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AN OVERVIEW OF SPECTRAL-
LINE PROFILES EXPECTED FROM IONS IN PLASMA

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An Overview of Spectral-
Line Profiles Expected from Ions in
Plasma.*

Bruce W. Shore

September 1974

Abstract

This report summarizes the mechanisms which broaden spectral lines in plasmas and discusses the magnitudes of relevant parameters for Lyman- α -like lines of hydrogenic-ion impurities in a plasma. The report includes a bibliography on recent relevant theoretical work on line profiles.

This work was performed under the auspices of the United States Atomic Energy Commission.

1. Introduction

Spectral-line profiles are widely used as tools for diagnosing density and temperature conditions in plasmas. Texts edited by Huddleston (1965) and Lochte-Holtgreven (1968), the books of Griem (1964, 1974), and review articles by Cooper (1966), Robinson and Lenn (1967) and Burgess (1972), all review this subject.

The theory of spectral line shapes has been studied and reported at great length. The NBS publishes extensive bibliographies of the subject (Fuhr, Roszman and Wiese, 1972, 1974). The text of Sobel'man (1972) contains the best complete discussion; thorough reviews have been written by Margenau and Lewis (1959), Baranger (1962), Breene (1964), Van Regemorter (1965), Traving (1968), Cooper (1969), Ben-Reuven (1969), and Myerscough and Peach (1971). Lines of hydrogen have been examined in particular detail (e.g. Griem, Kolb and Shen, 1959; Griem 1960; Pfennig, Trefftz and Vidal 1966; Kepple and Griem 1968; Bacon, Shen, and Cooper 1969; Vidal, Cooper and Smith 1970; Pfennig 1971; Bacon 1972, 1973; Pfennig 1972; Kepple 1972; Lisitsas and Sholin 1972; Lisitsas 1972, Bottcher 1973). Tables of hydrogen line-broadening parameters have been published (Hill, 1964; 1967; Edmonds, Schlüter and Wells 1967; Vidal, Cooper and Smith 1973). There has also been some work on hydrogen-like ions (Griem 1960; Kortén and Berg 1970; Kepple 1972). The NBS bibliography contains additional references to the theory of ion lines and to observations.

The following sections outline the basic results of line-broadening theory, expressed in simplest form. Detailed corrections, as presented in the extensive literature on this subject, all fit within the broad framework presented below. Specifically I consider an idealized Lyman- α ($2p \rightarrow 1s$) transition for a one-electron ion, nuclear charge Z , mass M_Z , immersed in a plasma of free electrons (mass m) and ions of charge z , mass M . I take the electron density N_e and ion density N_Z each to be equal to N and I take their common temperature to be T .

The basic quantity of interest I take to be the relative spectral-line intensity at frequency ω emerging from an optically thin emission source. This function I denote by $\frac{dF^{\text{line}}}{d\omega}(\omega)$, normalized to unit total line strength:

$$\int d\omega \frac{dF^{\text{line}}}{d\omega}(\omega) = 1 \quad (1.1)$$

The profile function $dF^{\text{line}}/d\omega$ may be viewed as the convolution of three independent contributions which I shall term Doppler, Lorentz, and Static distributions.

2. Doppler Distribution

The distribution of frequencies, about the central value ω_0 , caused by the thermal distribution of radiator velocities, is given by the formula

$$\frac{dF^D}{d\omega}(\omega) = \frac{1}{W_D} P^D\left(\frac{\omega - \omega_0}{W_D}\right) \quad (2.1)$$

where $P^D(x)$ is the Doppler profile

$$P^D(x) = \frac{1}{\sqrt{\pi}} \exp(-x^2) \quad (2.2)$$

Here W_D the Doppler spread, is the mean frequency times the ratio of radiator and photon speeds:

$$WD = \omega_o \sqrt{\frac{2kT}{M_z c^2}} \quad (2.3)$$

The Doppler profile applies when the emitted radiation, regarded from a reference frame at rest with the atomic emitter, is uncorrelated with any any previous direction of photon absorption. Numerous authors have considered the modifications which occur when spectral lines are formed as a result of radiation scattering, thereby retaining a frequency memory (cf. Henyey 1941; Field 1959; Hummer 1962; Finn 1967; Jeffries 1968).

3. Lorentz Distribution

A combination of natural (e.g. radiative) decay and interruptions (or impacts) yields profiles of the form

$$\frac{dF^L}{d\omega}(\omega) = \frac{1}{(W_N + W_I)} P^L\left(\frac{\omega - \omega_o + \Delta}{W_N + W_I}\right) \quad (3.1)$$

where $P^L(x)$ is the Lorentz profile

$$P^L(x) = \frac{1}{\pi} [x^2 + 1]^{-1}, \quad (3.2)$$

Δ is an impact-produced line shift, and the Lorentz width

$$W_L = W_N + W_I \quad (3.3)$$

is the sum of a natural (decay) width W_N and an interruption (impact) width W_I . The Lorentz profile requires modification when one treats transitions

between degenerate levels (this occurs for Lyman α). Nevertheless, the gross qualitative behavior of formula (3.2) persists.

3A. Decay Width

For an isolated cold atom the natural width is the Einstein transition rate $A_{2p \rightarrow 1s}$ (i.e. The inverse of the 2p lifetime):

$$\begin{aligned} W_N &= A_{2p \rightarrow 1s} \\ &= \frac{2e^2}{mc^3} \omega^2 \frac{1}{3} f_{1s \rightarrow 2p} \end{aligned} \quad (3.4)$$

Here $f_{1s \rightarrow 2p} \approx 0.41$ is the (dimensionless) oscillator strength. This transition rate is readily calculated, with great accuracy, for one-electron hydrogenlike ions. The presence of perturbers will distort the bound orbitals and so affect this probability. Such effects have not ordinarily been included in profile calculations.

When one deals with hot atoms (i.e. atoms which contain one or more electron holes in closed shells) then decay becomes possible by Auger-electron emission. This Auger-decay rate adds to the radiative rate to give a total natural width.

3B. Impact Width

Any process which acts to shorten the emitted sinusoidal wavetrain will contribute to the interruption width. Thus collisions with perturbing electrons and ions will, through impact, excite and/or ionize the radiator thereby shortening the effective lifetime. Ionization by intense electro-

magnetic fields has this same effect. The interruption width W_I is approximately the total rate of interruptions, as dictated by the total interruption cross-section σ_s caused by species s :

$$W_I \approx \sum_s \langle N_s v_s \sigma_s \rangle \quad (3.5)$$

Here $\langle \dots \rangle$ denotes a thermal average over all types of collisions. Refined analyses of line-broadening theory shows that impact widths and shifts should in fact vary with frequency across the profile. (cf. Ross, 1966; Smith and Hooper, 1967; Klein, 1969; Lee, 1971; Futrelle, 1972). Widths which neglect this this dependence are termed impact approximation widths; incorporation of frequency dependence is termed a "unified theory".

The theoretical description of interruptions (collisions) is much more difficult than the description of radiative decay; estimates of impact widths are therefore much less accurate than the natural width. A simple classical estimate can be obtained by assuming that a perturber at position $R(t)$ shifts the radiated frequency according to the power law

$$\omega(R) - \omega_0 = \frac{C_\mu}{[R]^\mu} \quad (3.6)$$

For hydrogenic spectral lines the dominant perturbation is a linear Stark effect produced by passing electrons or ions: The perturber monopole interacts with the permanent hydrogenic dipole moment, producing an inverse-square law ($\mu=2$) frequency shift. By assuming a classical trajectory $R(t)$ for the perturber (e.g. a straight line), one can calculate the classical phase shift

$$\eta(\tau) = \int_0^\tau dt [\omega(t) - \omega_0] = \int_0^\tau dt \frac{C_\mu}{[R(t)]^\mu} \quad (3.7)$$

and thence compute a width and shift by carrying out a thermal average over trajectories (i.e. over phase shifts):

$$W - i\Delta = \langle v(1 - \exp[i\eta(\infty)]) \rangle_{av} \quad (3.8)$$

The classical thermal average consists of an integral over impact parameter ρ together with an integral over a Maxwellian velocity distribution. If we replace v by the mean thermal velocity $\langle v \rangle$ then the thermal average of a function $F \equiv F(\rho, v)$ becomes

$$\langle vF \rangle \approx N \int d\rho \, 2\pi\rho \langle v \rangle F(\rho, \langle v \rangle) \quad (3.9)$$

In turn, the impact width is approximately given by

$$W = N^2 \pi \langle v \rangle [\rho_W]^2 \quad (3.10)$$

where ρ_W , the Weisskopf radius, is the impact parameter which yields unit phase shift:

$$[\rho_W]^{\mu-1} = \frac{\alpha_\mu C_\mu}{\langle v \rangle} \quad (3.11)$$

Here α_μ is a numerical constant bounded by π and 1. For the linear Stark effect appropriate to hydrogen the preceding approximations yield the Weisskopf radius

$$\rho_W = \frac{\pi C_2}{v} \approx \frac{e^2}{\hbar} \frac{\pi \langle r \rangle}{\langle v \rangle} \quad (3.12)$$

where $\langle r \rangle$ is the atomic excited-state radius. The cross-section for scattering by a dipole potential diverges; on incorporating impact parameter cutoff at a maximum value ρ_m (usually taken as the lesser of the Debye length and the interparticle separation) one obtains the approximation (cf. Sobel'man, Eq. 36.35)

$$\begin{aligned} W &\approx N \cdot 2\pi \langle v \rangle [\rho_w]^2 [1 + \ln(\rho_m/\rho_w)] \\ &= N \cdot \frac{e^4}{\hbar^2} 2\pi^3 \frac{\langle r \rangle^2}{\langle v \rangle} [1 + \ln(\rho_m/\rho_w)] \end{aligned} \quad (3.13a)$$

More refined quantum estimates have the form (cf. Sobel'man, Eq. 38.33)

$$W \approx N \frac{e^4}{\hbar^2} 10 \langle v \rangle [\rho_w]^2 [1 + 3\ln(4\rho_m/\rho_w)] \quad (3.13b)$$

Quantum treatment of impacts differs in some details from the classical theory, but it arrives at essentially the same result. The quantum phase shift δ replaces the classical phase $\eta/2$; the cross-section is expressible in terms of the scattering matrix S whose diagonal elements have the form $S = \exp(2i\delta)$. Then Eq. (3.8) becomes

$$\begin{aligned} W - i\Delta &= \langle v(1 - S_{up}^+ S_{low}) \rangle_{av} \\ &= \langle v(1 - \exp[-2i(\delta_{up} - \delta_{low})]) \rangle_{av} \end{aligned} \quad (3.14)$$

where subscripts identify quantities associated with upper and lower levels of the radiative transition. Because the impact width depends upon differences in scattering phase shifts, it receives no contribution from the very large Coulomb elastic scattering cross-section. (One can regard this as a cancellation of two infinite elastic-scattering cross-sections.) The quantum

thermal average is often expressed in terms of orbital angular momentum; Eq. (3.9) is then replaced by

$$\langle vF \rangle \approx N \sum_{\ell} (2\ell+1) \frac{\pi \hbar^2}{m \langle v \rangle} F(\ell, \langle v \rangle) \quad (3.15)$$

If we now take the cross-section to have maximum value (obtained with $\delta_{\text{low}}=0$, $\delta_{\text{up}}=\pi$) out to a cutoff angular momentum

$$\hbar \ell_w = m \langle v \rangle \rho_w = \pi \langle d \rangle m e^2 / \hbar \quad (3.16)$$

and to vanish for larger ℓ , then we recover the classical result (3.10).

Alternatively, one can employ the Bethe-Born approximation for the monopole-dipole scattering cross-section. Using an average Gaunt factor $\bar{g} \approx 0.2$ and using a sum rule for the totality of excitation oscillator strength one obtains an approximation

$$W \approx N \cdot \frac{e^4}{\hbar^2} \cdot \frac{\langle r \rangle^2}{\langle v \rangle} \cdot 10 \bar{g} \quad (3.17)$$

which differs little from the classical result.

At present the most elaborate computations of impact widths use either a classical-path perturber (e.g. Griem, 1967; Voslamber 1969; Smith, Cooper, and Vidal 1969; Gersten and Foley, 1969) or a close-coupling quantum approximation (e.g. Bely 1969; Barnes 1971; Tran-Mihn and Van Regemorter 1972).

4. Static Distribution

Line radiation originates in an ensemble of radiating atoms, each subject to a slightly different quasi-static electric microfield $\underline{\epsilon}$. The resulting statistical distribution of frequency shifts follows a pattern which can be written

$$\begin{aligned} \frac{dF^S}{d\omega}(\omega)d\omega &= \frac{dF^S}{d\epsilon}(\omega[\epsilon]) \frac{d\omega[\epsilon]}{d\epsilon} d\epsilon \\ &= \frac{1}{W_S} \frac{d\omega[\epsilon]}{d\epsilon} P^S\left(\frac{\omega[\epsilon] - \omega_0}{W_S}\right) d\epsilon \end{aligned} \quad (4.1)$$

where the static width, W_S is the shift caused by the mean electric field $\langle \underline{\epsilon} \rangle$:

$$W_S = \omega[\langle \underline{\epsilon} \rangle] - \omega_0 \quad (4.2)$$

For example, the linear stark effect which occurs with an atomic dipole moment d , is characterised by the shift

$$\omega[\epsilon] - \omega_0 = \hbar^{-1} d \cdot \underline{\epsilon} \approx \hbar^{-1} \langle d \rangle \epsilon \quad (4.3)$$

Hence the static width is

$$W_S \approx \hbar^{-1} \langle d \rangle \langle \epsilon \rangle \quad (4.4)$$

and the static profile can be written

$$\frac{dF^S}{d\omega}(\omega) d\omega = \frac{1}{\langle \epsilon \rangle} P^S\left(\frac{\epsilon}{\langle \epsilon \rangle}\right) d\epsilon \quad (4.5)$$

For spectral lines which are broadened by independent perturbers according to an inverse-power-law in perturber separation R ,

$$\omega[R] - \omega_0 = \frac{C}{R^\mu} = W_S \left[\frac{\langle R \rangle}{R} \right]^\mu \quad (4.6)$$

the profile is written

$$\begin{aligned} \frac{dF^S}{d\omega}(\omega) \cdot d\omega &= \frac{1}{W_S} \frac{d\omega[R]}{dR} P^S\left(\frac{\omega[R] - \omega_0}{W_S}\right) dR \\ &= \frac{1}{\langle R \rangle} \left[n \left(\frac{\langle R \rangle}{R} \right)^{n-1} P^S\left(\frac{\langle R \rangle^n}{R^n} \right) \right] dR \end{aligned} \quad (4.7)$$

Alternatively one can write

$$\frac{dF^S}{d\omega}(\omega) \cdot d\omega = \frac{1}{\langle \epsilon \rangle} \left[\frac{\mu}{2} \left(\frac{\epsilon}{\langle \epsilon \rangle} \right)^{\frac{\mu-2}{2}} P^S\left(\frac{\epsilon^{\mu/2}}{\langle \epsilon \rangle^{\mu/2}} \right) \right] d\epsilon \quad (4.8)$$

where $\epsilon = ze/R^2$ is now the microfield of an ion, charge z , a distance R from the radiator. The linear Stark effect is the special case $\mu = 2$.

Equation (4.4), written in terms of mean interparticle distance $\langle R \rangle$, becomes

$$W_S = Zz \frac{e^2}{\hbar} \frac{\langle r \rangle}{\langle R \rangle^2} = Zz \frac{e^2}{\hbar} \cdot \langle r \rangle \cdot \left[\frac{4\pi}{3} N \right]^{2/3}. \quad (4.9)$$

The static profile function $P^S(x)$ can only be obtained approximately.

In the nearest-neighbor approximation it is

$$P^{SN}(\beta) = \frac{3}{2} \cdot \beta^{-5/2} \cdot \exp\left[-\beta^{-3/2}\right] \quad (4.10)$$

In the Holtsmark approximation it is

$$\begin{aligned} P^{SH}(\beta) &= \frac{2}{\pi} \beta^{-1} \int_0^\infty dx \sin(x) \cdot x \cdot \exp\left[-(x/\beta)^{3/2}\right] \\ &= \frac{2\beta}{\pi} \int_0^\infty dx \cdot x \cdot \sin(\beta x) \cdot \exp(-x^{3/2}) \end{aligned} \quad (4.11)$$

For more general approximations the profile may depend not only upon β (which is $\epsilon/\langle \epsilon \rangle = \langle d \rangle \epsilon / W_S$) but also upon other functions of ϵ , z , Z , N , and T

(cf. Hooper, 1966; Smith and Cooper, 1968). In any case Eq. (4.2) defines the static width which sets the scale of the profile.

5. Convolution of Mechanisms

As a result of simultaneous independent action of these three mechanisms, the resulting line profile becomes a convolution of three profiles:

$$\frac{dF^{\text{line}}}{d\omega}(\omega) = \int d\omega' \int d\omega'' \frac{dF^S}{d\omega}(\omega'') \frac{dF^L}{d\omega}(\omega' - \omega'') \frac{dF^D}{d\omega}(\omega - \omega') \quad (5.1)$$

or, assuming linear stark effect with $\omega'' - \omega_0 = h^{-1} \langle d \rangle \epsilon$, the profile is

$$\begin{aligned} \frac{dF^{\text{line}}}{d\omega}(\omega) = & \int \frac{d\omega'}{W_D} P^D\left(\frac{\omega - \omega' - \omega_0}{W_D}\right) \int \frac{d\epsilon}{\langle \epsilon \rangle} P^S\left(\frac{\epsilon}{\langle \epsilon \rangle}\right) \\ & \times \frac{1}{W_L} P^L\left(\frac{\omega' - \langle d \rangle \epsilon h^{-1} + \Delta}{W_L}\right) \end{aligned} \quad (5.2)$$

Thus the intensity at a frequency increment $d\omega$ removed from ω_0 is expressible as

$$\frac{dF^{\text{line}}}{d\omega}(\omega_0 + d\omega) = \frac{1}{(W_N + W_I)} \int_{-\infty}^{\infty} dx P^D(x) \int_0^{\infty} d\beta P^S(\beta) \cdot P^L\left(\frac{d\omega + x W_D - \beta W_S}{W_N + W_I}\right). \quad (5.3)$$

Here x is the frequency shift in units of Doppler spread W_D , and β is the linear Stark shift in units of the static width W_S (or equivalently, β is the microfield in units of the mean field $\langle \epsilon \rangle$).

Strong lines, particularly resonance lines, are further affected by self-absorption. This effect requires detailed treatment of line formation in radiative transfer, and hence it makes the central portions of strong lines unsuitable for plasma diagnostics.

The light emerging from an emission source undergoes further redistribution of frequencies in the succession of spectrographic and/or photographic devices which ultimately produce a recorded line profile. The resulting distortion is expressible as

$$\frac{dF^{\text{obs}}}{d\omega}(\omega) = \int \frac{d\omega'}{W_{\text{ins}}} \left\{ \frac{dF^{\text{line}}}{d\omega}(\omega) + \frac{dF^{\text{con}}}{d\omega}(\omega) \right\} P^{\text{ins}}\left(\frac{\omega - \omega'}{W_{\text{ins}}}\right) \quad (5.4)$$

where $P^{\text{ins}}(x)$ is an instrumental profile function, W_{ins} is a characteristic instrumental width, and $\frac{dF^{\text{con}}}{d\omega}(\omega)$ represents the contribution from background continuum radiation.

The Doppler and Lorentz functions $P^{\text{D}}(x)$ both fall monotonically from maximum values at $x = 0$. The static function $P^{\text{S}}(x)$ rises from zero at $x = 0$ to a maximum at x_s , then falls monotonically. For the nearest-neighbor distribution appropriate to the linear Stark effect the maximum is $x_s = (3/5)^{2/3} \approx 0.711$; the Holtsmark distribution has a maximum at $x_s \approx 3$. Figure 1 shows the profiles of these separate distributions; plus signs mark the points where the function $P(x)$ has half its maximum value. Doubling these abscissae gives the full-width at half-maximum:

$$\text{Doppler: } w_D = 2\sqrt{\ln 2} \quad W_D = 1.66 W_D$$

$$\text{Lorentz: } w_L = 2W_L$$

$$\text{Holtsmark: } w_{\text{SH}} \approx 6W_s.$$

6. Physical Parameters

Preceding sections noted the mechanisms which broaden spectral lines. The present section notes the magnitudes of characteristic lengths and times (or frequencies) which occur in these mechanisms.

A. Basic Units

I shall employ atomic units, setting to unity the electron charge electron mass and the Dirac constant of action: $e = m = \hbar = 1$. The resulting units include the following (powers of 10 in parentheses):

1 a.u. Length	= Radius of first Bohr orbit of hydrogen	$= a_0 = \hbar^2 / m e^2$	$= 5.29167(-11) \text{ m}$ $= 0.53 \text{ \AA}$
1 a.u. Speed	= Sommerfeld fine structure constant (α) times speed of light (c)	$= \alpha c$	$= 2.18765(+6) \text{ m sec}^{-1}$
1 a.u. Time	= ratio of unit length to unit speed	$= a_0 / \alpha c$	$= 2.41888(-17) \text{ sec}$
1 a.u. Frequency	= ratio of unit speed to unit length	$= \alpha c / a_0$	$= 4.134138(+16) \text{ sec}^{-1}$
1 a.u. Energy	= Twice the ionization energy of Bohr-model hydrogen (or two Rydberg units)	$= e^2 / a_0$	$= 4.35944(-18) \text{ J}$ $= 27.21 \text{ eV}$
1 a.u. Temperature	= Twice the Rydberg energy divided by Boltzmann constant (k)	$= e^2 / a_0 k$	$= 3.15779(+5) \text{ }^\circ\text{K}$
1 a.u. Area	= square of the Bohr radius	$= a_0^2$	$= 2.80018(-21) \text{ m}^2$ $= 28 \text{ Megabarns}$
1 a.u. Volume	= cube of the Bohr radius	$= a_0^3$	$= 1.48176(-31) \text{ m}^3$
1 a.u. Density	= inverse of unit volume	$= a_0^{-3}$	$= 6.74873(+30) \text{ m}^{-3}$ $= 6.75(+24) \text{ cm}^{-3}$

1 a.u. Power	= twice the power needed to ionize hydrogen at the rate of once per Bohr period	= 1.8023(-1) W
1 a.u. Power Density	= 1 a.u. power per 1 a.u. volume	= 1.2163(+30) W m ⁻³
1 a.u. Electric Field Density	= field at first Bohr orbit of hydrogen	= 5.14216(+11) V m ⁻¹

B. Radiator

These units prove particularly convenient when treating valence electrons of neutral system, whose orbit size, period, and energy are of the order of unity with these scales. The radiating ion defined in Section I has the following characteristic physical parameters:

$\langle r \rangle$	= Bohr radius of 2p orbit, nuclear charge Z	= $2^2/Z$
$\langle v \rangle$	= Bohr speed of 2p orbit, charge Z	= $Z/2$
ω_0	= Lyman α frequency	= $\frac{Z^2}{2} \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = Z^2 \times 3/8$
ω_{13}	= separation between Ly α and Ly β	= $Z^2 \times 5/72$
λ_0	= Lyman α wavelength	= $2\pi/\alpha\omega_0 = 16\pi/3\alpha Z^2$
$f_{1 \rightarrow 2}$	= oscillator strength, 1s $\rightarrow$ 2p, charge Z	= 0.41
$A_{2 \rightarrow 1}$	= radiative decay rate, 2p level	= $(2/3)\alpha^3 \omega_0^2 f_{1 \rightarrow 2} = 128\alpha^3 f_{1 \rightarrow 2}/27Z^4$
τ	= radiative lifetime of 2p level	= $1/A_{2 \rightarrow 1} = 6.62(+7)Z^4$

C. Static Perturbations

The surrounding statistical distribution of ions and electrons have the following mean (static) characteristics:

$$\begin{aligned}
 \langle R \rangle &= \text{mean interparticle distance (ions and electrons from radiator)} &= [3/4\pi N]^{1/3} \\
 \langle \epsilon \rangle &= \text{mean electric field of ions at nucleus (Holtzmark field)} &= z/\langle R \rangle^2 \\
 W_S &= \text{mean stark energy of radiator} = \langle r \rangle \langle \epsilon \rangle = z \langle r \rangle / \langle R \rangle^2 & \text{(Holtzmark width)} \\
 \omega_e &= \text{electron plasma-frequency} &= \sqrt{4\pi z^2 N} = z\sqrt{3}/\langle R \rangle^{3/2} \\
 \omega_z &= \text{ion plasma-frequency} &= \sqrt{4\pi z^2 N/zM} = \omega_e / \sqrt{M_z}
 \end{aligned}$$

D. Dynamic Perturbations

Finally, the plasma ions and electrons have the following thermal (dynamic) characteristics (I take the hydrogen mass to be $M = 1836$ and the mass of the radiating atom to be $M_z = 2ZM$, and the plasma-ion mass to be $M_z = zM$):

$$\begin{aligned}
 \langle v_e \rangle &= \text{mean thermal speed of electrons} &= \sqrt{3kT} \\
 \langle v_z \rangle &= \text{mean thermal speed of ions} &= \langle v_e \rangle / \sqrt{M_z} \\
 \langle v_z \rangle &= \text{mean thermal speed of radiators} &= \langle v_e \rangle / \sqrt{2ZM} \\
 W_D &= \text{Doppler spread of Lyman-}\alpha \text{ line} &= \omega_0 \sqrt{2kT/M_z c^2} \\
 & &= \alpha \omega_0 \langle v_e \rangle / \sqrt{3ZM} \\
 \lambda_T &= \text{Thermal De Broglie wavelength of electrons} &= 2\pi / \langle v_e \rangle \\
 \rho_w &= \text{Electron-Weisskopf radius (impact parameter for strong collisions by electrons)} &= \pi \langle r \rangle / \langle v_e \rangle \\
 \Omega_e &= \text{electron impact limiting frequency} &= \langle v_e \rangle / \rho_w = \langle v_e \rangle^2 / \pi \langle r \rangle \\
 \Omega_z &= \text{ion impact limit} &= \Omega_e / M_z
 \end{aligned}$$

7. Examples

Figures 2-3 display the density variation of these full-widths for several illustrative cases. The graphs display the following lines:

- (a) The Doppler width (computed as $2 \ln 2 W_D$ using Eq. 2.3).
- (b) The Natural width of the $2p \rightarrow 1s$ transition (computed as $2W_N$ using Eq. 3.4).
- (c) The Impact width of the $2p$ level (computed as $2W_W$ using Eqs. 3.10 and 3.12).
- (d) The Static width of the $2p$ level (computed as $6W_S$ using Eq. 1.9).
- (e) Instrumental widths, taken to be the inverse of resolving power 10^4 and 10^7 .

In addition, the graphs are bounded at top and right (shown as shading) by the following constraints.

- (f) The assumption of a linear Stark effect grows less satisfactory as the static width grows larger. The Inglis-Teller limit, when this width equals the spacing between lines thereby merging the lines, is shown as a vertical line at the $H\alpha$ frequency.
- (g) The notion of impacts becomes ill-defined when the perturber separation $\langle R \rangle$ becomes comparable to the orbit size $\langle v \rangle$. I have bounded all graphs at a maximum density N such that $\langle v \rangle = \langle R \rangle$.

Figure 2 shows these quantities for hydrogen, $Z=1$, in a hydrogenic plasma $z=1$ for a typical experimental temperature of 10^4 °K. The thermal energy KT is 0.032 atomic units, so that perturber-electrons move relatively slowly on an atomic scale.

Figure 3 repeats this display for $Z=10$ (still with hydrogenic plasma) and a temperature 10^6 °K chosen to give the same relative energies of atomic electrons and perturber electrons.

Figure 4 repeats this display for $Z=17$ (and $z=1$) and a temperature of 1 keV. On an atomic scale this choice corresponds to a fourfold increase in the relative energies of thermal and bound electrons.

Although detailed description of line profiles require elaborate computations, Figures 2-4 indicate the general properties of the profile. At low density (e.g., less than 10^{15} cm⁻³ for $Z=1$ at 10^4 °K and less than 10^{22} for $Z=17$ at 1 keV). The line width is primarily set by thermal Doppler broadening. At higher densities the static Stark width dictates the line width. The far wings of the line, several half-widths from line center, are unaffected by Doppler broadening.

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Figure Captions

- Figure 1 Comparison of Doppler (Eq. 2.2), Lorentz (Eq. 3.2), nearest-neighbor (Eq. 4.10) and Holtsmark (Eq. 4.11) distributions.
- Figure 2 Full Widths at half-maximum of $2p \rightarrow 1s$ ($\lambda = 1215 \text{ \AA}$, $\hbar\omega = 10.2 \text{ eV}$) for $Z = 1$ and $T = 10^4 \text{ [}^\circ\text{K]} = 0.032 \text{ [au]}$. Ordinate is log of density $N \text{ [cm}^{-3}\text{]}$, abscissa is log of width w in eV and in Angstroms.
- Figure 3 Full widths of $2p \rightarrow 1s$ ($\lambda = 12 \text{ \AA}$, $\hbar\omega = 1.0 \text{ keV}$) for $Z = 10$ and $T = 10^6 \text{ [}^\circ\text{K]} = 0.032Z^2 \text{ [au]}$.
- Figure 4 Full widths of $2p \rightarrow 1s$ ($\lambda = 4.2 \text{ \AA}$, $\hbar\omega = 2.9 \text{ keV}$) for $Z = 17$ and $kT = 1 \text{ keV} = 0.13Z^2 \text{ [au]}$.

Figure 1

$Z = 1$ $T = 10^4$ [°K] = 0.032 [au]

Figure 2

$$Z = 10 \quad T = 10^6 \text{ [}^\circ\text{K]} = 0.032 Z^2 \text{ [au]}$$

Figure 3

$$Z = 17 \quad T = 1 \text{ [keV]} = 0.13 Z^2 \text{ [au]}$$

Figure 4

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