$

$$\text{Im } \Omega = 0 \quad \text{on} \quad G^* A^* B^* C^* D^*$$

Use of Conformal Mapping

It is necessary to map the upper-half z plane to the upper-half w plane with a unit semicircle removed requiring the slit $0 < z < ai$ to be mapped on the circumference of the unit semicircle.

To do that, we map first the upper-half plane ζ with the unit semicircle removed to the upper-half plane w requiring the circumference to be mapped on the slit $-1 < w < 1$ as is indicated in Figure 2. This mapping

is well known and is given by (9)

$$w = \frac{1}{2}(\zeta + \frac{1}{\zeta}) \quad (8)$$

As a second step we investigate the mapping of the w -plane to the z -plane as is also indicated in Figure 2. Actually we will map the z -upper half plane onto the w -upper half plane.

We easily can see that the region $G^*A^*B^*C^*D^*$ can be considered as the interior of a degenerate polygon with vertices

$$A_1 = G^* = D^*, \quad A_2 = A^*, \quad A_3 = B^*, \quad A_4 = C^*$$

and corresponding interior angles

$$\alpha_1 = -\pi, \quad \alpha_2 = \frac{\pi}{2}, \quad \alpha_3 = 2\pi, \quad \alpha_4 = \frac{\pi}{2}$$

Thus, we can apply a Schwarz-Christoffel transformation in order to map the interior of this polygon onto the w -upper half plane with the vertices of the polygon A_j , $j=1,4$ mapped on the points u_j of the real axis of the w plane.

This tranformation is given by the formula

$$z = f(w) = A \int \prod_{j=2}^3 (w - u_j)^{\frac{\alpha_j}{\pi} - 1} dw + B \quad (9)$$

Using the mentioned values of α_j , $j=1, \dots, 4$ and also choosing

$$u_1 = -\infty, \quad u_2 = -1, \quad u_3 = 0, \quad u_4 = 1$$

Eq. (9) is reduced to

$$z = A \sqrt{w^2 - 1} + B \quad (10)$$

The constants A and B are easily calculated by the imposed conditions

$$a1 = f(0) \quad ; \quad 0 = f(\pm 1)$$

The values of constants being known, Eq. (10) finally gives

$$z = a \sqrt{w^2 - 1} \quad (11)$$

Now combination of Eqs. (8) and (11) gives

$$z = \frac{a}{2} \left(\zeta - \frac{1}{\zeta} \right) \quad (12)$$

The above transformation maps conformally the physical upper half plane- z onto the upper half plane- w with the unit semicircle removed. The inverse transformation is taken from Eq. (12) as

$$\zeta = \frac{1}{a} \left(z \pm \sqrt{z^2 + a^2} \right) \quad (13)$$

By requiring the point at infinity in the z -plane to go to the point at infinity in the ζ plane we get from (13) the appropriate inverse transformation as

$$\zeta = \frac{1}{a} \left(z + \sqrt{z^2 + a^2} \right) \quad (14)$$

Solution in the ζ -plane

In the upper half ζ -plane with the unit semicircle removed, (Figure 2) we are seeking a complex potential $\chi(\zeta)$ satisfying the conditions

$$\text{Im } \chi(\zeta) = 0 \quad \text{on} \quad GABCD$$

$$\chi'(\zeta) = \frac{d\chi}{d\zeta} = \frac{d\Omega}{dz} \frac{dz}{d\zeta} = \tau_{\infty} \frac{a}{2} \quad \text{at infinity} \quad (15)$$

It is readily verified that an analytic function satisfying these conditions is

$$\chi(\zeta) = \frac{\tau_{\infty} a}{2} \left(\zeta + \frac{1}{\zeta} \right) \quad (16)$$

Solution in the Physical Plane

Using Eqs. (14) and (16) we find the complex potential $\Omega(z)$ for the physical plane to be given by

$$\Omega(z) = \tau_{\infty} \sqrt{z^2 + a^2} \quad (17)$$

Therefore the elastic stress distribution is given by

$$\tau = \tau_x - i\tau_y = \Omega'(z) = \tau_{\infty} \frac{z}{\sqrt{z^2 + a^2}} \quad (18)$$

We observe that this complex conjugate vector has exactly the form of Westergaard's stress function apart from an appropriate modification due to the orientation of the crack along the imaginary axis.

Now using the fact that the square root of a complex number can be written as

$$\sqrt{c + di} = \pm \left\{ \sqrt{\frac{c + \sqrt{c^2 + d^2}}{2}} + i \operatorname{sgn}(d) \sqrt{\frac{-c + \sqrt{c^2 + d^2}}{2}} \right\} \quad (19)$$

and also making use of the symmetry of the problem with respect to the y-axis (in order to stay with one sign, say +), we can obtain the stress fields.

We find from Eqs. (18) and (19) after some algebraic manipulations, the stress distribution for the region $x > 0$, $y > 0$ to be given by

$$\tau_x = \frac{\sqrt{2} \tau_{\infty} \left\{ x \sqrt{(x^2 - y^2 + a^2) + \sqrt{(x^2 - y^2 + a^2)^2 + 4x^2 y^2}} + y \sqrt{-(x^2 - y^2 + a^2) + \sqrt{(x^2 - y^2 + a^2)^2 + 4x^2 y^2}} \right\}}{2 \sqrt{(x^2 - y^2 + a^2)^2 + 4x^2 y^2}}$$

$$\tau_y = - \frac{\sqrt{2} \tau_{\infty} \left\{ y \sqrt{(x^2 - y^2 + a^2) + \sqrt{(x^2 - y^2 + a^2)^2 + 4x^2 y^2}} - x \sqrt{-(x^2 - y^2 + a^2) + \sqrt{(x^2 - y^2 + a^2)^2 + 4x^2 y^2}} \right\}}{2 \sqrt{(x^2 - y^2 + a^2)^2 + 4x^2 y^2}} \quad (20)$$

It is easily verified that the above solution formula has the following expected properties:

$$(i) \quad \tau_y = 0 \quad \text{on} \quad \{y = 0, x\} \quad ; \quad \tau_y \rightarrow 0 \quad \text{as} \quad x^2 + y^2 \rightarrow \infty$$

$$\tau_x = 0 \quad \text{on} \quad \{0 \leq y < a, x = 0\} \quad ; \quad \tau_x \rightarrow \tau_\infty \quad \text{as} \quad x^2 + y^2 \rightarrow \infty$$

i.e. the boundary conditions are satisfied.

$$(ii) \quad \tau_y = 0 \quad \text{on} \quad \{x = 0, y > a\}$$

i.e. symmetry is also satisfied

$$(iii) \quad \tau_x \rightarrow \infty \quad \text{as} \quad x^2 + (y - a)^2 \rightarrow 0$$

$$\tau_y \rightarrow -\infty \quad \text{as} \quad x^2 + (y - a)^2 \rightarrow 0, \quad \text{except in case (ii).}$$

i.e. the asymptotic behavior at the crack tip is satisfied.

The solution formula (20) also holds for an infinite linear elastic, isotropic, homogeneous plate with a central crack of length $2a$, under mode III loading. This is so, since the complete symmetry with respect to x-axis leaves the boundary conditions unchanged.

Since the stress field is described by the solution formula (20), the next step will be to describe the displacement field. The only non-vanishing displacement $u_z(x, y, z) = w(x, y)$ is given by the formula

$$w = \frac{1}{G} \operatorname{Re}[\Omega(z)] \quad (21)$$

and using Eq. (17) we get

$$w = \frac{1}{G} \operatorname{Re}[\tau_\infty \sqrt{z^2 + a^2}]$$

and finally

$$w = \frac{\tau_\infty}{G\sqrt{2}} \sqrt{x^2 - y^2 + a^2 + \sqrt{(x^2 - y^2 + a^2)^2 + 4x^2y^2}} \quad (22)$$

where G is the shear modulus.

This formula gives the displacement field for each point of the domain with $x > 0$, $y > 0$. Taking advantage of the symmetry with respect to the y -axis, we deduce the displacement field for each point of the initial semi-infinite domain.

As in the case of stresses, Eq. (22) also holds for an infinite, linear elastic, isotropic, homogeneous plate with a central crack of length $2a$ under mode III loading.

By examining Eq. (22) we deduce

$$(i) \quad w = 0 \quad , \quad \text{for} \quad \{x = 0, y > 0\}$$

$$(ii) \quad w = \frac{\tau_{\infty}}{G} \sqrt{a^2 - y^2} \quad , \quad \text{for} \quad \{x = 0, y < a\}$$

The above results were expected. Actually Sneddon, et al (5) used (i) condition as part of his mixed boundary conditions to solve the problem. At the end he came up with condition (ii).

COMPARISON WITH THE PREVIOUS SOLUTION

As we mentioned in the introduction, approximate expressions for the mode III problem near to the crack tip have been previously obtained. These are of the form (10)

$$\tau_x = \frac{K_{III}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \quad , \quad \tau_y = -\frac{K_{III}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \quad (23)$$

where $K_{III} = \tau_{\infty} \sqrt{\pi a}$ is the stress-intensity faction and r, θ are the polar coordinates as indicated in Figure 3(a).

Introducing the dimensionless variables

$$X = \frac{x}{a} \quad , \quad Y = \frac{y}{a} \quad , \quad T_x = \sqrt{2} \frac{\tau_x}{\tau_{\infty}} \quad , \quad T_y = \sqrt{2} \frac{\tau_y}{\tau_{\infty}}$$

and thus $R = \frac{r}{a}$ we get from Eq. (20)

$$T_x = \frac{X \sqrt{(X^2 - Y^2 + 1) + \sqrt{(X^2 - Y^2 + 1)^2 + 4X^2 Y^2}} + Y \sqrt{-(X^2 - Y^2 + 1) + \sqrt{(X^2 - Y^2 + 1)^2 + 4X^2 Y^2}}}{\sqrt{(X^2 - Y^2 + 1)^2 + 4X^2 Y^2}}$$

$$T_y = - \frac{Y \sqrt{(X^2 - Y^2 + 1) + \sqrt{(X^2 - Y^2 + 1)^2 + 4X^2 Y^2}} - X \sqrt{-(X^2 - Y^2 + 1) + \sqrt{(X^2 - Y^2 + 1)^2 + 4X^2 Y^2}}}{\sqrt{(X^2 - Y^2 + 1)^2 + 4X^2 Y^2}} \quad (20*)$$

and from Eq. (23)

$$T_x = \frac{1}{\sqrt{R}} \cos \frac{\theta}{2} \quad ; \quad (T_y = - \frac{1}{\sqrt{R}} \sin \frac{\theta}{2}) \quad (23*)$$

See Figure 3(b).

For convenience, we denote $TX1$, $TY1$ the approximate solution, Eqs. (23*) and $TX2$, $TY2$ our exact solution, Eqs. (20*).

For the comparison of these solutions near to the crack tip, three individual examples were considered, $\theta = 0^\circ$, $\theta = 60^\circ$ and $\theta = 90^\circ$. It is easily calculated that for these three particular cases the stress distribution is given by the formulas:

$$(i) \quad \theta = 0^\circ : \quad TX1 = \frac{1}{\sqrt{R}} \quad ; \quad TY1 = 0$$

$$TX2 = \frac{\sqrt{2} (R+1)}{\sqrt{R} \sqrt{R+2}} \quad ; \quad TY2 = 0$$

$$(ii) \quad \theta = 60^\circ : \quad TX1 = \frac{\sqrt{3}}{2 \sqrt{R}}$$

$$TX2 = \frac{\frac{\sqrt{3}}{2} R \sqrt{\frac{R}{2} - 1 + \sqrt{R^2 + 2R + 4}} + (1 + \frac{R}{2}) \sqrt{1 - \frac{R}{2} + \sqrt{R^2 + 2R + 4}}}{\sqrt{R} \sqrt{R^2 + 2R + 4}}$$

$$\begin{aligned}
 TY1 &= -\frac{1}{2\sqrt{R}} \\
 TY2 &= -\frac{(1+\frac{R}{2})\sqrt{\frac{R}{2}-1+\sqrt{R^2+2R+4}}-\sqrt{\frac{3}{2}}R\sqrt{1-\frac{R}{2}+\sqrt{R^2+2R+4}}}{\sqrt{R}\sqrt{R^2+2R+4}}
 \end{aligned}$$

$$\begin{aligned}
 (iii) \quad \theta = 90^\circ : \quad TX1 &= \frac{1}{\sqrt{2R}} \\
 TX2 &= \frac{R\sqrt{R+\sqrt{R^2+4}}+\sqrt{-R+\sqrt{R^2+4}}}{\sqrt{R}\sqrt{R^2+4}} \\
 TY1 &= -\frac{1}{\sqrt{2R}} \\
 TY2 &= -\frac{\sqrt{R+\sqrt{R^2+4}}-R\sqrt{-R+\sqrt{R^2+4}}}{\sqrt{R}\sqrt{R^2+4}}
 \end{aligned}$$

It can be easily shown that in all cases

$$\left. \begin{matrix} TX2 \\ TY2 \end{matrix} \right\} = \left. \begin{matrix} TX1 \\ TY1 \end{matrix} \right\} + G(R), \quad \text{where} \quad \lim_{R \rightarrow 0} \frac{G(R)}{R} = 0$$

The plots for the stresses on the above three individual cases are presented in Figures 4, 5 and 6 correspondingly.

The increment DR for these plots was taken equal to 0.01 and the initial distance from the crack-tip was taken equal to 0.000001. Data for increments $DR = 0.001$ and $DR = 0.0001$ were also obtained by the computer. A few selected values of R and the corresponding values of stresses for the first two cases are presented in Tables 1 and 2.

It is seen that relatively large differences exist even at distances equal to 5 percent of the crack length. For example, at an r/a of 0.1, the exact solution gives τ_{xz} values that are 7.5 percent greater than the one term solution. These differences increase to 30% for distances of $r/a = 0.5$. Analogous plots can be determined for Mode I and II solutions

as indicated in Appendix I. Here, again, differences of the same order given in Figures 4, 5, 6 and Tables 1, 2 would exist.

Significance to Finite Element Approaches

Having exact solutions might provide better matching functions for finite element analyses. To demonstrate this, it is sufficient to give a brief chronology of finite element approaches.

- (1966-1970) Conventional finite element approaches⁽¹¹⁻¹³⁾ were used without a singularity restriction at the crack-tip. Hilton and Sih⁽¹⁴⁾ showed these could give large errors.
- (1969-1973) Improved finite element approaches used on embedded singularity^(15,16) which restricted the solution near the crack-tip to a one-term asymptotic expansion.
- (1970-1973) Further improvement was made by using an embedded singularity with higher order elements^(17,18), e.g., the four-term eigenvalue displacement function rather than the one-term.

Considering the last two, Wilson⁽¹⁸⁾ demonstrated that if the restriction is made at large distances from the crack-tip, large computational errors in stress intensity could result using a one-term element as compared to a three-term one. This is not too surprising since, over a decade ago⁽¹⁹⁾, it was pointed out photoelastically that isochromatics matched the Williams 6-term solution closely while the one-term singularity underestimated the stresses by as much as a factor of two. Even when the singularity restriction is close to the crack tip, Wilson⁽¹⁸⁾ showed that the stress intensity calculated using a one-term element could differ by four percent from one calculated using a four-term element. The error could be even larger if additional terms were utilized. Furthermore, one wonders if such errors might not be magnified when the singularities are embedded in a plastic region that is large compared to the size of the core region containing the singularity.

Implications to Elastic-Plastic Solutions

There are small-scale yielding solutions, as exemplified by Rice⁽²⁰⁾, which rely on the elastic singularity to characterize the amplitude of the elastic-plastic field. These invariably have used one-term solutions. There are the computational procedures used by Hilton and Hutchinson⁽¹⁶⁾ and by Hilton and Sih⁽¹⁴⁾ where the embedded plastic singularity is used as a matching function near the crack-tip and the rest of the elastic-plastic region is allowed to follow from the standard finite element format. It is significant that both of these approaches rely on one-term solutions as matching functions. Now, using the later approach, Hilton and Sih shows agreement with the small scale yielding result for $\sigma_{\infty}/\sigma_{ys}$ up to 0.5. However, one wonders about this result since, at an r/a of 0.2 where the elastic-plastic boundary is depicted⁽¹⁴⁾, the exact solution for the stresses* deviates by about 12 percent from the one-term solution. This would represent a considerably larger deviation in the position of the elastic-plastic boundary.

We are led to the possibility that most elastic-plastic solutions for large scale yielding (and even those approaching small-scale yielding) will be in error if the exact solution is not appropriately incorporated into the numerical computation.

SUMMARY AND CONCLUSIONS

An exact convenient solution for the Mode III linear elastic problem was obtained by the conformal mapping technique. The solution describes the stress and displacement field in any point of the infinite medium. The importance of the solution exists in the fact that a successful and easy comparison may be made with well-known, one-term approximate solutions. In

* See Appendix I.

Appendix I, the above ideas are extended to Mode I and II crack problems. By comparison of these solutions to previous ones, we conclude:

1. The asymptotic behavior of the stress field is well illustrated.
2. A neighborhood of the crack tip defined by a critical radius $R^* = R^*(\theta)$, within which approximate and exact solutions are nearly the same, can be easily determined. For example, seven percent deviations in the stress field exist at $R^* = 0.1$, where the distance from the crack tip is normalized on crack length.
3. In several approaches used to solve both elastic and elastic-plastic problems (e.g. finite element analyses), approximate one-term elastic or plastic singularities have been used as matching solutions. Some of these approaches are clearly incorrect, and intuitively, all are questionable.
4. We suggest that the analytical solutions presented herein might provide more accurate matching solutions for computational schemes.
5. The method used (Schwarz-Christoffel transformation) is applicable to other Mode III problems where the geometry of the discontinuity varies. (See Appendix II). It is shown that the V-notch case collapsed to the edge-cut gives identical stress functions as found above for Mode III.

Acknowledgements: The support of the Atomic Energy Commission under contract AT-11-1-2212 is gratefully acknowledged. Thanks are also due to Professor George Stephanopoulos for valuable discussions.

REFERENCES

1. I. N. Sneddon "The distribution of stress in the neighborhood of a crack in an elastic solid" Proc. Royal Soc. London, Vol. A-187, 1946.
2. G. R. Irwin "Analysis of stress and strains near the end of a crack transversing plate" Transactions Am. Soc. Mech. Engrs., J. App. Mech. 1957.
3. G. R. Irwin "Fracture" Handbuch der Physik, Vol. VI, Springer, Berlin, 1958.
4. M. L. Williams "On the stress distribution at the base of a stationary crack" Transactions, Am. Soc. Mech. Engrs., J. Appl Mech. 1957.
5. I. N. Sneddon and M. Lowengrub "Crack problems in the classical theory of elasticity" J. Wiley and Sons, New York 1968.
6. G. R. Irwin "Fracture Mechanics" in Structural Mechanics, Pergamon Press, New York, N.Y. 1960.
7. H. M. Westergaard "Bearing pressure and cracks" Transactions Am. Soc. Mech. Engrs., J. App. Mech. 1939.
8. Muskhelishvili, N. I. (1953a) Some basic problems of the mathematical theory of elasticity, Groningen, P. Noordhoff Ltd.
9. H. Kober, Dictionary of Conformal Representations, Dover Publications, 1952.
10. Fracture toughness testing and it's applications. ASTM STP 381, Philadelphia 1965, p. 66.
11. J. L. Swedlow, M. L. Williams and W. H. Yang, Proc. First Int. Conf. Frac. 1, 1966, p. 259.
12. A. S. Kobayashi, D. E. Maiden, B. J. Simon, ASME Paper No. 69-WA/PVP-12, 1969.
13. S. K. Chan, I. S. Tuba and W. K. Wilson, Engrg. Fracture Mech., 2, No. 1, 1970, p. 1.
14. P. D. Hilton and G. C. Sih, in Mechanics of Fracture, 1, ed. by G. C. Sih, Noordhoff International Publishing, Leyden, 1973, p. 426.
15. W. K. Wilson, "On combined mode fracture mechanics," University of Pittsburgh, Ph.D. Dissertation, 1969.
16. P. D. Hilton and J. W. Hutchinson, Engrg. Fracture Mech., in press, 1974.
17. E. Byskov, Int. J. Fracture Mech., 6, No. 2, 1970, p. 159.

18. W. K. Wilson, in Mechanics of Fracture, 1, ed. by E. C. Sih, Noordhoff International Publishing, Leyden, 1973, p. 484.
19. W. W. Gerberich, Proc. Soc. Exp. Stress Anal., XIX, No. 2, 1962, p. 359.
20. J. R. Rice and G. Rosengren, J. Mech. Phys. Solids, 16, 1968, p. 13.

TABLE 1: COMPARISON OF MODE III STRESSES, ($\theta = 0^\circ$)

R	TX1, One-term	TX2, Exact	TY1, One-term	TY2, Exact
.000001	.1000000000 E+04	.1000000750 E+04	0	0
.000101	.9950371902 E+02	.9951125627 E+02	0	0
.000501	.4467670516 E+02	.4469349068 E+02	0	0
.001001	.3160697706 E+02	.3163070105 E+02	0	0
.005001	.1414072162 E+02	.1419370477 E+02	0	0
.010001	.9999500037 E+01	.1007434806 E+02	0	0
.050001	.4472091234 E+01	.4638081001 E+01	0	0
.100001	.3162201849 E+01	.3394659004 E+01	0	0
.200001	.2294151301 E+01	.2558403751 E+01	0	0
.500001	.1414212148 E+01	.1897365584 E+01	0	0
.990001	.1005037308 E+01	.1635742015 E+01	0	0

TABLE 2: COMPARISON OF MODE III STRESSES, ($\theta = 60^\circ$)

R	TX1, One-term	TX2, Exact	TY1, One-term	TY2, Exact
.000001	.8660254038 E+03	.8660260533 E+03	-.50000000 E+03	-.49999962 E+03
.000101	.8617274844 E+02	.8617927603 E+03	-.49751859 E+02	-.49748090 E+02
.000501	.3869116163 E+02	.3870569983 E+02	-.22338352 E+02	-.22329960 E+02
.001001	.2737244507 E+02	.2739299493 E+02	-.15803488 E+02	-.15791629 E+02
.005001	.1224622415 E+02	.1229215659 E+02	-.70703608 E+01	-.70438968 E+01
.010001	.8659821058 E+01	.8724775739 E+01	-.49997500 E+01	-.49624041 E+01
.050001	.3872944617 E+01	.4018157000 E+01	-.22360456 E+01	-.24262089 E+01
.100001	.2738599095 E+01	.2943852316 E+01	-.15811309 E+01	-.14673963 E+01
.200001	.1936486832 E+01	.2226181694 E+01	-.11180311 E+01	-.96377915 E+00
.500001	.1224743647 E+01	.1677270654 E+01	-.70710607 E+00	-.49174192 E+00
.990001	.8903878402 E+00	.1486763594 E+01	-.50251865 E+00	-.25276019 E+00

APPENDIX I

Currently, the authors have determined the exact stress and displacement fields for the Mode I and Mode II problems, in a semi-infinite cracked plate, using the previously developed method.

For completeness reasons we present here exact formulas of the stress field for the Mode I and Mode II problems in an infinite plate with a central crack of length $2a$. For this determination the Westergaard's stress function was used. It is easily shown that the exact expressions of the stress field $\{x, y\} \geq 0$ are:

Mode I

$$\begin{aligned} T_{xx} &= \frac{X\sqrt{A+B} + Y\sqrt{-A+B}}{B} - Y \frac{A\sqrt{-A+B} + 2XY\sqrt{A+B}}{B^3} \\ T_{yy} &= \frac{X\sqrt{A+B} + Y\sqrt{-A+B}}{B} + Y \frac{A\sqrt{-A+B} + 2XY\sqrt{A+B}}{B^3} \\ T_{xy} &= Y \frac{A\sqrt{A+B} - 2XY\sqrt{-A+B}}{B^3} \end{aligned} \quad (24)$$

Mode II

$$\begin{aligned} T_{xx} &= 2 \frac{Y\sqrt{A+B} - X\sqrt{-A+B}}{B} - Y \frac{A\sqrt{A+B} - 2XY\sqrt{-A+B}}{B^3} \\ T_{yy} &= Y \frac{A\sqrt{A+B} - 2XY\sqrt{-A+B}}{B^3} \\ T_{xy} &= \frac{X\sqrt{A+B} + Y\sqrt{-A+B}}{B} - Y \frac{A\sqrt{-A+B} + 2XY\sqrt{A+B}}{B^3} \end{aligned} \quad (25)$$

where

$$T_{xx} = \sigma_x \sqrt{2}/(\sigma_\infty \text{ or } \tau_\infty), \quad T_{yy} = \sigma_y \sqrt{2}/(\sigma_\infty \text{ or } \tau_\infty), \quad T_{xy} = \tau_{xy} \sqrt{2}/(\sigma_\infty \text{ or } \tau_\infty)$$

$$X = \frac{x}{a} \quad , \quad Y = \frac{y}{a}$$

$$A = X^2 - Y^2 - 1$$

$$B = \sqrt{(X^2 - Y^2 - 1)^2 + 4X^2Y^2}$$

Using Eqs. (24) and (25) and the corresponding approximate expressions from the one-term solutions, plots analagous to Figures 4, 5 and 6 may be obtained.

APPENDIX II

As was mentioned in the conclusions, the method is a powerful tool for getting analytical solutions for various linear geometries of the discontinuity. An example is illustrated here for a semi-infinite domain where now the discontinuity is a V-notch of angle $\pi - 2\theta$ and depth a (see Fig. 7).

We assume again linear elasticity, isotropy, homogeneity and the same condition at infinity. Following the same approach we map the above domain (considered as a degenerate polygon with interior angles π , $\pi - \theta$, $\pi + 2\theta$, $\pi - \theta$), onto the w -plane of Fig. 2.

It can be proved, using Eq. (9) and the properties of Beta and Gamma functions that this transformation is expressed finally as:

$$z = \frac{a \sqrt{\pi} (1 - i \tan \theta)}{\tan \theta \Gamma\left(\frac{\theta}{\pi} + \frac{1}{2}\right) \Gamma\left(1 - \frac{\theta}{\pi}\right)} \int_0^w \frac{w^{\frac{2\theta}{\pi}}}{(1 - w^2)^{\frac{\theta}{\pi}}} dw + a i \quad (26)$$

where Γ denotes the gamma function.

Now the mapping (9) is used to get the solution in the convenient ζ -plane (Fig. 2) using appropriate conditions at infinity. Going back to the physical plane and omitting the details of the successive steps we get finally

for the complex potential the form

$$\Omega(z) = \frac{\tau_{\infty} a \sqrt{\pi} (1 - i \tan \theta) i^{\frac{2\theta}{\pi}}}{\tan \theta \Gamma(\frac{\theta}{\pi} + \frac{1}{2}) \Gamma(1 - \frac{\theta}{\pi})} w(z) \quad (27)$$

and for the "complex conjugate stress vector" an intensity of the field

$$\Omega'(z) = \tau_{\infty} i^{\frac{2\theta}{\pi}} \left\{ \frac{1 - [w(z)]^2}{[w(z)]^2} \right\}^{\frac{\theta}{\pi}} \quad (28)$$

where $w(z)$ is the inverse transformation of Eq. (26).

The imaginary and real parts of $\Omega'(z)$ represent the stresses and the problem is solved in principle.

Actually for various values of θ , only numerical integration of Eq. (26) will give explicitly the inverse transformation $w(z)$. It is possible for certain values of θ to express the inverse transformation in terms of incomplete Beta functions. Finally it is remarkable to see that for the limiting case $\theta = \frac{\pi}{2}$, i.e. when the notch will shrink to a crack ($\tan \theta \rightarrow \infty$, $\Gamma(1) = 1$, $\Gamma(\frac{1}{2}) = \sqrt{\pi}$), the above analysis results into formulas which were independently derived above, Eqs. (17,18).

For completeness, it should be mentioned that when the discontinuity consists of orthogonal line segments the desirable mapping is expressed in terms of Elliptic integrals. Also in the case of an elliptic discontinuity a "transformation of boundaries in parametric form" could be introduced.

(a)

(b)

APPENDIX B-2

The Effect of Prior Austenitic Grain Size On
Hydrogen Assisted Failure of Type 4340 Steel

by

W. W. Gerberich and J. F. Lessar

Introduction

As of the present, there is much evidence to support the theory that environmentally enhanced embrittlement of martensitic steel involves atomic or molecular hydrogen (1-13).

The effect of test and material parameters such as temperature, environment, state of stress, and yield strength have already been investigated (5-17). However, these variables may either overshadow or be mistaken for the effects of other variables. For example, there is a strong tempering effect in high strength steels subjected to stress corrosion cracking or hydrogen embrittlement conditions. Is this due to a yield strength effect or a change in the precipitate morphology? In maraging steels, small changes in alloy additions or aging treatments can drastically change growth kinetics even though the yield strength remains relatively constant (18). Is this due to an electrochemical effect at the crack tip or a trapping effect for hydrogen diffusion? Another possible variable is the grain size which may not show up in martensitic steels since the prior austenitic grain size is usually constant (ASTM 6-8). However, assuming there is a difference, is it due to greater segregation of tramp elements or differences in hydrogen diffusion at the smaller grain boundary area presented by the larger grains?

There have been relatively few investigations as to the effect of prior austenitic grain size upon crack growth rate (da/dt) and threshold stress intensity (K_{TH}). One investigation by Proctor and Paxton emphasized the effect of prior austenitic grain size on the time to failure for stress corrosion cracking (SCC) of type 4340 steels (20). This involved only a narrow grain size range and no direct measure of crack growth kinetics. Nevertheless, their data clearly demonstrated that grain refinement led to higher tensile strengths and larger failure times during stress corrosion cracking. However, Carter's work (19) indicated that Si additions to type 4340 steel led to higher strengths and longer failure times (decreased da/dt) even though the grain size became coarsened. This observed opposite behaviors in modified 4340 steel during SCC was admittedly on two different types of material, but nevertheless raises the fundamental question of what the grain size effect really is. Furthermore, recent derivations on grain size effects on cracking thresholds and crack growth kinetics (1) indicated $K_{TH} \propto d^{1/2}$ and $da/dt \propto d^{-3/2}$ for elastically predominant behavior. It was of some practical and theoretical interest, then, to verify the grain size effect.

In an effort to improve upon previous investigations, the electrochemical effects were minimized by

going to a study of specimen that had been cathodically charged with hydrogen. A single temper of 4340 steel (300°C), which gave a yield strength near 200,000 psi (1320 MPa) and well defined Stage I crack growth kinetics was chosen as the base line material. The range of grain sizes studied was expanded to approximately five times that of Proctor and Paxton. Thus, the object of the study was to evaluate threshold stress intensity, yield strength, crack velocity, and mode of fracture for a 4340 steel varying in grain size from 20μ to in excess of 140μ .

Experimental

Double Cantilever Beam (DCB) specimens, as depicted in Figure 1, were used for this study (15, 21, 22). This specimen design was selected for its simplicity and the information that it yields. Because of its unique design, the DCB specimen has a useful stress intensity range from the plane strain fracture toughness, K_{IC} , to approximately $0.1 K_{IC}$. As an added benefit, usually two or more tests can be obtained from one specimen.

The monitoring of the crack velocity (da/dt) and the applied stress intensity (K), can be done by several methods. In this investigation, an Instron Tensile testing machine was used to record the applied load (P) on the specimen as a function of time (Figure 2).

On a fine scale this curve was not smooth but jagged; indicating discontinuous crack propagation, characteristic of materials that can fail by hydrogen assisted cracking (5-3, 14, 20, 23). Using the load versus time curve the instantaneous applied stress intensity (K) and crack length can be found as of function of time. The crack growth rate (da/dt) can then be calculated as of function of K. At a crack growth rate equal to zero, the applied stress intensity (K) is equal to the materials' threshold stress intensity for hydrogen assisted cracking (K_{TH}).

The double cantilever beam (DCB) specimens were machined from a commercial billet of 4340 type steel (Table 1). Light microscopy examination of material from this billet showed the material to contain an estimated 0.45 volume percent of large, parallel, stringer type inclusions. Microprobe analysis determined the inclusions to be composed of Manganese and Sulfur in an almost stoichiometric composition. The DCB specimens were machined so as to align the inclusions along the specimen length. A compliance curve was then experimentally determined for this specimen type and material (Figure 1); a computer curve-fit of compliance (c) as of function of crack length (a) being given by:

$$c = v/P = 10^{-6} \text{ in/lb} (24.36 - 14.12a \text{ in}^{-1} + 10.23a^2 \text{ in}^{-2} + 1.469a^3 \text{ in}^{-3}) \quad (1)$$

Here, (v) represents the specimen deflection at the loading point and (P) is the applied load necessary to produce such a deflection.

The applied stress intensity was calculated from the load (P) versus time (t) curve (Figure 2), at constant displacement (v), using the following equations (21):

$$v/P = c = f(a) \quad (2a)$$

$$a = f(c) = \ln (7.235 \times 10^{-2} + 5.414 \times 10^4 c \text{ lb/in} - 3.547 \times 10^8 c^2 \text{ lb}^2/\text{in}^2 + 9.243 \times 10^{11} c^3 \text{ lb}^3/\text{in}^3) \quad (2b)$$

$$G = (P^2/2B) dc/da \quad (2c)$$

$$K = (EG)^{1/2} \quad (2d)$$

where (a) is the crack length (inch), (v) is the deflection (inch), (P) is the applied load (pound), (G) is the strain energy release rate (psi - in) and (K) is the applied stress intensity (psi - in^{1/2}). Equation (2b) was obtained by curve-fitting $a = f(c)$ with the same experimental compliance data used to curve fit equation 1, $c = f(a)$. Crack tip velocity was calculated from the load versus time curve. At different loads P_1 and P_2 , corresponding to times t_1 and t_2 ,

crack lengths a_1 and a_2 were calculated. The average crack velocity, da/dt , was then estimated over this increment.

For heat treatment, an inert argon atmosphere furnace was utilized to attain varying grain sizes. Various austenitizing times at 900°C, 1000°C, and 1100°C gave the results shown in Figure 3. Specimens austenitized at 1100°C were furnace cooled to 900°C before quenching to avoid possible quench crack formation, and all specimens were tempered at 300°C for three hours. As a material characterization, the 0.2 percent offset yield strength, tensile strength, and elongation at fracture were determined using tensile specimens machined such that the stringer inclusions were aligned normal to the specimen thickness. After heat treatment to the desired grain size, the specimens were surface ground to remove any possible decarburized surface material. The specimens were tested on a 10,000 lb Instron at a cross-head speed of 0.05 in/min with an extensometer attached to the specimen. The results, as indicated in Table 1, are normal except for relatively low fracture stresses and strains. It is possible that the inclusions were responsible for low fracture stresses, although Wood, et al (24), observed low fracture stresses and strains for similar steels austenitized at high temperatures.

Prior to testing, the DCB specimens were cathodically charged in a four weight percent sulfuric acid solution plus a poison (6, 3, 9). The poison was composed of 2 grams of yellow phosphorous in 40 ml of carbon-disulfide with the poison being added at a ratio of 10 ml of poison to 900 ml of the dilute acid. The charging current density was calculated according to (8):

$$H_2 = 13.3 + 2.7 \log I \quad (3)$$

where H_2 is the hydrogen content (ppm) and I is the current density (amp/in²).

Following hydrogen charging the specimen were cadmium plated in a sodium cyanide - cadmium oxide plating solution. The plating thickness was in all cases, greater than 0.001 inches. The specimen were then baked at 150°C for seven hours to assure a uniform hydrogen concentration. In all cases, the charging current was adjusted to give a uniform hydrogen content estimated to be approximately 2.5ppm after baking.

Several crack tip velocity versus stress intensity curves were plotted for each grain size studied. On each of these curves the best line representative of Mode I cracking was drawn through the experimental data points. Taking points lying on the lines representing Mode I cracking, an average Mode I cracking velocity (da/dt) versus applied stress intensity (K)

was calculated for each of the grain sizes studied. Points representative of Mode II cracking were determined in a similar fashion.

Results and Discussion

Effect on Yield Strength

Yield strength plotted versus average grain size in Figure 4 show that as the grain size decreases the yield strength increases. Experimental data obtained by Proctor and Paxton (20) show similar behavior. A plot of yield strength versus grain size ($d^{-1/2}$), in Figure 5, shows that the data from Proctor and Paxton is higher than the results observed for the materials used in this study. However, the slope of the straight lines through the data points are essentially the same. The 300°C temper, in this study, as compared to Proctor and Paxton's 200°C temper would only partially explain the downward shift in yield strength. In fact, data from Gerberich, et al (25), and Carter (19)* also fell on the curve for the present investigation even though Carter's data represented a 200°C temper condition. One suspects that there is something inherently different in Proctor and Paxton's 4340 steel as compared to other studies. The yield strength difference is certainly not an interstitial effect since the carbon

*The two grain sizes reported from Carter (19) here were of low silicon content, and well within the range encountered for commercial 4340 steel.

content of Proctor and Paxton's steel was 0.33 wt % C while the average of the others was 0.42 wt % C. The obvious difference was in the way the materials were heat treated, since Proctor and Paxton used a thermal cycling technique to obtain their ultra-fine grained material. It is well documented (26, 27, 28) that thermal cycling using the reverse austenite $\rightleftharpoons$ martensite transformation will produce a thermal-mechanical like strengthening, similar to that produced by ausforming. Because of the defect density inherited from the austenite, martensite produced by thermal cycling would have higher strengths for a commensurate grain size produced by conventional treatments.

Effect on K_{TH}

The observed effect of prior-austenitic grain size on K_{TH} is given numerically in Table 1 and graphically in Figure 6. K_{TH} appears to first decrease and then increase with increasing grain size. However, the previous data point (25) for cathodically charged fine grained 4340 produced a low threshold; hence, the dotted line at the fine grain sizes. Also plotted here are the data from Proctor and Paxton (20) and Carter (19) for K_{ISCC} observed in aqueous solutions. One could almost say that below 40 μ , there seems to be no grain size dependence while above 40 μ , K_{TH} increases with increasing grain size. The change of

K_{TH} with prior austenitic grain size changes may be attributed to several origins; two possibilities are the influence of grain size on yield strength, and a change in the fracture unit as the grain size increases.

A direct comparison of K_{TH} to grain size is probably misleading since yield strength is known to have a drastic effect on the threshold. In Carter's study (19), for relatively large silicon additions, yield strength increased with increasing grain size. In all other data, the $d^{-1/2}$ dependence indicated in Figure 5 was followed. An attempt was made to normalize the yield strength effect with a relationship from a previous investigation (1). It was proposed that for nearly elastic behavior, the threshold could be given by:

$$K_{TH} = \ln \left(\frac{3(2\gamma d)^{1/2} C_{CR} RT}{2(1 + \nu) C_0 \bar{V}_H} \right) \quad (4)$$

where C_{CR} was the critical hydrogen concentration, C_0 was the initial concentration, T was temperature, $\bar{V}_H$ was the partial molar volume of hydrogen in iron, ν was Poisson's ratio and d was the average grain size. This is a reasonable estimate as long as the plastic zone size is very small, i.e., less than the grain size. Furthermore, it was found that threshold stress intensities could be predicted for a large number of 4340 type alloy steels if $(C_{CR}/C_0) = 460,000 \text{ psi}/\sigma_{YS}$. By

using this previously determined correlation, one can normalize the yield strength by plotting: $K_{TH}^2 / \{\ln(C_{CR}/C_0)\}^2$ versus grain size, d . Since the rest of the terms in Eq. 4 should be nearly constant at room temperature, one should obtain a direct proportionality. Such a plot is shown in Figure 7. If one disregards Proctor and Paxton's data for the moment, one finds that there is a tendency for the normalized threshold parameter to increase with grain size. This is fairly significant considering that yield strength both increases (19) and decreases (present investigation) with increasing grain size in the two sets of data where the trend is observed.

Considering Proctor and Paxton's data, it is the two conditions with the finest grain sizes that deviate from the other data. These were the thermally-cycled materials and it is known that this and similar thermal-mechanical treatments can increase fracture resistance (26-28). For example, Ault, et al (28), have shown that the resistance of 4340 steel to SCC may be increased by ausforming. Comparing their 245,000 psi (1670 MPa) yield strength conventional steel to their 265,000 psi (1830 MPa) ausformed steel, the time to failure increases by about a factor of eight. Making the same comparison in Proctor and Paxton's data, one again finds a factor

of eight increase in time to failure for conventional to thermally-cycled 4340 steel. Thus, the reverse trend indicated in Figure 7 is quite probably a microstructural effect associated with thermal-cycling but not fine grain size, per se.

Effect on da/dt

A major interest in this study was to determine the effect of prior austenitic grain size on the crack growth rate. Plotted in Figure 3 are the averaged values of crack velocity versus applied stress intensity for the grain sizes studied. The crack velocities observed are the same magnitude as those observed by Peterson (29) for 4340 steel in 3.5 percent NaCl.

Of the three modes of crack growth (8, 19, 30-33), only stage I and stage II regions were observed. The stage I regions for the 50 μ , 90 μ , and 140 μ grain sized steels were linear up to the stage II region and had almost identical slopes. The curve representing the smallest grain size material differed from curves of the larger grain sized materials. The larger grain sized materials showed a moderately sharp transition from stage I to stage II crack growth, while the smallest grain size material displayed a gradual transition from stage I to stage II. The stage II crack velocities appeared to decrease slightly with increasing grain size.

The observed stage II velocities agreed with those reported by Carter (3, 19) and Sullivan (34) for 4340 steel of similar strength.

For comparison of crack velocities to a theoretical approach (1), data from 4340 steels tested in both aqueous solutions (19) and the hydrogen-charged condition (25, present investigation) were collected. Crack growth rates at an applied stress intensity in the vicinity of $30,000 \text{ psi-in}^{1/2}$ ($33 \text{ MPa-m}^{1/2}$) were determined where possible. For several of Carter's alloys (19), the range of stress intensity had to be increased to about $35,000 \text{ psi-in}^{1/2}$ ($33.5 \text{ MPa-m}^{1/2}$) in order to obtain reported crack velocities. In all cases except one, these represented stage I crack growth kinetics. These crack velocities, as observed at nearly constant stress intensity, were plotted as a function of grain size in Figure 9. Although there is some scatter, it appears as though grain size has a strong influence, with da/dt inversely proportional to the square of the grain size. This would be opposite to the effect suggested by Proctor and Paxton (20), as inferred from their time to failure observations. But, again, one should not make a direct comparison because of the aforementioned thermal cycling complication.

The theoretical comparison was made to a correlation derived for stage I growth kinetics (1). There,

it was suggested that the pressure tensor gradient at the crack tip was the driving force for hydrogen diffusion, which led to a kinetic growth dependence given by:

$$\left(\frac{da}{dt}\right)_I = \frac{2(1+Z') C_O D_A \bar{V}_H K}{3d^{3/2} RT (C_{CR} - C_O)} \quad (5)$$

Here, D_A is the apparent diffusivity of hydrogen in steel, K is the applied stress intensity and the other symbols are as above. Although the crack growth rate data do not suggest a linear dependence with K , the trend is in the right direction. More significant to the present study, the strong grain size prediction is observed in Figure 9.

In an attempt to normalize the crack growth rate data, it was decided to keep all parameters on the right hand side of Eq. 5 constant except grain size and applied stress intensity. First, it was assumed that $2(1+Z') \bar{V}_H / 3RT$ was constant, which is reasonable for a single class of high-strength steels tested at room temperature. Secondly, it was assumed that D_A was essentially independent of grain size and microstructure in the steels considered. This is a far more tenuous assumption since it is known that large changes in alloy content can promote precipitation and hence increasing numbers of trapping sites. This, in turn,

can decrease apparent diffusivity by more than an order of magnitude. Nevertheless, the alloy content was not drastically changing. This, in conjunction with nearly constant tempering temperatures in the range of 200-300°C, would indicate that the microstructures, except for the grain sizes, were not too different. Perhaps the exceptions are the two highest silicon bearing 4340 heats investigated by Carter (19). The only other variable besides K is $C_o/(C_{CR} - C_o)$. Taking the same empirical correlation as was used for the threshold, a normalization was affected by keeping $K C_o/(C_{CR} - C_o)$ constant. This was achieved by calculating the value of $C_{CR}/C_o = 430,000 \text{ psi}/\sigma_{ys}$ from the measured yield strengths. Then, K was picked to give $K C_o/(C_{CR} - C_o)$ to be 20,070 $\text{psi-in}^{1/2}$ (22 MPa- $\text{m}^{1/2}$) in the case of cathodically-charged steels and 35,130 $\text{psi-in}^{1/2}$ (38.5 MPa- $\text{m}^{1/2}$) in the case of those tested in aqueous solutions. The choice of these two constant values was dictated by the range of da/dt versus K data available. A plot of da/dt versus grain size for constant $K C_o/(C_{CR} - C_o)$ should result in a $d^{-3/2}$ dependence if all other parameters are constant. As indicated in Figure 10, a d^{-1} to d^{-2} dependence is roughly observed. The data are sufficiently scattered so as to rule out any substantial difference between the two environmental conditions. Furthermore, the data are not inconsistent

with the $d^{-3/2}$ dependence predicted by Eq. 5.

Fractography

Following testing, the fracture surface of each specimen was examined to determine the mode (s) of failure. In the heat treatment and prior austenite grain size range investigated, the failure mode was, for all grain sizes, intergranular with small areas of tear as shown in Figure 11. Intergranular failure has been found to be common for high strength steels undergoing hydrogen assisted cracking (6, 8, 9). The failure mode appeared independent of crack length for the stress intensity range studied. Although intergranular failure was observed for all of the grain sizes, failure differences were observed for the different grain sizes.

The fracture surfaces of the smaller grain sized materials (20-50 μ) displayed random grain sized areas of tear as shown in Figures 11a and 11b. The size and random occurrence of the tear areas may indicate that grain orientation may determine whether intergranular failure or failure by tear is the energetically favored failure mode for the individual grain. Failure near or between inclusions was intergranular with small areas of tear; Figures 12a and 12b illustrate this phenomena. The smaller grain sizes, in general, displayed a greater

amount of failure by tear than was observed for the larger grain sized materials.

Tear failure, occurring in the materials with grains 90 microns or larger, was restricted to near or between the stringer inclusions. Both the inclusions and areas of tear are seen in Figures 12c and 12d. The tear failure nucleated at an inclusion would continue from inclusion to neighboring inclusion if the separation distance was not too great. At large inclusion separations the failure mode would change from ductile tear, at the inclusion, to intergranular failure. Very few grain sized areas of tear were observed in the larger grain sized materials. The absence of the grain sized tear areas could be explained by the additional energy necessary for complete ductile tear of the larger grains. The additional energy needed to tear the larger grains would probably negate any possible grain orientation effect as was postulated for the smaller grain sized materials. This could explain the decrease in the amount of tear and absence of grain sized tear regions in the larger grained, lower yield strength materials. Normally, as the yield strength decreases the amount of failure by tear increases. Since this did not occur, it would seem that the effect of increased grain size on decreased crack growth kinetics was due to a diffusion

limiting process and not an enhanced cohesive energy at the grain boundaries.

Conclusions

1. Prior austenite grain size has a mild effect on threshold stress intensities (K_{ISCC} , K_{TH}) in stress corrosion and hydrogen charged tests of 4340 steels. The effect is most pronounced for grain sizes greater than 40μ where a doubling of the grain size will increase K_{TH} by approximately 25 percent.

2. Thresholds normalized on a yield strength-concentration dependent parameter, increased less rapidly than the $d^{1/2}$ proportionality predicted from elasticity considerations.

3. Prior austenitic grain size has a large effect on crack growth kinetics (da/dt), with an inverse grain size squared dependence being observed at nearly constant applied stress intensities.

4. Crack growth rates, normalized to a constant stress/intensity-concentration term, agree reasonably well with a $d^{-3/2}$ dependence predicted by a pressure tensor gradient model of hydrogen diffusion.

5. The primary failure mode for all grain sizes was intergranular failure. The smaller grained sized materials (20μ and 50μ) contained grain sized areas of ductile tear. A smaller amount of ductile tear

was observed in the larger grained materials (90 μ and 140 μ) and this only occurred at inclusions.

6. The larger grained materials had the lower growth rates, but the least ductile rupture, it is tentatively concluded the kinetics are diffusion limited and not due to an enhanced cohesive energy at the grain boundaries of large grained materials.

ACKNOWLEDGMENT: This work was supported by The United States Atomic Energy Commission under Grant AT-11-1-2212.

REFERENCES

1. W. W. Gerberich, Hydrogen in Metals, ed. by I. M. Bernstein and A. W. Thompson, Am. Soc. for Metals, 115, 1974.
2. W. W. Gerberich and Y. T. Chen, Scripta Met., 1974, vol. 8, p. 243.
3. A. E. Schuetz and W. D. Robertson, Nace Technical Committee Report, 1957, publication 57-17, vol. 13, p. 437.
4. W. A. Van Der Sluys, Eng. Frac. Mech., 1969, vol. 1, p. 447.
5. C. S. Kortovich and E. A. Steigerwald, Eng. Frac. Mech., 1972, vol. 4, p. 637.
6. H. H. Johnson, J. G. Morlet and A. R. Troiano, Trans. AIME, 1953, p. 523.
7. G. L. Hanna, A. R. Troiano and E. A. Steigerwald, Trans. ASM, 1964, vol. 57, p. 653.
8. C. R. Barth and E. A. Steigerwald, Met. Trans., 1970, vol. 1, p. 3451.
9. E. A. Steigerwald, F. W. Schaller and A. R. Troiano, Trans. AIME, 1960, vol. 218, p. 832.
10. C. F. Barth, E. A. Steigerwald and A. R. Troiano, Corrosion, 1969, vol. 25, no. 9, p. 353.
11. G. Sandoz, Met. Trans., 1972, vol. 3, p. 1159.
12. C. D. Beachem, Met. Trans., 1972, vol. 3, p. 437.
13. C. St. John and W. W. Gerberich, Met. Trans., 1973, vol. 4, p. 539.
14. B. F. Brown, Met. Review, 1963, vol. 13, p. 171.
15. B. F. Brown, The Theory of SCC in Alloys, ed. by J. C. Scully, p. 156, W. J. Laney & Son, Leeds, England, 1971, C.D.
16. B. F. Brown and C. D. Beachem, Corr. Sci., 1965, vol. 5, p. 745.

17. W. A. Van Der Sluys, Trans. ASME, 1967.
18. A. J. Stavros and H. W. Paxton, Met. Trans., 1970, vol. 1, p. 3049.
19. C. S. Carter, Corrosion, 1969, vol. 25, no. 10, p. 423.
20. R. P. M. Proctor and H. W. Paxton, Trans. ASME, 1962, vol. 62, p. 989.
21. H. V. Hyatt, Corrosion, 1970, vol. 26, no. 11, p. 487.
22. C. Y. Freed and J. M. Krafft, Journal of Materials, 1966, vol. 1, no. 4, p. 770.
23. A. R. Troiano, Trans. ASME, 1960, vol. 52, p. 54.
24. W. E. Wood, E. R. Parker and V. F. Azchay, "An Investigation of Metallurgical Factors Which Affect the Fracture Toughness of Ultra High Strength Steel", Report #LBL - 1474, Univ. of Cal., Berkeley, 1973.
25. W. W. Gerberich, Y. T. Chen and C. St. John, "A Short Time Diffusion Correlation for Hydrogen Induced Crack Growth Kinetics," submitted to Met. Trans., 1974.
26. B. Hyall and G. Krauss, Jr., Trans. ASME, 1968, vol. 61, p. 169.
27. T. J. Koppenaal and E. Gold, Met. Trans., 1972, vol. 3, p. 2965.
28. R. T. Ault, K. O. McDowell, P. L. Hendricks and T. M. S. Ronald, Trans. ASME, 1967, vol. 60, p. 79.
29. M. H. Peterson, B. F. Brown, R. L. Newbegin, and R. E. Groover, Corrosion, 1967, vol. 23, p. 1942.
30. S. Mostovoy, H. R. Smith, R. G. Lingwall and E. J. Ripling, Eng. Frac. Mech., 1971, vol. 13, p. 291.
31. C. S. Carter, Eng. Frac. Mech., 1971, vol. 3, p. 1.
32. H. L. Dunegan and A. S. Tetelman, Eng. Frac. Mech., vol. 2, p. 307.

33. G. E. Morris and R. W. Stockle, Scripta Met., 1972, vol. 6, p. 651.
34. A. H. Sullivan, Eng. Prog. Mech., 1974, vol. 4, p. 65.

NOMENCLATURE

K_{TH}	Threshold Stress Intensity
K	Applied Stress Intensity
K_{IC}	Plane Strain Fracture Toughness
d	Grain Size
ν	Poisson's Ratio
$\bar{V}_H$	Partial Molar Volume of Hydrogen in Iron
da/dt	Crack Growth Rate
R	Gas Constant
C_{CR}	Critical Hydrogen Concentration
C_0	Initial Hydrogen Concentration
T	Temperature
DCB	Double Cantilever Beam Type Specimen
D_A	Apparent Diffusivity of Hydrogen in Steel

LIST OF FIGURES

Table 1	Material Properties
Figure 1	Specimen Design and Compliance
Figure 2	Typical Load Versus Time
Figure 3	Grain Size Versus Time and Temperature
Figure 4	Grain Size Versus Yield Strength
Figure 5	Yield Strength Dependence on Inverse - Square Root of the Grain Size
Figure 6	Threshold Stress Intensity Versus Grain Size
Figure 7	$\frac{k^2 T}{C_0^2} \frac{1}{(\ln \frac{C_0}{C_0})^2}$ Versus Grain Size
Figure 8	Crack Velocity Versus Applied Stress In- tensity
Figure 9	Crack Velocity Versus Grain Size for Constant Applied Stress Intensity
Figure 10	Crack Velocity Versus Grain Size for Normalized Applied Stress Intensity
Figure 11	Specimen Fractography; Fracture Surface
Figure 12	Specimen Fractography; Inclusions

MATERIAL PROPERTIES - 4340 STEEL
COMPOSITION: 0.43C, 0.77Mn, 0.015P, 0.070Si, 1.73Ni, 0.89Cr, 0.23Mo
WT. %

γ -TEMP.	γ -TIME	GRAIN-SIZE	HARDNESS	YIELD STRENGTH	ULTIMATE STRENGTH	ELONG. STRAIN	K _{TH}
C°	hr	ASTM#	Rc	1000 PSI	1000 PSI	in/in	1000 PSI-in ^{1/2}
900°C	1	8.3	52	212-217	242-256	0.0206- 0.0327	28.3
1000°C	1	5.5	51-52	197	210-221	0.0116- 0.0156	24.1
1100°C	4	3.9	50-51	190-192	225-229	0.0232- 0.0258	26.1
1100°C	16	2.8	49-50	174-181	210-216	0.0198- 0.0245	31.3

* 300°C TEMPER

TABLE I

FIGURE 1

FIGURE 2

FIGURE 3

FIGURE 4

FIGURE 5

FIGURE 6

FIGURE 7

FIGURE 8

FIGURE 9

FIGURE 10

(a)

40μ

(b)

40μ

(c)

40μ

(d)

80μ

Figure 1

Figure 12

APPENDIX C

A Theoretical Review of Stress Relaxation Testing

by

E. C. Aifantis and W. W. Gerberich

A THEORETICAL REVIEW OF STRESS RELAXATION TESTING

By

E. C. Aifantis and W. W. Gerberich

ABSTRACT

Several theoretical approaches to stress relaxation testing are reviewed with the finding that all are internally consistent. The interesting results are that self-consistency identities prove that a constant dislocation mobility and a constant internal structure stress are commensurate with the assumptions that $(\partial\sigma/\partial \ln \dot{\epsilon}_p)_{\epsilon_p}$ is constant and $(\partial\sigma/\partial \epsilon_p)$ is independent of strain rate. The first of two self-consistency identities leads back to the Gilman-Johnston dislocation mobility analysis. The second identity leads to Feltham's empirical equation for stress relaxation testing, viz.

$$\sigma_0 - \sigma = S \log_{10}(1 + \nu t)$$

where σ_0 and σ refer to stresses at time zero and time, t , and S , ν are constants related to the slope and curvature of the stress relaxation data. It is shown that this consistent behavior can be easily utilized to interpret dislocation mobility constants at large strains. On the other hand, at small strains where "anomalous" stress relaxation has been observed, experimental data on Polycrystalline Fe show an inconsistency with the above approaches. The conclusion is that the mobile dislocation density is not constant at strains near yield.

A THEORETICAL REVIEW OF STRESS RELAXATION TESTING

by

E.C. Aifantis and W. W. Gerberich

Department of Chemical Engineering and Materials Science
University of Minnesota
Minneapolis, Minnesota 55455

INTRODUCTION

In the analysis of plastic deformation phenomena in solids, details of the interactions of solute additions with dislocations has received much attention^(1,2). Specifically, dynamic strain ageing^(3,4), solute strengthening^(5,6) or solute softening^(7,8) may play dominant roles with respect to the dynamics of dislocation motion. To assess some of the factors such as thermal activation and strain rate sensitivity, either change of strain rate or stress relaxation tests have been used. Both of these can give such parameters as strain rate sensitivity, $d\sigma/d\ln\dot{\epsilon}$, the thermal stress for dislocation motion, σ^* , and the exponent on the dislocation velocity function, m^* .

Trouton and Rankine⁽⁹⁾ and Feltham^(10,11) gave some of the early empirical laws on stress relaxation testing while Guin and Pratt⁽¹²⁾ as well as Lloyd, et al⁽¹³⁾ indicated how dynamical dislocation properties could be established. Finally, Sleeswyk, et al⁽¹⁴⁾ demonstrated that stress relaxation testing must be self-consistent by showing that separate values of $(d\sigma/d\ln\dot{\epsilon})_p$ derived from two independent experimental variables were equal. The following paper reviews these last two results and illustrates that (i) the dislocation dynamics argument is self-consistent and (ii) the empirical law of Feltham⁽¹¹⁾ is verified. Also, comparison to dislocation dynamics data for Fe at 77°K substantiates the approach and leads to a rapid interpretation

of stress relaxation testing. A caution is added for the initial stages of yielding, where stress relaxation testing is shown to be inconsistent.

REVIEW

Stress Relaxation Following Lloyd, et al⁽¹³⁾

Consider a typical relaxation curve as illustrated in Figure 1. The specimen is loaded in a displacement-controlled machine and then stopped at a constant displacement. Having arrived at a stress-strain point indicated by σ_0, ϵ_0 in a time dictated by $\dot{\epsilon}_0$, the stress then relaxes according to $\sigma = \sigma(t)$, some logarithmic function. The characteristic of the relaxation experiment is that the strain remains constant, or

$$\epsilon_{\text{total}} = \epsilon_p + \epsilon_e = \text{const.} \quad (1)$$

where ϵ_p and ϵ_e are the plastic and elastic components of the total strain. On differentiation, Eq. (1) becomes

$$\dot{\epsilon}_t = \dot{\epsilon}_p + \dot{\epsilon}_e = 0 \quad \text{or} \quad \dot{\epsilon}_p = -\dot{\epsilon}_e \quad (2)$$

This simply means that any relaxation in the elastic stress must be matched by a corresponding plastic deformation in the specimen, the rates of elastic and plastic deformation being equal. One can write the elastic strain rate as

$$\dot{\epsilon} = \frac{\dot{\sigma}}{E} \quad (3)$$

where $\dot{\sigma}$ is the applied stress rate and E is the combined modulus of elasticity of the machine and the specimen. Obviously, for an infinitely stiff machine, E would be the modulus of elasticity of the specimen if the initial loading were into the elastic range. Loading into the plastic range, one defines E by⁽¹⁴⁾

$$E = \frac{1 + \epsilon_{p0}}{C} \quad (4)$$

where ϵ_{p0} is the prior plastic strain as indicated in Figure 1 and C is defined by

$$C = \frac{A_0}{l_0 S} \quad (5)$$

Here, A_0 is the cross-sectional area of the undeformed specimen, l_0 is the initial gage length and S is the combined stiffness of the undeformed specimen and the testing machine. Using the well-known formulation of Johnston and Gilman⁽¹⁵⁾, one defines the plastic strain rate in terms of mobile dislocation density, ρ_m , by

$$\dot{\epsilon}_p = \phi b \rho_m \bar{v} \quad (6)$$

where ϕ is an orientation factor of about $\frac{1}{2}$, b is the burgers vector and $\bar{v}$ is the average dislocation velocity. From experimental observations⁽¹⁵⁾, one can empirically define dislocation velocity by

$$\bar{v} = B(\sigma^*)^{m^*} \quad (7)$$

where B and m^* are material constants for a given test temperature and σ^* is the effective or thermal component of the flow stress. Combining Eqs. (2), (3), (6) and (7) leads to

$$0.5 b \rho_m B(\sigma^*)^{m^*} = -\frac{1}{E} \frac{d\sigma}{dt} \quad (8)$$

The total stress required for plastic flow is made up of a thermal component, σ^* , and a temperature and strain-rate insensitive component, σ_I , or

$$\sigma = \sigma_I + \sigma^* \quad (9)$$

Thus, in order to solve Eq. (8), it is useful to make two assumptions that

have been experimentally verified^(12,13), viz.

- i) ρ_m remains constant during relaxation.
- ii) σ_I remains constant during relaxation.

The second assumption is useful, since now

$$\frac{d\sigma}{dt} = \frac{d\sigma^*}{dt} \quad (10)$$

and one can treat Eq. (8) as a simple first order ordinary differential equation. Using (10), one can write Eq. (8) as

$$\frac{d\sigma^*}{(\sigma^*)^{m^*}} = -0.5 \rho_m B E dt \quad (11)$$

which, upon integration, gives

$$\sigma^* (1 - m^*) = -0.5 (1 - m^*) b \rho_m B E t - (1 - m^*) a' \quad (12)$$

where a' is a constant of integration. One can determine a' from Eq. (12) at time, $t = 0$, giving

$$a' = \frac{\sigma_0^* (1 - m^*)}{m^* - 1} \quad (13)$$

where σ_0^* is the effective stress at $t = 0$. For convenience, a new constant given by

$$a = a' (m^* - 1) \quad (14)$$

is introduced so that the formalization of the initial value problem from Eqs. 12, 13 and 14 is

$$\sigma^* = (Kt + a)^{-n} \quad (15)$$

where the constants are:

$$K = 0.5(m^* - 1)b \rho_m B E \quad (15a)$$

$$a = \sigma_0^* (1 - m^*) \quad (15b)$$

$$n = 1/(m^* - 1) \quad (15c)$$

Differentiating Eq. (15) and taking the log leads to

$$\log \left(- \frac{d\sigma}{dt} \right) = \log(nK) + (-n - 1) \log(Kt + a) \quad (16)$$

which is essentially what Lloyd, et al⁽¹³⁾ arrived at except that they omitted a constant in their derivation. This can be rewritten as

$$\log \left(- \frac{d\sigma}{dt} \right) = K^* + (-n - 1) \log(t + A^*) \quad (17)$$

where the new constants are:

$$K^* = \log(nK) + (-n - 1) \log K \quad (17a)$$

$$A^* = a/K \quad (17b)$$

The stress-relaxation data may be plotted either as $\log(-d\sigma/dt)$ versus $\log t$ or $\log(t + A^*)$ as indicated in Figure 2. It is obvious that you can determine K^* , A^* , n , a and K from such plots and hence σ^* and m^* which characterize dislocation dynamics during relaxation. In order to do the latter, σ^* must be separated from σ via Eq. (9) knowing σ_I . One can find σ_I by either waiting long times for $d\sigma/dt = 0$, where $\sigma \rightarrow \sigma_I$, or by finding a "zero relaxation rate" condition as described by Lloyd and Embury⁽¹⁶⁾. This simply consists of incrementally unloading a sample until no relaxation occurs in a one hour time period.

Thus, one can describe dislocation dynamics knowing the stress relaxation rate as a function of time and assuming ρ_m to be constant. Of course, this is a fairly cumbersome task and, as yet, the original assumptions have not been justified. To do this, rewrite Eq. (16) in a slightly different form

so that it can be compared to the second approach to stress relaxation, i.e.

$$\log\left(-\frac{d\sigma}{dt}\right) = K^{**} + (-n-1) \log(vt+1) \quad (18)$$

where the new constants are:

$$K^{**} = \log(nK) + (-n-1) \log a \quad (18a)$$

$$v = K/a \quad (18b)$$

With Eq. (18), it is next possible to establish some self-consistency identities with the approach of Sleeswyk, et al⁽¹⁴⁾.

Stress Relaxation Following Sleeswyk, et al⁽¹⁴⁾.

The main assumption here is that

$$\left(\frac{\partial \sigma}{\partial \ln \dot{\epsilon}_p}\right)_{\epsilon_p} = \text{const.} \quad (19)$$

which simply means that in a change of strain rate test, the stress increment due to a strain rate change is independent of the plastic strain level. This would be true if the mobile dislocation density were constant, as was assumed by Lloyd, et al for their approach⁽¹³⁾. Also, this assumption has been repeatedly verified by strain rate tests in bcc metals.

The second assumption Sleeswyk, et al⁽¹⁴⁾ made was that the work hardening that occurred during stress relaxation was independent of the strain rate and only a function of plastic strain. Sargent, Jones and Conrad⁽¹⁸⁾ had experimentally found this to be true, as expressed by

$$\left(\frac{\partial \sigma}{\partial \epsilon_p}\right)_{\dot{\epsilon}_p} = f(\epsilon_p) \quad (20)$$

It follows then that any stress increment may be written as

$$d\sigma = \left(\frac{\partial \sigma}{\partial \ln \dot{\epsilon}_p} \right)_{\epsilon_p} d \ln \dot{\epsilon}_p + \left(\frac{\partial \sigma}{\partial \epsilon_p} \right)_{\dot{\epsilon}_p} d \epsilon_p \quad (21)$$

which, with algebraic manipulation, becomes

$$\left(\frac{\partial \sigma}{\partial \ln \dot{\epsilon}_p} \right)_{\epsilon_p} = \frac{d\sigma}{d \ln \dot{\epsilon}_p} \left[1 - \left(\frac{\partial \sigma}{\partial \epsilon_p} \right)_{\dot{\epsilon}_p} \frac{d \epsilon_p}{d \sigma} \right] \quad (22)$$

From Eqs. (3) and (4), one observes

$$\frac{d \epsilon_p}{d \sigma} = \frac{d \dot{\epsilon}_p}{d \dot{\sigma}} = -\frac{1}{E} = -\frac{C}{1 + \epsilon_p} \quad (23)$$

so that Eqs. (22) and (23) give

$$\left(\frac{\partial \sigma}{\partial \ln \dot{\epsilon}_p} \right)_{\epsilon_p} = \frac{d\sigma}{d \ln \dot{\epsilon}_p} \left[1 + \frac{C}{1 + \epsilon_p} \left(\frac{\partial \sigma}{\partial \epsilon_p} \right)_{\dot{\epsilon}_p} \right] \quad (24)$$

For a given stress relaxation test, the terms from Eqs. (19), (20) and (23) are constant so that Eq. (24) may be written as

$$\frac{d\sigma}{d \ln \dot{\epsilon}_p} = \alpha = \text{const.} \quad (25)$$

Integrating Eq. (25) gives

$$\sigma_0 - \sigma = \alpha (\ln \dot{\epsilon}_{p_0} - \ln \dot{\epsilon}_p) \quad (26)$$

where σ_0 and $\dot{\epsilon}_{p_0}$ correspond to time, $t = 0$. From Eqs. (2), (3) and (26), one finds

$$\exp\left(\frac{\sigma_0 - \sigma}{\alpha}\right) d\sigma = -K_0 dt \quad (27)$$

where $K_0 = E \dot{\epsilon}_{p_0} = \text{const.} \quad (27a)$

This is the same as Eq. (8) from Sleeswyk, et al⁽¹⁴⁾. From here on, we will treat the analysis somewhat differently so as to demonstrate the self-consistency of this analysis with Lloyd's⁽¹³⁾. It is obvious that the log of

Eq. (27) gives

$$\log\left(-\frac{d\sigma}{dt}\right) = \log K_0 + \frac{\sigma - \sigma_0}{\alpha} \quad (28)$$

If one now compares this with the previous analysis, Eq. (18), it is seen that two self-consistency identities must exist, viz.

$$(I) \quad K^{**} = \log K_0$$

$$(II) \quad \frac{\sigma - \sigma_0}{\alpha} = (-n-1) \log(\nu t + 1)$$

Self-Consistency Identities.

First, examine (I). From (I), Eqs. (18a) and (27a), it is seen that

$$K_0 = E \dot{\epsilon}_{P_0} = n K a^{(-n-1)} = e^{K^{**}} \quad (29)$$

It is the two center terms that are of interest and if Eqs. (15a), (15b) and (15c) are invoked, it follows that

$$0.5 b \rho_m B \sigma_0^{*m} = \dot{\epsilon}_{P_0} \quad (30)$$

One immediately recognizes this as the dislocation dynamics Eq. (6) defined at the initial condition, $t = 0$. This is in keeping with the initial assumptions and is at least consistent with the start of the stress relaxation test. Next, consider the second self-consistency requirement. From II, one can write

$$\sigma_0 - \sigma = S \log_{10}(1 + \nu t) \quad (31)$$

$$\text{with} \quad S = 2.3 \alpha (n+1) \quad (31a)$$

It is significant that Eq. (31) is exactly the empirical formula proposed by Feltham^(10,11) to apply throughout the stress relaxation test. In fact, this has been repeatedly verified⁽¹²⁾.

DISCUSSION

Comparison to Experimental Data.

To ascertain whether the above consistency produced quantitatively viable results, it was of interest to compare both the calculated values of S and ρ_m to experimental values. Although it was a little difficult to obtain sufficient data on exactly the same type of material, there were several sources of data on alpha iron tested at 77°K^(11,14,19). Even though the interstitial contents of the various materials used were not constant, it was felt that a first order comparison would result.

First, consider the constant, S . From Sleeswyk, et al⁽¹⁸⁾, data for iron at 77°K are:

$$\left(\frac{\partial \sigma}{\partial \ln \dot{\epsilon}}\right)_{\epsilon_p} = 1.47 \text{ kg/mm}^2 \quad \text{for tension}$$

$$1 + \frac{C}{1+\epsilon_p} \left(\frac{\partial \sigma}{\partial \epsilon_p}\right)_{\dot{\epsilon}_p} = 1.07$$

These values, when inserted into Eqs. (24) and (25), give $\alpha = 1.37$. In order to obtain S from Eq. (31a), a value for n and hence m^* was necessary. One can obtain m^* from the change of strain rate data of Conrad and Frederick⁽¹⁹⁾ who found $(1/T)(d\sigma/d \log \dot{\epsilon}) \approx 16 \times 10^{-3} \text{ kg/mm}^2 \text{ } ^\circ\text{K}$ at 77°K. They also found $\sigma_{77^\circ\text{K}}^* \approx 48 \text{ kg/mm}^2$ and from

$$m^* = \frac{\partial \ln \dot{\epsilon}}{\partial \ln \sigma} = \frac{\sigma^*}{T} \left[\frac{1}{T} \frac{d\sigma}{d \ln \dot{\epsilon}} \right]^{-1} \quad (32)$$

one finds that $m^* \approx 39$. This, in conjunction with Eqs. (15c), (31a) and $\alpha = 1.37$, gives $S = 3.24 \text{ kg/mm}^2$. This compares quite favorably to the actual constant of 3.13 kg/mm^2 reported by Feltham⁽¹¹⁾ for stress relaxation testing of alpha Fe at 77°K. This not only demonstrates that the stress

relaxation approach is self-consistent, but also shows that one can obtain the same strain rate sensitivity constants as from change of strain rate tests.

Next, consider the mobile dislocation density, ρ_m . Before this can be determined, the stress relaxation equation must be put in a convenient form. From Eqs. (17b), (18b) and II, one can show that stress relaxation should be given by

$$\frac{d\sigma}{d \log(t+A^*)} = -\alpha \quad (33)$$

which is the relationship found by others^(12,14). Thus, a linear plot of stress versus $\log(t+A^*)$ should result from a stress relaxation test. Data of Guin and Pratt⁽¹²⁾ on Molybdenum illustrate the point in Figure 3. This allows one to evaluate Eq. (27) by integration, giving

$$[\sigma_0 - \alpha \log(E \frac{\dot{\epsilon}_{p0}}{\alpha})] - \sigma = \alpha \log(t+A^*) \quad (34)$$

Here A^* is the constant of integration but must be equivalent to the A^* in Eq. (33) since α , E and $\dot{\epsilon}_{p0}$ are constants. One can next combine Eqs. (26) and (34) to obtain

$$\frac{\dot{\epsilon}_p(t+A^*)E}{\alpha} = 1 \quad (35)$$

which, evaluating at $t = 0$, gives

$$\dot{\epsilon}_{p0} A^* E = \alpha \quad (36)$$

It is now possible to evaluate ρ_m using two different approaches. First, take A^* from Sleeswyk, et al⁽¹⁴⁾ to be about 0.7 seconds and determine $\dot{\epsilon}_{p0}$ from $\alpha = 1.37$ and $E = 2.1 \times 10^4 \text{ kg/mm}^2$. From Eq. (36), this gives $\dot{\epsilon}_{p0}$ to be $9.3 \times 10^{-5} \text{ sec}^{-1}$ which is consistent with the strain rate reported by Sleeswyk, et al⁽¹⁴⁾. With $\tau^* = 24 \text{ kg/mm}^2$ and $m^* \approx 39$ at 77°K, the

dislocation velocity would be

$$\bar{v}_{77^\circ\text{K}} \approx v_0 \left(\frac{24}{32}\right)^{39} \approx 10^{-5} \text{ cm/sec} \quad (37)$$

where $v_0 = 1 \text{ cm/sec}$ at $\tau_0^* = 32 \text{ kg/mm}^2$. Thus, with

$$\dot{\epsilon}_{p_0} = 9.3 \times 10^{-5} \text{ sec}^{-1} \quad \sigma^* = 48 \text{ kg/mm}^2$$

$$\bar{v} \approx 10^{-5} \text{ cm/sec} \quad m^* = 39$$

$$\phi \approx 0.5 \quad A^* = 0.7 \text{ sec}$$

$$b = 2.5 \times 10^{-8} \text{ cm}$$

one calculates a $\rho_m \approx 7.5 \times 10^8/\text{cm}^2$ from Eq. (6). A second way to calculate ρ_m is to combine Eqs. (15a), (15b) and (17b), giving

$$A^* = \frac{\sigma_0^* (1 - m^*)}{0.5 (m^* - 1) b \rho_m B E} = \frac{\sigma_0^*}{0.5 (m^* - 1) b \rho_m \bar{v} E} \quad (38)$$

With the above values, this leads to a calculated $\rho_m \approx 6.9 \times 10^8/\text{cm}^2$ which again demonstrates the self-consistency of the approach. These values can be compared to direct observations, via thin film transmission microscopy, of ρ_m in iron. For stresses near yielding at room temperature and -120°C , Saka, Noda and Imura⁽²⁰⁾ found that $\rho_m \approx 1 - 4 \times 10^9/\text{cm}^2$. Considering that their results were independent of test temperature and that they were observing the most "active" areas in the foil, a calculated average of $7 \times 10^8/\text{cm}^2$ is in excellent agreement.

Approach to Stress Relaxation Testing.

Since the above approaches are internally consistent, it is acceptable to combine Eqs. (30), (36) and (38) to give

$$\alpha = \frac{\sigma_o^*}{m^* - 1} \quad (39)$$

From the above values, this gives a calculated α of 1.26 which is in reasonable agreement with the observed value of 1.37. Considering the multiple sources of data, this difference is not significant. This brings us full circle and allows a fairly simple procedure to evaluate dislocation mobility and stress relaxation parameters from a single test. The approach is to determine the slope

$$\frac{d\sigma}{d \log (t + A^*)} = -\alpha$$

- i) A^* is obtained by choosing a constant so that the data fit a straight line.
- ii) α is the slope as shown in Figure 3.
- iii) By using a "zero relaxation rate" approach⁽¹⁶⁾, establish σ_I and hence σ_o^* .
- iv) obtain m^* directly from Eq. (39).

A Further Test of Self-Consistency.

To see if this approach were general, we decided to evaluate pure polycrystalline Fe near 77°K at a prior strain near the yield stress where Feltham⁽¹¹⁾ had reported anomalously high stress relaxation. A specimen of pure Fe, with 0.15% Ti added to remove residual carbon, was cooled to 83°K and loaded to a prior plastic strain, ϵ_{p_o} , of 0.0026 and then relaxed. Strain versus time was also recorded and an $\dot{\epsilon}_{p_o} \approx 6 \times 10^{-4} \text{ sec}^{-1}$ was determined. The resulting data, as plotted in Figure 4, were treated as above and it was found that:

$$\begin{array}{ll} A^* \approx 3 \text{ sec.} & \alpha = -d\sigma/d \log (t + A^*) = 4.75 \text{ kg/mm}^2 \\ \sigma_o^* = 49.2 \text{ kg/mm}^2 & S = 12 \text{ kg/mm}^2 \end{array}$$

$$m^* = 11.3 \quad \rho_m \approx 1.2 \times 10^5 / \text{cm}^2 \quad \text{to} \quad 9.1 \times 10^5 / \text{cm}^2$$

Here, the two values of ρ_m refer to Eqs. (38) and (6). Considering the data above the dotted line, it is seen that σ_0^* is a reasonable value. However, stress relaxation as measured by α or S is approximately four times as high and the time constant is four times as long as was obtained at larger pre-strains. The values below the dotted line must be regarded as fictitious. In the first place, the values of both m^* and ρ_m are low. Secondly, the two calculations of ρ_m were not internally consistent. It becomes apparent that the approaches discussed above are inconsistent for small strains near yield.

Besides Feltham's⁽¹¹⁾ observation that stress relaxation was anomalous and irregular for strains up to 0.003, Sleeswyk, et al⁽¹⁴⁾ found that the term in the brackets in Eq. (24) was constant for $\epsilon_{p_0} > 0.02$ but varied by about a factor of four at smaller pre-strains. This would simply indicate that ρ_m or $(\partial\sigma/\partial\ln\dot{\epsilon}_p)\epsilon_p$ are not constant at low strains nor are they comparable to the values found at larger pre-strains. Hence, the self-consistency argument breaks down.

CONCLUSIONS AND SUMMARY

1. One approach to stress relaxation testing is to assume that dislocation mobility, ρ_m , and the internal structure stress, σ_i , remain constant during relaxation.
2. A second approach assumes $(\frac{\partial \sigma}{\partial \ln \dot{\epsilon}})_{\epsilon_p}$ to be constant and $(\frac{\partial \sigma}{\partial \epsilon_p})$ to be independent of strain rate.
3. These two analyses lead to self-consistency identities which
 - (a) verifies the Gilman-Johnston dislocation mobility equation⁽¹⁵⁾;
 - (b) derives the empirical formula for stress relaxation testing originally observed by Feltham^(10,11)

$$\sigma_0 - \sigma = S \log_{10} (1 + vt)$$

where S and v are constants, t is time, σ is applied stress and σ_0 is the stress at time zero.

4. The validity of these self-consistency identities was further verified by experimental data where it is shown that for iron at large pre-strains:

<u>Calculated</u>	<u>Observed</u>
$S = 3.24 \text{ kg/mm}^2$	$S = 3.13 \text{ kg/mm}^2$
$\alpha = 1.26$	$\alpha = 1.37$
$\rho_m \approx 7 \times 10^8/\text{cm}^2$	$\rho_m \approx 10^9/\text{cm}^2$

5. A simple approach to stress-relaxation testing allows the strain-rate sensitivity parameter, $\partial \ln \dot{\epsilon} / \partial \ln \sigma$, to be directly calculated from a single test.
6. Stress relaxation testing is inconsistent with the assumption of a constant mobile dislocation density at small strains near the yield point of polycrystalline iron.

ACKNOWLEDGEMENT

This work was supported by the United States Atomic Energy Commission under Grant AT-11-1-2212.

REFERENCES

1. Asilomar Conference, 2nd Int. Conf. on Strength of Metals and Alloys, Vol. I, II, III, Am. Soc. for Metals, Metals Park, Ohio (1970).
2. J. P. Hirth and J. Lothe, Theory of Dislocations, McGraw-Hill, New York (1968).
3. P. G. McCormick, Acta Met., 20, (1972), p. 351.
4. K. Mukherjee, C. D'Antonio, R. Macaig and G. Fisher, J. Appl. Phys., 29, (1968) p. 5434.
5. R. L. Fleischer, Acta Met., 11, (1966) p. 203.
6. W. C. Leslie, R. J. Sober, S. G. Babcock and S. J. Green, Trans. ASM, 62, (1969), p. 690.
7. B. Harris, Phys. Stat. Sol., 18, (1966), p. 715.
8. U. Hildebrandt and W. Dickenscheid, Acta Met. 19, (1971), p. 49.
9. F. T. Trouton and A. O. Rankine, Phil. Mag., 8, (1904), p. 538.
10. P. Feltham, Phil. Mag., 6, (1961), P. 209.
11. P. Feltham, Phil. Mag., 6, (1961), p. 847.
12. F. Guin and P. L. Pratt, Phys. Stat. Sol., 6, (1964), p. 111.
13. D. J. Lloyd, P. J. Worthington and J. D. Embury, Phil. Mag., 22, (1970), p. 1147.
14. A. W. Sleeswyk, G. H. Boersma, G. Hut and D. J. Verel, 2nd Int. Conf. on Strength of Metals and Alloys, I, Am. Soc. for Metals, (1970) p. 204.
15. W. G. Johnston and J. H. Gilman, J. Appl. Phys., 30, (1959), p. 129.
16. D. J. Lloyd and J. D. Embury, Met. Sci. J., 4, (1970), p. 6.
17. A. W. Sleeswyk, Proc. Xth Int. Cong. Refrig., 1, (1960), p. 135.
18. G. A. Sargent, G. Jones and H. Conrad, Scripta Met., 3, (1969), p. 481.
19. H. Conrad and S. Frederick, Acta Met., 10, (1962), p. 1013.
20. H. Saka, K. Noda and J. Imura, Crystal Lattice Defects, 4, (1973), p. 45.

List of Figures

1. Schematic of stress, strain, time relationship for a stress-relaxation test.
2. Several approaches to the determination of stress-relaxation constants from rate data.
3. Typical stress-relaxation data for Molybdenum tested after a relatively large pre-strain (From Guin and Pratt, (14)).
4. "Anomalous" stress-relaxation data for pure polycrystalline iron tested after a small pre-strain.