-1-

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SOME OBSERVATIONS ON THE "BAND PASS" METHOD
OF VIBRATION ANALYSIS

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ABSTRACT

The general characteristics of band pass filters in relation to vibration analysis are described. Comparisons are made on several steady state wave shapes between the components passed for various filter settings of the SKL Filter No. 302 and the components which would be passed by a hypothetical filter having the same attenuation characteristics but without phase shift.

Observations are made on the differences shown by the comparisons and on the use of a band filter as an instrument for analysis of vibration records.

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SOME OBSERVATIONS ON THE "BAND PASS" METHOD
OF VIBRATION ANALYSIS

The following information is the result of a study instigated for the information of Sandia Corporation personnel who receive data from the Field Test Organization in the form of the results of the "Band Pass" Method of Vibration Analysis. The band pass mechanism studied in this paper is the SKL Variable Electronic Filter No. 302. This is the same model filter as used first by Division 5226 of the Field Test Organization in analyzing the records by the band pass method. It is not the filter that is planned for ultimate use by Division 5226 but most of the observations hold for the proposed filter and it is hoped that a similar study can be made later on the "14 Band Pass Filter" which will soon be put into use. It is well to note that for the most part, this study has been made on steady state vibrations whereas the actual records which are to be analyzed by this method are not steady state but contain shock transients whose amplitudes may be an order of magnitude larger than the amplitude of the approximately steady state conditions caused by vibrations alone. The effect of this deviation from assumed conditions is hard to estimate. It seems apparent that a study of the effects of such deviations should be made.

First, let us refresh our memory by defining what we mean by a band pass filter. A band pass filter is a device which allows a certain limited portion of the frequency spectrum to be transmitted through the filter while those portions of the spectrum outside of the band are attenuated relative to the signal inside of the band. In this regard, the limits of a filter are specified as those points on either side of the center frequency which are attenuated a given amount, usually 3 db, with respect to the amplitude of the center frequency. An important figure describing the characteristics of a band pass filter is the amount of attenuation per octave outside of the band pass range. Quite obviously, if we wish to determine the size of the component vibrations occurring in a certain frequency band, we do not want to read appreciable amounts of vibrations occurring in adjacent bands. This indicates that we would like a high rate of attenuation per octave so that, for instance, a vibration containing a component one-half an octave from one side of the filter setting would have that component attenuated so highly that it would not appear on the record. On the other hand, we are faced with the dilemma that we must pay for an increase in rate of attenuation by accepting an increase in the phase shift of the signal. Any practical filter is therefore a compromise between the amount of the attenuation desired and the amount of phase shift that can be tolerated. This situation is expressed in the relations governing phase shift as a function of the attenuation slope*. It should be noted that the form of the equation expressing this phenomenon is such that the "changes in the slope of the attenuation characteristics at frequencies close to the reference frequency have far more effect upon phase shift than do changes in the slope of the characteristics at more remote frequencies." The region of frequencies close to the reference frequency is in

*Terman Radio Engineers Handbook p. 219.

general the frequency in which we are most interested and practically it is very difficult to design the attenuation characteristics so that they lead to a minimum phase shift.

With this as a background, let us look at the particular filter which was used in the experiments reported here. The instruction manual indicates that the characteristics of the filter are as shown in Figure 1. We note that for the sharpest tuning, the amplitude of the center point frequency has been attenuated by 3.5 db and if we use the filter as indicated, the band will have a constant ratio of low frequency "cut off" to high frequency "cut off" of 2:3 or .8:1.2. Table I shows the band widths and filter settings as proposed by Mr. Washburn of Division 5226 for this filter. Note that the above statements hold for all bands except the 2-20 band where only one section of the filter is used and where that section is used as a low pass filter. Figure 2 shows the composite response for the various settings.

There is a very pertinent question of what type of expansion we should use to give us the theoretical components of the input wave which we wish to investigate. An excellent discussion of this problem is given by Guillemin*. Briefly, this reference points out that a Fourier Analysis of a wave does not necessarily give the best results for the magnitude of the various components and that many other expansions in orthogonal sets are not only possible but may perhaps give a more accurate result. The fact remains however, that the methods of the Fourier Analysis are the simplest and most widely known of the methods of analyses and that the differences between the coefficient of the initial terms derived by, let us say, Fejer Polynomials and those derived by a Fourier expansion are almost identical. Therefore, we will use the simple Fourier series as a good approximation and check theoretically the response of the band pass filter to any function for which we may obtain a Fourier series.

To accomplish this, we shall follow the procedure given below:

- (1) Write the Fourier Series for the function which is to be fed into the filter.
- (2) Calculate the amplitude of the fundamental and of each of the various harmonics.
- (3) From the graph of the response of the band pass filters, determine the per cent of the various harmonics (and the fundamental) which will be passed for each band pass setting.

*E. A. Guillemin, "The Mathematics of Circuit Analysis," John Wiley and Sons, 1949.

685-3

- (4) Plot and graphically add all those components of appreciable magnitude passed by the filter. This gives the wave shape of the vibration passed by the filter disregarding the phase shift actually present in the filter.
- (5) Set up the desired function from a function generator of some type and feed this function into the band pass filter, recording the output of the filter together with the input function on a dual beam scope.

After following these steps, we may make comparisons between the theoretical response of a band pass filter having the attenuation as shown by the SKL instruction book but without phase shift, and the actual response of the filter as taken from an oscilloscope.

The above procedure has been followed for square, triangular and saw-tooth waves, and is demonstrated in Figures 3 through 8.

In addition, the response of the filter is shown by Figures 9 through 11 to (1) a triangular pip repeated at regular intervals made by feeding the filter with only the top portion of a triangular wave, (2) a series of short pulses that are sections of a sine wave, and (3) a pulse with a rapid rise time followed by an exponential decay. These wave shapes could be analyzed in a manner similar to the square, triangular and a saw-tooth wave but the calculations are somewhat long and tedious so this was not done.

The series used for the square wave is:

$$y = \frac{4}{\pi} (\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x \dots),$$

for the triangular wave is

$$y = \frac{8}{\pi^2} (\sin x + \frac{1}{3^2} \sin 3x + \frac{1}{5^2} \sin 5x \dots),$$

and for the saw-tooth wave is

$$y = \frac{2}{\pi} (\sin x - \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x - \frac{1}{4} \sin 4x + \frac{1}{5} \sin 5x \dots),$$

The figures show that in general the results are close to what might have been expected. A careful study of the differences lead to the following observations:

1. As predicted by the manufacturers literature, there is evidence of considerable phase shift. It may be particularly noted that in the case of the 30

to 45 cycle band analysis of the square wave, the phasing of the fundamental and the third harmonic has been changed just enough so that the waves are additive at their maximum values when they should have been of opposite signs. This has the effect of giving a larger amplitude to the wave than would have been reported without phase shift. It should be noted, however, that with perfectly random steady state vibrations of many frequencies, that phase shift would have the effect only of shifting the time at which the maxima took place.

2. Because of the not too steep slope of the sides of the response curve, i.e. attenuation of 18 db per octave, vibrations of relatively large amplitude were recorded in bands where none should exist, i.e. 2-20, 30-45 and 45-67 cycle bands for the square and triangular waves and on the 2-20, and 30-45 cycle band for the saw-tooth wave. In general, the same condition will appear in all band pass filters to a greater or lesser extent. This fact should be kept in mind when analysis is made of a vibration record to keep from duplicating information from adjacent bands.
3. It is obvious from the shape of the response curve that a single calibration will hold for only one or two points on the response curve. This means that the calibration may be set for the center frequency in which case an amplitude of a certain value on either side of the center implies a higher input signal than is actually indicated, or the calibration may be set for an average value over the band in which case an amplitude of a certain value indicates a smaller input signal in the central part of the band and a larger input signal in the outer ends of the bands than is actually shown. The decision of which system to use is academic but the ultimate user of the information should know which system is being used.
4. The question of the "ringing" of the filter when a pulse or wave with a steep wave front is fed to it is worthy of some discussion. Having adopted the premise that the coefficients of the initial terms give a reasonable approximation for the magnitude of the initial terms, we may compare the shape of the wave front generated by feeding a 25 cycle square wave into the filter. Figure 12 shows the results when one or both sections are used for low pass operation with a high cut-off frequency of 500 cycles. Figure 13 shows the results when a single rectangular pulse of the same duration as the waves of the 25 cycle square wave is fed to the filter. Figure 14 shows the wave shape for the first 180° of a square wave developed by using the first ten terms of the Fourier Series for a square wave

$$y = \frac{4}{\pi} \left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} \dots \dots \frac{\sin 19x}{19} \right).$$

Comparison of the results for the waves, Figures 12 and 14, indicates a small amount of overshoot when only one filter is used and a larger amount when both are used. Notice that the droop of the wave is also more pronounced when two filters are used instead of one.

-685-5

8

In the case of the pulse excitation, theoretically all frequencies are present in the form of a continuous spectrum and there should not be the long lasting overshoot; however, inspection of Figures 12 and 13 show that results are obtained which are very similar to those gotten by passing a wave through the filter. This demonstrates that ringing is actually present. That this is to be expected is shown by the theoretical response of the filter to a step function as shown by Figure 2.113 in the instruction book for the SKL Filter.

5. Finally, we should investigate the possible magnitudes for the various components of the composite vibration wave. A proof is given in Appendix I that the height of any component of a wave of periodic nature can never be more than approximately 27 per cent larger than the composite wave. We may note, however, that it is possible, though not very probable, that the band width may contain several components whose magnitudes approach the maximum value and that as a result, the signal passed by a particular band width may be larger than the theoretical maximum of a single component. A further complication is added by phase shift characteristics of the filter. Such a complication was demonstrated in the case of the analysis of the 25 cycle square wave in the 30 to 45 cycle band width. Here both the fundamental and the third harmonic were passed by the filter set caused in addition of amplitudes resulting in a roughly triangular wave shape instead of a subtraction of maxima which would have given an almost trapezoidal wave shape of lower amplitude. Even with these complications, if any component of a vibration wave shows amplitudes greater than 120 per cent of the composite wave, it is recommended that the equipment be recalibrated and that the data be rechecked for possible errors in reduction.

While this paper is somewhat brief, it does bring forth some of the problems and limitations involved in the reduction of shock and vibration records by using a band pass filter. It is hoped that this paper will also prove an incentive for the ultimate users of the test results to study the methods used in vibration analysis so that the results of such analyses will prove more useful.

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9

TABLE I

<u>VIBRATION</u>		<u>ANALYSIS</u>	<u>PASS BANDS AND FILTER SETTINGS</u>	
Q = 2.5	$f_1/f_2 = 1$		3db points f/f_2 0.8 - 1.2	
<u>Pass Band</u>		<u>Band Center</u>	<u>Sections in Cascade</u>	
3db points			Dial Settings	
c.p.s.		c.p.s.	(f_c of L.P.)	(f_c of H.P.)
2 - 20		low pass connections	$f_c = 20$	-
20 - 30		25	$f_2 = 25$	$f_1 = 25$
30 - 45		37.5	$f_2 = 37.5$	$f_2 = 37.5$
45 - 68		57	$f_2 = 57$	$f_1 = 57$
68 - 102		85	$f_2 = 85$	$f_1 = 85$
100 - 150		125	$f_2 = 125$	$f_1 = 125$
150 - 225		187.5	$f_2 = 187.5$	$f_1 = 187.5$
225 - 337		281	$f_2 = 281$	$f_1 = 281$
335 - 502		418	$f_2 = 418$	$f_1 = 418$
500 - 750		625	$f_2 = 625$	$f_1 = 625$
750 - 1125		937	$f_2 = 937$	$f_1 = 937$

685-7

10

FIGURE 1

//

KEUFFEL & ESSER CO., N. Y. NO. 359-110
Logarithmic, 2 X 3 Cycles.
MADE IN U.S.A.

685-9

0°

25 CYCLE SQUARE WAVE
INPUT FUNCTION

2-20 ~ BAND

20-30 ~ BAND

30-45 ~ BAND

COMPUTED WAVE SHAPES FOR VARIOUS BAND PASS SETTINGS
SHOWN IN FIG. 2 (NO PHASE SHIFT)

359-14 KEUFFEL & ESSER CO.
Millimeters, 5 mm. lines accented, cm. lines heavy.
MADE IN U. S. A.

FIGURE 3(a)

685-10

FIGURE 3(b)

685-11

14

- 15 & 16 -

2-20 ~
20-30 ~

30-45 ~
45-68 ~

68-102 ~
100-150 ~

150-225 ~
225-337 ~

FIGURE 4

ANALYSIS OF 25 CYCLE SQUARE WAVE BY FILTER

685-12

FIGURE 5(b)

2-20 ~
20-30 ~

30-45 ~
45-68 ~

68-102 ~
100-150 ~

150-225 ~
225-337 ~

FIGURE 6

ANALYSIS OF 25 CYCLE TRIANGULAR WAVE BY FILTER

45-68 ~ BAND

68-102 ~ BAND

100-150 ~ BAND

FIGURE 7(b)

685-17

20

- 23 -

2-20 ~
20-30 ~

30-45 ~
45-68 ~

68-102 ~
100-150 ~

150-225 ~
225-337 ~

FIGURE 8

ANALYSIS OF 25 CYCLE SAW TOOTH WAVE BY FILTER

685-18

2-20 ~
20-30 ~

30-45 ~
45-68 ~

68-102 ~
100-150 ~

150-225 ~
225-337 ~

FIGURE 9(a)

ANALYSIS OF 8% OF THE UPPER HALF OF A TRIANGULAR WAVE BY FILTER

685-19

22

337 - 500 ~
500 - 750 ~

750 - 1125 ~

FIGURE 9(b)

689-20

23

- 26 -

2-20 ~
20-30 ~

30-45 ~
45-68 ~

68-102 ~
100-150 ~

150-225 ~
225-337 ~

FIGURE 10(a)

ANALYSIS OF TOP OF SINE WAVE BY FILTER

685-21

24

337 - 502 ~
500 - 750 ~

750 - 1125 ~

FIGURE 10(b)

685-22

25

- 28 -

PULSE WAVE SHAPE
1000 CYCLE SINE WAVE

2-20 ~
20-30 ~

30-45 ~
45-68 ~

68-102 ~
100-150 ~

FIGURE II(a)

ANALYSIS OF PULSE (EXPANDED TIME SCALE) BY FILTER

685-23

26

150-225 ~
225-337 ~

337-500 ~
500-750 ~

750-1000 ~

750-1000 SHOWING TWO
PULSES 1/25 SEC. APART
FOR COMPARISON TO CLIPPED
TRIANGULAR AND SINE WAVE
PULSES.

FIGURE II(b)

685-24

(A) 2 SECTIONS OF FILTER SET AT
500 CYCLE LOW PASS OPERATION
FOR 25 CYCLE SQUARE WAVE

FIGURE 12 (B) SAME AS FIG. TO LEFT EXCEPT
ONLY ONE FILTER.

(A) REACTION OF FILTER TO 20
MILLESECOND SQUARE PULSE,
SETTING AS DIRECTLY ABOVE
FIG. 12 (A)

FIGURE 13

(B) REACTION OF FILTER TO 20
MILLESECOND SQUARE PULSE,
SETTING AS DIRECTLY ABOVE
FIG. 12 (B)

685-25

28

$$y = \frac{4}{\pi} \left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} - \frac{\sin 19x}{19} \right)$$

FIGURE 14

685-26

29

APPENDIX I

In vibration theory, it is assumed that the Fourier series representing acceleration contains no constant term. This must be approximately so since the net distance relative to the rest of the structure traveled by the part on which the pickup is mounted is zero, therefore, the net area under the velocity curve ($s = \int v \, dt$) must be zero. Since the displacement is zero many times during any appreciable part of the vibration record, the velocity must pass through zero many times, (though not simultaneously with the displacement curve). Thus, we are finally led to the conclusion that the area under the acceleration curve balances out since this is a necessary condition for the velocity curve to pass many times through zero. The constant term of a Fourier series

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx,$$

is the net area under the acceleration curve. Therefore, we conclude that the constant term is approximately zero. The Fourier series for accelerations is, therefore, expressible as a series of sine and cosine terms alone. A further property of Fourier series is that, by making use of half range expansions, we may develop any wave shape in a series of either sine or cosine terms alone. If we "normalize" the wave by multiplying its amplitude by a suitable constant so as to limit the maximum amplitude in either the positive or negative direction or in both directions to unity, we may make the statement that $|f(x)| \leq 1$. If this is the case,

68-5-27

$$|a_n| = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) |\cos nx| dx$$

$$\leq \frac{1}{\pi} \int_{-\pi}^{\pi} |\cos nx| dx = \frac{4}{\pi}$$

and in like manner

$$|b_n| \leq \frac{4}{\pi}$$

We have thus demonstrated that for any wave whose maximum height is normalized to unity, the maximum value of the coefficients of any term in a sine or cosine Fourier series is $\frac{4}{\pi} = 1.27 +$; therefore, no individual component, i.e. harmonic, of the Fourier series representing a function has an amplitude which is more than approximately 27 per cent larger than the composite wave.

As a matter of general interest, a square wave of unit height is one wave which has such a maximum coefficient in its Fourier series, the sine series for the wave being

$$f(x) = \frac{4}{\pi} \left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \dots \right)$$

and the cosine series being

$$f(x) = \frac{4}{\pi} \left(\cos x - \frac{\cos 3x}{3} + \frac{\cos 5x}{5} - \dots \right)$$

In each case the Fourier coefficient of the fundamental is $\frac{4}{\pi}$.