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EXTRACTION OF SYNCHROCYCLOTRON BEAMS
NEAR THE MAXIMUM ENERGY

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THE MAXIMUM ENERGY

Warren Fenton Stubbins

July 23, 1956

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ABSTRACT

A statement of the requirements for regenerative extraction of particles from a cyclotron in the nonlinear magnetic field is followed by an analysis for determining the regenerator shape and position and estimating the rate of increase of radial oscillation amplitude. The effect of the regenerator on axial motion is considered briefly. The regenerator field is computed for several regenerator positions as an illustrative example.

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I INTRODUCTION

In 1950 Tuck and Teng¹ proposed the removal of synchrocyclotron beams by means of magnetic field perturbations placed near the outer radius of the cyclotron. The first perturbation the "peeler," consists of a region of field decreasing radially. This is followed at some angle with a region of radially increasing field, the regenerator. An exact analysis of this system was made by LeCouteur² and Judd³ for beam extraction occurring in a linear portion of the cyclotron's magnetic field. Later, LeCouteur⁴ extended this analysis to the region where the cyclotron field was rapidly decreasing with radius. Crewe and LeCouteur⁵ applied the linear analysis and extracted a portion of the circulating beam of the University of Liverpool synchrocyclotron. LeCouteur and Lipton⁶ considered a numerical analysis of the deflection with only a region of increasing field as the perturbation. Crewe and Kruse⁷ applied this analysis directly to the University of Chicago synchrocyclotron and removed a portion of the circulating beam with a regenerator alone.

In order to determine the required field perturbation for extracting a portion of the beam of the 184-inch synchrocyclotron, it has been necessary to extend these analyses. This extension, which is intimately tied to the physical process, applies to a system using only a regenerator.

II METHOD OF CALCULATION

The determination of the required perturbation for extracting the beam of a synchrocyclotron is made in seven steps, as follows:

- (a) From the Lorentz equation, obtain the nonlinear equations of motion for the radial and axial oscillations.
- (b) From these nonlinear equations and the magnetic field shape in the region for extraction, determine the frequencies of radial and axial motion for various amplitudes of oscillations.
- (c) Using the radial frequencies, compute by a simple matrix analysis the increase in the radial amplitude and the necessary change in radial momentum to satisfy the extraction requirements.
- (d) Determine the field perturbation from the necessary change in radial momentum.
- (e) From the field perturbation, determine the resulting change in axial momentum.

- (f) Compute the growth of the axial motion from the impulses of the regenerator.
- (g) From the set of values of radial and axial growth as functions of the regenerator position and the initial amplitudes of oscillations, select a suitable arrangement for the particular cyclotron.

III THE EXTRACTION REQUIREMENTS

The extraction process begins when the precession of the center of rotation of the particles about the center of the cyclotron is arrested. The maximum of the radial extent of the particles then occurs repeatedly at one azimuth. As the radial oscillation amplitude increases, the particles move outward; a channel may be placed in this region. If the radial gain per turn is large enough and the beam is not greatly attenuated by axial blowup, the particles may "step over" the channel wall and give efficient beam removal.

The extraction requirements, simply stated, are:

- (a) The precession must be arrested
- (b) Sufficient gain per turn must be obtained
- (c) Losses owing to axial blowup must be minimized.

To understand the objective and method of calculation one needs to note that under (a) the particles repeatedly cross the synchronous radius at the same two points in the cyclotron. These points may be designated by their angles, and the regenerator and channel positions referred to them. The synchronous radius R is the radius at which the particles circulate when they have no radial oscillation amplitude, and it is also the radius at which the regenerator field starts. Radial displacement is measured from the synchronous radius. The synchronous radius suitable for deflection in the cyclotron is just inside the radius at which normal beam destruction occurs.

IV. THE NONLINEAR EQUATIONS OF MOTION

In the static magnetic field of the cyclotron, the force on a moving charged particle is given by the Lorentz equation,

$$\vec{F} = q\vec{v} \times \vec{B} = \frac{d}{dt} \vec{p}, \quad (1)$$

where q is the charge, $\vec{v}$ is the velocity, $\vec{B}$ is the magnetic field, $\vec{p}$ is the momentum, and t is the time. Component equations of Eq. (1) are written in cylindrical coordinates for particular convenience in cyclotron problems. In the extraction process, the particle energy may be considered as unchanging since the energy gain over the few turns is small compared with the total energy. Thus the particle speed is a constant during the extraction process. The speed of particles is expressed as the square root of the sum of the squares of the azimuthal, radial, and axial velocities. This is used to replace time, as the independent variable, by azimuthal angle, θ . We denote the synchronous radius by R and define the radius of the particle measured from the center of the cyclotron as $r = R(1 + \rho)$ and the axial displacement by $z = RZ$. We combine the component equations and obtain to squared terms the following equations:

$$\frac{d\rho^2}{d\theta^2} + (1 - n)\rho = \left(2n + 1 + \frac{m}{2}\right)\rho^2 - \left(\frac{m}{2} + \frac{n}{2}\right)Z^2 + \frac{1}{2}\left(\frac{d\rho}{d\theta}\right)^2 - \frac{1}{2}\left(\frac{dZ}{d\theta}\right)^2, \quad (2)$$

$$\frac{d^2Z}{d\theta^2} + nZ = -(2n + m)\rho Z + \frac{dZ}{d\theta} \frac{d\rho}{d\theta}, \quad (3)$$

where $Z = z/R$, $\rho = r - R/R$ and R is the synchronous radius. Also

$$n = -\frac{R}{B_Z(0)} \frac{\partial B_Z}{\partial r}, \quad m = -\frac{R^2}{B_Z(0)} \frac{\partial^2 B_Z}{\partial r^2}. \quad (4)$$

V. FREQUENCIES OF OSCILLATION

Using the nonlinear Eqs. (2) and (3) and the magnetic field data in Fig. 1, we determine the frequencies of oscillation of the radial and axial motions. We use this information in following the particles through their paths within the cyclotron field and through the regenerator field.

The path may be divided into two parts, that part in which the particles are beyond the synchronous orbit, and that part in which the particles are between the synchronous orbit and the machine center. These parts are defined by the two points at which the synchronous orbit is crossed by the particle orbit.

The method of determining the required perturbation may be illustrated by the following example. In Fig. 1 we select a synchronous orbit just inside

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Fig. 1. The magnetic field and the n value vs radius in the extraction region.

the $n = 0.2$ point at 79.8 inches. The n value is 0.155 and it is greater beyond, and less towards the center of the cyclotron. The dotted lines show an approximation to the change in n as the radius of the orbit increases and decreases. The change in n is related to m which is defined in equation (4) above. The approximations that we use are

$$n = 0.155 \text{ at } r = 79.8 \text{ inches;}$$

$$\frac{m}{R} = 0.055 \text{ per in. for } 78 \text{ in.} \leq r \leq 79.8 \text{ in.}$$

$$= 0.138 \text{ per in. for } r > 79.8 \text{ in.}$$

$$= 0 \quad \text{for } r < 78 \text{ in.}$$

The frequency of oscillation as a function of the amplitude of the particle motion and the nonlinearities of the magnetic field is found by the approximation technique of Kryloff and Bogoliuboff.⁸ We compute these frequencies separately for each portion of the cycle.

For the frequency of radial motion in the median plane, $Z = 0$, we obtain the following equations: (Throughout this paper "frequency" is used in this sense.)

$$\frac{d\phi}{d\theta} = 0.919 - .019a \text{ for } r > R, \quad (5)$$

$$\frac{d\phi}{d\theta} = 0.919 + .009a \text{ for } r < R \text{ and for } a \leq 1.8 \text{ in.}$$

Where a is the amplitude of oscillation and equals R_p we obtain

Amplitude (in.)	$\frac{d\phi}{d\theta} (r > R)$	$\frac{d\phi}{d\theta} (r < R)$
0.25	0.914	0.922
0.5	0.909	0.924
1.	0.900	0.928
1.5	0.891	0.933
2	0.881	0.936
2.5	0.871	0.936

If $\frac{d\phi}{d\theta} = \omega_r$, the radial motion whose approximation is a sinusoid of the angle ϕ is related to the angle in the cyclotron by $\phi = \omega_r \theta$.

VI. RADIAL MOTION

We now consider the oscillation of the particles in the radial direction and with no axial motion. In accordance with the extraction requirements the particles must cross the synchronous radius R at two specific points.

When the particles are beyond R or within R we use the frequency appropriate to their positions. The analysis is repeated for several amplitudes of radial oscillation and for several positions of the regenerator.

Figure 2 shows the motion of the particles without and with the perturbation.

When the particles move freely, their periods are greater than that of the cyclotron and they precess. The half periods are different owing to the different radial positions of the particles.

To satisfy the extraction requirement the period must be reduced to that of the cyclotron. Since the inward motion is not affected, the half period associated with it will remain the same. The half period is given by

$$\frac{\pi}{\omega_r (r < R)} \quad \text{Requiring that the particle cross the synchronous orbit at}$$

$0, 2\pi$, etc. as it moves outward enables us to determine the value of θ_2 :

$$\theta_2 = \pi \left(2 - \frac{1}{\omega_r (r < R)} \right) \quad (6)$$

This value of course varies with the amplitude because $\omega_r (r < R)$ is a function of the amplitude. The values for $\theta = 0, 2\pi$, etc. correspond to one of the unique points of the motion, and $\theta_2, \theta_2 + 2\pi$, etc., correspond to the other. We now ask what change in radial momentum must be accomplished, impulse-wise, at θ_1 , to bring the particles to the synchronous radius at θ_2 ; this is the function of the regenerator.

To study this we consider the motion of the particle in the three intervals suggested by Fig. 2. For convenience we use a simple matrix calculation over these intervals.

The motion of a particle over an interval may be expressed in the following matrix notation,

$$\begin{pmatrix} p_1 \\ p_1' \end{pmatrix} = \begin{pmatrix} \cos \omega_r \theta_1 & \frac{1}{\omega_r} \sin \omega_r \theta_1 \\ -\omega_r \sin \omega_r \theta_1 & \cos \omega_r \theta_1 \end{pmatrix} \begin{pmatrix} p_0 \\ p_0' \end{pmatrix} \quad (7)$$

$\omega_r = \omega_r (r > R)$

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Fig. 2. The radial motion vs cyclotron angle θ . (a) Motion without regenerator, (b) motion with regenerator causing period of radial motion to be one turn in the cyclotron.

where ρ_0 and $\rho_0' = \frac{d\rho_0}{d\theta}$ are the values at the beginning of the interval and ρ_1 and ρ_1' are the values at the end of the interval.

Because of the action of the regenerator, the initial conditions for the second interval are different from that at the end of the first interval. The regenerator has changed ρ_1' without changing ρ_1 . Thus for the second interval we have

$$\begin{pmatrix} \rho_2 \\ \rho_2' \end{pmatrix} = \begin{pmatrix} \cos \omega_r (\theta_2 - \theta_1), \frac{1}{\omega_r} \sin \omega_r (\theta_2 - \theta_1) \\ -\omega_r \sin \omega_r (\theta_2 - \theta_1), \cos \omega_r (\theta_2 - \theta_1) \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_1' + \delta\rho_1' \end{pmatrix}, \quad (8)$$

$$\omega_r = \omega_r (r > R)$$

where $\delta\rho_1'$ is the change in the slope by the regenerator.

Over the third interval we have a similar expression,

$$\begin{pmatrix} \rho_3 \\ \rho_3' \end{pmatrix} = \begin{pmatrix} \cos \omega_r (\theta_3 - \theta_2), \frac{1}{\omega_r} \sin \omega_r (\theta_3 - \theta_2) \\ -\omega_r \sin \omega_r (\theta_3 - \theta_2), \cos \omega_r (\theta_3 - \theta_2) \end{pmatrix} \begin{pmatrix} \rho_2 \\ \rho_2' \end{pmatrix}, \quad (9)$$

$$\omega_r = \omega_r (r < R).$$

Where ρ_3 and ρ_3' are the initial values for the next cycle. We note that we have the range of $\omega_r (\theta_3 - \theta_2) = \pi$, thus the third matrix has the values of

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

and

$$\begin{pmatrix} \rho_3 \\ \rho_3' \end{pmatrix} = \begin{pmatrix} -\rho_2 \\ -\rho_2' \end{pmatrix}. \quad (10)$$

This merely changes the sign of the position and slope. We evaluate the effect of the regenerator by considering the conditions at 0 and θ_2 and noting the change of sign.

At the beginning of the initial interval, we have $\rho_0 = 0$, $\rho_0' = \rho_0'$, and at the end of the second interval we have $\rho_2 = 0$, $\rho_2' = \rho_2'$. If we determine the ratio of $\frac{\rho_2}{\rho_0'}$ we have the ratio of the amplitudes before and

after passing through the regenerator. This gives us the radial increase per turn; we call it α .

We may also ask what $\delta(\rho')$ is required to give $\rho_2 = 0$. Putting these conditions in the matrix equations and simplifying the expressions, we obtain

$$a = - \frac{\sin \omega_r \theta_1}{\sin \omega_r (\theta_2 - \theta_1)}, \quad \omega_r = \omega_r (r > R), \quad (11)$$

$$\delta(\rho') = - \frac{\rho_0 \sin \omega_r \theta_2}{\sin \omega_r (\theta_2 - \theta_1)}, \quad \omega_r = \omega_r (r > R). \quad (12)$$

Considering various values of θ_1 -- i.e., regenerator position -- and the different amplitudes of radial oscillations, one may compute the radial increase per turn, a , and the required change in radial momentum.

One sees from the equations and from Fig. 2. that the smaller the interval $\theta_2 - \theta_1$ the greater the increase a and regenerator strength.

Figure 3 shows the values of a and $\delta(\rho')$ for various regenerator positions and amplitudes of oscillation. The regenerator must have the correct strength at the radius at which the particle passes through it to cause the particle to cross the synchronous orbit at θ_2 . The radius at θ_1 is close to the amplitude of oscillation but must be calculated for each case.

VII REGENERATOR FIELD

Knowing the required change in particle motion, we may determine the magnetic field perturbation to accomplish this in the following manner.

The radial momentum is given by

$$m \frac{dr}{dt} = mR \frac{d\rho}{dt}, \quad (13)$$

and since

$$\frac{d\rho}{dt} = \frac{d\theta}{dt} \frac{d\rho}{d\theta} = \omega_0 \frac{d\rho}{d\theta},$$

we have

$$m \frac{dr}{dt} = mR \omega_0 \frac{d\rho}{d\theta}. \quad (14)$$

The change in the radial momentum is

$$mR \omega_0 \delta(\rho') = mR \omega_0 \left[\left(\frac{d\rho}{d\theta} \right)_{\text{out}} - \left(\frac{d\rho}{d\theta} \right)_{\text{in}} \right]. \quad (15)$$

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Fig. 3. Gain per turn and regenerator strength vs radial displacement into regenerator for several regenerator positions.

The force exerted by the regenerator multiplied by the time of action equals the change in radial momentum. The force is eBv and the time is $\frac{R(1+\rho)\theta}{v}$, where v is the velocity of the particle, B is the regenerator field e is the electronic charge, and θ is the angle over which the regenerator extends.

The force times the time is then

$$eBR(1+\rho)\theta = mR\omega_0\delta(\rho'),$$

so that

$$\delta(\rho') = \frac{eB(1+\rho)\theta}{m\omega_0} \quad (16)$$

Multiplying and dividing by B_0 , the magnetic field at R , we obtain

$$\delta(\rho') = \frac{eB_0}{m} \frac{1}{\omega_0} \frac{B(1+\rho)\theta}{B_0}$$

Now

$$\frac{eB_0}{m} = \omega_0,$$

so that we have

$$\delta(\rho') = \frac{B(1+\rho)\theta}{B_0} \quad (17)$$

Thus the regenerator field and angle are

$$B\theta = \frac{\delta(\rho') B_0}{(1+\rho)} \quad (18)$$

We have used a simple impulse computation to obtain the value $B\theta$ which is the integrated regenerator field over the angle of the regenerator. This approximation must be examined more carefully for a long regenerator. When B_0 is in gauss, $B\theta$ is in gauss-radians.

If we consider a regenerator placed at $120^\circ = \theta_1$, from Fig. 3 we obtain

$$\begin{aligned} R\delta(\rho') &= a(R\rho) + b(R\rho)^2 \\ &= 0.695(R\rho) + 0.02(R\rho)^2, \end{aligned} \quad (19)$$

and from Fig. 1 we find, at $R = 79.8$ in. and $B_0 = 21,730$ gauss,

$$B\theta = \frac{0.695(R\rho) + 0.02(R\rho)^2}{R(1+\rho)} B_0 \quad (20)$$

We may consider this in table form.

$R\rho$ (in.)	$R\delta(\rho')$	$B\theta$ (kilogauss-degrees)
0.5	0.36	5.58
1	0.715	11.0
1.5	1.08	16.5
2	1.47	22.3
2.5	1.70	25.6

VIII EFFECT OF REGENERATOR FIELD ON AXIAL MOTION

The axial momentum is given by

$$m \frac{dz}{dt} = m R \frac{dZ}{dt} = m R \omega_0 \frac{dZ}{d\theta} \quad (21)$$

The change in axial momentum is

$$m R \omega_0 \delta(Z') = m R \omega_0 \left[\left(\frac{dZ}{d\theta} \right)_{out} - \left(\frac{dZ}{d\theta} \right)_{in} \right] \quad (22)$$

The impulse time is the same as for the radial computation and the force is given by $F_z = evB_r$, where B_r is the radial component of the magnetic field perturbation. Using the relation, $\nabla \times \vec{B} = 0$, we obtain

$$B_r = \frac{\partial B_z}{\partial r} z = \frac{\partial B_z}{\partial r} RZ.$$

The change in momentum is then

$$m R \omega_0 \delta(Z') = Re \frac{\partial B_z}{\partial r} R(1 + \rho) \theta Z;$$

thus we have

$$\delta(Z') = \frac{Re(1 + \rho) Z}{m \omega_0} \frac{\partial B_z}{\partial r} \theta \quad (23)$$

Again multiplying and dividing by B_0 , we obtain

$$\delta(Z') = \frac{eB_0}{m} \frac{1}{\omega_0} \frac{R(1+\rho)Z}{B_0} \frac{\partial B_z}{\partial r} \theta,$$

$$\delta(Z') = \frac{R(1+\rho)Z}{B_0} \frac{\partial B_z}{\partial r} \theta, \quad (24)$$

where we now recognize B_z as the regenerator field calculated above (21) from

$$B_z = B = \frac{1}{\theta} \frac{a(R\rho) + b(R\rho)^2}{R(1+\rho)} B_0 \quad (25)$$

$$\frac{\partial B}{\partial r} = \frac{1}{R} \frac{\partial B}{\partial \rho} = \frac{B_0}{\theta R^2} \frac{d}{d\rho} \left(\frac{(aR)\rho + (bR^2)\rho^2}{1+\rho} \right)$$

$$= \frac{B_0}{\theta R^2} \frac{aR + 2bR^2\rho + bR^2\rho^2}{(1+\rho)^2} \quad (26)$$

Thus we obtain

$$\delta(Z') = \frac{Z}{R} \frac{aR + 2bR^2\rho + bR^2\rho^2}{(1+\rho)} \quad (27)$$

For a 1-in. axial amplitude this disturbance is about twice as strong as that occurring in the radial motion from the same field perturbation. It may be noticed that this disturbance is a function of axial amplitude, and for a given axial amplitude will be rather constant as the particle radius changes, because the slope of the regenerator field changes slowly. It will, however, vary with regenerator position. The axial amplitude at which the particle crosses the regenerator is very important, and particles of large amplitude of axial oscillation will be lost.

IX. AXIAL MOTION

The growth of the axial motion may be examined by solving Eqs. (2) and (3) for specific amplitudes and regenerator positions and strengths. LeCouteur and Lipton⁶ recommend that the numerical method of Runge-Kutta be used for integrating the equations, and suggest that an approximation extended by them from the linear analysis is a good guide to the rate of axial amplitude increase.

Although the impulses to the axial motion are large, the period of the impulse and the period of axial motion differ. This difference and the damping of axial amplitudes by the radially decreasing magnetic field aid in retaining a good fraction of particles within the necessary bounds. The freedom of adjusting the aximuthal position of the regenerator and the radius of the synchronous orbit--i. e., the radial start of the regenerator--is used to optimize the efficiency of extraction.

X. SELECTION OF FINAL ARRANGEMENT

The selection of the regenerator position may now be made according to the following criteria:

- (a) For a given channel requirement, starting angle, wall thickness, and entrance radius, what is the minimum permitted gain per turn?
- (b) For the distribution, known or assumed, of axial oscillation amplitudes, what is the amount of beam loss one is willing to accept?
- (c) What field perturbations may be produced in the cyclotron gap by the means at one's disposal?

Efficient extractions may be more easily achieved by changing the value of the synchronous radius at which the regenerator starts, but if one is required to reduce the synchronous radius, then the energy of the extracted beam is reduced.

XI INFLUENCE OF THE CHANNEL ON THE BEAM

As the particle amplitude is increased by the regenerator action it approaches the magnetic channel, and prior to entering it, the particle experiences the field perturbations from the channel. This will be serious as the radial oscillation amplitudes become large, because the uncorrected field is appreciable only near the channel wall itself.

The impulse of the uncorrected channel perturbation may be computed as above. The computation enables one to modify the regenerator field to account for the channel effect, and, thus, the maximum effort of corrective shimming for the channel is not required. Reduction of the effort is desirable because the corrective shimming is difficult and makes the channel less effective.

The particle motion is studied over four regions, with the perturbation at the channel dividing into two regions what was called the first region in Part VI. The extraction requirements may be achieved in a manner which compensates for the disturbance of the channel.

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